Final Exam - BUS 37904

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Summary of equations for the model:

Realized quality of product j:

$$\xi_{jt} = \vartheta_j + \nu_{jt}, \;\; \nu_{jt} \sim N(0, \sigma_\nu^2) \tag{1}$$

Realized utility net of latent utility draw ϵ_{jt}:

$$u_{jt} = \gamma - \mathbf{exp}(-\rho \xi_{jt}) - \alpha p_{jt} \tag{2}$$

Expected utility conditional on state vector:

$$u_j(x_t) = E\left[\gamma - \mathbf{exp}(-\rho \xi_{jt}) - \alpha p_{jt} | x_t\right] \tag{3}$$

Match value is drawn from normal distribution:

$$\vartheta_{ij} \sim N(\bar{\vartheta}_j, \tau_j^2) \tag{4}$$

Consumers have rational expectations:

$$\pi_{jt0} \sim N(\mu_{jt}, \tau^2)$$

Expected Value Function:

$$v(\pi_t) = \sum_{k=1}^{K} \omega_k \left(\int \max\left\{ v_j(p_k, \pi_t) + \epsilon_j \right\} g(\epsilon) d\epsilon \right) \tag{5}$$

Value Function as a Chebyshev polynomial:

$$v(x) = T(x)^T \theta = \sum_k \theta_k T_k(x) \tag{6}$$

Combine Chebyshev approximation with quadrature and we get:

$$\begin{aligned} \int v(x') p(x'|x, j) dx' &= \sum_q \omega_q \left(\sum_k \theta_k T_k(x_q) \right) \\ &= \sum_k \theta_k \left(\int T_k(x') p(x'|x, j) dx' \right) \\ &= \sum_k \theta_k E_k(x, j) \end{aligned}$$

Part 1:

After making a decision, the consumer realizes that the real match quality for that draw was ξ_{jt}. This will allow the consumer to update his beliefs about the match quality distribution. When updating the individual takes the expectation of the match quality as:

Realized quality, ν represents product quality fluctuation:

$$\xi_{jt} = \vartheta_j + \nu_{jt}, \quad \text{where: } \nu_{jt} \sim N(0, \sigma_\nu^2)$$

Match value, or perceived value is ϑ_{ij}. It's perceived variation comes from ϵ_τ which is a composite of product quality fluctuation ν_{jt} and perception error ε_{jt}:

$$\begin{aligned}
&\vartheta_{jt} \sim N(\vartheta_j, \tau_j^2) \\
\implies &\vartheta_{jt} = \xi_{jt} + \varepsilon_{jt} \\
&\quad = \vartheta_j + \underbrace{\nu_{jt} + \varepsilon_{jt}}_{\epsilon_{\tau,j,t}} \\
&\quad = \vartheta_j + \epsilon_{\tau,j,t}
\end{aligned}$$

Consumers have rational expectations which come from $\mu_{jt} \sim N(\mu(t), \sigma_\mu^2(t))$. To make the rational Bayesian update tractable, we will assume that initial quality prior is normally distributed, and also that $\epsilon_{\tau,j,t} \sim N(0, \tau_j^2)$. Then, consumers can rationally update their beliefs based on the experience surprise using the following rule:

$$\mathbb{E}[\mu_{jt}|I(t)] = \mathop{\mathbb{E}}_{t-1}[\mu_{jt}|I(t-1)] + D_{jt}\beta_j(t)(\vartheta_{jt} - \mathbb{E}[\vartheta|I(t-1)])$$

Where $\beta_j(t) = \frac{\sigma_{\mu,j}^2(t)}{\sigma_{\mu,j}^2(t) + \sigma_\tau^2}$.

Which means we can express the expectation updating rule as follows:

$$\mu_j(t) = \mu_j(t-1) + D_{jt}\beta_j(t)(-\mu_j(t-1) + \epsilon_{\tau,j})$$

And the perception error variance is given by:

$$\sigma_{\mu,j}^2(t) = \frac{1}{\frac{1}{\sigma_{\mu,j}^2(0)} + \frac{\sum_{s=0}^{t} D_{jt}}{\sigma_\tau^2}}$$

