FUZZY NOISE REMOVAL AND EDGE DETECTION ON HEXAGONAL IMAGE

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Abstract

Traditionally images are digitized, processed and displayed in a rectangular grid. But rectangular grid has many inherent ambiguities such as continuity, inter-pixel distance, etc. These ambiguities restrict rectangular grid to obtain optimal results in image processing. This study considers a particular topic of grayscale morphology image processing based on fuzzy discipline for hexagonally sampled images. The proposed research presents a methodology for hexagonally sampled images that consist of processing, and display of processed images on hexagonal grid. Image processing includes fuzzy morphological operations with different sizes, shapes and directional fuzzy structuring elements with an application of noise removal and edge detection. Performance evaluation conducted to demonstrate that hexagonal grid structure coupled with fuzzy morphological image processing is more robust than the rectangular counterpart in many applications.

Keywords: *hexagonal grid, fuzzy morphology, edge detection, noise removal*

Introduction

Edge detection and noise removal are essential pre-processing steps in the field of image processing, computer vision, robotics and automation [1]. Traditionally, images are digitized and stored in a rectangular arrangement due to the fact that conventional image capture and display devices are based on rectangular grid pixels. Moreover, numerous types of sampling systems are feasible, some are listed by Whitehouse [2]. Among them

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hexagonally sampled image is the most prominent one [3]. It offers less storage, less computation time, increased coding efficiency, less quantization error, equidistant property and consistent connectivity [4]. An astonishing observation of this study is that hexagonal images are more visually pleasing to human eyes. Mersereau [5] pointed out that a regular hexagonal scheme is best for sampling circular signals. On a regular hexagonal grid, sampling points are equidistant from each of their six nearest pixels as shown in Fig. 1. Moreover, circular mask and curved edges performs better in hexagonal grid than rectangular grid as illustrated in Fig. 2. Recently, image acquisition and visualization devices equipped with an ever-increasing number of rectangular pixels per inch are introduced. But at the same time it demands more computation power and storage space. Thus, hexagonal grid systems can be a smart alternative for image processing such as medical imaging, remote sensing, industrial inspection, etc. instead of rectangular grid images.

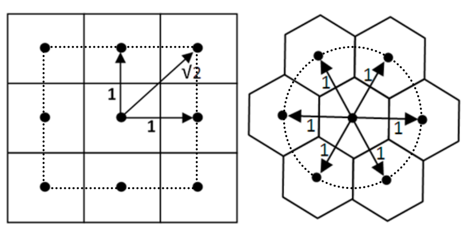


Figure 1. Inter-pixel distance in rectangular grid and hexagonal grid. Circular mask is well suited in hexagonal grid than rectangular grid.

(a) (b)

Figure 2. Curved edges on: (a) hexagonal grid; (b) rectangular grid.

In the early 1990's, Her, Wuthrich *et al*. and many others first proposed re-sampling method to convert an image from a rectangular grid structure to a hexagonal counterpart [6-9]. This method focused on two parts. The first was re-sampling an image to a hexagonal grid image. Then, display of the hexagonal image again used the brick wall approach. Moreover, Her [9] proposed a new 3 coordinates system to represent hexagonal data and used proposed method to represent the displacement from each of the major axes of the hexagon. In 2005, a simulated hyper pixel method [4] has been introduced to propose a framework for practical grid hexagonal-image processing. This method was also based on image re-sampling to generate hexagonally sampled images from the rectangular grid images.

The word "fuzzy" means "vague." Fuzziness occurs when the border of information is not crispy or clear-cut. Fuzzy sets first introduced by Lotfi A. Zadeh [10] as an extension of the classical set theory. It can handle the idea of partial truth and false. Classical set theory allows the membership of an element to either fully belong or not to the set. In contrast, fuzzy set theory allows the gradual transition of the membership of elements in a set. Grayscale image has ambiguity within pixels due to inherent vagueness in an image. An image can be fuzzy due to many factors such as, ambiguity in gray values, spatial (i.e., geometrical) ambiguity, etc. In digital image, pixels with different intensity levels can visually appear to share the same brightness. There occurs an ambiguity in labeling the pixel either bright or dark. Spatial ambiguity can be caused for fuzziness in object border within an image. In a digital image, a pixel is either an edge or a no-edge. But the edges are not always precisely defined and thus ambiguity arises. Moreover, for fuzzy morphological operator, a 3 by 3 rectangular structuring element (SE) is the closest fit for a circular SE (as shown in Fig. 2). However, using such an element to erode curved objects may introduce undesirable discontinuities. The consistent pixel connectivity in hexagonal images is an attractive feature allowing for better circular SE definition.

