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Lee Middleton and Jayanthi Sivaswamy

Hexagonal Image Processing

A Practical Approach

With 116 Figures

Springer

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To my parents,
Lee

To Munni and to the loving memory of Appa,
Jayanthi

Foreword

The sampling lattice used to digitize continuous image data is a significant determinant of the quality of the resulting digital image, and therefore, of the efficacy of its processing. The nature of sampling lattices is intimately tied to the tessellations of the underlying continuous image plane. To allow uniform sampling of arbitrary size images, the lattice needs to correspond to a regular - spatially repeatable - tessellation. Although drawings and paintings from many ancient civilisations made ample use of regular triangular, square and hexagonal tessellations, and Euler later proved that these three are indeed the only three regular planar tessellations possible, sampling along only the square lattice has found use in forming digital images. The reasons for these are varied, including extensibility to higher dimensions, but the literature on the ramifications of this commitment to the square lattice for the dominant case of planar data is relatively limited. There seems to be neither a book nor a survey paper on the subject of alternatives. This book on hexagonal image processing is therefore quite appropriate.

Lee Middleton and Jayanthi Sivaswamy well motivate the need for a concerted study of hexagonal lattice and image processing in terms of their known uses in biological systems, as well as computational and other theoretical and practical advantages that accrue from this approach. They present the state of the art of hexagonal image processing and a comparative study of processing images sampled using hexagonal and square grids. They address the hexagonal counterparts of a wide range of issues normally encountered in square lattice-based digital image processing - data structures for image representation, efficient pixel access, geometric and topological computations, frequency domain processing, morphological operations, multiscale processing, feature detection, and shape representation. The discussions of transformations between square and hexagonal lattice-based images and of hybrid systems involving both types of sampling are useful for taking advantage of both in real-life applications. The book presents a framework that makes it easy to implement hexagonal processing systems using the square grid as the base, e.g., to accommodate existing hardware for image acquisition and display, and gives sample computer code for some commonly encountered computations.

This book will serve as a good reference for hexagonal imaging and hexagonal image processing and will help in their further development. I congratulate the authors on this timely contribution.

Professor Narendra Ahuja
August, 2004

Preface

The field of image processing has seen many developments in many fronts since its inception. However, there is a dearth of knowledge when it comes to one area namely the area of using alternate sampling grids. Almost every textbook on Digital Image Processing mentions the possibility of using hexagonal sampling grids as an alternative to the conventional square grid. The mention, however, is usually cursory, leading one to wonder if considering an alternative sampling grid is just a worthless exercise. Nevertheless, the cursory mention also often includes a positive point about a hexagonal grid being advantageous for certain types of functions. While it was curiosity that got us interested in using hexagonal grids, it was the positive point that spurred us to study the possibility of using such a grid further and deeper. In this process we discovered that while many researchers have considered the use of hexagonal grids for image processing, most material on this topic is available only in the form of research papers in journals or conference proceedings. In fact it is not possible to find even a comprehensive survey on this topic in any journal. Hence the motivation for this monograph.

In writing this book, we were mindful of the above point as well as the fact that there are no hardware resources that currently produce or display hexagonal images. Hence, we have tried to cover not only theoretical aspects of using this alternative grid but also the practical aspects of how one could actually perform hexagonal image processing. For the latter, we have drawn from our own experience as well that of other researchers who have tried to solve the problem of inadequate hardware resources.

A large part of the work that is reported in the book was carried out when the authors were at the Department of Electrical and Electronic Engineering, The University of Auckland, New Zealand. The book took its current shape and form when the authors had moved on to the University of Southampton (LM) and IIIT-Hyderabad (JS). Special thanks to Prof. Narendra Ahuja for readily agreeing to write the foreword. Thanks are due to the anonymous reviewers whose feedback helped towards making some key improvements to the book.

Lee: Thanks are first due to Prof. Mark Nixon and Dr John Carter who were understanding and provided me time to work on the book. Secondly thanks go to my, then, supervisor Jayanthi for believing in the idea I came to her office with. Thirdly, I would like to thank the *crew* at Auckland University for making my time there interesting: adrian, anthony, bev, bill, brian, brad, bruce, colin, david, dominic, evans, evan, geoff (×2), jamie, joseph, nigel, russell m, and woei. Finally, thanks go to Sylvia for being herself the whole time I was writing the manuscript.

