Asset returns, idiosyncratic and aggregate risk exposure*

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Abstract

We present a class of tractable incomplete-market models, where agents face both aggregate risk and financial market participation costs. Tractability relies on the assumption of a periodic utility function which is linear beyond a threshold, following a contribution of Fishburn (1977) in decision theory. We prove equilibrium existence and derive theoretical results about asset prices and consumption choices. The model is able to match US data and in particular to quantitatively reproduce a low safe return and a high equity premium, together with a realistic exposure of households to both idiosyncratic and aggregate risks.

Keywords: Incomplete markets, risk sharing, consumption inequalities.

JEL codes: E21, E44, D91, D31.

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1 Introduction

Infinite-horizon incomplete-insurance market models with credit constraints are known to be difficult to solve in presence of aggregate shocks. These models generate a large amount of heterogeneity, reflected by a time-varying distribution of agents' wealth with continuous support, such that numerical methods are needed to approximate the equilibrium (Krusell and Smith, 1998). The existence of simple recursive equilibria in such environments is still an open question (Miao, 2006). These economies are nevertheless appealing. First, they can account for a significant heterogeneity across households in consumption, wealth or income as is observed in the data. Second, incomplete markets can contribute to the explanation of certain asset price properties, which are otherwise hard to rationalize in complete market environments.

In this paper, we present a class of incomplete market models allowing for theoretical investigations of equilibrium allocations and asset prices. More precisely, we prove the existence of an equilibrium with aggregate shocks, heterogeneous levels of idiosyncratic risk and stock market participation costs, where we can analytically analyze the main determinants of the risk allocation and of asset prices. The model is based on two assumptions. First, we assume that the periodic utility function is linear beyond a certain threshold, while strictly concave before, though being globally smooth. This utility function was first introduced in decision theory by Fishburn (1977) to analyze risk for "below-target returns".[1] We show that this utility function provides tractability in incomplete-market models in an interesting way when compared to alternatives. In particular, incomplete market models have often relied on

[1] This captures the idea that investors are averse to risk for low returns (below a given target), while they care much less about risk for high returns.

quasi-linearity in the labor supply to reduce the dimension of the state space (Scheinkman and Weiss, 1986; Lagos and Wright, 2005; Challe, LeGrand and Ragot, 2013, among others). This assumption has nevertheless the drawback that the consumption of agents with infinite labor elasticity is constant and independent of wealth and income. The infinite Frisch elasticity of labor supply is also far too high at household level (see Hall, 2010, for a recent survey). Linearity in the periodic utility function may thus be an attractive alternative choice to study consumption dynamics. Our second assumption is that the supply of securities is not too large. This implies that credit constraints bind after a small number of periods, generating an equilibrium with a small number of heterogeneous agents. Our economy therefore features a "small-trade" equilibrium, where prices can be analytically studied, as in no-trade equilibria (sse below for references), but where we can also investigate consumption allocations as well as the role of security volumes and liquidity. Further financial frictions, such as participation costs, can also be introduced.

We show that our model is able to reproduce realistic household risk exposure and asset prices. It is known that models with only incomplete insurance markets fail to reproduce the equity premium, for a realistic calibration (Krusell and Smith, 1998 and Krusell, Mukoyama and Smith, 2011), while limited participation and preference heterogeneity can help in reproducing relevant aspects of asset prices (Guvenen, 2009). In this paper, we prove that incomplete markets together with heterogeneous idiosyncratic risk exposure and participation costs can generate a low return for the safe asset, a high equity premium and realistic household risk exposures, in a model where agents have identical preferences. Indeed, the stock market participation cost for high idiosyncratic risk agents generates market segmentation. These high-risk agents do not hold stocks, but trade safe bonds to self-insure against their idiosyncratic risk, which generates a low bond return. Low-idiosyncratic-risk agents participate in the stock market, and thus face a greater exposure to aggregate risk, which implies a high stock return. The combination of heterogeneous individual risk exposure with participation costs is therefore a key ingredient of our model. In this setup, we show that a higher volume of securities decreases asset prices and improves consumption smoothing, whereas a higher level of idiosyncratic risk generates both a decrease in the bond interest rate and an increase in stock prices. Interestingly, these features are consistent with the trends observed in the aftermath of the 2008 crisis. We finally take advantage of the tractability of our framework to estimate key parameters and show that the model is able to match the data on consumption allocations and asset prices. As in the data, the model generates a higher volatility of the consumption growth rate for low-income households than for high-income households.[2] High-income households are also found to bear a larger fraction of the aggregate risk than low income households, while the latter face a larger total risk than the former (Parker and Vissing-Jorgensen, 2009).

The paper contributes to the theoretical literature on incomplete-market models. In this literature, analytical tractability can be obtained in a no-trade equilibrium, as in Constantinides and Duffie (1996) or Krusell, Mukoyama and Smith (2011). In these economies, assets can be priced, even in absence of trade. In our model, trades do additionally occur at the equilibrium. Our assumption of linearity in the utility function is reminiscent of several papers that consider linearity in consumption or leisure utility or in the production function, so as to reduce ex-post heterogeneity, such as Scheinkman and Weiss (1986), Lagos and Wright (2005), Kiyotaki and Moore (2005, 2008), Miao

[2]See De Giorgi and Gambetti (2012), Gârleanu, Kogan and Panageas (2012), Meyer and Sullivan (2013), among others for empirical evidence.

and Wang (2015) or Dang, Holmstrom, Gorton, and Ordoñez (2014) among others. Finally, this paper generalizes some previous works (Challe, Le Grand and Ragot 2013; Challe and Ragot 2014 or Le Grand and Ragot 2015).