Fuzzy image processing has three key steps: image fuzzification, modification of membership values and image defuzzification (see Fig. 3). The fuzzification and defuzzification steps are needed because we do not possess fuzzy hardware. Thus, the coding of grayscale pixels to fuzzify and decoding of the results to defuzzify are indispensable to process images with fuzzy techniques. The key step of fuzzy image processing lies in the modification of membership value. After the image data are transformed from gray-level hexagonal grid plane to the membership plane (fuzzification), appropriate fuzzy techniques are then followed to modify the membership values. The proposed method applied fuzzy morphology to modify the membership values.

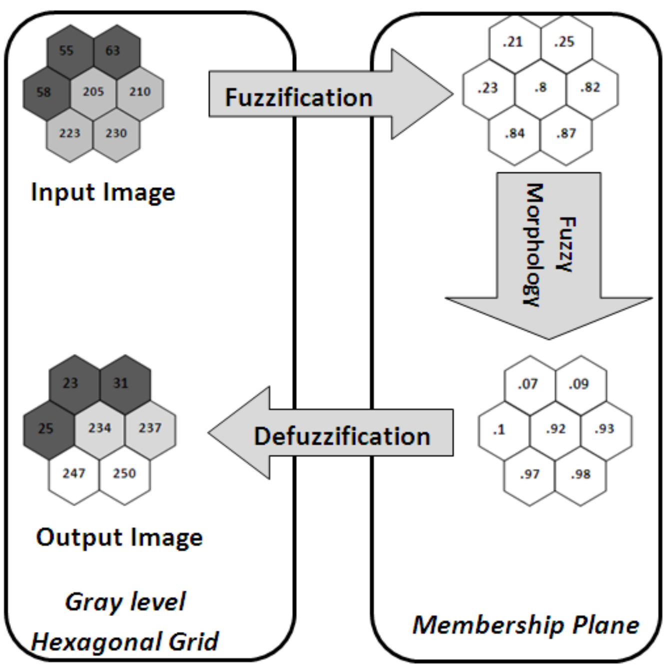


Figure 3. Steps of Fuzzy Image Processing.

Fuzzy Morphology uses the concepts of fuzzy set theory. Fuzzy mathematical morphology is the extension of grayscale morphological operations to fuzzy sets. End of 80s, Bloch et al. [11], Di Gesu [12], Sinha *et al*.[13], De Baets [14] and many others first proposed mathematical morphology concepts for fuzzy image. The fuzzy dilation and fuzzy erosion of A by B (SE) are defined by:

$$Dc\ (A, B)(y) = sup_x C(B(x - y), A(x)), \tag{1}$$

$$E_I\ (A, B)(y) = inf_x I(B(x - y), A(x)), \tag{2}$$

where C is a Conjunctor and I is an Implicator. De Beats [14] idea was to fuzzify the underlying logical operations and used Boolean Conjunctor and Boolean Implicator to obtain successful fuzzification.

The most renowned concept of fuzzy morphology is the alpha-morphology. It is founded on the level sets of fuzzy membership degree function and first introduced by Bloch *et al*. [11]. Moreover, Sinha and Dougherty [13] presented inclusion operation to formulate dilation and erosion as a logical operation. Then, De Baets [14] further investigated on Inclusion operation approach. But, there are still some mathematical requirements need to be satisfied by fuzzy morphology. Recently, Hidalgo et al. [15] proposed some comparison of edge detectors based on fuzzy morphology. Moreover, he introduced fuzzy uninorm concept [16].

There have been done sufficient numbers of research on fuzzy morphology and hexagonal image processing separately. However none of these studies provides a framework for fuzzy morphology on hexagonally sampled images. This paper addresses significant key issues when considering fuzzy morphology on hexagonal image such as hexagonal fuzzy mask, optimal membership function, etc. The proposed algorithm possesses several advantages using hexagonal images with fuzzy morphology for noise removal and edge detection. These are due to the continuity and inter-pixel distance of the hexagonal pixels generating more consistent contours. These features make fuzzy morphological processing within the hexagonal framework computationally efficient. Thus, fuzzy morphology and hexagonal image together make a strong case for image processing and can serve as a viable alternative to rectangular image processing.

Research Objectives

To overcome the limitations of grayscale morphology image processing on rectangular grid images, a systematic design framework as shown in Fig. 4 employing fuzzy morphology with different sizes and directional fuzzy hexagonal structuring elements on hexagonally sampled images is proposed. In our framework, we applied fuzzy morphological image processing with various sizes and directional fuzzy hexagonal structuring elements on hexagonally sampled images. To further exploit the advantages of hexagonal images for broader applications, the proposed research develops an efficient method to remove noises and detect edges by using fuzzy morphological operator. In summary, our objectives are the following:

I. We offered a complete methodology for hexagonally sampled images which contained conversion of rectangular to hexagonal images, processing, and display of processed images on hexagonal grid. Image processing includes fuzzy morphological gradient operator to detect edges and fuzzy morphology to remove noises.

II. Develop fuzzy hexagonal structuring elements with different sizes and directions and select the combination of optimum structuring element for proposed study.