Jayanthi: Thanks to Richard Staunton for many helpful comments and discussions, to Prof. Mark Nixon for the hospitality. I am also grateful to Bikash for clarifications on some of the finer points and to Professors K Naidu, V U Reddy, R Sangal and other colleagues for the enthusiastic encouragement and support. The leave from IIIT Hyderabad which allowed me to spend concentrated time on writing the book is very much appreciated. The financial support provided by the DST, Government of India, the Royal Society and the British Council partly for the purpose of completing the writing is also gratefully acknowledged. Finally, I am indebted to Prajit for always being there and cheering me on.

Contents

1

Introduction

The perceptual mechanisms used by different biological organisms to negotiate the visual world are fascinatingly diverse. Even if we consider only the sensory organs of vertebrates, such as the eye, there is much variety. From the placement of the eyes (lateral as in humans or dorsal as in fish and many birds) to the shape of the pupil, and the distribution of photoreceptors. The striking aspect about nature is the multiplicity in the designs and solutions devised for gathering visual information. This diversity also continues in the way the visual information is processed. The result of this multiplicity is that the visual world perceived by different organisms is different. For instance, a frog's visual world consists only of darting objects (which are all, hopefully, juicy flies), whereas a monkey's and a human's visual world is richer and more colourful affording sight of flies, regardless of whether they are immobile *or* airborne.

In contrast, computer vision systems are all very similar, be it in gathering visual information or in their processing. The sensing systems are designed similarly, typically based on square or rectangular arrays of sensors which are individually addressed to access the visual information. Just about the only thing that differs among sensors is the spectrum of light information that can be captured. Furthermore, the information is processed using algorithms evaluated and tested over a considerable amount of time since, like most sciences, computer vision requires the repeatability of the performance of algorithms as a fundamental tenet. Hence, from both an algorithmic and physical view all computer vision systems can be said to *see* the world with the *same eyes*.

This monograph endeavours to study the effect of changing one aspect of the sensing methodology used in computer vision, namely the sampling lattice. The change considered is from a square to a hexagonal lattice. Why hexagons? Two simple reasons: geometry and nature. Hexagonal lattices have been of interest to humans for over two millennia. Geometers from Pythagorean times have studied hexagons and found them to have special properties, including membership in the exclusive set of three regular polygons with which one can tile the plane, the other two being a square and a triangle. A honeycomb is the best 2-D example of a hexagonal lattice in nature and has fascinated people, including scientists, and been studied for a long time. This has led to the well known honeycomb conjecture. This conjecture, put simply, states that the best way to partition a plane into regions of equal area is with a region that is a regular hexagon. This conjecture has existed at least since Pappus of Alexandria but has eluded a formal proof until very recently, when Prof. Thomas Hales [1,2] proved it elegantly in 1999. In gathering information about the visual world, we believe the task at hand is similar to the problem underlying the honeycomb conjecture: capture the visual information with a set of identical sensors arranged in a regular grid structure on a planar surface. Taking cues from science and nature it is then interesting to ask what happens when you use a hexagonal (instead of a square) lattice to gather visual information. To use the previous analogy, this is viewing the world with *different eyes*. This alternative view of the visual world may present researchers with some advantages in representation and processing of the visual information. Furthermore, such a study may illuminate the importance of the role which the sensors play in computer vision.

1.1 Scope of the book

As stated, the aim of this monograph is to study the effect of changing the sampling lattice from a square to a hexagonal one. Based on lattice geometry, the hexagonal lattice has some advantages over the square lattice which can have implications for processing images defined on it. These advantages are as follows:

- *Isoperimetry.* As per the isoperimetric theorem, a hexagon encloses more area than any other closed planar curve of equal perimeter, except a circle. This implies that the sampling density of a hexagonal lattice is higher than that of a square lattice.
- *Additional equidistant neighbours.* Every hexagon in the lattice and hence a hexagonal pixel in an image has six equidistant neighbours with a shared edge. In contrast, a square pixel has only four equidistant neighbours with a shared edge or a corner. This implies that curves can be represented in a better fashion on the hexagonal lattice and following an edge will be easier.
- *Uniform connectivity.* There is only one type of neighbourhood, namely N_6, possible in the hexagonal lattice unlike N_4 and N_8 in the square lattice. This implies that there will be less ambiguity in defining boundaries and regions.