Once consumer updated, in the discrete choice model, the consumer makes his decision based on his priors on the match values for each product and enters the utility function as:

$$u = \gamma - \exp(-\rho(\mu_j(t-1) + \epsilon_{\tau,j})) - \alpha p_{jt}$$

Note that if there is no individual experience fluctuation ($\varepsilon_{jt} = 0$) we are left with ν_{jt} which is the product quality fluctuation. Then the utility will use as argument $\mu_j(t-1) + \nu_{jt} \sim \varepsilon_{jt}$ which is an approximation of the present product quality based on consumer prior beliefs.

$$u(\mathbf{p}, \pi) = \gamma - \exp(-\rho \underbrace{(\mu_j(t-1) + \nu_{j,t})}_{\sim \xi_{jt}}) - \alpha p_{jt}$$

Under this model, we can get rid of prices as a state variable from the value function. This is because prices are i.i.d and therefore previous prices do not reveal any information about future prices. This helps the modelling of the decision process since that way the average surprise of prices in every state will be zero, which implies no learning and therefore we can get rid of it as a state variable. If prices were not i.i.d over time then consumer would be able to update beliefs about future prices and therefore the decision process would be more complicated to formulate.

Part 2:

See the attached code.

Part 3:

Part A: Plot the value functions and CCP's for each product j on given a specific price vector and holding the beliefs for the other products constant. Discuss the results.

After holding prices constant, we see that the value function at the beginning fluctuates a lot since the consumer is learning about quality attributes over time, but then stabilizes at somewhere around 0.5 which is what we would spect after he realizes the average levels of mean quality.

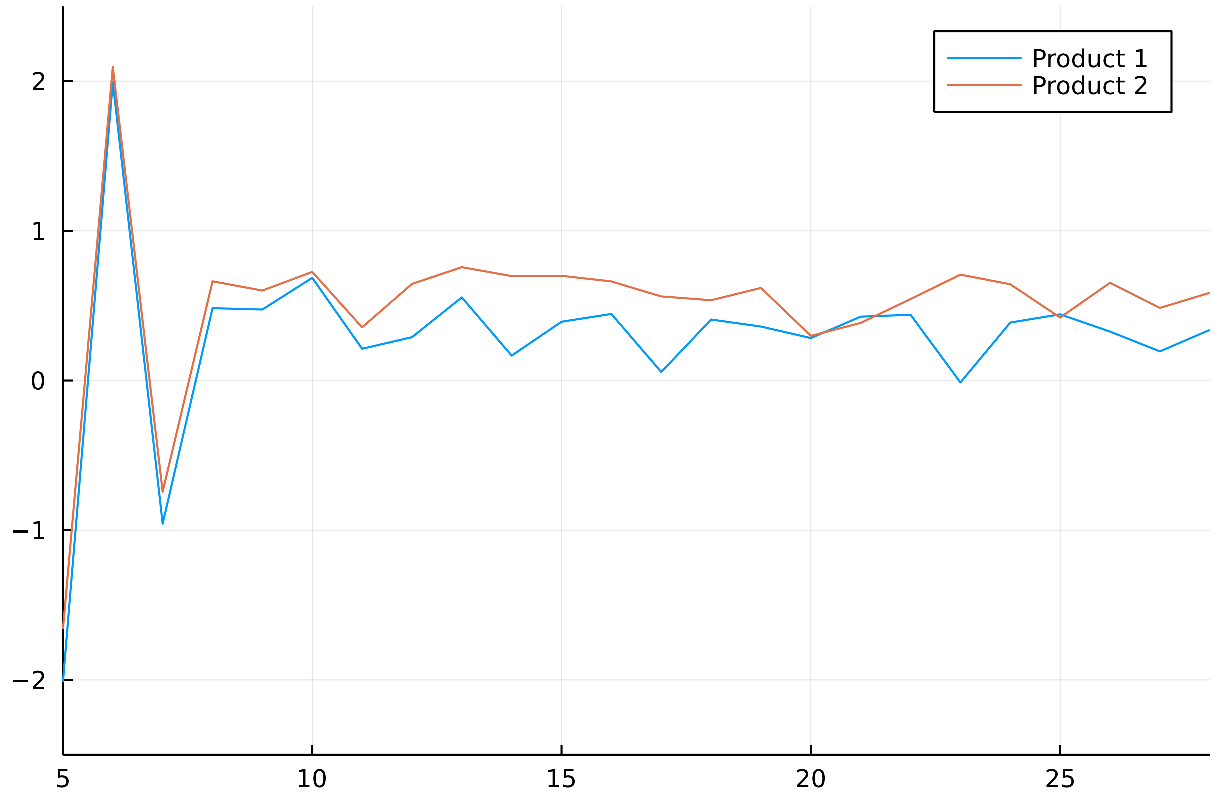

Figure 1: Value function over time

To see the evolution of the mean quality beliefs here I plot the time series and it stabilizes for both products close to the real value.