III. Evaluate the performance of the proposed algorithm by comparing the Mean Square Error (MSE), Signal to Noise Ratio (SNR), and the ratio of edge pixel to image size with traditional rectangular grid pixels edge detection and noise removal techniques.

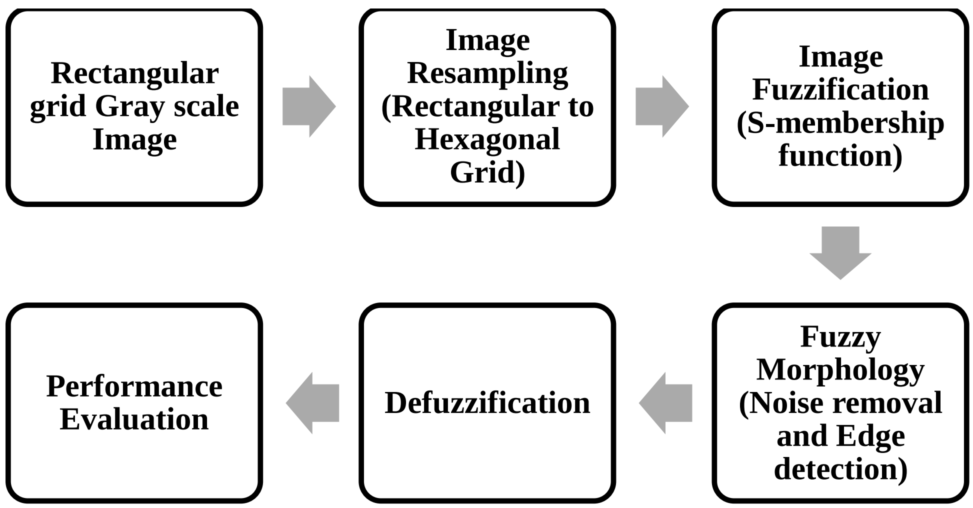


Figure 4. System diagram for performing fuzzy image processing on hexagonally. sampled grid.

Methodology

The proposed research is grouped into three subsections. First, we presented an image resampling using simulated hexagonal grid method followed by image fuzzification. Secondly, we employed fuzzy morphology on hexagonally sampled images with the application of noise removal and edge detection algorithm. Afterward, defuzzification is utilized to display results on conventional image display device. Finally, performance evaluation is conducted to compare the MSE, SNR and the ratio of edge pixel to image size of the proposed algorithm with traditional rectangular grid technique.

1. *Image Resampling*

We converted rectangular grid image to hexagonal grid by using simulated hexagonal grid method. Simulated hexagonal grid can be created through clusters of rectangular sub-pixels. Many rectangular grids can be combined together to create clusters of sub-pixels (as shown in Fig. 5). For example, d = 7; is composed of 120 rectangular grids to create one hexagonal pixel. A cluster of sub-pixels will closely resemble the shape of hexagon.

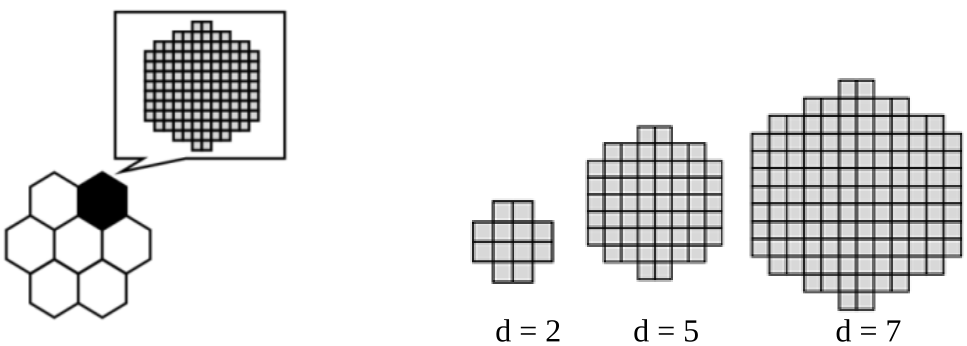
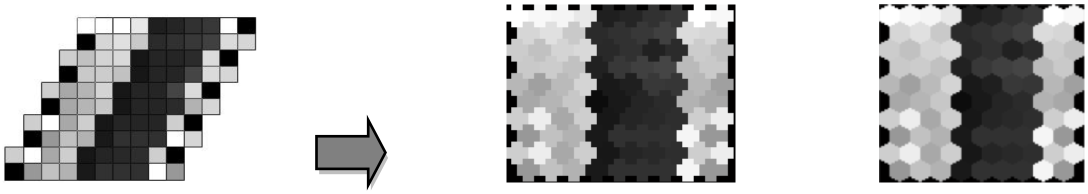


Figure 5. Different values for simulated hexagonal grid.