The paper is also related to the vast quantitative literature on asset prices with heterogeneous agents. Our contribution consists in explaining asset prices and household risk exposure with just two assumptions, i.e., limited insurance markets and participation costs. Guvenen (2009) shows that asset price properties can be rationalized in a model with exogenous limited participation and household heterogeneity in intertemporal elasticities of substitution. Constantinides and Ghosh (2014) build on Constantinides and Duffie (1996) to construct a no-trade equilibrium with Epstein-Zin preferences. Chien, Cole and Lustig (2011, 2012) consider an incomplete market model featuring exogenous trading restrictions, which can easily be simulated. Gomes and Michaelides (2008) and Favilukis (2013) show that the equity premium can be reproduced in a model with preference heterogeneity (in intertemporal elasticity of substitution or in bequest motive) together with incomplete markets or limited participation.

The remainder of the paper is organized as follows. In Section 2, we present the model and derive our existence result. In Section 3, we present the intuition underlying our model in simplified versions of our framework. In Section 4, we perform a quantitative exercise to show that the model can reproduce household risk exposures and asset returns. Section 5 discusses the key assumptions of the model. Section 6 concludes.

2 The model

The model relies on three assumptions: (i) incomplete insurance markets, (ii) stock market participation costs and (iii) concave-linear utility function.

2.1 Risks and securities

Time is discrete and indexed by $t = 0, 1, \ldots$ The economy is populated by two types of infinitely-lived and ex-ante different agents (the ex-ante heterogeneity will be made clearer later on). Each population of type $i = 1, 2$ is of size 1 and is distributed on a segment J_i according to a measure ℓ_i.[3] We call these populations "type-1" and "type-2" agents.

2.1.1 Aggregate risk

There is a single aggregate shock $(z_t)_{t\geq 0}$, which can take n different values in the set $Z = \{z_1, \ldots, z_n\}$. We assume that the aggregate risk process $(z_t)_{t\geq 0}$ is a time-homogeneous first-order Markov chain whose transition matrix is denoted $\Pi = (\pi_{kj})_{k,j=1,\ldots,n}$. The probability π_{kj} of moving from state k to state j is thus constant. For every date $t \geq 0$, $z^t \in Z^{t+1}$ denotes a possible history of aggregate shocks up to date t, which is defined as a possible realization of the $(t+1)-$tuple $(z_0, \ldots, z_t)$.

[3] Among others, Feldman and Gilles (1985) have identified issues when applying the law of large number to a continuum of random variables. Green (1994) describes a construction of the sets J_i and of the non-atomic measures ℓ_i to ensure that our statements hold. Feldman and Gilles (1985), Judd (1985), and Uhlig (1996) also propose other solutions to this issue. From now on, we assume that the law of large numbers applies.

2.1.2 Asset markets

Agents can hold two types of assets –a risky stock and a riskless bond. We choose to confine our attention to the simplest market structure with limited participation.

The risky asset. Agents can trade shares of a Lucas tree with a constant mass V_X. The tree dividend is stochastic and the payoff in state k be y_k ($k = 1, \ldots, n$). At any date t, we denote by P_t the (endogenous) price of one "stock" or "risky asset" (i.e., a share of the tree).

The bond. Agents can also purchase riskless bonds of maturity one. Purchased at date t at price Q_t, these bonds pay off one unit of the consumption good at the next date in all states of the world. The total supply of bonds is constant and equal to V_B. These bonds –or safe assets– are issued by the government and funded by taxes on productive agents. In the absence of government consumption, taxes and the issuing of new bonds exactly cover the payoffs of maturing bonds. Moreover, the tax τ_t on productive agents of both types is assumed to be proportional to the productive agent's income.[4] Hence, a balanced government budget constraint at any date t implies that the tax rate is

$$\tau_t = \frac{(1 - Q_t)V_B}{\omega_t^1 \eta_t^1 + \omega_t^2 \eta_t^2}. \tag{1}$$

As we will see later on, participation in the bond market is free, while participation in the stock market may require the payment of a periodic participation cost.

[4] A time-varying bond supply would not change the conclusions of the paper. Moreover, a lump-sum tax would not quantitatively change the results, as will be clear after the presentation of the equilibrium structure.

2.1.3 Idiosyncratic risk

Agents face an idiosyncratic risk in addition to the aforementioned aggregate risk. This individual risk can neither be avoided nor insured. We call this a productivity risk even though it may cover many other individual risks (such as the risks of unemployment, income, health, etc.) that are likely to affect their productivity (see Chatterjee, Corbae, Nakajima and Rios-Rull, 2007 for a quantitative discussion). At any point in time, type-i agents can either be *productive* (denoted herein by p) earning income $\omega^i(z_t)$ or *unproductive* (denoted by u), earning income δ^i.[5] Both incomes may depend on the agent type i. To simplify the exposition, we assume that δ^i does not depend on z_t, but all our results can be easily extended to stochastic incomes δ^i. We assume that, regardless of the aggregate state, $\omega^i(z_t)$ is greater than δ^i for both agent types. Moreover, type-1 agents when productive have a higher income than type-2 agents. We refer to type-1 agents as "high-income" agents and to type-2 as "low-income" agents. These assumptions are summarized in Assumption B below.

