# Edge-detection method using binary morphology on hexagonal images

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This paper proposes a new edge-detection and noise-removal method for hexagonally sampled images by using binary morphology edge detection on hexagonally sampled images. This is achieved by applying binary morphological operators on hexagonal images. This approach presents a framework for conversion of hexagonally sampled images and processing to detect edges on a hexagonal grid. The proposed study shows that a method combining horizontal, 60° and 120° directional hexagonal structuring elements achieves results superior to those of a rectangular image. The experimental results showed that the proposed method was more efficient for binary hexagonal image edge detection and noise removal than the commonly used rectangular image.

**Keywords:** Hexagonal grid, Binary morphology, Edge detection, Noise removal

## Introduction

Edge detection and noise removal are essential preprocessing steps in image processing and computer vision.[1] Several techniques enable edge detection, among which employing a morphological gradient is one of the conventional.[2] Traditionally, images are digitised and stored in a rectangular arrangement because conventional image capture and display devices are based on a rectangular grid. However, Whitehouse and Phillips[3] proposed numerous types of feasible sampling system, among which the hexagonally sampled image is the most prominent[4] because it requires less storage, less computation time, and exhibits an equidistant property and consistent connectivity.[5] For morphological operators, a $3\times3$ rectangular grid structuring element (SE) is the closest fit for a circular SE. However, using such an element to erode curved objects can introduce unwanted discontinuities because the elements in the SE are at various distances from the centre. The consistent connectivity of pixels in hexagonal images is an attractive feature that allows for superior circular SEs. Moreover, computation of neighbourhood is simple in a hexagonal lattice because it only has six neighbourhood pixels. By contrast, a rectangular lattice has four and eight neighbourhood pixels. To overcome these limitations, high-resolution rectangular grids are often used; however, using high-resolution images has become expensive. Binary morphology on a hexagonal lattice can be a superior solution to overcome these difficulties. The main problem that limits the use of hexagonal images is the lack of hardware for hexagonal images. Thus, a resampling method was developed to convert rectangular-grid images to hexagonal-grid images.[6,7] A thorough review of morphological operators (such as dilation, erosion, opening and closing) can be found in Ref. 8. Numerous morphological edge-detection algorithms[9,10] have been proposed for rectangular images to detect edges with noise. This paper proposes a novel approach to noise-removal and edge-detection algorithms by applying binary morphology on hexagonal images.

## Research objectives

Numerous studies have been conducted on mathematical morphology and hexagonal image processing. However, there has been little discussion about combining mathematical morphology and hexagonal image processing with edge-detection and noise-removal applications. One reason might be a lack of hexagonal pixel display devices. To alleviate this problem, we proposed a new noise-removal and edge-detection method for hexagonally sampled images by applying binary morphology. In summary, our objectives were as follows:

- To develop a method for processing hexagonally sampled images, including the conversion of rectangular to hexagonally sampled images, processing and display of processed images on a hexagonal grid. Image processing includes binary morphological operations on hexagonal grid images to detect edges and remove noises. Moreover, to develop simulated hexagonal grids and apply to display hexagonal images.
- To develop binary hexagonal SEs with specific sizes and directions. A better combination is then determined for the proposed method.
- To evaluate the performance of the proposed algorithm, this study employs quantitative comparison and mean square error (MSE).

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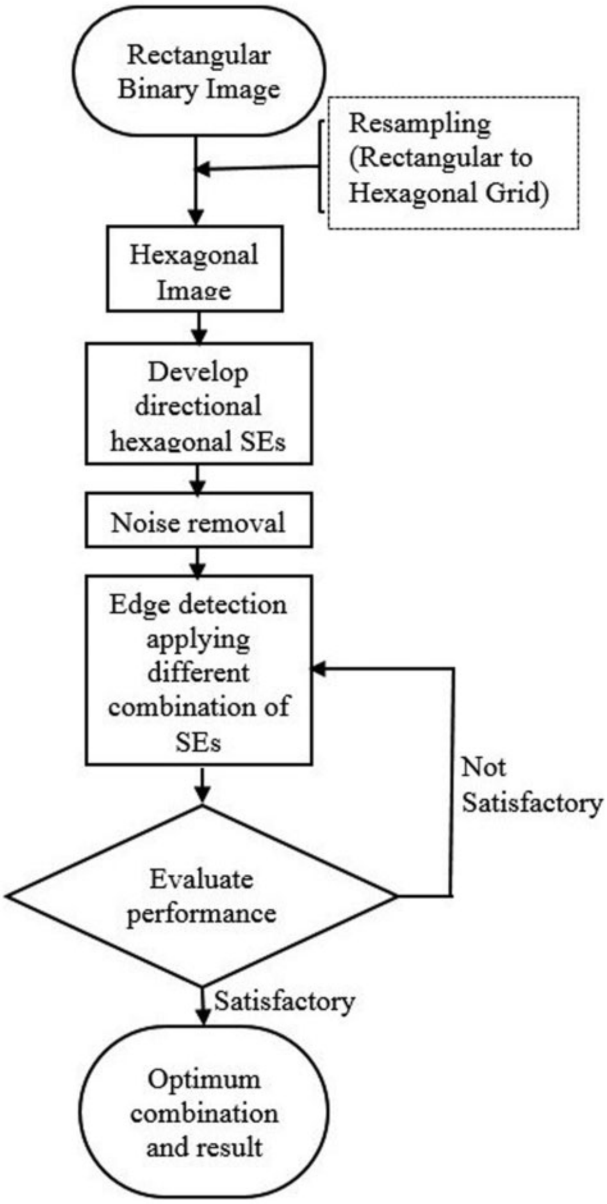



