PyE Práctica 5

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April 2025

Ejercicio 1

a) $\int_{-\infty}^{\infty} kx^2dx = 1 \Rightarrow \int_{-1}^{0} kx^2dx = 1 \Rightarrow k\int_{-1}^{0} x^2dx = k[\frac{x^3}{3}]_{-1}^{0} = k(0-(\frac{1}{3})) = \frac{k}{3} \Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3$

b) $E(X) = \int_{-1}^{0} x3x^2dx = 3\int_{-1}^{0} x^3 = 3[\frac{x^4}{4}]_{-1}^{0} = 3(0-\frac{1}{4}) = -\frac{3}{4}$
$E(X^2) = \int_{-1}^{0} x^2 3x^2 = 3\int_{-1}^{0} x^4 = 3[\frac{x^5}{5}]_{-1}^{0} = 3(0-(-\frac{1}{5})) = \frac{3}{5}$
$V(X) = E(X^2) - E(X)^2 = \frac{3}{5} - \frac{9}{16} = \frac{3}{80}$

c)

$$F(X) = \begin{cases} 0 & \text{si } x < -1 \\ \int_{-1}^{x} 3t^2dt & \text{si } -1 \leq x \leq 0 \\ 1 & \text{si } x > 0 \end{cases}$$

$\int_{-1}^{x} 3t^2dt = 3\int_{-1}^{x} t^2dt = 3[\frac{t^3}{3}]_{-1}^{x} = x^3 + 1$

$$F(X) = \begin{cases} 0 & \text{si } x < -1 \\ x^3 + 1 & \text{si } -1 \leq x \leq 0 \\ 1 & \text{si } x > 0 \end{cases}$$

d) $x^3 + 1 = 0{,}5 \Rightarrow x^* = \sqrt[3]{-0{,}5}$

Ejercicio 2

a) $f(y) = \frac{d}{dy}(1 - e^{-y^2}) = 2ye^{-y^2}$

$$f(y) = \begin{cases} 0 & \text{si } y < 0 \\ 2ye^{-y^2} & \text{si } y \geq 0 \end{cases}$$

b) $P(Y \geq 200) = 1 - P(Y \leq 200) = 1 - F(200) = e^{-200^2} = e^{-40000} \approx 0$

Ejercicio 3

a) $P(T < 200/T > 150) = \frac{P(150<T<200)}{P(T>150)}$
Calculamos la función de distribución: $F(t) = \int_{100}^{t} \frac{100}{x^2}dx = [-\frac{100}{x}]_{100}^{t} = 1 - \frac{100}{t}$
Luego: $P(150 < T < 200) = F(200) - F(150) = (1 - \frac{100}{200} - (1 - \frac{100}{150})) = \frac{1}{6}$
$P(T > 150) = 1 - F(150) = \frac{100}{150} = \frac{2}{3}$
$P(T < 200|T > 150) = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{4}$

b) $P(T \leq 150) = F(150) = 1 - \frac{100}{150} = \frac{1}{3}$
$P(\text{Al menos uno falla}) = 1 - P(\text{Todos fallan}) = 1 - (\frac{2}{3})^3 = \frac{19}{27}$

c) Buscamos: $P(T < 150)^n \leq 0{,}5 \Rightarrow (\frac{2}{3})^n \leq 0{,}5$
$nlog(\frac{2}{3}) \leq log(0{,}5) \Rightarrow n \geq \frac{log(0{,}5)}{log(\frac{2}{3})} \approx 1{,}709$
Resultado: Se necesitan mínimo 2 tubos.

Ejercicio 4

$P(x > \frac{1}{3}/\text{Tipo A}) = \int_{\frac{1}{3}}^{1} 4(1-x)^3dx = \int_{\frac{2}{3}}^{0} 4u^3(-du) = \int_{0}^{\frac{2}{3}} 4u^3du = 4[\frac{u^4}{4}]_0^{\frac{2}{3}} = (\frac{2}{3})^4 = \frac{16}{81}$
La probabilidad de clasificar en B siendo A es de: $\frac{16}{81}$

Ejercicio 5

a)
$$f(X) = \begin{cases} \frac{1}{3} & \text{si } 1 \leq x \leq 4 \\ 0 & \text{En otro caso} \end{cases}$$

b)
$$F(X) = \begin{cases} 0 & \text{si } x < 1 \\ \frac{x-1}{3} & \text{si } 1 \leq x \leq 4 \\ 1 & \text{si } x > 4 \end{cases}$$

c) $P(X > 2) = 1 - F(2) = 1 - \frac{2-1}{3} = \frac{2}{3}$

d) $P(2 < x < 3) = \frac{3-2}{3} = \frac{1}{3}$

e) $P(X < 1{,}5) = F(1{,}5) = \frac{1{,}5-1}{3} = \frac{1}{6}$

Ejercicio 6

a)

$$f(X) = \begin{cases} \frac{1}{5} & \text{si } 20 \leq x \leq 25 \\ 0 & \text{En otro caso} \end{cases}$$

$$F(X) = \begin{cases} 0 & \text{si } x < 20 \\ \frac{x-20}{5} & \text{si } 20 \leq x \leq 25 \\ 1 & \text{si } x > 25 \end{cases}$$

Luego: $P(X \leq 23) = \frac{23-20}{5} = \frac{3}{5}$

b) $E(X) = \frac{a+b}{2} = \frac{20+25}{2} = \frac{45}{2}$ minutos