For each type-i agent $j \in J_i$ at any date t, the function $\xi_t^{i,j}(z^t)$ characterizes the current status of the agent's productivity, taking the value 1 when the agent is productive and 0 when unproductive. Therefore, agent j earns a total income of $(1-\xi_t^{i,j}(z^t))\delta^i + \xi_t^{i,j}(z^t)\omega^i(z_t)$. We assume that for each agent, the productivity risk process $(\xi_t^{i,j}(z^t))_{t\geq 0}$ is a two-state Markov-chain with transition matrix $T_t^i = \begin{pmatrix} \alpha_t^i(z^t) & 1-\alpha_t^i(z^t) \\ 1-\rho_t^i(z^t) & \rho_t^i(z^t) \end{pmatrix}$. For example, when productive in period $t-1$, the probability of remaining productive in the next period t for

[5] Our idiosyncratic productivity risk is reminiscent of Kiyotaki and Moore (2005, 2008), Kocherlakota (2009) and Miao and Wang (2015), although the model and the paper scope are very different.

any type-i agent is denoted $\alpha_t^i(z^t) \in (0,1)$. Time-varying transition rates are introduced for the sake of generality.[6] For every date t, $\xi^{i,j,t} \in \{0,1\}^{t+1} = E^{t+1}$ denotes an history of individual shocks for agent $j \in J_i$ up to date t.

We call $\eta_t^i \in (0,1)$ the share of productive agents among type-i population. Initial values η_0^1 and η_0^2 being given, the laws of motion of productive shares are

$$\eta_t^i(z^t) = \alpha_t^i(z^t)\eta_{t-1}^i(z^{t-1}) + (1-\rho_t^i(z^t))(1-\eta_{t-1}^i(z^{t-1})), \text{ for } i=1,2 \text{ and } t \geq 1. \tag{2}$$

To obtain a tractable framework, we impose the following constraint:

Assumption A (Population shares) *The probability of remaining productive in the next period depends solely on the current aggregate state: $\alpha_t^i(z^t) = \alpha^i(z_{t-1})$.*

The shares of unproductive and productive agents depend only on the current state of the world z_t: $\eta_t^i(z^t) = \eta^i(z_t)$ for $t \geq 0$ and $i = 1,2$.

This assumption simplifies the dynamics of the population structure, but does not guarantee analytical tractability, since in general, it does not prevent the wealth distribution from being continuous. Assumption A includes the standard case where the transition probability α_t^i and the share η_t^i ($i = 1,2$) are constant and equal to α^i and η^i. The assumption also obviously includes the stochastic case. Furthermore, Assumption A implies that the primitives of our model are the probabilities α_t^i and the shares η_t^i, while the transition rates $(\rho_t^i)_{t\geq 0}$ adjust for the law of motion in equation (2) to hold. In the constant case, the probability ρ_t^i is constant, while in the stochastic case, it depends on the current and previous aggregate states: $\rho_t^i(z^t) = \rho_t^i(z_{t-1}, z_t)$.

[6]Krusell and Smith (1998) provide estimations of time-varying transition rates for employment risk.

2.2 Agents' preferences

Agents' preferences are a crucial feature of the model tractability, enabling us to derive our small-trade equilibrium with a finite number of states.

Description of preferences. The periodic utility function $\tilde{u}$ is continuous, strictly increasing and globally concave. It is strictly concave for low values of consumption and has possibly 2 linear parts. This assumption can formally be written through conditions imposed on marginal utility $\tilde{u}'$:

$$\tilde{u}'(c) = \begin{cases} u'(c) & \text{if } c \leq c_1^*, \\ \lambda^2 & \text{if } c_2^* \leq c \leq c_3^*, \\ \lambda^1 < \lambda^2 & \text{if } c_4^* \leq c \leq c_5^*. \end{cases} \tag{3}$$

When agents consume a low amount, they value their consumption with the marginal utility $u'(\cdot)$, which is the derivative of a function $u : \mathbb{R}^+ \to \mathbb{R}$ assumed to be twice derivable, strictly increasing, and strictly concave. When agents consume a higher amount, their marginal utility is constant for two consumption intervals, on which it is equal to λ^i for $i = 1, 2$.[7] Figure 1 plots the shape of such a periodic utility function.

We now formulate our next assumption.

Assumption B (Income processes) *We assume that in any state $k = 1, \ldots, n$, we have $c_2^* < \omega^2(z_k) < c_3^*$, $c_4^* < \omega^1(z_k) < c_5^*$ and $\delta^i < c_1^*$ for $i = 1, 2$. This notably implies that $\delta^i < \omega^i(z_k)$ for both types $i = 1, 2$ and $\omega^1(z_k) > \omega^2(z_k)$ ($\forall k = 1, \ldots, n$).*

Assumption B states that the income of productive agents lies in the set

[7] There is no further restriction on $\tilde{u}$, except that it has to be continuous, strictly increasing and globally concave.

