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ABSTRACT

The Spiral Architecture has been developed as a fast way of indexing a hexagonal pixel-based image. In combination with spiral addition and spiral multiplication, methods have been developed for hexagonal image processing operations such as translation and rotation. Using the Spiral Architecture as the basis for our operator structure, we present a general approach to the computation of adaptive coarse scale Laplacian operators for use on hexagonal pixel-based images. We evaluate the proposed operators using simulated hexagonal images and demonstrate improved performance when compared with rectangular Laplacian operators such as Marr-Hildreth.

Index Terms—hexagonal images, spiral architecture, coarse scale

1. INTRODUCTION

The concept of using hexagonal pixels for image representation has been researched for many years and has recently received renewed interest. To date, researchers have investigated the use of hexagonally structured architectures for image structure and addressing [4], and a theoretical framework for signal modelling and transforms on a hexagonal lattice has been recently developed in [6]. In particular, in 1996 Sheridan [8] introduced the Spiral Architecture for hexagonal image representation and since then much work has been carried out on the application of existing edge detection techniques based on the Spiral Architecture. However, these edge detection techniques are typically simple extensions of existing rectangular operators, such as Prewitt and Sobel [5], rather than designed to exploit the specific advantages of the hexagonal structure.

Image processing operators developed for use on rectangular grids (rectangular pixel-based images) may be dominated by the inherent preferred directions along the *x*- and *y*-axes in a Cartesian structure, leading to the inheritance of anisotropic properties. Such anisotropy is reflected in the spectral properties of the operators, and improvements can be achieved by developing operators that consider "circularity" [2, 7]. Traditional methods of enhancing operator circularity include the use of compass operators to rotate feature detection masks to successfully detect diagonal edges. However, this alone does not overcome the problem of accurately detecting curved features. The use of a hexagonal sampling lattice provides a more accurate representation of curved structures than the traditional rectangular sampling lattice. Characteristics such as equidistance of all pixel neighbours and improved spatial isotropy of spectral response facilitate the implementation of circular symmetric kernels that is associated with an increase in accuracy when detecting edges, both straight and curved [7].

In this paper we present an efficient design procedure for the development of hexagonal Laplacian operators, based on the spiral architecture, that can be applied directly to hexagonal images for coarse scale edge detection. The structure of each spiral operator corresponds to a layer (λ) in the spiral architecture: at each layer λ, each spiral operator contains 7^{λ} point values.

2. SPIRAL IMAGE REPRESENTATION

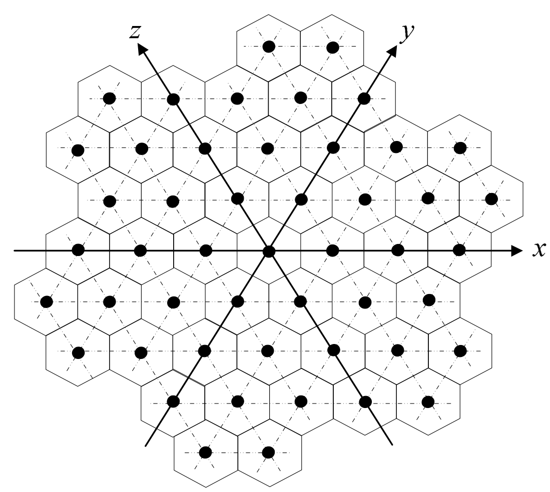

Figure 1: Spiral image representation

We represent the hexagonal image by a spiral array of samples of a continuous function u of image intensity on a domain Ω. Figure 1 represents part of a hexagonal image with nodes placed at the centre of each hexagonal pixel. These nodes are the reference points for the development of tri-directional derivative operators using an element based method. Interconnecting pixel nodes produces edges that

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form triangular elements, i.e. finite elements. This creates a mesh of equilateral triangular elements that overlays the image domain; an example of a section of mesh is illustrated in Figure 1. On any node in the mesh, say node i, we associate a piecewise linear basis function ϕ_i which has the properties $\phi_i = 1$ at node i and $\phi_i = 0$ at any node $j \neq i$. ϕ_i is thus a "tent-shaped" function with support restricted to a small spiral neighbourhood centred on node i consisting of only those elements that have node i as a vertex. We then approximately represent the image u over a spiral neighbourhood Ω_i^λ by a function

$$U = \sum_{j \in D_i^\lambda} U_j \phi_j$$

in which the parameters $\{U_j\}$ are the image intensity values, and D_i^λ is the set of nodes in Ω_i^λ. This gives a piecewise linear representation on the spiral neighbourhood Ω_i^λ.