To test the methods for image conversion and fuzzy morphology, this study created an 8 by 8 synthetic gray scale rectangular grid image (as shown in Fig. 6). From Fig. 6 we can see that, D7 simulated hexagonal conversion method performed better than D2. The geometric properties of D7 hexagonal pixel are similar to hexagon shape and it showed equivalent inter-pixel distance and smooth curved edges. Moreover, rectangular grid may perform well only for horizontal and vertical direction. Proposed D7 hexagonal grid performed not only for horizontal and vertical lines but also oblique edges. Horizontal and vertical edges are relatively few in natural scenes, which make the ability in representing oblique edges important. In real application to find oblique edges is important.



(a) (b) (c)

Figure 6. Image conversion from rectangular to simulated hexagonal grid: (a) rectangular synthetic image (b) hexagonal image (D=2) (c) hexagonal image (D=7).

2. Image Fuzzification

The first step of fuzzy image processing is image fuzzification (as shown in Fig. 3). In the lack of fuzzy hardware, fuzzification and defuzzification steps are required to process fuzzy images. For image fuzzification we required to employ membership function where the membership values lie between 0 and 1. The selection of membership function depends on the application. In image processing, heuristic membership functions [17] where widely used to define certain properties (such as lightness or darkness of a pixel value). Among heuristic membership functions, *S*-function is the most prominent one. The shape of *S*-function depends on the parameters *a, b,* and *c.* The parameters *a, b* and *c* are specified to ensure the membership function maximizes the information contained in the image. In this study we applied *S*-membership function as given below:

$$\mu(x) = \begin{cases} 0 & x<a, \\ 2\,[(x-a)/(c-a)]^2 & a < x < b, \\ 1 - 2\,[(x-c)/(c-a)]^2 & b < x < c, \\ 1 & x > c, \end{cases} \tag{3}$$

where, b is any value between a and c. For $a = X_{min}$, $c = X_{max}$, the membership function plot is shown in Fig. 7 for normalized set.

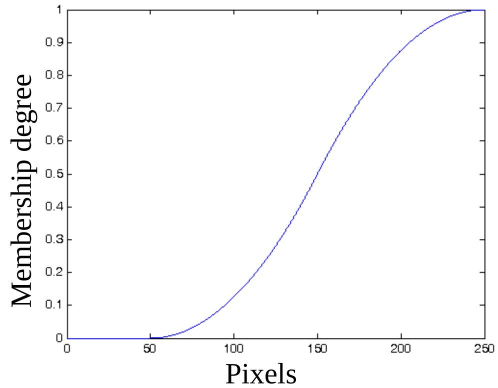


Figure 7. *S*-membership functions.

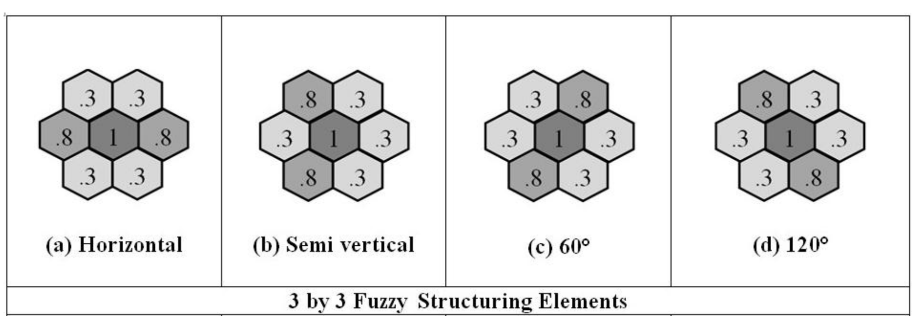
The fuzzified image, as shown in Fig. 8, can be created from *S*-membership function with improved quality. In membership function $\mu = 1$ represents maximum brightness. This study applied *S*-membership function on both standard test image and synthetic ones. The enhanced image is analyzed in terms of its output quality. Subjective evaluation had done by applying numerous parameters. From the experimental results, we choose proposed *S*-membership. The proposed S-membership function not only fuzzified image but also improved the quality of image. Now, grayscale image can be viewed as two dimensional fuzzy set with hexagonal grid, where membership value range from 0 to 1 as shown in Fig. 8.


(a) (b)

Figure 8. Image fuzzification (a) test image pepper, and (b) fuzzified pepper.