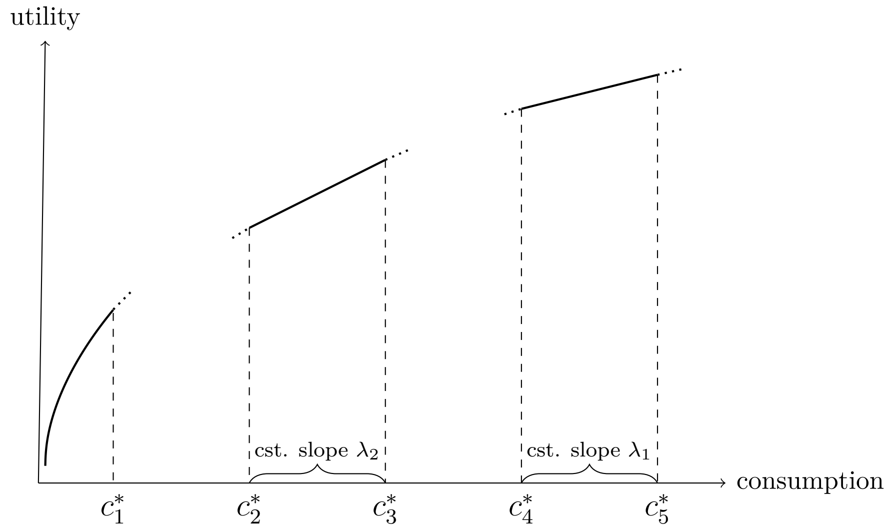

Figure 1: Shape of the periodic utility function

where the utility function is linear, and that the income of unproductive agents lies in the set where the utility is strictly concave. A straightforward corollary is that in absence of trade, unproductive agents are worse off than productive ones (for both types): $u'(\delta^i) > \lambda^i$ for $i = 1, 2$.

Consequence on the equilibrium. To push the interpretation further, we need to slightly anticipate the equilibrium construction below. In our equilibrium, unproductive agents of both types consume a low amount that they value at the strictly concave part of the utility function. Productive agents of type i consume a higher amount that they value with a linear utility of slope λ^i. This assumption of constant marginal utility for productive agents helps generating a limited heterogeneity equilibrium. Indeed, it implies that the individual history of productive agents does not matter for the pricing of securities since their marginal utility depends only on their type and not on their wealth or past saving choices.

Interpretation. The utility function introduced in equation (3) can be seen as a generalization of Fishburn (1977), who considers a concave-linear utility function. Fishburn's utility function is linear above a given threshold and strictly concave below it. In a portfolio choice problem, the agent endowed with such a utility, is risk-neutral for large payoffs and risk-averse for low ones. Loosely speaking, this functional form reflects the asymmetry in risk perception. Payoff realizations that are lower than a given threshold are perceived as actual risks, while payoffs greater than the threshold are perceived as being "nice surprises". The concave-linear utility function attributes therefore different statuses to under- and over-performances. As explained by Fishburn (1977, p. 123), this concave-linear functional form is "motivated by the observation that decision makers in investment contexts frequently associate risk with failure to attain a target return". In this paper, we introduce this utility function in an incomplete-market framework, especially to gain tractability. Furthermore, we generalize this assumption and assume that there are two thresholds and two linear parts. Note that it would be equivalent to assume that each agent type is endowed with a proper Fishburn concave-linear utility, with a common concave part but different linear parts. We discuss this assumption further in Section 5. Importantly, this assumption does not imply that agents are risk-neutral. As is explicit in Section 2.6, the concave part directly affects asset pricing and the behavior of productive agents.

2.3 Agent's program

Timing. At the beginning of every period, the agent observes her current productivity status and the dividend payoff. The agent therefore knows the history of both aggregate and idiosyncratic shocks up to that date.

Allocations. Due to the timing of the agent's program, agents' choices –

consumption levels and demands for stocks and bonds– at date t are mappings defined over the state space of possible shock histories $Z^t \times E^t$. We call $(c_t^{i,j} : Z^t \times E^t \rightarrow \mathbb{R}^+)_{t \geq 0}$ the consumption plan for a type-i agent $j \in J_i$, whose consumption levels are assumed to be positive. The stock demand plan is $(x_t^{i,j} : Z^t \times E^t \rightarrow \mathbb{R})_{t \geq 0}$ while $(b_t^{i,j} : Z^t \times E^t \rightarrow \mathbb{R})_{t \geq 0}$ is the demand plan for bonds.

Participation costs. Participation in the stock market is costly for agents, even though trading riskless bonds is free. Trading stocks requires type-i agents to pay in every period a lump-sum participation cost χ_i, $i = 1, 2$. This periodic participation cost will generate endogenous stock market limited participation. The participation cost is paid in every period when the agent purchases stocks but there is no cost for liquidating a stock portfolio.[8] Participation costs are consistent with many empirical studies, such as Mankiw and Zeldes (1991) or more recently Vissing-Jorgensen (2002). Participation costs are a frequent device to understand limited participation in the stock market and the equity premium.[9]

We state an assumption regarding stock market participation costs so as to pin down asset market structure. Consistently with empirical facts presented in Section 4, we set participation costs, such that type-1 agents trade stocks, while type-2 agents do not. More precisely:

Assumption C (Participation costs) *We assume that $\chi_2 \geq \overline{\chi}_2$ is large enough for type-2 agents not to trade stocks, while type-1 agents do not pay*

[8]Removing the participation cost would not impair equilibrium existence. However, it would change market participation structure and limit the ability of the model to jointly reproduce asset prices and consumption inequalities, as shown in the quantitative exercise of Section 4.