The goal of this monograph is then to verify the above and to understand the overall impact of changing the sampling lattice underlying a digital image, from both a theoretical and a practical perspective. Towards this goal, we will first seek out answers from what has been done in this field by other researchers in a period that spans roughly 40 years. We will also seek to further our understanding by developing a framework for hexagonal image processing and studying specific issues using the framework. For the sake of brevity, the terms square image and hexagonal image will be used throughout to refer to images sampled on a square lattice and hexagonal lattice, respectively.

In general, we will examine the entire gamut of issues pertaining to processing hexagonally sampled images. These start from fundamental ones such as appropriate data structures, definitions of neighbourhoods and distance functions which are essential for examining and developing processing methodologies in both the spatial and frequency domains. Applications using some of these methodologies are also of interest as they are the end goal of any image processing system. The coverage is intended to be comprehensive enough to help develop more extensive studies as well as applications. However, an exhaustive coverage is neither intended nor possible, given the current state of development of this field.

1.2 Book organisation

This monograph is divided into eight chapters and four appendices.

Chapter 2 provides an overview of the relevant background in both biological and computer vision. The latter focusses exclusively on hexagonal image processing and summarises the work that has been reported in the literature up till now.

Chapter 3 is concerned with developing a comprehensive framework for hexagonal image processing. The approach to the development concentrates on aspects required for an efficient framework: addressing and processing. Fundamental aspects of processing in both spatial and frequency domains using the developed framework, are examined and discussed. Overall this chapter is quite theoretical in nature. For non-mathematically inclined readers, the key point to examine is how the addressing scheme works, as this is central to the remaining chapters in the book.

Chapter 4 provides many examples of processing hexagonally sampled images. The proposed framework is employed for this and the examples cover most traditional problems in the field of image processing. In the spatial domain, this includes edge detection and skeletonisation. In the frequency domain, this includes the development of an algorithm for the fast computation of the discrete Fourier transform and linear filtering. Further examples included in the chapter are operations using image pyramids and mathematical morphology.

Several applications of the proposed hexagonal framework are illustrated in Chapter 5. A biologically-inspired application involves finding interesting points in an image. The rest of the applications presented are applicable to problems in content-based image retrieval. This includes one which uses a search methodology to find the shape of an object and discriminates shapes of objects based on the local energy. The applications discussed in this chapter employ the fundamental approaches outlined in Chapter 3.

The practical aspects of processing hexagonal images is investigated in Chapter 6. Two alternatives to the conventional systems which process square sampled images, are considered. These are, namely, a complete system for hexagonal image processing and a mixed system where some of the processing uses square sampled images while others use hexagonally sampled images. These alternative systems require solutions for image acquisition and visualisation which are developed and presented along with accompanying code (in Python).

To help understand the effect of changing the sampling lattice from a square to a hexagonal one, a comprehensive comparison between processing images sampled using these lattices is provided in Chapter 7. The comparison is performed from a computational perspective as well as based on visual quality analysis. Several of the examples illustrated in Chapter 3 are also compared in both square and hexagonal image processing frameworks. Conclusions are drawn about the relative merits and demerits of processing within the two frameworks.

The final chapter provides a discussion of the future directions for hexagonal image processing that merit immediate attention.

Appendix A provides derivations and proofs of various results which are mentioned throughout the book. Appendix B provides a derivation of the arithmetic tables required in the proposed framework. The Bresenham algorithms for drawing lines and circles on the hexagonal lattice are given in Appendix C. Finally, Appendix D provides some useful code for resampling and visualisation of hexagonal images and for performing all the arithmetic called for in the proposed framework.

2

Current approaches to vision

Many advances in the physical sciences have come from examination of the world around us. By performing experiments our understanding of the governing laws of the universe expands and thus we are able to build systems to take advantage of our newly acquired knowledge. The early understanding of the nature of light came from understanding and trying to mimic our eyes. The models were incorrect but they provided the foundation for all future research which truly was performed standing upon giants' shoulders. So it was in the early days of the science of image processing. The primary motivation was to recreate the abilities of the visual system in modern computing devices.