3. *Fuzzy Morphology*

The performance of fuzzy morphological operation mainly depends on the size, shape and directions of structuring element. Thus, we applied two different sizes (3 by 3 and 5 by 5) and four directional (horizontal, semi vertical, 60°, and 120°) fuzzy hexagonal structuring elements (as shown in Fig. 9). The SE in grayscale morphology serves as a template to selected portion of an image. Conversely, fuzzy structuring element itself is a function or image, which has greater capability to affect the image through direction, size and shape. Due to the nature of fuzzy sets, fuzzy morphological operations are more sensitive to details within an image, allowing to fine tune standard morphology operations. In fuzzy morphology the choice of the best SE shape has a great impact on the result obtained. It requires selecting according to the image structure. For edge detection, directional SE's are required to get optimal results. In natural images, horizontal and vertical edges are relative few in comparison with oblique counterparts. In the proposed hexagonal grid, vertical direction was the combination of 60° and 120° to the horizontal. This was more efficient to detect oblique contours. Thus, we applied four directional fuzzy hexagonal SE (Horizontal, semi vertical, 60° and 120°) and two sizes (3 by 3 and 5 by 5). The most basic fuzzy morphological operations are dilation and erosion. Fuzzy erosions and dilations produce results identical to the nonlinear minimum and maximum. Opening and closing are a sequential combination of dilation and erosion.



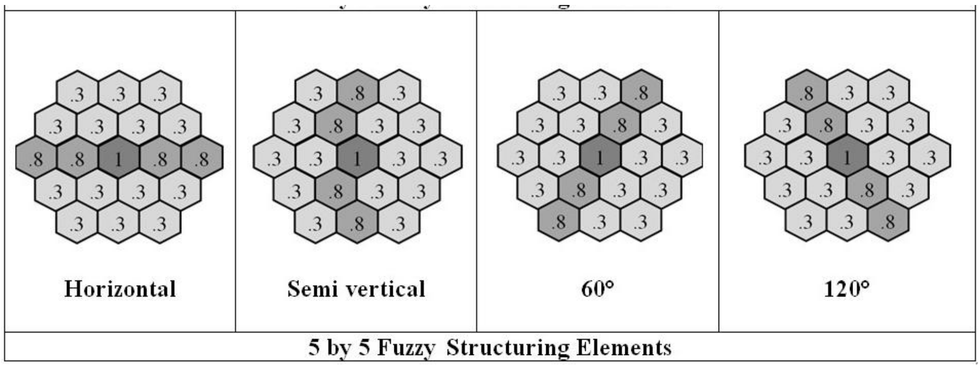
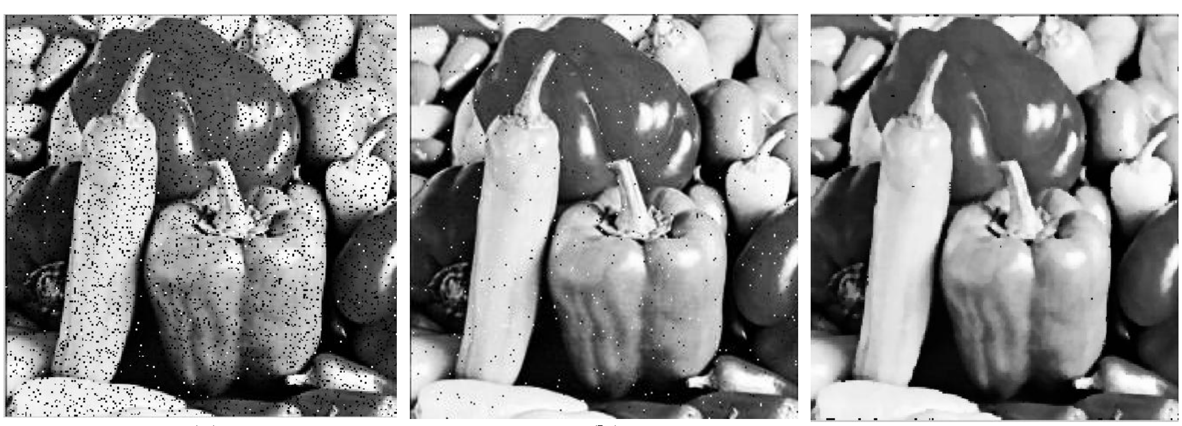


Figure 9. Directional fuzzy hexagonal structuring elements.

4. Noise Removal and Edge Detection

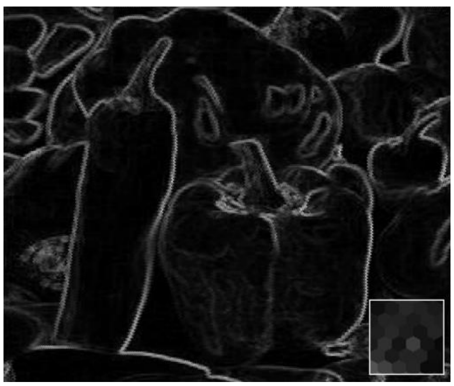
To evaluate the proposed method of noise removal, we added salt and pepper noises on standard test image named Pepper. This study proposed opening followed by closing by applying various combination of SE. The optimal combination selected was of horizontal, 60° and 120° fuzzy 3 by 3 structuring elements to remove noise (as shown in Fig. 10). The SNR values of noisy image and after applying noise removal method on rectangular and hexagonal grid were 10.13, 19.97 and 27.09 dB respectively. The proposed noise removal method performed well by using fuzzy morphology on hexagonal image as shown in Fig. 10 and SNR values.