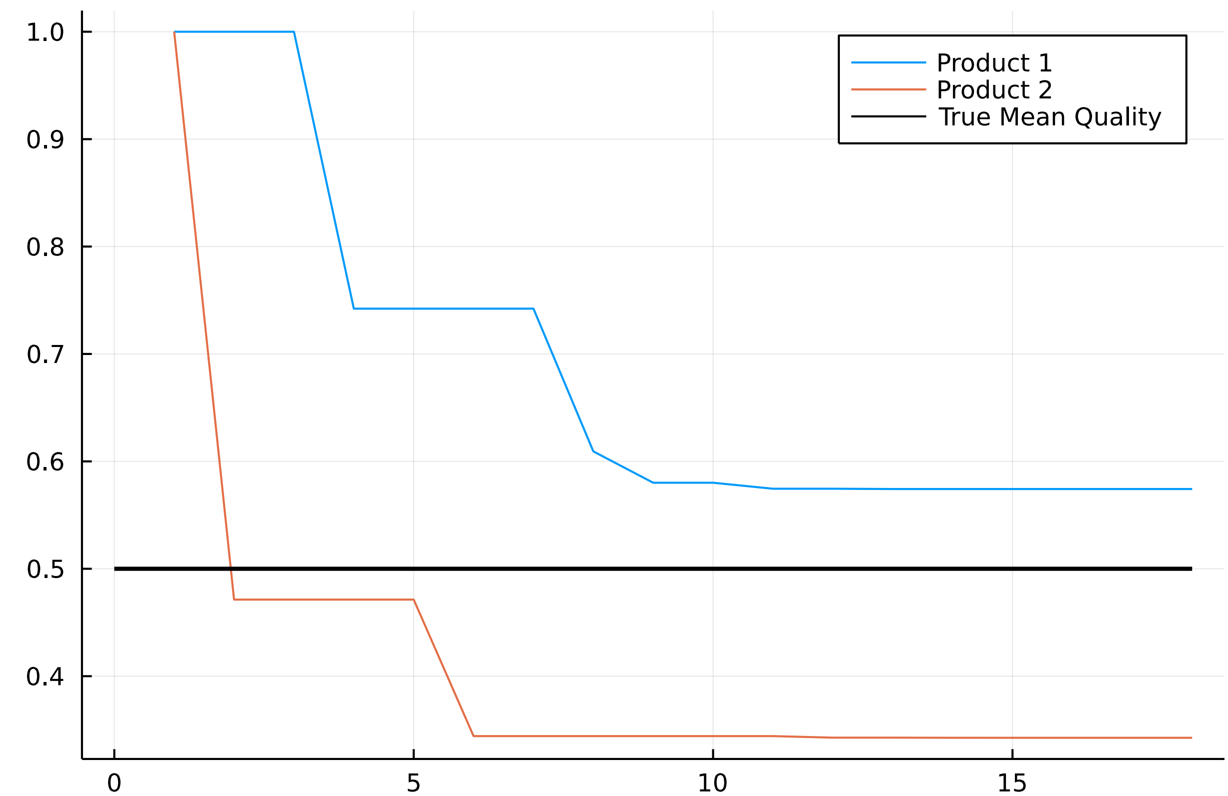

Figure 2: Mean quality consumer beliefs

Part B: Simulate choice sequences (a0; : : : ; aT)(n) for t = 0; : : : ; T: Condition on an initial prior and the true (but initially unknown to the consumer) match values ij , and draw prices pt; latent utility terms t; and signal noise terms t: Predict the consumer's choices based on the choice-specific value functions and update the prior accordingly. Average over N choice sequences (choose a large N) to predict the choice probabilities

Figure 1 shows the evolution of the probability estimates as t increases. As can be seen these probabilities stabilize for both products over time, which also implies that the probability of the outside option stabilizes.

