PyE Práctica 4

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Ejercicio 1

a) $R_x = [2, 12]$

b) Prob puntual y dist. acumulada:

x	**S**	**p(x)**	**F(x)**
2	(1,1)	1/36	1/36
3	(1,2), (2,1)	2/36	3/36
4	(1,3), (2,2), (3,1)	3/36	6/36
5	(1,4), (2,3), (3,2), (4,1)	4/36	10/36
6	(1,5), (2,4), (3,3), (4,2), (5,1)	5/36	15/36
7	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	6/36	21/36
8	(2,6), (3,5), (4,4), (5,3), (6,2)	5/36	26/36
9	(3,6), (4,5), (5,4), (6,3)	4/36	30/36
10	(4,6), (5,5), (6,4)	3/36	33/36
11	(5,6), (6,5)	2/36	35/36
12	(6,6)	1/36	1

c)
- $E(X) = \sum_{i=2}^{12} x_i p(x_i) = 2\frac{1}{36} + 3\frac{2}{36} + 4\frac{3}{36} + 5\frac{4}{36} + 6\frac{5}{36} + 7\frac{6}{36} + 8\frac{5}{36} + 9\frac{4}{36} + 10\frac{3}{36} + 11\frac{2}{36} + 12\frac{1}{36} = 7$
- $E(X^2) = \sum_{i=2}^{12} x_i^2 p(x_i) = 4\frac{1}{36} + 9\frac{2}{36} + 16\frac{3}{36} + 25\frac{4}{36} + 36\frac{5}{36} + 49\frac{6}{36} + 64\frac{5}{36} + 81\frac{4}{36} + 100\frac{3}{36} + 121\frac{2}{36} + 144\frac{1}{36} = \frac{329}{6}$

d) $V(X) = E(X^2) - (E(X)^2) = \frac{329}{6} - 49 = \frac{35}{6}$

e) $\sigma_x = \sqrt{\frac{35}{6}} \approx 2,415$

Ejercicio 2

a)
- $p(1), p(2) = 0$
- $p(3) = \frac{1}{3}$
- $p(4) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

- $p(5) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$
- $p(6) = 1 - \frac{2}{3} = \frac{1}{3}$

b) $P(3 < T \leq 5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = F(5) - F(3)$

c)
- $E(T) = \sum_{i=3}^{6} x_i p(x_i) = 3\frac{1}{3} + 4\frac{1}{6} + 5\frac{1}{6} + 6\frac{1}{3} = \frac{9}{2}$
- $E(T^2) = \sum_{i=3}^{6} x_i^2 p(x_i) = 9\frac{1}{3} + 16\frac{1}{6} + 25\frac{1}{6} + 36\frac{1}{3} = \frac{131}{6}$
- $V(T) = E(T^2) - (E(T))^2 = \frac{131}{6} - \frac{81}{4} = \frac{19}{12}$
- $\sigma_t = \sqrt{\frac{19}{12}} = \approx 1,26$

Ejercicio 3

a)
- $p(1) = \frac{1}{8}$
- $p(2) = \frac{3}{8} - \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$
- $p(3) = \frac{3}{4} - \frac{3}{8} = \frac{3}{8}$
- $p(4) = 1 - \frac{3}{4} = \frac{1}{4}$

b)
- $P(1 \leq X \leq 3) = F(X = 3) = \frac{1}{8} + \frac{1}{4} + \frac{3}{8} = \frac{3}{4}$
- $P(X < 3) = F(X = 3) = \sum_{i=1}^{2} p(x_i) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$
- $P(X > 1,4) = F(4) = \frac{1}{4} + \frac{3}{8} + \frac{1}{4} = \frac{7}{8}$

c)
- $E(X) = \sum_{i=1}^{4} x_i p(x_i) = 1\frac{1}{8} + 2\frac{1}{4} + 3\frac{3}{8} + 4\frac{1}{4} = \frac{11}{4}$
- $E(X^2) = \sum_{i=1}^{4} x_i^2 p(x_i) = 1\frac{1}{8} + 4\frac{1}{4} + 9\frac{3}{8} + 16\frac{1}{4} = \frac{17}{2}$
- $V(X) = E(X^2) - (E(X))^2 = \frac{17}{2} - (\frac{22}{8})^2 = \frac{15}{16}$
- $\sigma_x = \sqrt{\frac{15}{16}} \approx 0,968245836$

Ejercicio 4

a) (Dist. Binomial)

- $P(Y = 0) = \binom{3}{0}(0,05)^0(0,95)^3 = 0,8574$
- $P(Y = 1) = \binom{3}{1}(0,05)^1(0,95)^2 = 0,1354$
- $P(Y = 2) = \binom{3}{2}(0,05)^2(0,95)^1 = 0,0071$
- $P(Y = 3) = \binom{3}{3}(0,05)^3(0,95)^0 = 0,0001$

b) $P(Y > 1) = F(3) - F(1) = 0,0071 + 0,0001 = 0,0072$

Ejercicio 5

a) $R_x = \mathbf{N}$

b) $P(Z = 5) = (0,95)^4(0,05) = 0,040725312$ (Dist. Geométrica)