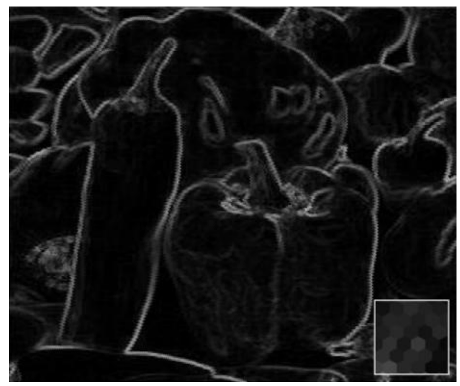
(a) (b) (c)

Figure 10. The proposed noise removal method: (a) noisy image (10.13 dB), (b) rectangular grid (19.97 dB), (c) hexagonal grid (27.09 dB).

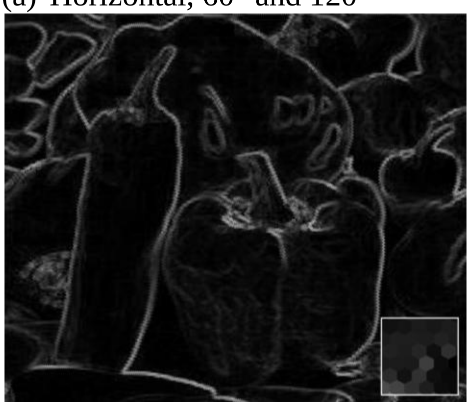
A morphological gradient is the difference between the dilation and the erosion of a given image by applying same SE. It is a well-known operator for edge detection. Most basic morphological gradient operator is shown by: Edge = $(A \oplus B) - (A \Theta B)$. The fuzzy dilation and erosion formula are given in Equations 1 and 2. The result of the dilated images subtracted from eroded images using fuzzy structuring elements on fuzzy images is shown in Figs. 11, and 12 (in lower right portion of white box showed 8 times zoomed hexagonal pixels).



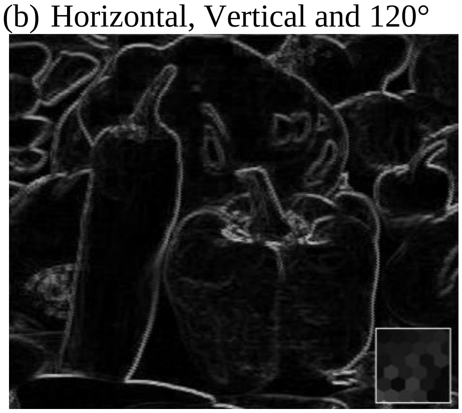
(a) Horizontal, 60° and 120°



(b) Horizontal, Vertical and 120°



(c) Horizontal, Vertical and 60°



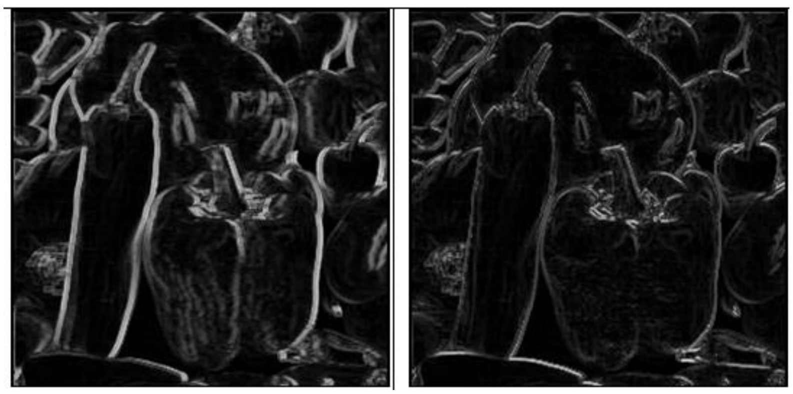
(d) Vertical, 60° and 120°

Figure 11. Edge detection by applying different directional 3 by 3 fuzzy SE on hexagonal grid.