**1 System diagram for performing image processing on a hexagonally sampled grid**

## Methodology

The research was grouped into two sections. First, we employed an image resampling method to convert images from a rectangular grid to a hexagonal grid; subsequently, we applied noise removal, followed by edge detection, by using a binary morphological gradient method. Second, the proposed method was compared using the rectangular grid counterpart and performance evaluation, as shown in Fig. 1.

### Resampling (rectangular to hexagonal)

This study employed a simulated hexagonal grid to convert an image from a rectangular to a hexagonal grid structure. Numerous rectangular grids combined created clusters of subpixels, as shown in Fig. 2. According to the D7 method of resampling, a cluster of subpixels closely represented the shape of a hexagon. A small inset with enlarged view was added at the lower left side of two test images to show the hexagonal grid, as shown in Fig. 2.

### Hexagonal SEs

The performance of morphological operations heavily depends on the size, shape and direction of an SE. In this paper, eight new solid hexagonal SEs are proposed to remove noise and detect edges from hexagonally sampled images, as shown in Fig. 3.

### Hexagonal binary morphological operator

The most basic morphological image processing operations are dilation and erosion. This study performed binary dilation and erosion on hexagonally sampled images. This was accomplished by applying eight new hexagonal SEs to apply dilation and erosion in standard test images. Opening and closing was a sequential combination of dilation and erosion.

### Noise removal and edge detection

To evaluate the proposed method of noise removal, we added salt and pepper noises in a standard test image. Salt and pepper noise was added to the test image titled 'Face' with a probability of 5%. This paper proposes opening followed by closing with various combinations of SEs. The better combination to remove noise was defined by horizontal, 60°, 120° and $3\times3$ SEs, as shown in Fig. 4.

A morphological gradient is the difference between the dilation and erosion of a given image when the same SE is applied. The most basic morphological gradient operator can be expressed as follows:

$$\mathrm{Edge}=(A\oplus B)-(A\Theta B) \quad (1)$$

The result of the dilated images subtracted from eroded images, using hexagonal SEs on hexagonal binary images, is shown in Fig. 5.

In this experiment, we applied combinations of hexagonal SEs and evaluated the obtained results. The performances of the $3\times3$ hexagonal SEs were superior to those of their $5\times5$ counterparts. The smaller circular-shape hexagonal SEs achieved superior performance in identifying curved edges. By contrast, larger SEs introduced unwanted thickness and discontinuities. We also applied $3\times3$ directional SEs on a rectangular image, as shown in Fig. 6, which indicates numerous unwanted discontinuities in the rectangular images.