[9]The impact of participation costs on asset prices has for instance been studied in Basak and Cuoco (1998), Heaton and Lucas (1999), Polkovnichenko (2004), Gomes and Michaelides (2008), Guvenen (2009), Walentin (2010) or Favilukis (2013).

participation costs: $\chi_1 = 0$.

Equation (16) in Section 2.6 provides an explicit formula for $\overline{\chi}_2$, based on the observation that type-2 agents will never trade stocks if the stock return, net of participation cost, is lower than the bond return, in all states of the world. We could have introduced a non-zero participation cost for type-1 agents to discuss more general market structures.[10] However, for simplicity's sake, we focus on the minimal setup that allows our model to be consistent with empirical facts.

Budget and borrowing constraints. At each date t, the choices of a type-i agent $j \in J_i$ are limited by a budget constraint in which total resources made up of income, stock dividends, and security-sale values are used to consume, pay taxes, and purchase securities, as follows:[11]

$$c_t^{i,j} + P_t\, x_t^{i,j} + Q_t b_t^{i,j} + \chi_i 1_{x_t^{i,j}>0} = (1-\xi_t^{i,j})\delta^i + \xi_t^{i,j}(\omega_t^i - \tau_t^i) + (P_t + y_t)x_{t-1}^{i,j} + b_{t-1}^{i,j}, \quad (4)$$

where $1_{x_t^{i,j}>0}$ is the indicator function.

In addition, agents face borrowing constraints. They can neither produce any share of stocks nor short-sell the bond. This implies, for any agent $j \in J_i$ at date $t \geq 0$, that:[12]

$$x_t^{i,j}, b_t^{i,j} \geq 0. \quad (5)$$

A *feasible allocation* is a collection of plans $(c_t^{i,j}, x_t^{i,j}, b_t^{i,j})_{t\geq 0}^{j\in J_i, i=1,2}$ such that equations (4) and (5) hold at any date t. We call $\mathcal{A}_i$ the set of feasible alloca-

[10] The finite-state equilibrium indeed partly simplifies the analysis of endogenous limited market participation.

[11] We drop the dependence on z^t and $\xi^{i,j,t}$ to lighten notations.

[12] It would be possible to have strictly negative (but not too loose) borrowing constraints on bonds and stocks, while preserving the equilibrium existence. However, the set $\mathcal{V}$ of admissible security volumes defined below in Proposition 1 would be different.

tions for a type-i agent.

Agent's program. The program of a type-i agent $j \in J_i$ consists in finding the feasible allocation that maximizes her intertemporal utility subject to a transversality condition ruling out exploding paths. Instantaneous utilities are discounted by a time preference parameter $\beta \in (0,1)$. The operator $E_0[\cdot]$ is the unconditional expectation over aggregate and idiosyncratic shocks. The initial financial asset endowments are denoted by $x_{-1}^{i,j}$ and $b_{-1}^{i,j}$. The agent's program can be expressed as ($j \in J_i$):

$$\max_{(c_t^{i,j}, x_t^{i,j}, b_t^{i,j})_{t\geq 0} \in \mathcal{A}_i} E_0 \left[\sum_{t=0}^{\infty} \beta^t \tilde{u}(c_t^{i,j}) \right] \tag{6}$$
$$\text{s.t. } \lim_{t\to\infty} \beta^t E_0 \left[\tilde{u}'(c_t^{i,j}) x_t^{i,j} \right] = \lim_{t\to\infty} \beta^t E_0 \left[\tilde{u}'(c_t^{i,j}) b_t^{i,j} \right] = 0,$$
$$\{x_{-1}^{i,j}, b_{-1}^{i,j}, \xi_0^{i,j}, z_0\} \text{ are given.}$$

Agents' risk-sharing is limited along three dimensions. First, as in the Bewley-Huggett-Aiyagari literature, individual risk is uninsurable because no asset is contingent on the productivity status. Second, agents face participation and borrowing constraints. Finally, the set of securities may not complete the insurance market.

2.4 Equilibrium definition

We start with security market clearing conditions, stating that aggregate demand should be equal to total supply, which amounts to V_X for stocks and V_B for bonds. We define the probability measure $\Lambda_t^i : \mathcal{B}(\mathbb{R})^2 \times \mathcal{B}(E^t) \to [0,1]$ describing the distribution of type-i agents as a function of their security holdings and the history of their individual status.[13] As an example, $\Lambda_t^i(X, B^S, I)$

[13]For any metric space X, $\mathcal{B}(X)$ denotes the borel sets of X.