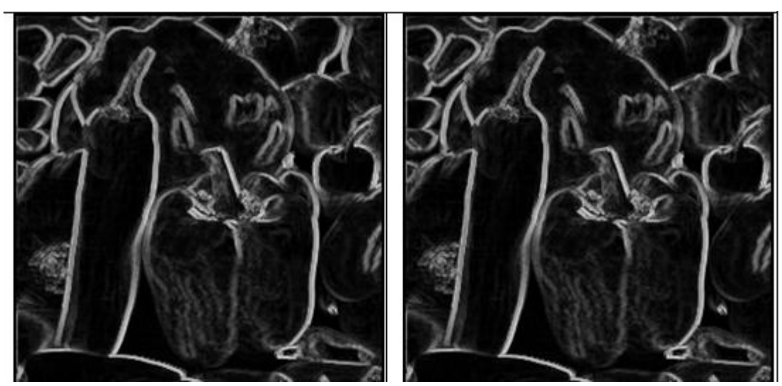
In this experiment, we applied different combination of fuzzy structuring elements and evaluated results with ground truth. The performances of 3 by 3 fuzzy structuring elements were better than 5 by 5 SE. The proposed study used real test images

(Lena, Pepper and Barbara) and synthetic images as inputs. Thus smaller circular shape fuzzy structuring elements performed better to find curve edges. Conversely, bigger SE introduced some unwanted thickness and discontinuities as shown in Fig. 12.

We also applied 3 by 3 fuzzy rectangular directional structuring elements on rectangular image as shown in Fig. 13. Bigger SE also introduced unwanted thickness on rectangular images. In Fig. 13 we can see the same combination of SE as we applied on hexagonal image.

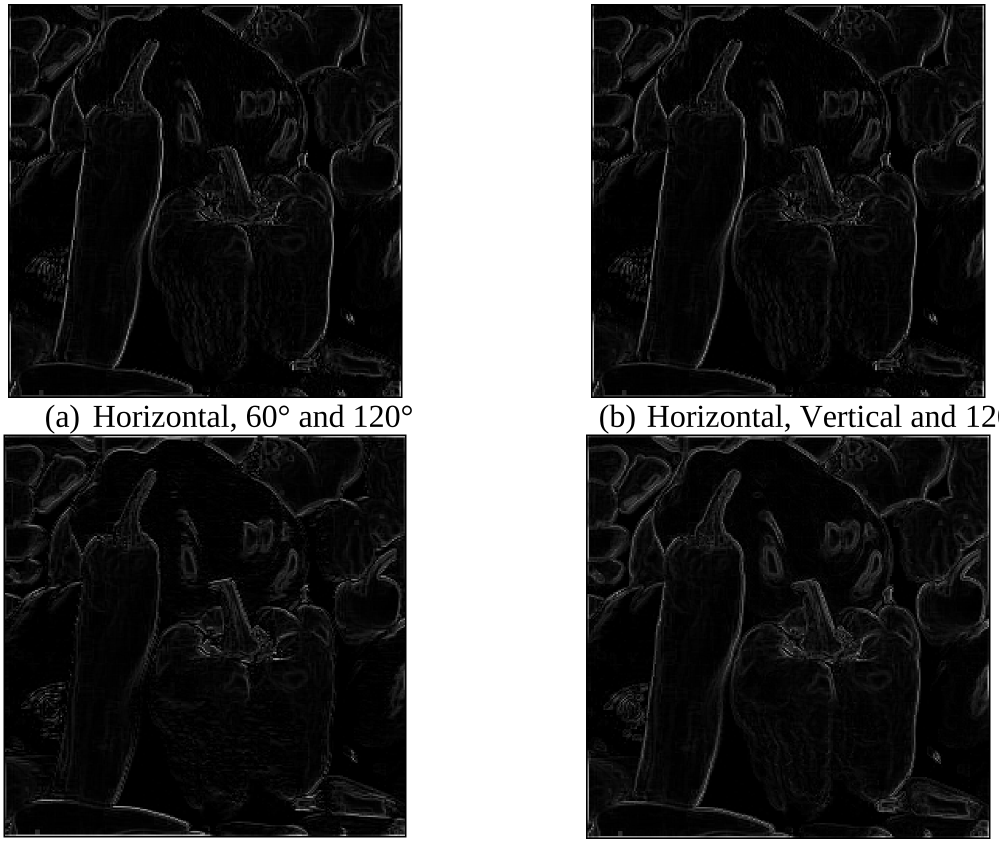


(a) Horizontal (b) Semi vertical



(c) 60° (d) 120°

Figure 12. Edge detection by applying 5 by 5 fuzzy SE on hexagonal grid.



(a) Horizontal, 60° and 120°   (b) Horizontal, Vertical and 120°

(c) Horizontal, Vertical and 60°   (d) Vertical, 60° and 120°

Figure 13. Edge detection by applying different directional 3 by 3 SE on rectangular grid.

Performance evaluation

In this section, different image quality measures are calculated for a proposed detected edge image with reference to a standard edge image (Ground truth or Gold Standard) as shown in table 1. There are various methods suggested to acquire Ground Truth [18]. Firstly, we used synthetic images and the corresponding ground truth to verify the proposed methodology. In real images, manual specification of edges can be used as a Ground Truth [19]. Secondly, real test images were inspected by human observers to outline by hand to acquire the Ground Truth. Ground Truth was manually created for Pepper test image by several persons on separate occasions. Then, the proposed method adopted the optimal GT as shown in Fig. 14. In GT brighter pixels represents edge, gray represents no-edge, and black as a don’t-count region. If evaluation reports an edge pixel in GT, then then it considered as a matched GT edge pixel.