## Performance evaluation

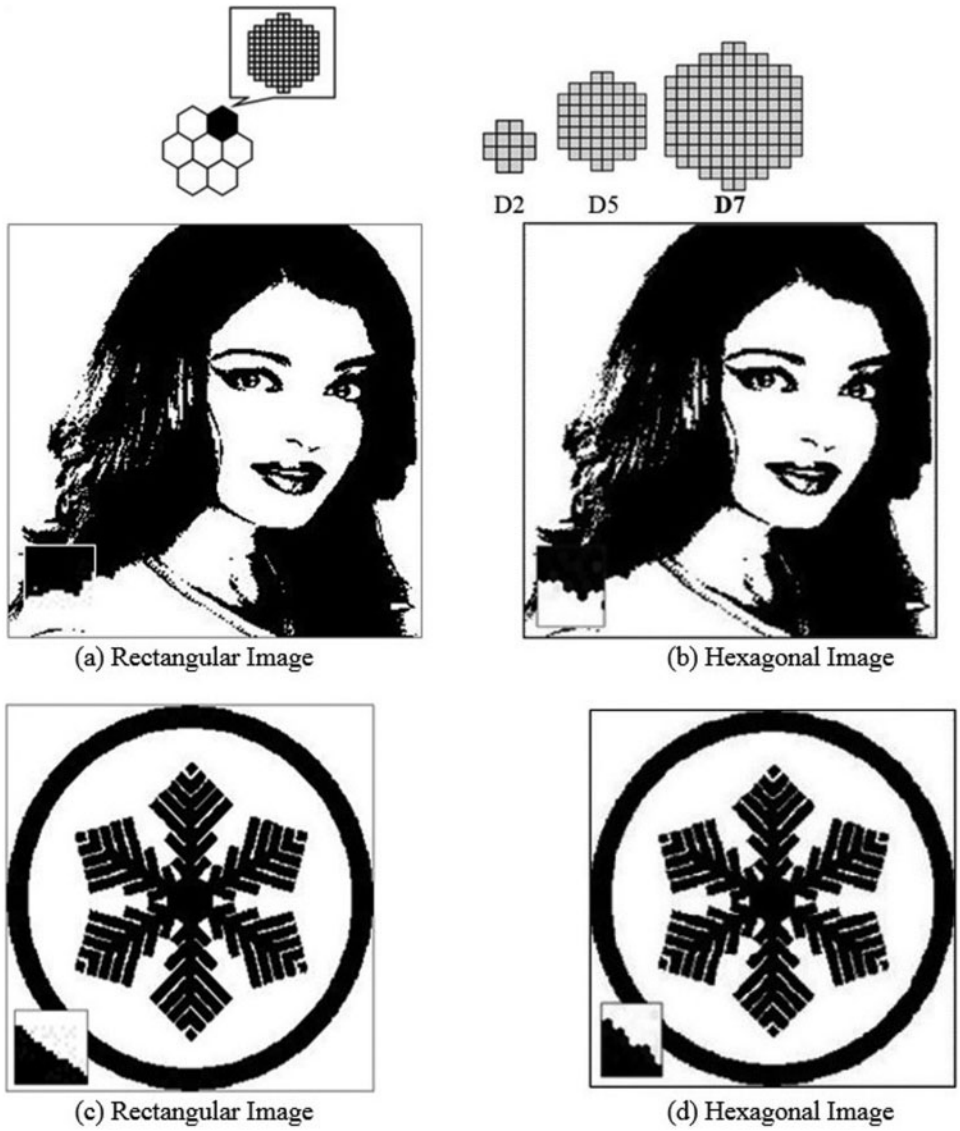
We applied MSE and the ratio of edge pixels to the image size to evaluate the proposed method. MSE indicated the average difference of the pixels throughout the image. A higher MSE represented a greater difference between the original and the processed image. The formula for the MSE calculation was expressed as follows:

$$\mathrm{MSE}=\frac{1}{N}\sum\nolimits_{\mathrm{i}}\sum\nolimits_{\mathrm{j}}\left(X_{\mathrm{ij}}-V_{\mathrm{ij}}\right)^2 \quad (2)$$

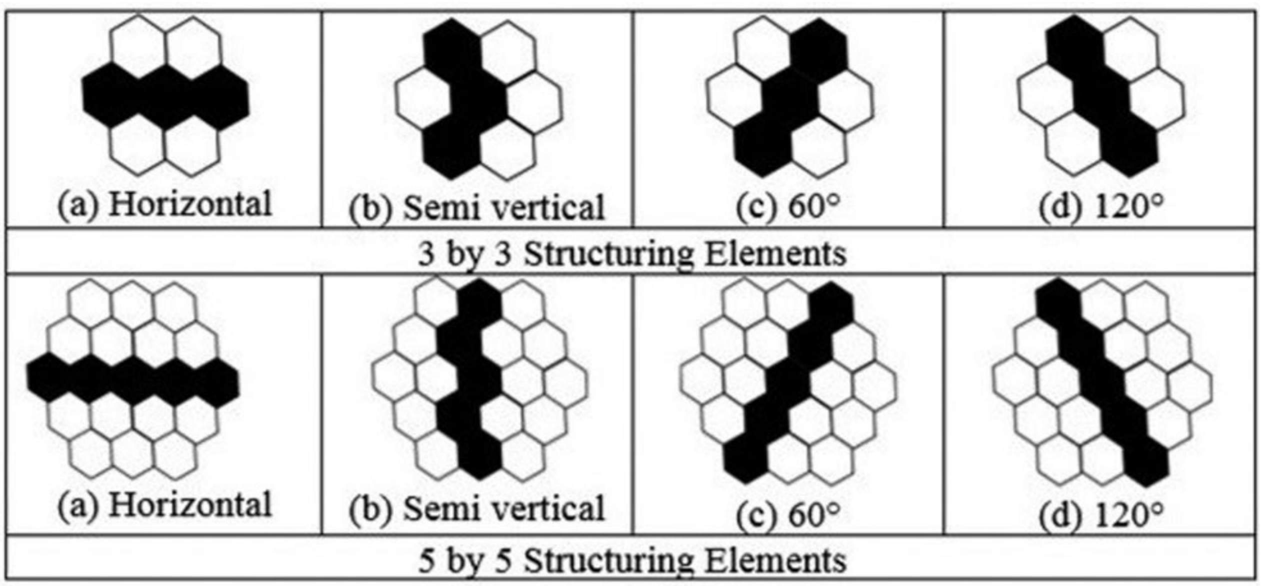
where $N$ is the size of the image, $X$ is the processed image and $V$ is the original image.

