BUS 37904 Problem Set 2

Franco Calle

Brief discussions with Chuhan Guo

University of Chicago

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Part 1:

1 Part I

The utility function in our model is:

$$u_j(x) = (1-j)\epsilon_0 + j[\delta + \epsilon_1 + r1(x = N)]$$

Where the state space is defined by the set of states $s = (x, \epsilon)$ with $\epsilon \sim EV(0,1)$ and $x \in \{0, 1, ..., N\}$.

And the transition probability is:

$$\begin{aligned} Pr(x|x, 0) = 1 \\ Pr(0|N, 1) = 1 \\ Pr(x+1|x, 1) = 1 \text{ for all x<N} \end{aligned}$$

2 Part II

Recall the additive separability of agent's utility:

$$U(x_t, a_t) = u(x_t, a_t) + \epsilon_t(a_t)$$

Then, plugging the flow utility in the value function we have:

$$V_\theta(x_t, \epsilon_t) = \max_{a_t \in (0,1)} \{u_j(x_t) + \epsilon_t(a_t) + \beta E[V(x_{t+1}, \epsilon_{t_1})|x_t, \epsilon_t, a_t]\}$$

and we have the expected value function as:

$$w(x) = \int \max_{a_t \in (0,1)} \left\{ u_j(x_t) + \epsilon + \beta \int w(x')f(x'|x, j)dx' \right\} g(\epsilon)d\epsilon$$

Where RHS is contraction as long as $0 < \beta < 1$.

Then, the conditional choice probability is:

$$p(a|x) = \frac{\exp(u(x_t, a) + \beta EV(x_t, a))}{\exp(u(x_t, 1) + \beta EV(x_t, 1)) + \exp(u(x_t, 0) + \beta EV(x_t, 0))}$$

Given that we have N states we can plug all of these in matrix form:

$$EV(a) = \Pi(a) \times \log(\exp(u(x_t, 1) + \beta EV(x_t, 1)) + \exp(u(x_t, 0) + \beta EV(x_t, 0)))$$

Where $\Pi(a)$ is the transition probability matrix conditional on a.

Finally, we can define the choice, specific value function as:

$$v_j(x) = u_j(x) + \beta \int w(w')f(x'|x, j)dx'$$

3 Part III

The difference $v_1(x) - v_0(x)$ is the additional value of being loyal to the brand in each state x. Meanwhile, $v_1(0) - v_0(0)$ is the differential value choosing the product vs the outside option when at state 0 there are no rewards accumulated. This is the right way to define the cost because if we din't consider $v_1(0) - v_0(0)$ the cost would be confounded by the intrinsic additional value of buying the brand rather than other. The switching cost in Hartman and Viard words would be how much greater the value of progressing further in the program is than the value of begining the program.

4 Part IV

Here I plot the costs for types starting from low 0.01 to high 1 in the x axis, with different values for β. We can see that as people discount value more agressively the cost increases as shown by the top line and each cost line monotonically decreases as type value increases.