(with $(X, B^S, I) \in \mathcal{B}(\mathbb{R})^2 \times \mathcal{B}(E^t)$) is the measure of agents of type i, with holdings in risky assets $x \in X$, in bonds $b \in B^S$, and with an individual history $\xi \in I$: $\Lambda_t^i(X, B^S, I) = \ell_i(\{j \in J_i : (x_t^{i,j}, b_t^{i,j}, \xi_t^{i,j}) \in (X, B^S, I)\})$. The market-clearing conditions can therefore be written as

$$\sum_{i=1,2} \int_{\mathbb{R}^2 \times E^t} x \Lambda_t^i(dx, db, d\xi) = V_X, \tag{7}$$

$$\sum_{i=1,2} \int_{\mathbb{R}^2 \times E^t} b \Lambda_t^i(dx, db, d\xi) = V_B. \tag{8}$$

Finally, by Walras' law, the good market clears when asset markets clear.

We can now define a sequential competitive equilibrium.

Definition 1 (Sequential competitive equilibrium) *A sequential competitive equilibrium is a collection of allocations* $(c_t^{i,j}, x_t^{i,j}, b_t^{i,j})_{t \geq 0}^{j \in J_i}$ *for* $i = 1, 2$ *and of price processes* $(P_t, Q_t)_{t \geq 0}$ *such that, for an initial distribution of stock and bond holdings and of idiosyncratic and aggregate shocks* $\{(x_{-1}^{i,j}, b_{-1}^{i,j}, \xi_0^{i,j})_{i=1,2}^{j \in J_i}, z_0\}$, *we have:*

1. *given prices, individual strategies solve the agents' optimization program in equation (6);*

2. *the security markets clear at all dates: for any* $t \geq 0$, *equations (7) and (8) hold;*

3. *the probability measures* Λ_t^i *evolve consistently with individual strategies in each period.*

2.5 Equilibrium existence

In standard economies featuring uninsurable idiosyncratic shocks, credit constraints and aggregate shocks, the equilibrium cannot be explicitly derived since it involves a continuous distribution of agents (typically of agents' wealth) with different individual histories. The usual strategy follows Krusell and Smith (1998) by computing approximate equilibria assuming a recursive structure. But, as pointed out by Heathcote, Stroresletten, and Violante (2009), the existence of such an equilibrium is still an open question.

In this paper, we prove the existence of an equilibrium and derive its theoretical properties under the assumption that the supply of both risky and riskless assets is not too large. In this case, unproductive agents (i.e., low-income agents) of both types remain credit-constrained even after selling off their entire portfolio. They will not participate in the financial markets while productive agents are trading securities.

More precisely, we construct an equilibrium where the portfolio chosen by each agent depends only on its type, its current productive status, and the aggregate state. In other words, at each date, all type-1 productive agents have the same (time-varying) portfolio and all type-2 productive agents have the same portfolio. In this economy, there are thus only four different portfolios at each point in time. This stems from the fact that productive type-1 agents will have the marginal utility λ^1 and productive type-2 agents the marginal utility λ^2. Assumption B guarantees that this is the case when securities are in zero supply. Proposition 1 below extends it to positive supplies and proves the existence of a small-trade equilibrium.

Proposition 1 (Equilibrium existence) *We assume that:*

$$\forall k \in \{1, \ldots, n\}, \ \beta \left(\alpha^1(z_k) + (1 - \alpha^1(z_k)) \frac{u'(\delta^1)}{\lambda^1} \right) < 1. \tag{9}$$

If security volumes (V_B, V_X) *belong to a set* $\mathcal{V} \subset \mathbb{R}_+ \times \mathbb{R}_+$ *defined in (**??**) –and containing* $(0, 0)$*–, then there exists an equilibrium with the following features:*

1. *the end-of-period security holdings of unproductive type-1 and type-2 agents is* 0 *for both the risky and the riskless assets;*

2. *the end-of-period security holdings of productive agents depend only on their type (*1 *or* 2*) and the current aggregate state;*

3. *the end-of-period holdings in stocks of type-2 agents is always* 0*;*

4. *the security prices depend only on the current aggregate state.*

The proof can be found in the Appendix. In the heterogeneous agent literature, several existence results can already be found. In Huggett (1993), agents trade short-lived riskless bonds in absence of aggregate shocks. Kuhn (2013) extends Huggett's result to permanent idiosyncratic shocks.[14] To our knowledge, Miao (2006) proves the sole existence result in an economy featuring asset trades, credit constraints, and idiosyncratic and aggregate risks with a continuum of agents. He considers an economy with general preferences in which agents can trade one short-lived asset, which are claims on capital. Our existence result concerns a setup with endogenous limited participation and both a short- and long-lived asset.[15]

[14]In a seminal paper, Duffie et al. (1994) consider endowment economies in which a finite number of ex-ante heterogeneous agents face aggregate risks and trade long-lived assets with borrowing constraints. They then prove the existence of ergodic equilibria with a recursive characterization, whose state space includes all endogenous variables (such as prices). In a similar vein, Becker and Zilcha (1997) prove the existence of a stationary equilibrium in a production economy with ex-ante heterogeneous agents facing aggregate risk. Krebs (2006) proves the existence of a no-trade equilibrium in a Krusell-Smith economy. Kubler and Schmedders (2002) prove existence of recursive equilibrium with a finite number of agents. These papers consider a finite number of households. This assumption helps in proving existence but makes the analysis of the properties of the equilibrium more difficult as all shocks are "aggregate". It may explain the wide use of Bewley-type model with a continuum of agents.