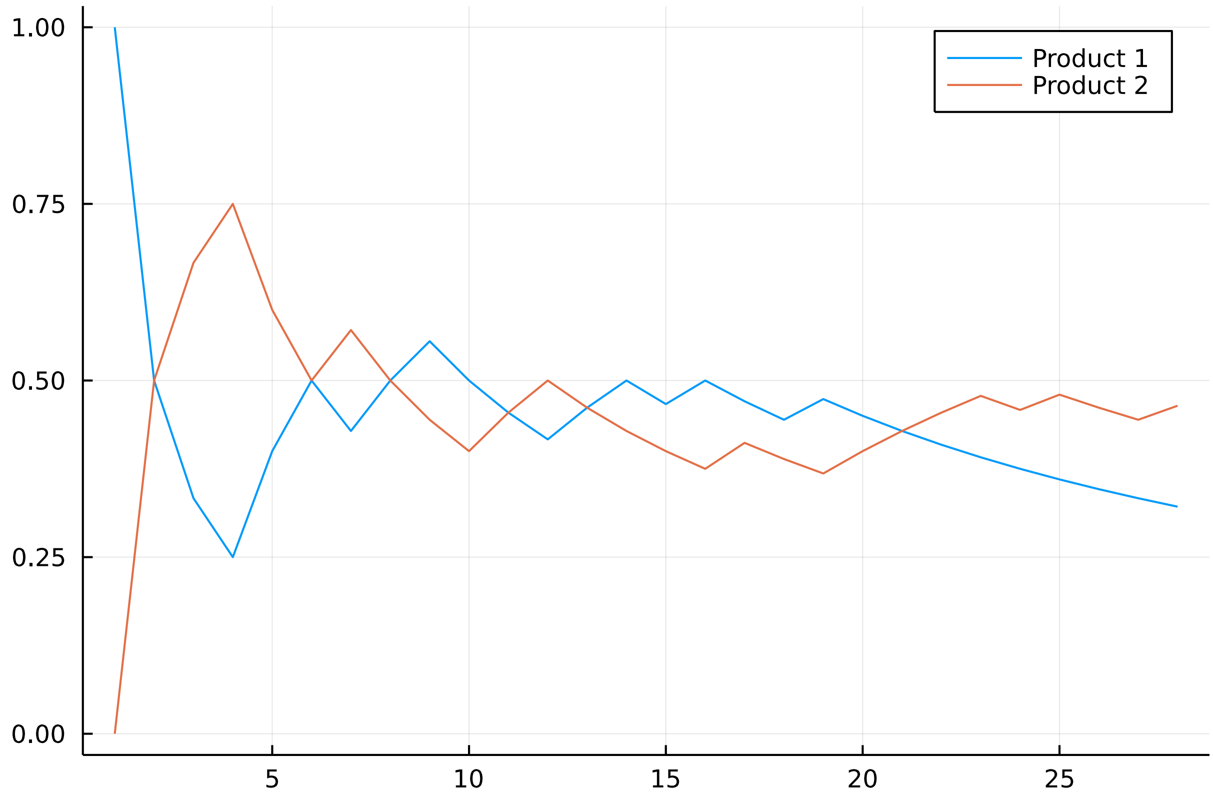

Figure 3: Probability Evolution

Part C: Show and explain how the choice probabilities evolve depending on the prior mean, the difference between the prior mean and the true match value ij ; and the prior variance, i.e. initial uncertainty.

As can be seen over time all probability estimates for choosing product 1 converge as time goes. All of the probabilities seem to stabilize around period 7. And the choice probabilities for people that have beliefs far away from the truth have initial probability estimates far away from the real propensity to consume.