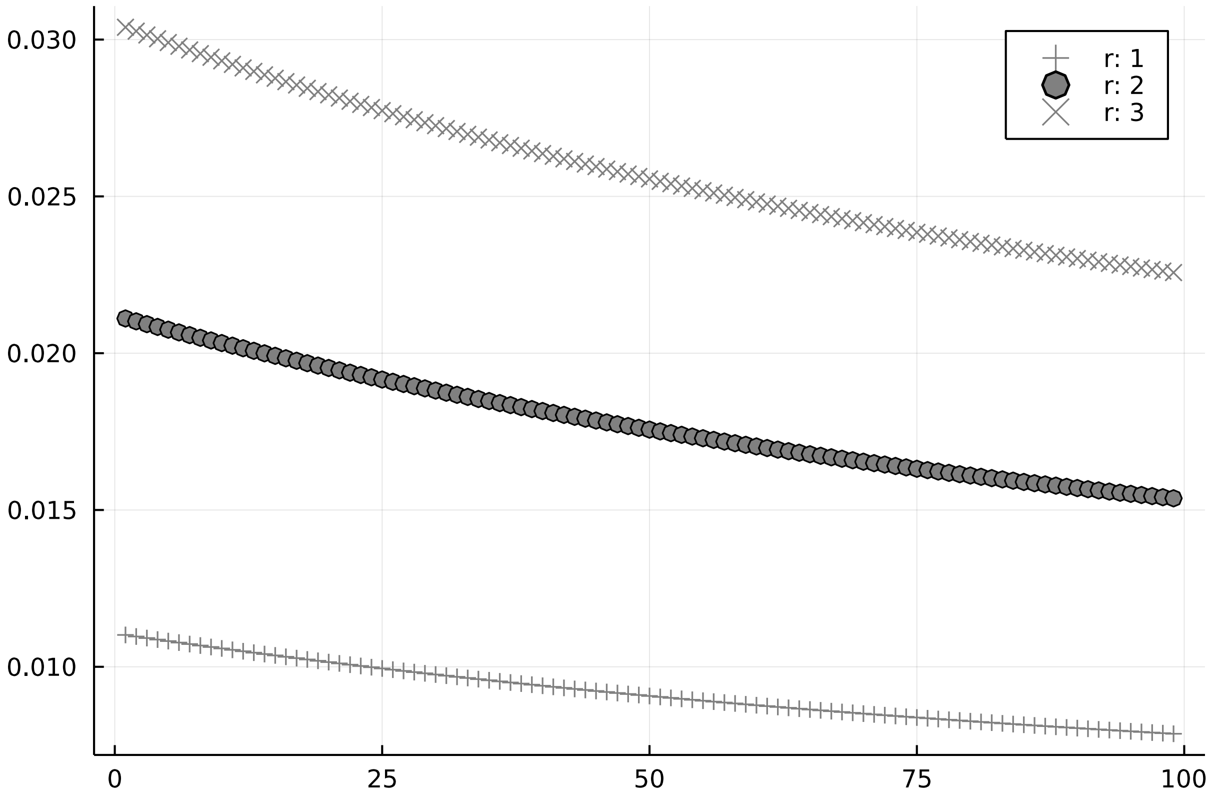


Figure 1: Cost for different types and discounts

However, if people discount very agressively $\beta = 0.8$, we can see that very low types have lower costs than some higher types, creating a non monotonic relation. In this case we would find a violation toHartmann and Viard argument.

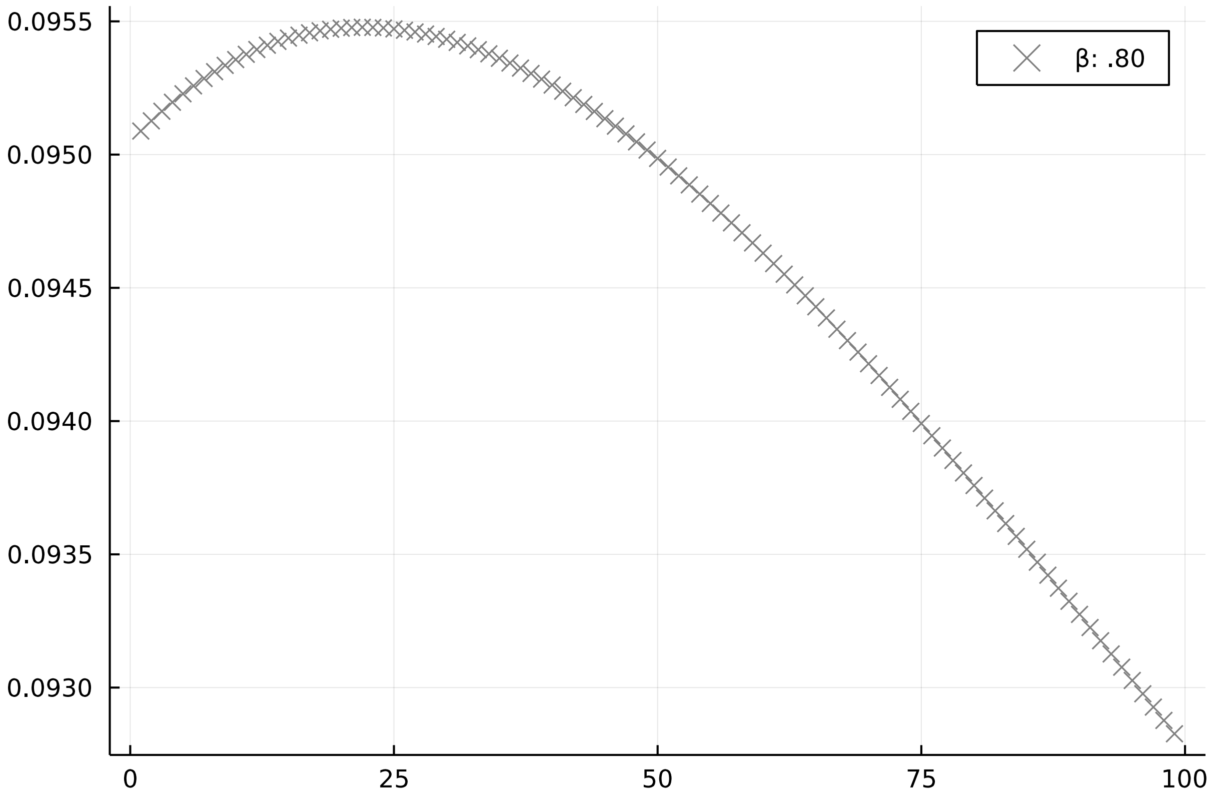


Figure 2: Cost for different types and agressive discount

5 Part V

Note that we have:

$$\begin{aligned}
Pr(a=j|x) &= \frac{\exp(v_j(x))}{\exp(v_0(x))+\exp(v_1(x))} \\
\implies \log(\sigma_1(x)) &= v_1(x) - \log(exp(v_0(x))+\exp(v_1(x))) \\
\implies v_1(x)-v_0(x) &= \log(\sigma_1(x)) - \log(\sigma_0(x))
\end{aligned}$$

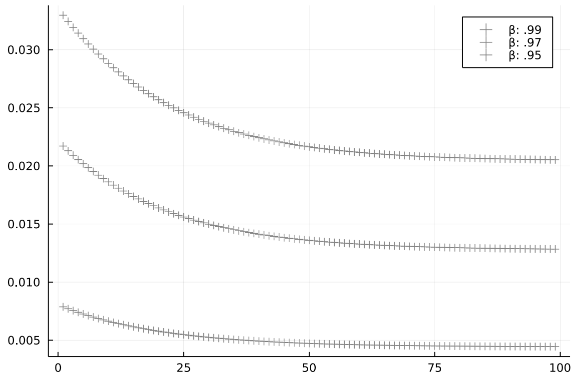
The third line is after subtracting and note that the common term cancels. Finally:

$$\begin{aligned}
\exp(c(x)) &= \exp(v_1(x)-v_0(x)-v_1(0)+v_0(0)) \\
&= \frac{\sigma_1(x)-\sigma_0(x)}{\sigma_1(0)-\sigma_0(0)}
\end{aligned}$$

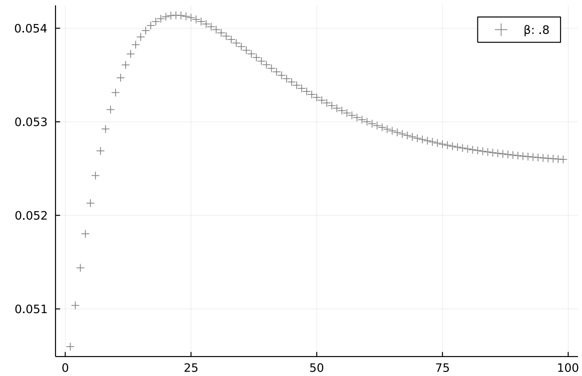
6 Part VI

Here I plot the costs obtained using different estimates. The mean and the median. It looks like when we use the median to measure the cost, the monotonicity over types is violated when $\beta = 0.8$.

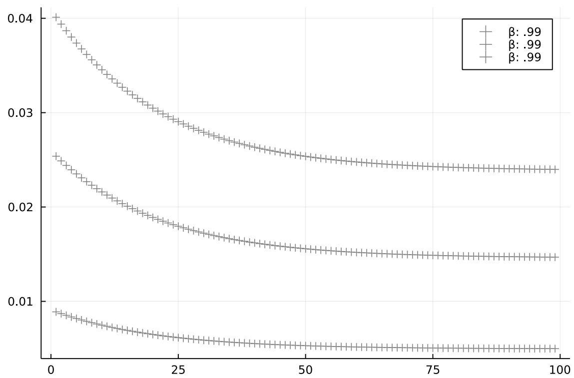
This does not happen however when using the average.



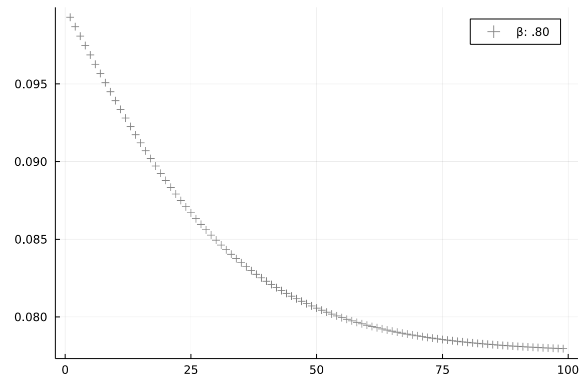
(a) Median Cost



(b) Median Cost Aggressive Discount



(c) Mean Cost



(d) Mean Cost Aggressive Discount

Figure 3: M-H sampled parameters for Linear Link