3. LAPLACIAN OPERATORS

In previous work [3] we have developed adaptive hexagonal image processing operators for direct use on hexagonal pixel-based images. In that work however, the operator adaptivity was achieved by creating an initial 7-point hexagonal mask, equivalent to the standard 9-point mask on a rectangular pixel-based image, and increasing the mask size at each stage by inclusion of a layer of immediately neighbouring values around the edge of the current mask. In this way a hierarchical sequence of approximately circular masks was developed, in which the size, S_i, of the i'th mask is given by $S_i = S_{i-1} + 6i$, and $S_0 = 1$ (i.e., we obtain masks of size 7, 19, 37, 65, etc.).

In this paper the adaptive spiral operators follow the structure of the spiral architecture and hence the Layer 1, Layer 2 and Layer 3 operators have the structure illustrated in Figure 2. It should be noted that although we show the Layer 1 operator structure, it is not appropriate to construct a Laplacian operator as small as Layer 1 due to its sensitivity to noise. Based on this spiral structure, the operators are formulated such that they correspond to a weak form in the finite element method [1]. A second order derivative in the direction of unit vector $\underline{b}$ is defined as $-\underline{\nabla} \cdot (\mathbf{B}\underline{\nabla} u)$, where $\mathbf{B}$ is a matrix of the form $\mathbf{B} = \underline{b}\underline{b}^T$. To obtain a weak form, $R(u)$, of this second order directional derivative, , it is multiplied by a test function $v \in H^1$, and the result integrated over the image domain Ω to give

$$R(u) = \int_\Omega \underline{\nabla} u \cdot \mathbf{B} \underline{\nabla} v d\Omega .$$

In the finite element method a finite-dimensional subspace $S^h \subset H^1$ is used for function approximation. In

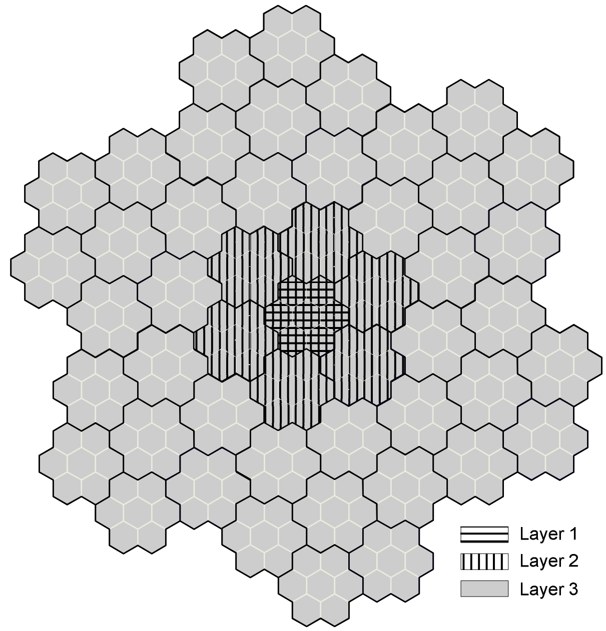


Figure 2: Layers of the Spiral Architecture corresponding to spiral operator neighbourhoods Ω_i^1, Ω_i^2 and Ω_i^3 respectively

our design procedure, the space S^h used to represent the image is defined by the finite element mesh of triangular elements and piecewise linear basis functions described in Section 2. For operator scaling, we use a finite dimensional test space $T_\lambda^h \subset H^1$ that explicitly embodies a scale parameter σ related to the Layer λ. T_λ^h comprises a set of Gaussian basis functions, ψ_i^λ $i = 1,..., N$ of the form

$$\psi_i^\lambda = \frac{1}{2\pi\sigma^2} e^{-\left(\frac{(x-x_i)^2 + \frac{1}{\sqrt{3}}((y-y_i)^2+(z-z_i)^2)}{2\sigma^2}\right)}$$

Each test function ψ_i^λ is restricted to have support over a neighbourhood Ω_i^λ, centred on node i, and the scaling parameter σ is explicitly related to the spiral neighbourhood size.