Ejercicio 6

a) Con reposición

- $P(D) = \frac{4}{20} = 0,2$
- $P(\bar{D}) = 1 - 0,2 = 0,8$
- $P(X = 0) = \binom{3}{0} 0,2^0 0,8^3 = 0,512$
- $P(X = 1) = \binom{3}{1} 0,2^1 0,8^2 = 0,384$
- $P(X = 2) = \binom{3}{2} 0,2^2 0,8^1 = 0,096$
- $P(X = 3) = \binom{3}{3} 0,2^3 0,8^0 = 0,008$
- $F(x) = P(X \leq x) = \sum_{i=0}^{x} p(X = x)$
- $F(0) = 0,512$
- $F(1) = 0,512 + 0,384 = 0,896$
- $F(2) = 0,512 + 0,384 + 0,096 = 0,992$
- $F(3) = 0,512 + 0,384 + 0,096 + 0,008 = 1$

b) Sin reposición

- $P(X = 0) = \frac{\binom{4}{0}\binom{16}{3}}{\binom{20}{3}} = 0,49122807$
- $P(X = 1) = \frac{\binom{4}{1}\binom{16}{2}}{\binom{20}{3}} = 0,421052631$
- $P(X = 2) = \frac{\binom{4}{2}\binom{16}{1}}{\binom{20}{3}} = 0,084210526$
- $P(X = 3) = \frac{\binom{4}{3}\binom{16}{0}}{\binom{20}{3}} = 0,003508771$
- $F(0) = 0,49122807$
- $F(1) = 0,49122807 + 0,421052631 = 0,912280701$
- $F(2) = 0,49122807 + 0,421052631 + 0,084210526 = 0,996491227$
- $F(3) = 0,49122807 + 0,421052631 + 0,084210526 + 0,003508771 = 0,999999998$

Ejercicio 7

- $P(\text{Aceptar el lote}) = P(X = 0) = \frac{\binom{3}{0}\binom{22}{5}}{\binom{25}{5}} = 0,495652173$
- $P(\text{Rechazar el lote}) = 1 - 0,495652173 = 0,504347826$
- Promedio de lotes rechazados con muestra de $100 = 100 \cdot 0,504347826 = 50,43478261$

Ejercicio 8

a) $P(X=x) = p(1-p)^{x-1}$

b) $P(X=5) = p(5) = p(1-p)^4$

c) *Demostración.*

$$f'(p) = \frac{d}{dp}[(1-p)^4 p] = (1-p)^4 - 4(1-p)^4 p$$

$$f'(p) = (1-p)^4[(1-p)-4p] = (1-p)^3(1-5p)$$

$$(1-p)^3 = 0 \Leftrightarrow p = 1$$

$$(1-5p) = 0 \Leftrightarrow p = \frac{1}{5}$$

Solo $\frac{1}{5}$ esta en (0,1), lo cual da el máximo.

QED

Ejercicio 9

a)
- $P(X=x) = \binom{x-1}{5-1} 0,8^5 0,2^{x-5}$
- $P(X=8) = \binom{7}{4} 0,8^5 0,2^3 = 0,0917504$

b) $E(X) = \frac{r}{p} = \frac{5}{0,8} = 6,25$

Ejercicio 10

a)
- $P(X=x) = e^{-3}\frac{3^x}{x!}$
- $P(X=0) = e^{-3}\frac{1}{1} = 0,0498$

b) $P(X \leq 2) = \sum_{i=0}^{2} e^{-3}\frac{3^i}{i!} = 0,0498 + 0,1494 + 0,2240 = 0,4232$

Ejercicio 11

Demostración.

$$P(X=x) = e^{-\lambda}\frac{\lambda^x}{x!}$$

$P(X \geq 1) < 0,5 \Rightarrow 1 - P(X=0) < 0,5 \Rightarrow 1 - e^{-\lambda} < 0,5 \Rightarrow e^{-\lambda} < 0,5 \Rightarrow -\lambda < \ln 0,5 \Rightarrow \lambda < -\ln 0,5 = 0,6931$

Finalmente, el promedio de grietas en 4 metros de alambre debe ser menor a 0,6931

QED

Ejercicio 12

a)
- $\lambda = 1 = E(X)$
- $P(X = 0) = e^{-1}\frac{1^0}{0!} = \frac{1}{e}$

b) $P(Z \leq 5) = \sum_{z=0}^{z=5} \binom{15}{z} (\frac{1}{e})^z (1 - \frac{1}{e})^{15-z} = 0,0010 + 0,00389 + 0,0366 + 0,0922 + 0,1610 + 0,2061 = 0,50079$

Ejercicio 13

a)
- $P(X \geq 1) = 1 - P(X = 0) = 0,4 \Rightarrow e^{-\lambda} = 0,6 \Rightarrow \lambda = -\ln 0,6 = 0,5108$
- $P(X \leq 2) = \sum_{x=0}^{2} e^{-\lambda} \frac{\lambda^x}{x!} = e^{-0,5108} + e^{-0,5108} \frac{0,5108^1}{1} + e^{-0,5108} \frac{0,5108^2}{2} = 0,6000 + 0,3065 + 0,0783 = 0,9848$

b)
- $P(Y = y) = \binom{10}{y} 0,4^y 0,6^{10-y}$
- $P(Y \leq 1) = P(Y = 0) + P(Y = 1) = 0,0436$

c) $E(Y) = np = 10 \cdot 0,4 = 4$