Modelización Numérica – Problemas Unidad 1 y 2

1. Sea una M/M/1 tal que $\lambda = 12$ clientes/segundo, y $T_s = 0{,}05$ segundo/cliente hallar:
   a) La utilización del sistema.
   b) La probabilidad que el sistema este ocioso.
   c) La probabilidad que en el sistema haya al menos 3 clientes y a lo sumo 5.
   d) La probabilidad que el número de clientes en el sistema sea como mínimo 3.
   e) La probabilidad que en el sistema haya como mínimo 1 cliente.

2. En una M/M/1 $\pi_1 = 0{,}24$. Hallar π_3.

3. En una M/M/1 $\pi_k = 0{,}008$, $\pi_{k+2} = 0{,}002$. Hallar π_{k-1}.

4. En una M/M/1 $\pi_{(k+1)^2+2} = 0{,}003$, $\pi_{(k+1)^2} = 0{,}009$. Hallar $\pi_{(k^2+2k)}$.

5. En una M/M/1 tal que $\pi_1 = 0{,}22$, graficar en un mismo par de ejes $\pi_n = f_{(n)}$.

6. Sea una M/M/1/4 talque $\lambda = 10$ clientes/segundo y $\mu = 16$ clientes/segundo, hallar: a) π_o.
   b) P_b (probabilidad de bloqueo).
   c) γ_I (rendimiento a la entrada).
   d) γ_o (rendimiento a la salida).
   e) % de rechazo.

7. Sea una M/M/1/N tal que $\pi_0 = 0{,}60$ siendo $\rho = 0{,}5$ hallar la capacidad del sistema (N).

8. Sea una M/M/1 que ha sido observado durante 20 segundos ($T_{obs} = 20$ segundos) hallándose los siguientes valores:

i	Ta_i	Ts_i	In	Out	W_i
1	2	3			
2	4	4			
3	11	2			
4	13	3			
5	15	3			

   a) Todos los tiempos son en segundos. Completar el cuadro y hallar el valor de N (número medio de clientes en el sistema)
   b) Realizar el diagrama de perfiles del sistema y de la cola.

9. Idem anterior con T_{obs} = 30 min

i	Ta_i	Ts_i	In	Out	W_i
1	6	5			
2	10	4			
3	18	5			
4	23	6			

Problema 1

$\rho = ?$

$\bar{T}_s = 0{,}05$ seg/cli

$\lambda = 12$ cli/seg

(a) $\bar{T}_s = \frac{1}{\eta}$

$\rho = \frac{\lambda}{\eta} = \lambda \cdot \frac{1}{\eta} \Rightarrow \lambda \cdot \bar{T}_s = 12$ cli/seg $\cdot$ 0,05 seg/cli

$\rho = 0{,}6$

(b) $\pi_0 = 1 - \rho$ $\quad \pi_0 = ?$

$\pi_0 = 1 - 0{,}6$

$\pi_0 = 0{,}4$

$P(3 \leq x \leq 5) = ?$

(c)

$P(3 \leq x \leq 5) = \pi_3 + \pi_4 + \pi_5$

$\pi_3 = \rho^3 (1 - \rho)$ $\quad \pi_4 = \rho^4 (1 - \rho)$

$\pi_3 = 0{,}6^3 (1 - 0{,}6) = 0{,}0864$

$\pi_4 = p\,\pi_3 = 0{,}6 \cdot 0{,}0864 = 0{,}05184$

(on est allé)

$\pi_5 = p\,\pi_4 = 0{,}6 \cdot 0{,}05184 = 0{,}031104$

$P(3 \leq x \leq 5) = 0{,}0864 + 0{,}05184 + 0{,}031104$

$P(3 \leq x \leq 5) = \boxed{0{,}1693}$

⑤ $P(3 \leq x)$?

