Sea un sistema M/M/1/4 tal que la tasa de arribos es 12 cli/seg y la tasa de servicio es 20 cli/seg.
Hallar:
A) π 0.
B) PB.
C) Y i.
D) Y o.
E) Porcentaje de clientes que logran ingresar al sistema.

Datos

M/M/1/4

$\lambda = 12$ cli/seg

$\mu = 20$ cli/seg

(A) $\pi_0 = ?$ $\qquad \pi_0 = \frac{1-\rho}{1-\rho^{N+1}}$

$$\rho = \frac{\lambda}{\mu} = \frac{12 \text{ cli/seg}}{20 \text{ cli/seg}} = 0{,}6$$

$$\pi_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-0{,}6}{1-0{,}6^{4+1}} = \frac{1-0{,}6}{1-0{,}6^5}$$

$$= \frac{0{,}4}{1-0{,}0776} = 0{,}4337$$

$$\boxed{\pi_0 = 0{,}4337}$$

(B) $PB = ?$

$$P_B = \Pi_N = \frac{\rho^N(1-\rho)}{1-\rho^{N+1}} = \frac{(0{,}6)^4(1-0{,}6)}{1-0{,}6^{4+1}}$$

$$= \frac{0{,}1296 \cdot 0{,}40}{1-0{,}6^5} = \frac{0{,}05184}{0{,}92224} = 0{,}05621$$

$$\boxed{PB = 0{,}05621}$$

(C) $\gamma_I = ?$

$$\gamma_I = \lambda - \tau = \lambda - \lambda P_B = \lambda(1-P_B)$$

$$= 12\ cli/seg\ (1-0{,}0562) = 12\ cli/seg\ (0{,}9438)$$

$$= 11{,}3256\ cli/seg$$

$$\boxed{\gamma_I = 11{,}3256\ cli/seg}$$

Ⓓ $\gamma_0 = ?$

$\gamma_0 = \lambda(1 - \pi_0) = 20 \text{ cli/seg} (1 - 0{,}4337)$

$= 20 \text{ cli/seg} (0{,}5663) = 11{,}326$

$\boxed{\gamma_0 = 11{,}326}$

Ⓔ $1 - 0{,}0562 = 0{,}9438 \rightarrow \boxed{94{,}38\%}$

OTRA forma

λ — 100%

γ — x

12 cli/seg — 100%

$11{,}3256 \text{ cli/seg}$ — x

$$\frac{11{,}3256 \text{ cli/seg} \cdot 100\%}{12 \text{ cli/seg}} = \frac{1132{,}56\%}{12} = \boxed{94{,}38\%}$$

Sea un sistema dependiente de estados tal que la probabilidad ociosa del sistema es 0,20 siendo λ i = 1/2 µ i + 1.

Hallar π 3

Datos

$\pi_0 = 0{,}20$

$\lambda_i = \frac{1}{2} \mu_{i+1}$

$\pi_3 = ?$

$$\pi_n = \frac{\lambda_0 . \lambda_1 \cdots \lambda_{n-1}}{\mu_1 . \mu_2 \cdots \mu_n} . \pi_0$$

$$\pi_3 = \frac{\lambda_0 . \lambda_1 . \lambda_2}{\mu_1 . \mu_2 . \mu_3} . \pi_0$$

$$\lambda_i = \frac{1}{2} \mu_{i+1} \left[\begin{array}{l} \lambda_0 = \frac{1}{2} \mu_{0+1} = \frac{1}{2} \mu_1 \\ \lambda_1 = \frac{1}{2} \mu_{1+1} = \frac{1}{2} \mu_2 \\ \lambda_2 = \frac{1}{2} \mu_{2+1} = \frac{1}{2} \mu_3 \end{array} \right.$$

$$\pi_3 = \frac{\frac{1}{2} \cancel{\mu_1} . \frac{1}{2} \cancel{\mu_2} . \frac{1}{2} \cancel{\mu_3}}{\cancel{\mu_1} . \cancel{\mu_2} \cancel{\mu_3}} . 0{,}20$$

$$\pi_3 = \left(\frac{1}{2}\right)^3 . 0{,}20 = \frac{1}{8} . 0{,}20 = \boxed{0{,}025}$$

Sea un sistema M/M/1/3 tal que la tasa de arribos es 14 cli/seg y la tasa medio de servicio es 0,05 seg/cli.