[15]In our setup it would also be possible to prove that the sequential competitive equilibrium is also a recursive competitive equilibrium in which the state variables are: current aggregate and idiosyncratic shocks and beginning-of-period security holdings for both agent types.

To prove existence, we start from first-order conditions, as in Coleman (1991), by following the steps of the proof of Theorem 4.15 in Stockey and Lucas (1989). Indeed, the Kuhn-Tucker theorem requires a Hermitian space of allocations, which is not the case for the set of bounded real sequences (which (c_t^i), (e_t^i), (x_t^i), and (b_t^i) belong to).

The equilibrium exists under two conditions. The first one, $\beta(\alpha^1(z_k)+(1-\alpha^1(z_k))\frac{u'(\delta^1)}{\lambda^1}) < 1$, ensures that stock prices are well-defined. If the condition does not hold, the stock price can possibly be infinite because agents are too patient or their desire to self-insure is too high.[16]

The second condition is that security volumes belong to a set $\mathcal{V}$, including the zero volume case $V_X = V_B = 0$. This assumption implies that security volumes should not be too high, which guarantees that agents, when becoming unproductive, are credit-constrained. A second consequence of this condition on $\mathcal{V}$ is that unproductive agents still value consumption with a strictly concave utility and productive agents with an affine one. It generalizes Assumption B to positive security volumes.

The equilibrium we consider presents four particular features. First, all unproductive agents are credit constrained. The intuition is as follows. Productive agents want to buy securities –both bonds and stocks– so as to hedge the risk of becoming unproductive in the next period. If the quantity of available securities is not too large, productive agents agree to pay a high price for these securities. But at that price, unproductive agents would like to short-sell them (but are prevented from doing so by credit constraints), because their

[16]Technically, this condition ensures that the mapping $P \mapsto \beta E_k\left[(\alpha_k^1 + (1-\alpha_k^1)u'(\delta_{k'}^1))(P_{k'} + y_{k'})\right]$, derived from the Euler equation, is a contraction with modulus strictly smaller than 1, where $E_k\left[X_{k'}\right] = \sum_{k'=1}^n \pi_{k,k'} X_{k'}$. The Banach fixed-point theorem then allows us to deduce the properties of the price. This condition does not appear in economies with only short-lived assets, which are simpler in this respect.

current income is low and they expect to be wealthier in the future. The second feature of our equilibrium is that the saving choices of productive agents only depend on the current aggregate state and on the agent's type. This property critically relies on the quasi-linearity of the utility function, which implies that all productive agents of each type have the same marginal utility and therefore the same demand for assets. The third aspect of our equilibrium is that only type-1 agents trade stocks, while type-2 do not. As already discussed, this result stems from stock market participation costs and in particular from Assumption C. The quasi-linearity of utility function also explains the fourth feature of our equilibrium, according to which security prices depend only on the current aggregate state.

We simplify our notations using the results of Proposition 1. Since security prices depend only on the current aggregate state, we call P_k the price of the risky asset and Q_k the price of the bond in state z_k ($k = 1, \ldots, n$). Bond holdings only depend on the aggregate states and unproductive agents do not hold any assets. We therefore call b_k^i the holdings in bonds of any productive type-i agent in state z_k ($k = 1, \ldots, n$). Since type-2 agents do not trade stocks ($x^2 = 0$), productive agents hold all stocks and $x_k^1 = \frac{V_X}{\eta_k^1}$. The equilibrium is therefore characterized by a finite sequence of $4 \times n$ variables $(b_k^1, b_k^2, P_k, Q_k)_{k=1,\ldots,n}$ instead of continuous distributions, as it is standard in incomplete market models.

2.6 Equilibrium structure

Due to the finite characterization of our equilibrium, we have a deeper understanding of the structure of the model. In the risky asset market, productive type-1 agents will always be the sole participants. In the bond market, both productive type-1 and productive type-2 agents can participate, even though an agent type may choose not to hold bonds in some states of the world. The next proposition summarizes this market structure.

Proposition 2 (Equilibrium properties) *There exist two distinct subsets* $I_i \subset \{1, \ldots, n\}$ $(i = 1, 2)$*, characterizing the states of the world in which only type-*i *agents trade bond, such that the* $4 \times n$ *variables* $(b_k^1, b_k^2, P_k, Q_k)_{k=1,\ldots,n}$ *defining the equilibrium are given by the following* $4 \times n$ *equations:*

$$P_k = \beta \sum_{j=1}^{n} \pi_{kj}(\alpha_k^1 + (1 - \alpha_k^1)\frac{1}{\lambda^1}u'(\delta^1 + (P_j + y_j)\,\frac{V_X}{\eta_k^1} + b_k^1))(P_j + y_j),\ k \in \{1, \ldots, n\}, \tag{10}$$

$$Q_k = \beta \sum_{j=1}^{n} \pi_{kj}(\alpha_k^1 + (1 - \alpha_k^1)\frac{1}{\lambda^1}u'(\delta^1 + (P_j + y_j)\,\frac{V_X}{\eta_k^1} + b_k^1)),\ k \in \{1, \ldots, n\} - I_2, \tag{11}$$

$$Q_k = \beta \sum_{j=1}^{n} \pi_{kj}(\alpha_k^2 + (1 - \alpha_k^2)\frac{1}{\lambda^2}u'(\delta^2 + b_k^2)),\ k \in \{1, \ldots, n\} - I_1, \tag{12}$$

$$V_B = \eta_k^1 b_k^1 \text{ and } 0 = b_k^2,\ k \in I_1, \tag{13}$$

$$V_B = \eta_k^2 b_k^2 \text{ and } 0 = b_k^1,\ k \in I_2, \tag{14}$$

$$V_B = \eta_k^1 b_k^1 + \eta_k^2 b_k^2,\ k \in \{1, \ldots, n\} - I_1 - I_2. \tag{15}$$