$P(3 \leq x) = 1 - [\pi_0 + \pi_1 + \pi_2]$

$\pi_0 = 0{,}4$

$\pi_1 = p\,\pi_0 = 0{,}6 \cdot 0{,}4 = 0{,}24$

(on est allé)

$\pi_2 = p\,\pi_1 = 0{,}6 \cdot 0{,}24 = 0{,}144$

$P(3 \leq x) = 1 - [0{,}4 + 0{,}24 + 0{,}144]$

$P(3 \leq x) = 1 - 0{,}784 = \boxed{0{,}216}$

(e) $P(1 \leq x) = ?$

$P(1 \leq x) = 1 - \Pi_0 = 1 - 0{,}4 = \boxed{0{,}6}$

Problema 2

$\Pi_1 = 0{,}24$ $\qquad \Pi_3 = ?$

$\Pi_1 = p(1-p) \Rightarrow 0{,}24 = p - p^2$

$\underbrace{p^2}_{a} - \underbrace{p}_{b} + \underbrace{0{,}24}_{c} = 0$

$p = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$p = \dfrac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot 0{,}24}}{2 \cdot 1}$

$p = \dfrac{1 \pm \sqrt{0{,}04}}{2} = \begin{cases} p_1 = 0{,}6 \\ p_2 = 0{,}4 \end{cases}$

$\Pi_{3_1} = p_1^3 (1 - p_1) = 0{,}6^3 (1 - 0{,}6) = \boxed{0{,}0864}$

$\Pi_{3_2} = p_2^3 (1 - p_2) = 0{,}4^3 (1 - 0{,}4) = \boxed{0{,}0384}$

Problem 3

$\pi_{k-1} = ?$

$\pi_k = 0{,}008$

$\pi_{k+2} = 0{,}002$

$$\pi_{k+2} = \rho^{k+2}(1-\rho)$$
$$\pi_k = \rho^{k}(1-\rho)$$

$$\frac{\pi_{k+2}}{\pi_k} = \frac{\rho^{k+2}(1-\rho)}{\rho^{k}(1-\rho)}$$

$$\frac{0{,}002}{0{,}008} = \rho^2$$

$$\sqrt{\frac{0{,}002}{0{,}008}} = \rho$$

$$0{,}5 = \rho$$

$$\pi_k = \rho\,\pi_{k-1}$$

$$\frac{\pi_k}{\rho} = \pi_{k-1}$$

$$\frac{0{,}008}{0{,}5} = \pi_{k-1} \Rightarrow \pi_{k-1} = 0{,}016$$

OTRA forma de obtener ρ

$$\pi_{k+2} = \rho\, \pi_{k+1}$$

$$\pi_{k+1} = \rho\, \pi_k$$

$$\pi_{k+2} = \rho(\rho \pi_k)$$ → Reemplazando en π_{k+1}

$$\pi_{k+2} = \rho^2 \pi_k$$

$$\frac{\pi_{k+2}}{\pi_k} = \rho^2$$

$$\sqrt{\frac{\pi_{k+2}}{\pi_k}} = \rho$$

$$\rho = \sqrt{\frac{9002}{0,008}} = 0,5$$

Problema 4

$\pi_{k+2k} = ?$

$$\pi(k+1)^2 + 2 = 9003$$

$$\pi(k+1)^2 = 0,009$$

$$\pi(k+1)^2 + 2 = \rho\, \pi(k+1)^2 + 1$$

$$\pi(k+1)^2 + 1 = \rho\, \pi(k+1)^2$$

$$\pi(K+1)^2+2 = p\,p\,\pi(K+1)^2$$

$$\frac{\pi(K+1)^2+2}{\pi(K+1)^2} = p^2$$

$$\sqrt{\frac{\pi(K+1)^2+2}{\pi(K+1)^2}} = p$$

$$p = \sqrt{\frac{0,003}{0,009}} = 0,577$$

$$(K+1)^2 = K^2+2K+1$$

$$(K+1)^2-1 = K^2+2K$$

$$\pi(K+1)^2 = p\,\pi(K+1)^2-1$$

$$\pi(K+1)^2 = p\,\pi K^2+2K$$

$$\frac{\pi(K+1)^2}{p} = \pi K^2+2K$$

$$\pi K^2+2K = \frac{0,009}{0,577} = 0,0156$$

Problema 2)