In the years since its inception the science of image processing has forked many times, each time with a resulting name change. Many of these forks disregarded the influence of the visual system when devising image processing algorithms. However, due to the rapid rise in computational power in recent times, it is possible to accurately model portions of the brain. This has led to a resurgence in research into image processing using the results from biological visual system.

It is with these ideas in mind that this chapter provides an overview of both biological and computer vision. The study of the visual system will lead to an architecture in which the brain performs its visual processing. This generic architecture will be then be applied to the study of conventional vision. Central to this thrust is the specific hexagonal arrangement implicit in the visual system's sensor array. Logically, this arrangement affects all other aspects of the visual system. In line with the historical perspective, the biological system will be discussed first followed by current computer vision approaches.

2.1 Biological vision

The complexities of the brain and all its subsystems have fascinated mankind for an extremely long time. The first recorded references to brain dissection and study date to Galen in the 2nd century AD though there is evidence, in papyri, that the Egyptians were also interested in brain function [3]. The awareness of a distinct subsystem associated with vision dates from the 18th century [4].

This section will provide a brief overview of some key aspects of the human visual system (HVS) which occupies two thirds of the human brain's volume. The first of part of the HVS is the eye. The eye performs a similar function to a smart sensor array in a modern camera. The key feature of the visual system is that it performs the processing of the information using a hierarchy of cooperative processes.

2.1.1 The human sensor array

The visual system, according to Kepler who founded the modern study of the eye in 1604 [5], begins when"the image of the external world is projected onto the pink superficial layer of the retina". Later, Descartes [6] studied the optics of the eye and Helmholtz [7] studied the retina. This early work promoted the view that the eye performed in the same way as a pinhole camera (or camera obscura). Advances made since then however, have led to the view held today that the eye is more sophisticated and functions like a mini-brain. We will now briefly explain the reasons for this view.

The eye is roughly spherical with a slightly protruding part that is exposed while the remaining part sits in the eye socket. The light enters the eye through the pupil behind the cornea and is projected by a lens onto the inner spherical surface at the rear part of the eye. Here, the light is converted into electrical signals in an array of interconnected nerve cells known as the retina. An interesting aspect of the retina is its structure. The superficial layers, which are transparent, consist of neurons while the photoreceptors are found at the deepest layer. In the thinnest part of the retina, called the fovea, the neurons are moved aside to let light pass through directly to the photoreceptors. There is also a region of the retina known as the optic disc where there is an absence of photoreceptors to permit neural wiring to carry information out to the brain. This gives rise to a blind spot.

There are two distinct sorts of photoreceptors, namely, rods and cones, with their nomenclature stemming from their shapes. Their functions are mutually complementary as summarised in Table 2.1.

A remarkable feature of the photoreceptive layer of the retina is that the rods and cones are distributed non-uniformly as illustrated in Figure 2.1. There is a radial distribution of these receptors: cones are concentrated in the central foveal region and, as one moves away from the centre, rods are found in abundance but gradually diminish in number. The foveal region, rich in cones, specialises in high resolution, colour vision under bright illumination such as during the day. This region however, is very small in extent. The region outside the fovea is rod-rich and hence contributes towards vision under low levels of illumination such as during night time. The field of view afforded by

Table 2.1. Differences between rods and cones.

Rods	Cones
high sensitivity	low sensitivity
more photopigment	less photopigment
high amplification	lower amplification
slow response	fast response
low resolution	high resolution
achromatic	chromatic (red, green, blue)
night vision	day vision

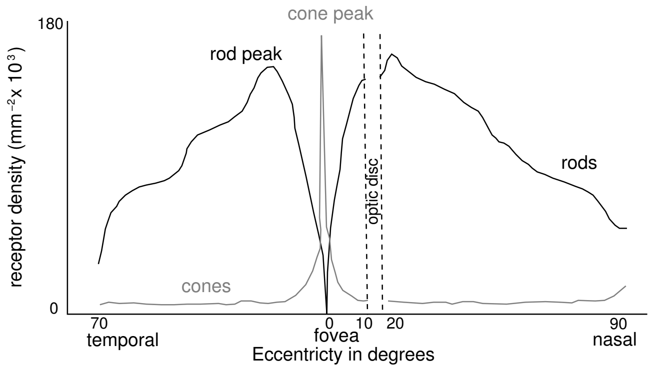

Fig. 2.1. Distribution of rods and cones in the retina (redrawn from Osterberg [8]).