Hallar:

A) π 0.

B) PB.

C) Y i.

Datos

$\lambda = 14$ cli/seg

$T_s = 0{,}05$ seg/cli

$\rho = \frac{\lambda}{\mu} \Rightarrow \lambda \cdot \frac{1}{\mu} \Rightarrow \lambda \cdot T_s = 14 \text{ cli/seg} \cdot 0{,}05 \text{ seg/cli}$

$\rho = 0{,}7$

(A) $\pi_n = \frac{\rho^n (1-\rho)}{1-\rho^{N+1}}$

$\pi_0 = \frac{\rho^0 (1-\rho)}{1-\rho^{3+1}} = \frac{1(1-\rho)}{1-\rho^4} = \frac{1-0{,}7}{1-(0{,}7)^4} = \frac{0{,}3}{1-0{,}2401}$

$= \frac{0{,}3}{0{,}7599} = 0{,}39478$

$\boxed{\pi_0 = 0{,}39478}$

(b) $P_B = \pi_N = \dfrac{\rho^N(1-\rho)}{1-\rho^{N+1}}$

$$P_B = \pi_3 = \frac{\rho^3(1-\rho)}{1-\rho^{3+1}} = \frac{(0{,}7)^3 \cdot (1-0{,}70)}{1-(0{,}70)^4} =$$

$$= \frac{0{,}343 \cdot 0{,}30}{1-0{,}2401} = \frac{0{,}1029}{0{,}7599} = 0{,}1354$$

$$\boxed{P_B = 0{,}13541}$$

(c) $\gamma = \lambda\,(1-P_B)$

$$\gamma = 14\ cl/sec\ (1-0{,}13541) = 14\ cl/sec\ (0{,}86459)$$

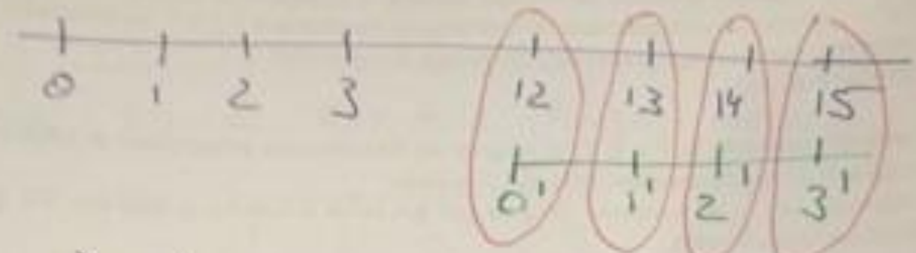
$$= 12{,}10426\ cl/sec$$

$$\boxed{\gamma = 12{,}10426\ cl/sec}$$

Sea un sistema dependiente de estados tal que π 12 = 0,1 siendo 3 λ i = µ i + 1.

Hallar π 15.

Lo que voy a plantear es un corrimiento



$$\pi_n = \frac{\lambda_0 \cdot \lambda_1 \cdot \lambda_2 \cdots \lambda_{n-1}}{\mu_1 \cdot \mu_2 \cdots \mu_n} \cdot \pi_0$$

$$\pi_{15} \equiv \pi_{3'} = \frac{\lambda_{0'} \cdot \lambda_{1'} \cdot \lambda_{2'}}{\mu_{1'} \cdot \mu_{2'} \cdot \mu_{3'}} \cdot \pi_0$$

$$3\lambda_i = \mu_{i+1}$$

$$\lambda_i = \frac{1}{3}\mu_{i+1}$$

$$\begin{cases} \lambda_0 = \frac{1}{3}\mu_{0+1} = \frac{1}{3}\mu_1 \\ \lambda_1 = \frac{1}{3}\mu_{1+1} = \frac{1}{3}\mu_2 \\ \lambda_2 = \frac{1}{3}\mu_{2+1} = \frac{1}{3}\mu_3 \end{cases}$$