Figure 14. Standard test image and its Ground truth prepared manually for evaluation.

We applied Mean Square Error (MSE), Signal to Noise Ratio (SNR) and the ratio of edge pixels to image size to evaluate proposed method. Mean Square Error (MSE) indicates the average difference of the pixels throughout the image. A higher MSE indicates a greater difference between the original and processed image. The formula for the MSE calculation is given as:

$$MSE = \frac{1}{N}\sum_i \sum_j \left(X_{ij} - \mathrm{V}_{ij}\right)^2 \quad (4)$$

Where *N* is the size of the image, *X* is the processed image, and *V* is the original image.

The ratio of edge pixels to image size measures the ratio between pixels of edges and the total image size. A greater ratio means the proposed edge detector found more edge pixels than other methods.

Table 1

Quantitative measure obtained by edge detectors in hexagonal grid and rectangular grid by using fuzzy hexagonal morphology for real test images Pepper

	Method	Pepper Test Image		
		MSE	**SNR**	**Ratio of edge pixels to image size**
Hexagonal Image	Combination of Horizontal, 60° and 120° SE	0.74	21.97	17.8%
	Combination of Horizontal, Vertical and 120° SE	1.92	19.85	17.01%
	Combination of Horizontal, Vertical and 60° SE	4.02	17.67	16.61%
	Combination of Vertical, 60° and 120° SE	7.05	15.22	16.01%
	Combination of 60° and 120° SE	7.31	15.06	15.85%
Rectangular Image	Combination of Horizontal, 60° and 120° SE	10.56	12.56	14.06%
	Combination of Horizontal, Vertical and 120° SE	11.36	12.43	14.01%
	Combination of Horizontal, Vertical and 60° SE	12.02	12.09	13.81%
	Combination of 60° and 120° SE	13.06	11.06	13.61%

The edge detectors and noise removal considered in this study was fuzzy morphology with hexagonal images. Fuzzy morphology heavily depends on the size and directions of fuzzy structuring elements. Thus hexagonal fuzzy structuring elements with different sizes and directions were developed. We applied different combination of fuzzy structuring elements on hexagonal image to detect edges and remove noises. In hexagonal images, vertical direction was the combination of 60° and 120° to the horizontal direction. These properties of hexagonal grid and SE together contributed to derive edges in oblique direction efficiently. Comparison of detected edges between rectangular and hexagonal images shows that shapes of peppers appear more clearly and less noisy on hexagonal image. Conversely, in rectangular image bigger SE element and more pixels needed to improve the performance but at the same it will increase computation cost and edges will be thicker. As seen in Table 1 the combination of horizontal, 60° and 120° hexagonal fuzzy structuring element method achieved better MSE, SNR and the ratio of edge pixels to image size values for Pepper test image and for most of the synthetic and real images tested in the experiment. The nature of hexagonally sampled images and fuzzy morphology are the main reason to get better result from the combination of horizontal, 60° and 120°. Moreover, angular resolution of hexagonal pixels is much better than rectangular pixels. Conversely rectangular grid (also shown in Fig.13 and Table 1) introduced unwanted discontinuities which lead to poor results. For low and high resolution image, rectangular circular structuring elements performed marginally. In rectangular image the corner points are further away from center pixel. Conversely, in hexagonal image inter-pixel distance are same. Overall the edge of the proposed method was more continuous, well-defined and clearer. In contrast, the rectangular grid edges were thicker and had some discontinuity for curved object. In Table 1, the ratio of the edge pixels to image size was also indicated that the proposed method showed quantitatively optimal result. MSE and SNR value clearly showed the superiority of proposed method.

Conclusion

Recently the development of CCD technology renewed interest in hexagonal image processing research area. Our aim was to investigate hexagonal grid with fuzzy morphology and compare with traditional rectangular grid. Hexagonal grid considered 3 axes of symmetry. The proposed study presented a complete methodology for conversion of hexagonally sampled images, processing and display of processed images on hexagonal grid. We applied *S*-membership function to fuzzified image and explored the influence of different directional fuzzy structuring elements on fuzzy gradients. Performance evaluation showed the combination of 3 by 3 horizontal, 60° and 120° SE is the optimum one. In conclusion, we found consistent and accurate gradient operators can be obtained by applying hexagonal lattices with fuzzy morphology. Current rectangular grid methods require increased number of pixels to improve angular resolution of image. Hence the proposed study solved the problem. Moreover, this study provided an application of edge detection and noise removal technique developed various sizes and directional fuzzy structuring elements and discovered the optimal combination. Results clearly showed hexagonal grid performed well for curved objects as well as horizontal and vertical lines. Hexagonal structuring elements are the optimal for fuzzy gradient based morphology edge detection and noise removal.

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