The ratio of edge pixels to image size measures the relative number between pixels of edges and the total image size. A greater ratio means that the proposed edge detector identified more edge pixels.

The most remarkable finding was that the combination of horizontal, 60° and 120° hexagonal SEs achieved




**2 Simulated hexagonal-grid test images**




**3 Structuring elements with various sizes and directions**

**4 The proposed noise removal method: (a) noisy image; (a) noise-free image**

(a) Horizontal, 60° and 120°

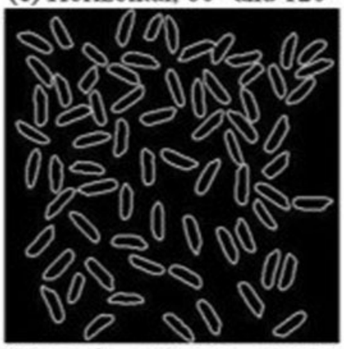
(b) Horizontal, Vertical and 120°

(c) Horizontal, Vertical and 60°

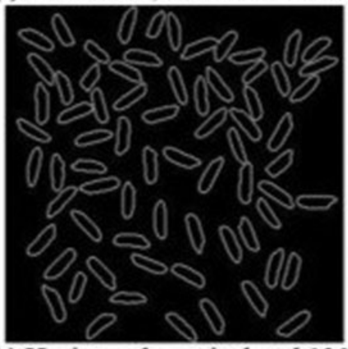
(d) Vertical, 60° and 120°

(e) Horizontal, 60° and 120°

(f) Horizontal, Vertical and 120°

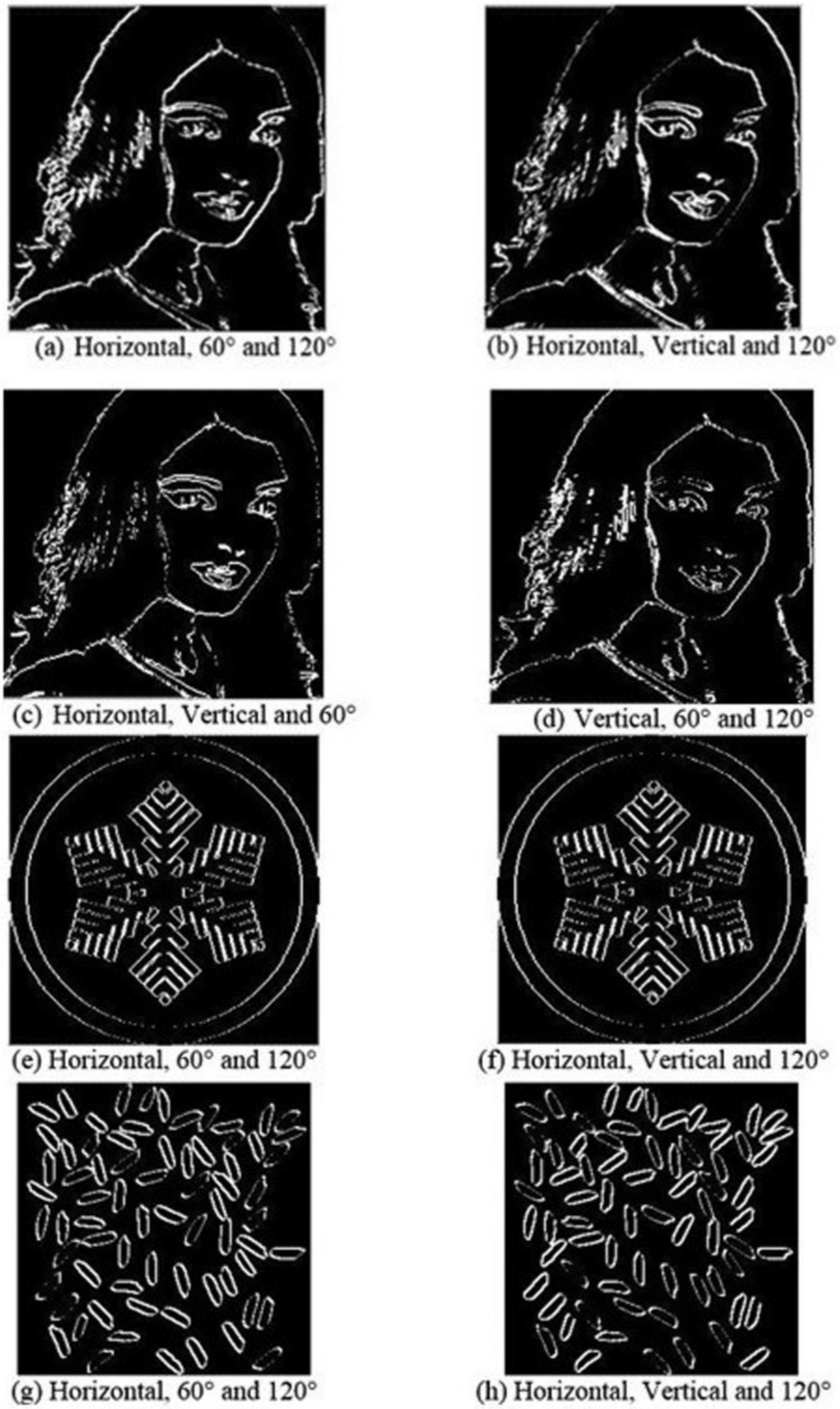


(g) Horizontal, 60° and 120°



(h) Horizontal, vertical and 120°

**5 Edge detection achieved by applying various directional 3 $\times$ 3 SEs on the hexagonal grid**




**6 Edge detection achieved by applying various directional $3 \times 3$ SEs on the rectangular grid**

a superior MSE and a higher ratio of edge pixels to the image size values for the Face test image and for most of the synthetic and real images tested in the experiment. The reason is that in hexagonal pixels, the vertical direction can be regarded as a combination of $60^\circ$ and $120^\circ$ to the horizontal direction because of the geometrical properties of the hexagonal shape of pixels. These properties of the hexagonal grid and the SE combined contributed to efficiently deriving edges in an oblique direction as well as the vertical and horizontal directions. The results obtained from the proposed analysis are shown in Table 1. A substantial difference was observed between the hexagonal and rectangular grids. The hexagonal grid achieved a lower MSE and a higher ratio of edge pixels to the image size (%). As shown in Table 1, the ratio of the edge pixels to the image size also indicated