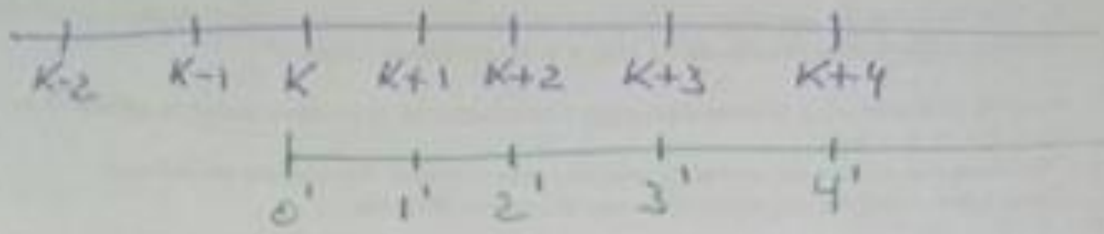
$$\pi_{15} = \pi_{3'} = \frac{\frac{1}{3}\cancel{\mu_1} \cdot \frac{1}{3}\cancel{\mu_2} \cdot \frac{1}{3}\cancel{\mu_3}}{\cancel{\mu_1} \cdot \cancel{\mu_2} \cdot \cancel{\mu_3}} \cdot 0{,}1$$

$$\pi_{15} \equiv \pi_{3'} = \left(\frac{1}{3}\right)^3 \cdot 0{,}1 = \frac{1}{27} \cdot 0{,}1 = \boxed{0{,}003703}$$

Sea un sistema dependiente de estados tal que π k = 0,16 siendo 3 λ i = 2 μ i + 1.

Hallar π k+4.

Lo que voy a plantear es un corrimiento



$$\pi_n = \frac{\lambda_0 \, \lambda_1 \, --- \, \lambda_{n-1}}{\mu_1 \, \mu_2 \, --- \, \mu_n} \cdot \pi_0$$

$$\pi_{K+4} \equiv \pi_{4'} = \frac{\lambda_{0'} \cdot \lambda_{1'} \cdot \lambda_{2'} \cdot \lambda_{3'}}{\mu_{1'} \cdot \mu_{2'} \cdot \mu_{3'} \cdot \mu_{4'}} \cdot \pi_{0'}$$

$$3\lambda_i = 2\mu_{i+1}$$

$$\lambda_i = \frac{2}{3}\mu_{i+1}$$

$$\lambda_0 = \frac{2}{3}\mu_{0+1} = \frac{2}{3}\mu_1$$

$$\lambda_1 = \frac{2}{3}\mu_{1+1} = \frac{2}{3}\mu_2$$

$$\lambda_2 = \frac{2}{3}\mu_{2+1} = \frac{2}{3}\mu_3$$

$$\lambda_3 = \frac{2}{3}\mu_{3+1} = \frac{2}{3}\mu_4$$

$$\pi_{K+4} \equiv \pi_{4'} = \frac{\frac{2}{3}\cancel{\mu_1}\,\frac{2}{3}\cancel{\mu_2}\,\frac{2}{3}\cancel{\mu_3}\,\frac{2}{3}\cancel{\mu_4}}{\cancel{\mu_1}\,\cancel{\mu_2}\,\cancel{\mu_3}\,\cancel{\mu_4}} \cdot 0{,}16$$

$$\pi_{K+4} \equiv \pi_{4'} = \left(\frac{2}{3}\right)^4 0{,}16 = \frac{16}{81} \cdot 0{,}16 = \boxed{0{,}031604}$$