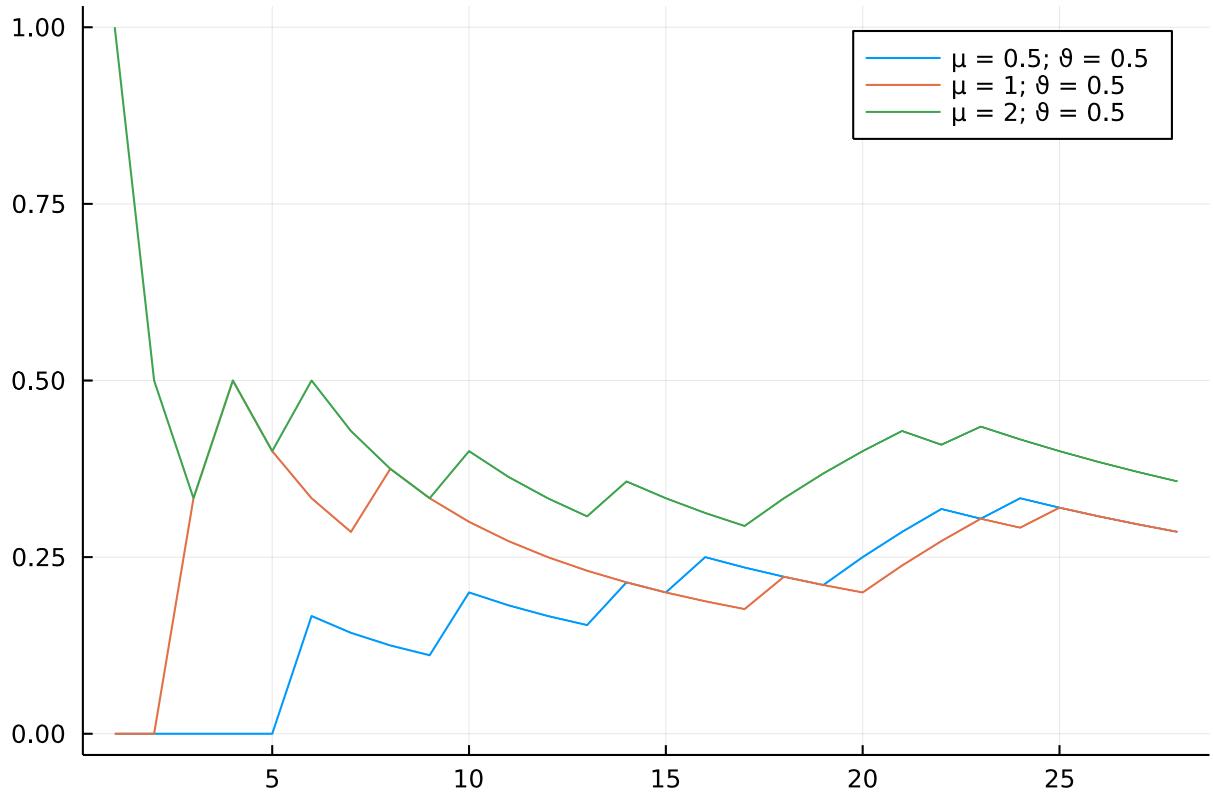

Figure 4: Probability of choosing product 1 evolution conditional on beliefs

Part D: Do you find evidence for experimentation?

Yes, however the experimentation process is a bit counter-intuitive because at the beginning there is not so much of it and then later at T=25 it increases to 0.55. The formula I'm following is:

$$\text{Inside Option}_t = \sum_{k=1}^{t} \text{inside option}_k$$

$$\text{Switch}_t = \sum_{k=1}^{t} \text{switch}_k$$

$$\text{Switch Rate}_t = \frac{\text{Switch}_t}{\text{Inside Option}_t}$$

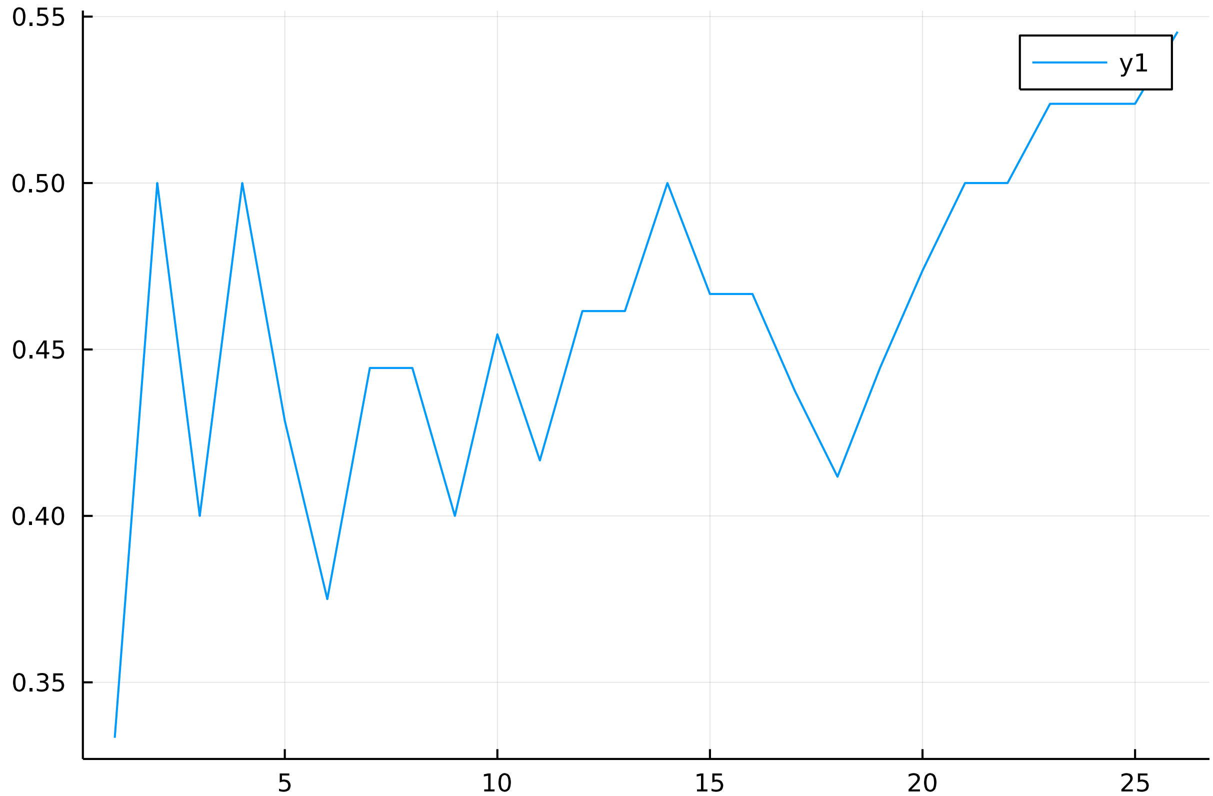

Figure 5: Experimentation

Part E: Compare the simulations for different discount factors, β = 0:998; 0:995; 0:99; 0; and for different levels of the risk aversion parameter ρ :

Here I plot the case where there are different discount values, it looks like discounting the future more heavily makes learning more difficult. The probabilities stabilize at period 15 for discount at 0.99, whereas for the cases where the consumer do not discount much the probability stabilizes faster over time.