**Table 1 Quantitative measure obtained by edge detectors in hexagonal grid and rectangular grid by using binary hexagonal morphology for real test image Face**

| Combination of structuring elements | | The Face test image | |
|---|---|---|---|
| | | MSE | Ratio of edge pixels to image size (%) |
| Hexagonal image | Horizontal, 60° and 120° SE | 0·97 | 15·78 |
| | Horizontal, vertical and 120° SE | 1·47 | 15·02 |
| | Horizontal, vertical and 60° SE | 3·02 | 14·79 |
| | Vertical, 60° and 120° SE | 5·05 | 14·23 |
| Rectangular image | Horizontal, 60° and 120° SE | 10·56 | 13·07 |
| | Horizontal, vertical and 120° SE | 11·36 | 12·93 |
| | Horizontal, vertical and 60° SE | 12·02 | 12·71 |

that the proposed method showed quantitatively favourable results. A comparison of the two grids revealed that, overall, the edge of the proposed hexagonal method was more continuous, well-defined and clearer.

## Conclusion

Recently, the development of charge-coupled device technology has renewed the interest in hexagonal image processing. Our aim was to investigate the hexagonal grid with binary morphology and compare it with the traditional rectangular grid. As shown in Table 1, the combination of horizontal, 60° and 120° hexagonal SEs achieved an improved MSE and a higher ratio of edge pixels to the image size values. In conclusion, from the performance evaluation and visual comparison, we determined that consistent and accurate gradient operators can be obtained by applying hexagonal lattices. The results clearly showed that the hexagonal grid performed adequately for curved objects as well as for horizontal and vertical lines.

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