$\pi_1 = 0,22$

$\pi_1 = p(1-p) \Rightarrow 0,22 = p - p^2$

$p^2 - p + 0,22 = 0$

$a \quad b \quad c$

$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$p = \frac{\pm \sqrt{1 - 4 \cdot 1 \cdot 0,22}}{2 \cdot 1}$

$p = \frac{1 \pm \sqrt{0,12}}{2} = \begin{cases} p_1 = 0,6732 \\ p_2 = 0,3267 \end{cases}$

$\pi_{1,1} = p_1(1-p_1) = 0,6732(1-0,6732)$

$\pi_{1,1} = 0,22$

$\pi_{1,2} = p_2(1-p_1) = 0,3267(1-0,3267)$

$\pi_{1,2} = 0,21$

Problema 6

M/M/1/4

$\lambda = 10$ cl./seg

$\mu = 16$ cl./seg

(A) $\Pi_0 = ?$

$$\Pi_0 = \frac{1-\rho}{1-\rho^{4+1}} \Rightarrow \Pi_0 = \frac{1-0,625}{1-(0,625)^{4+1}}$$

$$\Pi_0 = \frac{0,375}{1-(0,625)^5} = 0,41453$$

(B) $P_B = ?$

$$P_B = \Pi_4 = \frac{\rho^4(1-\rho)}{1-\rho^5}$$

$$P_B = \frac{0,625^4(1-0,625)}{1-0,625^5} = 0,06325$$

(C) $\gamma_I = ?$

$$\gamma_I = \lambda(1-P_B)$$

$$\gamma_I = 10 \text{ cl./seg } (1-0,06325)$$

$\gamma_I = 9,36748$ cl./sec

(D) $\gamma_0 = \pi (1 - \Pi_0)$ $\quad \gamma_0 = ?$

$\gamma_0 = 16$ cl./sec $(1 - 0,41453)$

$\gamma_0 = 9,36745$ cl./sec

(E) % clientes rechazados

$P_3 \cdot 100\%$

$9,6325 \cdot 100\% = 6,325\%$

Problema 7 $\quad N = ?$

$\Pi_0 = 0,6$

$\rho = 95$

$\Pi_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$

$96 = \frac{1 - 95}{1 - 0,5^{N+1}}$

$$0{,}6\left(1-0{,}5^{n+1}\right) = 0{,}5$$

$$1-0{,}5^{n+1} = \frac{0{,}5}{0{,}6}$$

$$-0{,}5^{n+1} = \frac{0{,}5}{0{,}6} - 1$$

$$-0{,}5^{n+1} = -\frac{1}{6}$$

$$\log 0{,}5^{n+1} = \log \frac{1}{6}$$

$$(n+1)\log 0{,}5 = \log 1 - \log 6$$

(log 1 → 0)

$$n+1 = \frac{-\log 6}{\log 0{,}5}$$

$$n = \frac{-\log 6}{\log 0{,}5} - 1$$

$$n = \frac{+0{,}77815}{+0{,}30103} - 1$$

$$n = \frac{0{,}47712}{0{,}30103}$$

$$n = 1{,}58495 \approx 2$$

Problem 8

$N = ?$

$T_{obs} = 20$ sec

i	T_{ai}	t_{si}	IN	OUT	W_i
1	2	3	2	5	3
2	4	4	5	9	5
3	11	2	11	13	2
4	13	3	13	16	3
5	15	3	16	19	4
					17

$$N = \lambda W$$

$$\lambda = \frac{N^{\circ} cl.}{T_{obs}} = \frac{5\ cl.}{20\ sec}$$

$$W = \frac{\sum_{i=1}^{n} W_i}{N^{\circ} cl.} = \frac{17\ sec}{5\ cl.}$$

$$N = \frac{5\ cl.}{20\ sec} \cdot \frac{17\ sec}{5\ cl.}$$

$$N = \frac{17}{20} = 0{,}85$$