high resolution and colour vision sensors is complemented by a combination of eye and head movements. The arrangement of the photoreceptors along the spherical retinal surface, is illustrated in Figure 2.2(a). Here, the larger circles correspond to the rods and the smaller circles to the cones. A significant fact to notice is that the general topology in this diagram is roughly hexagonal. This is because, as we shall see later, all naturally deformable circular structures pack best in two dimensions within a hexagonal layout such as found in honeycombs. An example of an enlarged portion of the foveal region of the retina, showing this behaviour, is given in Figure 2.2(b).

The signals from the photoreceptors are preprocessed by a neuronal assembly made of four major types of neurons: bipolar, horizontal, amacrine, and ganglion. Of these cells, the horizontal and amacrine are purely used as lateral connections joining remote regions. The lateral connections enable receptors to influence each other and help in contrast correction and adaptation to sudden changes in ambient illumination. The ganglion cells are specialised for processing different aspects of the visual image such as movement, fine spatial detail, and colour. Two of the widely studied types of ganglion cell are the magno and parvo cells. Functionally speaking, these two types of cells give rise to the formation of two distinct pathways (called the M and P pathways)

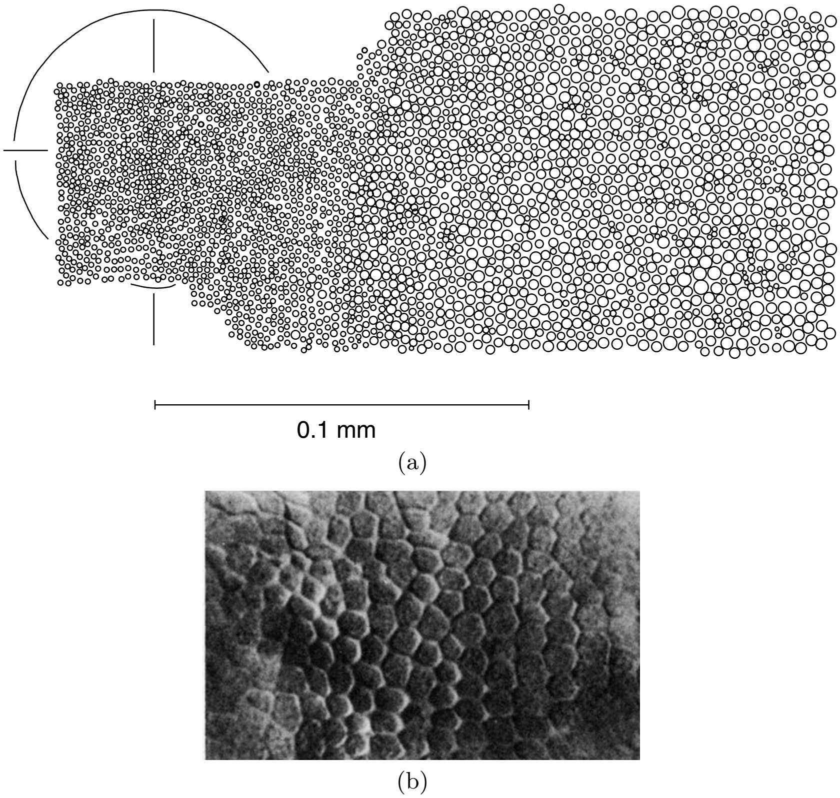

Fig. 2.2. (a) Arrangement of rods and cones in eye adapted from Pirenne 1967 [9] (b) A close up of the foveal region (reprinted from Curcio et al. [10]. Copyright 1987 AAAS).

through which visual information is passed to the brain and processed. The magno cells have large receptive fields due to their large dendritic arbours, and respond relatively transiently to sustained illumination. Thus, they respond to large objects and follow rapid changes in stimulus. For this reason it is believed that magno cells are concerned with the gross features of the stimulus and its movement. On the other hand, the more numerous parvo ganglion cells have smaller receptive fields and selectively respond to specific wavelengths. They are involved with the perception of form and colour and are considered responsible for the analysis of fine detail in an image. The ganglion cells are collected together in a mylenated sheath at the optic disk to pass the visual information to the next stage in the visual system in the brain.