PyE Práctica 4

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Ejercicio 1

a) $R_x = [2, 12]$

b) Prob puntual y dist. acumulada:

x	**S**	**p(x)**	**F(x)**
2	(1,1)	1/36	1/36
3	(1,2), (2,1)	2/36	3/36
4	(1,3), (2,2), (3,1)	3/36	6/36
5	(1,4), (2,3), (3,2), (4,1)	4/36	10/36
6	(1,5), (2,4), (3,3), (4,2), (5,1)	5/36	15/36
7	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	6/36	21/36
8	(2,6), (3,5), (4,4), (5,3), (6,2)	5/36	26/36
9	(3,6), (4,5), (5,4), (6,3)	4/36	30/36
10	(4,6), (5,5), (6,4)	3/36	33/36
11	(5,6), (6,5)	2/36	35/36
12	(6,6)	1/36	1

c)
- $E(X) = \sum_{i=2}^{12} x_i p(x_i) = 2\frac{1}{36} + 3\frac{2}{36} + 4\frac{3}{36} + 5\frac{4}{36} + 6\frac{5}{36} + 7\frac{6}{36} + 8\frac{5}{36} + 9\frac{4}{36} + 10\frac{3}{36} + 11\frac{2}{36} + 12\frac{1}{36} = 7$
- $E(X^2) = \sum_{i=2}^{12} x_i^2 p(x_i) = 4\frac{1}{36} + 9\frac{2}{36} + 16\frac{3}{36} + 25\frac{4}{36} + 36\frac{5}{36} + 49\frac{6}{36} + 64\frac{5}{36} + 81\frac{4}{36} + 100\frac{3}{36} + 121\frac{2}{36} + 144\frac{1}{36} = \frac{329}{6}$

d) $V(X) = E(X^2) - (E(X)^2) = \frac{329}{6} - 49 = \frac{35}{6}$

e) $\sigma_x = \sqrt{\frac{35}{6}} \approx 2,415$

Ejercicio 2

a)
- $p(1), p(2) = 0$
- $p(3) = \frac{1}{3}$
- $p(4) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

- $p(5) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$
- $p(6) = 1 - \frac{2}{3} = \frac{1}{3}$

b) $P(3 < T \leq 5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = F(5) - F(3)$

c)
- $E(T) = \sum_{i=3}^{6} x_i p(x_i) = 3\frac{1}{3} + 4\frac{1}{6} + 5\frac{1}{6} + 6\frac{1}{3} = \frac{9}{2}$
- $E(T^2) = \sum_{i=3}^{6} x_i^2 p(x_i) = 9\frac{1}{3} + 16\frac{1}{6} + 25\frac{1}{6} + 36\frac{1}{3} = \frac{131}{6}$
- $V(T) = E(T^2) - (E(T))^2 = \frac{131}{6} - \frac{81}{4} = \frac{19}{12}$
- $\sigma_t = \sqrt{\frac{19}{12}} = \approx 1,26$

Ejercicio 3

a)
- $p(1) = \frac{1}{8}$
- $p(2) = \frac{3}{8} - \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$
- $p(3) = \frac{3}{4} - \frac{3}{8} = \frac{3}{8}$
- $p(4) = 1 - \frac{3}{4} = \frac{1}{4}$

b)
- $P(1 \leq X \leq 3) = F(X = 3) = \frac{1}{8} + \frac{1}{4} + \frac{3}{8} = \frac{3}{4}$
- $P(X < 3) = F(X = 3) = \sum_{i=1}^{2} p(x_i) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$
- $P(X > 1,4) = F(4) = \frac{1}{4} + \frac{3}{8} + \frac{1}{4} = \frac{7}{8}$

c)
- $E(X) = \sum_{i=1}^{4} x_i p(x_i) = 1\frac{1}{8} + 2\frac{1}{4} + 3\frac{3}{8} + 4\frac{1}{4} = \frac{11}{4}$
- $E(X^2) = \sum_{i=1}^{4} x_i^2 p(x_i) = 1\frac{1}{8} + 4\frac{1}{4} + 9\frac{3}{8} + 16\frac{1}{4} = \frac{17}{2}$
- $V(X) = E(X^2) - (E(X))^2 = \frac{17}{2} - (\frac{22}{8})^2 = \frac{15}{16}$
- $\sigma_x = \sqrt{\frac{15}{16}} \approx 0,968245836$

Ejercicio 4

a) (Dist. Binomial)
- $P(Y = 0) = \binom{3}{0}(0,05)^0(0,95)^3 = 0,8574$
- $P(Y = 1) = \binom{3}{1}(0,05)^1(0,95)^2 = 0,1354$
- $P(Y = 2) = \binom{3}{2}(0,05)^2(0,95)^1 = 0,0071$
- $P(Y = 3) = \binom{3}{3}(0,05)^3(0,95)^0 = 0,0001$

b) $P(Y > 1) = F(3) - F(1) = 0,0071 + 0,0001 = 0,0072$

Ejercicio 5

a) $R_x = \mathbf{N}$

b) $P(Z = 5) = (0,95)^4(0,05) = 0,040725312$ (Dist. Geométrica)