Figure 6: Different Discount Values

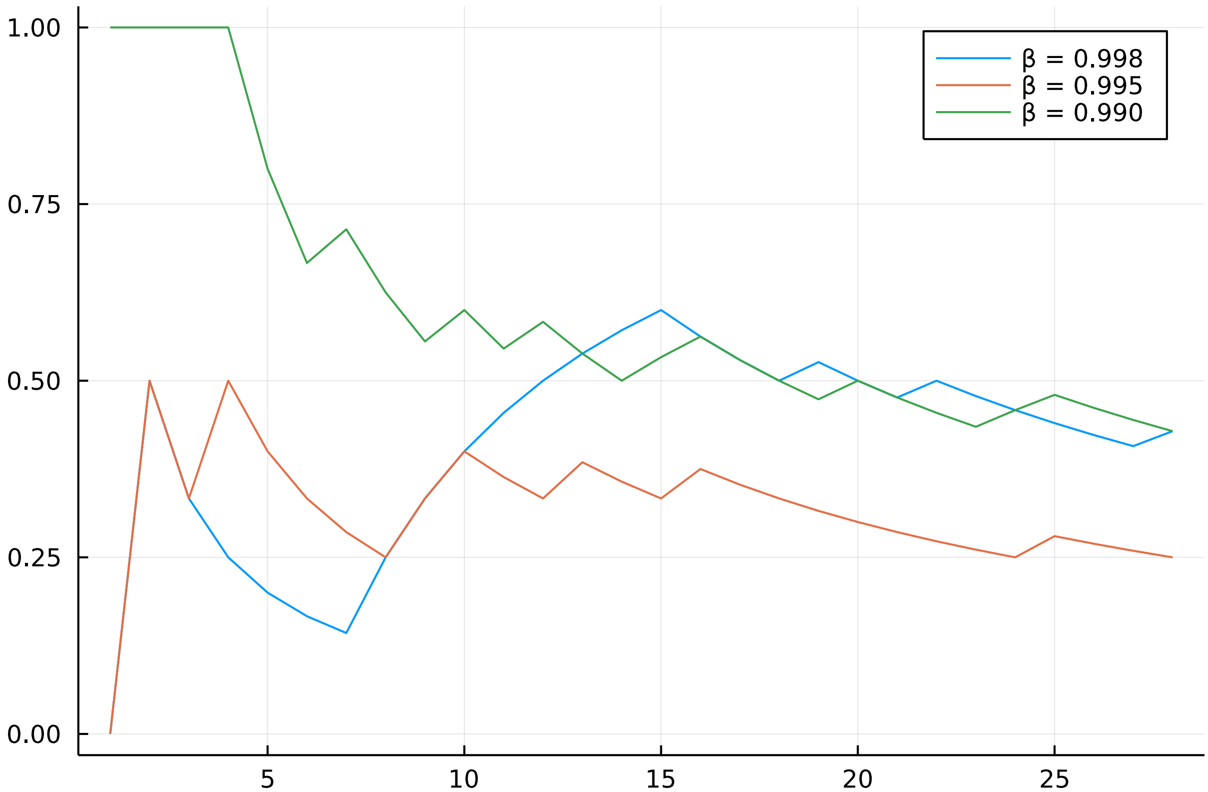

Figure 7: Different discount parameters

In the case of different risk aversion parameters I found that as consumers are more risk averse, they possibly explore less and therefore take longer to trend back to the real probabilities.

Figure 8: Different Risk Aversion Parameters

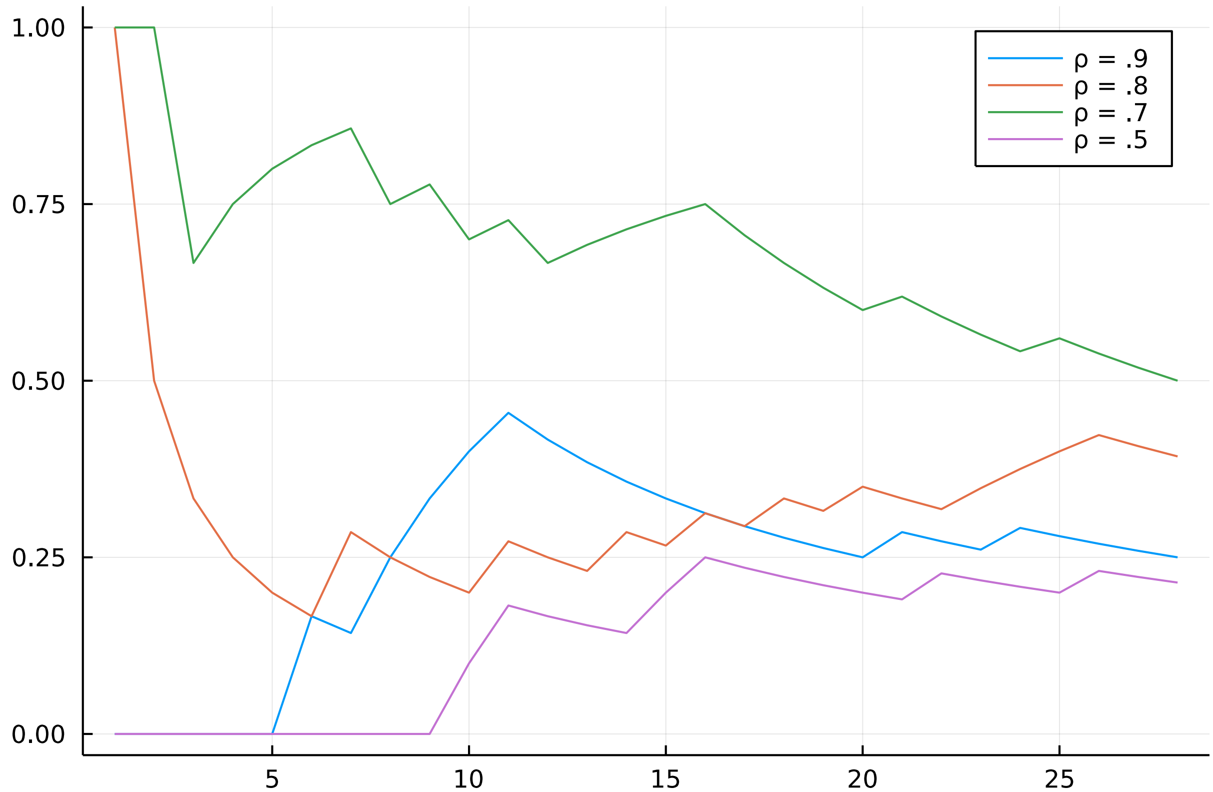

Figure 9: Different risk aversion levels