Zero-crossing methods are often based on the isotropic form of the second order derivative, namely $-\underline{\nabla} \cdot (\underline{\nabla} u)$, which yields the corresponding weak form

$$R(u) = \int_\Omega \underline{\nabla} u \cdot \underline{\nabla} v d\Omega$$

The image representation introduced in Section 2, together with the sets of test functions ψ_i^λ, i=1, …, N, is used to generate an approximate representation of the weak form of the operator by the functional

$$R_i^\lambda(U) = \int_\Omega \underline{\nabla} U \cdot \underline{\nabla} \psi_i^\lambda d\Omega$$

Similar to the traditional rectangular based Laplacian operators, the spiral Laplacian operator is comprised of a combination of the *x-*, *y-* and *z-* directional operator components. However, due to the rotational symmetries of the hexagonal structure, it is only necessary to construct one of these components, for example the *x*-directional component, and the others can be generated by simple rotation. Therefore, we need only compute

$$R_i^{\lambda} = \sum_{j=1}^{N} K_{ij}^{\lambda} U_j$$

where

$$K_{ij}^{\lambda} = \sum_{m|e_m \subset \Omega_i^{\lambda}} k_{ij}^{m,\lambda} \ ,$$

and $k_{ij}^{m,\lambda}$ is the element integral

$$k_{ij}^{m,\lambda} = \int_{e_m} \frac{\partial \phi_j}{\partial x} \frac{\partial \psi_i^{\lambda}}{\partial x} d\Omega$$

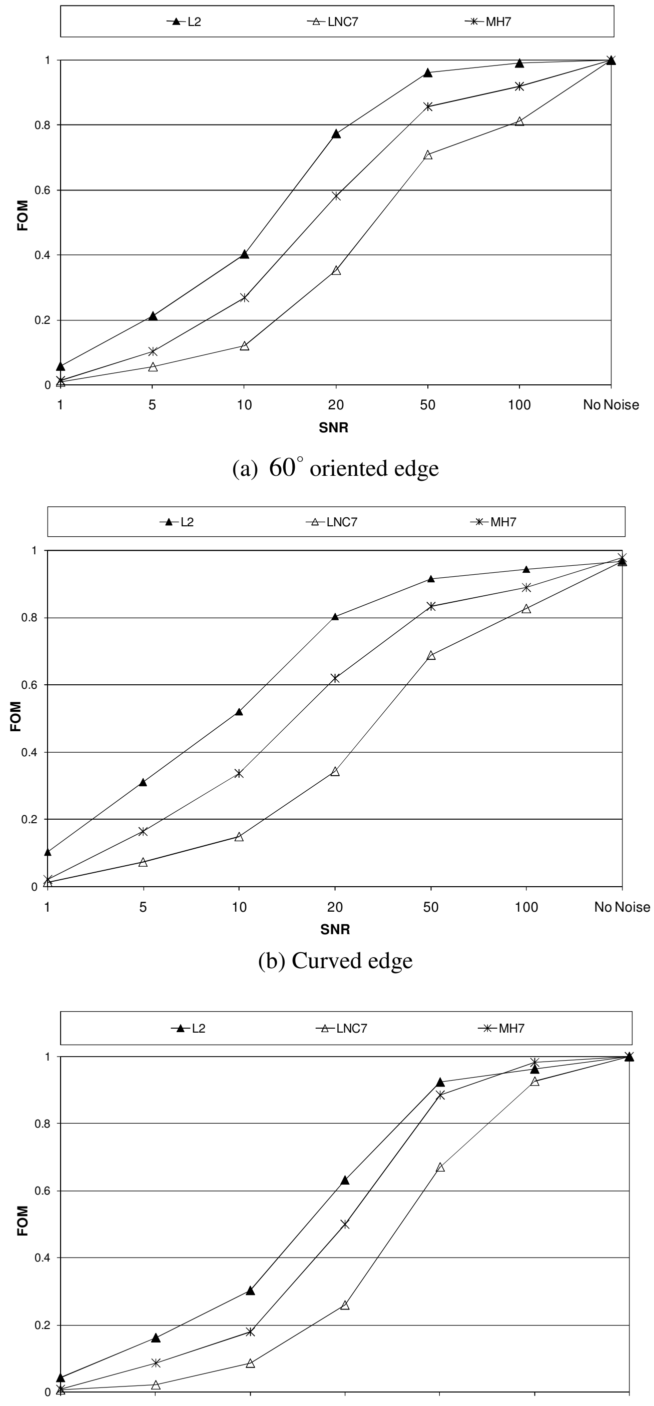
These integrals need to be computed only over elements in the spiral neighbourhood Ω_i^{λ}, rather than the entire image domain Ω, since ψ_i^{λ} has support restricted to Ω_i^{λ}. In order to calculate the values $k_{ij}^{m,\lambda}$ systematically, a local co-ordinate reference system for an equilateral triangle is implemented so that the integrals can be computed using standard numerical integration on a right-angled triangle. These values are then combined appropriately using a process of finite element assembly to obtain the second order derivative mask L_x (in the horizontal direction) over the spiral neighbourhood Ω_i^{λ}. The *y-* and *z-* directional masks (L_y and L_z respectively) may be found by anti-clockwise rotation of L_x through 60° and 120° respectively. These masks are then combined to create the Laplacian operator, *L*, in the following manner:

$$L = \frac{2}{3}\left(L_x + L_y + L_z\right)$$

Once convolved with the image, zero-crossings of the Laplacian operators are found in the same manner as used for rectangular Laplacian operators (i.e., by identifying changes in sign between neighbouring values).

4. PERFORMANCE EVALUATION

To process and evaluate hexagonal pixel-based images we create an environment that simulates the use of a hexagonal image sensor. We initially create hexagonal pixel-based images by resampling of rectangular pixel-based images as in [4], and then process and evaluate the images within a hexagonal environment. We compare the Layer 2 adaptive Laplacian operators from our proposed approach, based on the Spiral Architecture, (denoted as *L*2) with two Laplacian operators designed for use on standard rectangular-based images. The two rectangular based derivative operators used are the 7×7 Marr Hildreth operator (denoted as MH7)



(a) 60° oriented edge

(b) Curved edge

(c) Horizontal Edge

Figure 3. Figure of Merit results comparing the tri-directional spiral operators with equivalent standard use of square operators

and the 7×7 Laplacian near-circular operator [2] (denoted as LNC7). The Laplacian near-circular operator was specifically chosen for the comparative evaluation as the Laplacian Layer 2 operator can be considered to be the hexagonal equivalent of the conventional Laplacian near-circular operator. It should be noted that although a framework has been developed for coarse scale spiral operators, evaluation for the purposes of this paper consists of using 7×7 rectangular operators as they are equivalent in scale to the Spiral Architecture based Layer 2 operators, as $L2$, MH7 and LNC7 all have 49 mask values.

(a) Original Image (b) Spiral Image

Figure 4: Original input images

(a) MH7 (T=21) (b) LNC7 (T=36)

(c) L2 (T=23)

Figure 5: Edge maps corresponding to each of the Laplacian operators

In order to measure accurately the performance of the spiral Laplacian operators, we have modified the well-known Figure of Merit technique to accommodate the use of hexagonal pixel-based images. Figure 3 shows that the proposed $L2$ spiral Laplacian operator has increased accuracy over the equivalent rectangular operators for all evaluated edge directions. Further visual comparison of the Laplacian operators is presented in Figure 5. In Figure 4 we show the original image and the re-sampled hexagonal pixel image, and in Figure 5 we provide visually best edge maps at threshold (T) for each of the operators. Figure 5 illustrates that the operators have similar visual outputs. Although we provide evaluation for only the $L2$ operator, our design procedure may be applied to compute Laplacian operators at any spiral layer.

5. CONCLUSION

Over the last 40 years, the area of hexagonal imaging has been investigated with substantial evidence highlighting its superiority over conventional rectangular grids. We have presented a design procedure for spiral Laplacian operators for use on hexagonal images. The operator structure and enlargement is based on the Spiral Architecture, where each operator size corresponds to a layer in the Spiral Architecture. The performance evaluation presented demonstrates that the Spiral Laplacian operator applied to hexagonal images provides significantly better edge detection and localisation performance (as measured by FoM) than equivalent rectangular-based Laplacian operators applied to rectangular pixel-based images.

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