Our equilibrium is characterized by equalities (10)–(15). The first three sets of Euler equations provide security prices. Due to stock market limited participation, the risky asset price is only defined by the Euler equation (10) of productive type-1 agents, who hold all the stocks. Bonds may be traded by productive type-1 or type-2 agents possibly depending on the state of the world. Since our equilibrium features security prices that only depend on the current state of the world, there is one subset of states of the world, characterized by the index subset I_1, in which only type-1 agents bonds, while type-2 agents are excluded. In states of the world I_1: (i) the Euler equation (12) of type-2 agents does not hold, and (ii) the bond supply equals the demand of type-1 agents in equation (13). By the same token, there are states of the world, characterized by the subset index I_2, in which only type-2 agents hold bonds, while type-1 agents only hold stocks. This corresponds to the Euler equation (11) and the resource equality (14). Subsets I_1 and I_2 are possibly empty. For instance, if both are empty, it means that type-1 and type-2 agents always trade bonds.

We provide in Appendix the inequality conditions determining the financial market participation of both types of productive agents –see equations (**??**) and (**??**). Equations (**??**)–(**??**) in Appendix too, are the conditions ensuring that unproductive agents do not trade any assets. The latter equations obviously matter for the equilibrium existence and in fact implicitly define the set $\mathcal{V}$ of admissible security supplies V_X and V_B.

This system of equations can easily be simulated and estimated, as we do in Section 4. More importantly, this equilibrium enables us to derive theoretical insights about the household risk exposure and the equity premium throughout the business cycle.

Participation costs. We now derive explicitly the condition on the participation cost $\overline{\chi}_2$ of Assumption C for type-2 agents to never participate in the stock market. This cost is such that participating in the stock market is a dominated strategy, no matter the state of the world.

If type-2 agents participate in stock markets, their portfolio choice is denoted $\{\tilde{x}_k^2, \tilde{b}_k^2\}_{k=1,\ldots,n}$ given equilibrium prices $(P_k, Q_k)_{k=1,\ldots,n}$. Purchasing the quantity of stock $\tilde{x}_k^2$ is a dominated strategy in any state of the world k, if investing the same amount in bonds offers in every state a greater payoff. Due to participation cost, purchasing $\tilde{x}_k^2$ costs $P_k\tilde{x}_k^2 + \chi_2$ and pays off $\tilde{x}_k^2(P_j + y_j)$ in the next period when the state of the world is $j = 1, \dots, n$. Investing the same amount $P_k\tilde{x}_k^2 + \overline{\chi}_2$ in bonds pays off $\frac{P_k\tilde{x}_k^2+\overline{\chi}_2}{Q_k}$ units of consumption in all states of the next period. In consequence, if $\frac{P_k\tilde{x}_k^2+\chi_2}{Q_k} > \tilde{x}_k^2(P_j + y_j)$ for any k, j, type-2 agents never wish to trade stocks. We deduce the following expression for $\overline{\chi}_2$ that ensures Assumption C to hold:

$$\overline{\chi}_2 = \max_{k,j=1,\dots,n}(Q_k(P_j + y_j) - P_k)\tilde{x}_k^2. \tag{16}$$

Following the same steps as in Proposition 2, we obtain for the portfolio $\{\tilde{x}_k^2, \tilde{b}_k^2\}_{k=1,\dots,n}$:[17]

$$P_k \geq \beta \sum_{j=1}^{n} \pi_{kj}(\alpha_k^2 + (1-\alpha_k^2)\frac{1}{\lambda^2}u'(\delta^2 + (P_j + y_j)\tilde{x}_k^2 + \tilde{b}_k^2))(P_j + y_j), \ k \in \{1, \dots, n\}, \tag{17}$$

$$Q_k \geq \beta \sum_{j=1}^{n} \pi_{kj}(\alpha_k^2 + (1-\alpha_k^2)\frac{1}{\lambda^2}u'(\delta^2 + (P_j + y_j)\tilde{x}_k^2 + \tilde{b}_k^2)), \ k \in \{1, \dots, n\} - I_2, \tag{18}$$

where (17) and (18) hold with equality if $\tilde{x}_k^2 > 0$ and $\tilde{b}_k^2 > 0$ respectively.

3 Intuitions through simpler setups

Our model features heterogeneous uninsurable individual risk, aggregate risk and positive security volumes. We now examine, in turn, the role of the different model features in explaining asset returns, heterogeneity in consumption levels, and consumption growth.

[17] We have used the fact that credit constraints bind for unproductive type-2 agents after this deviation, which is true in the equilibrium under consideration.

Throughout this section –and only in this section–, we further simplify our setup so as to make mechanisms as transparent as possible. In particular, we make the following two assumptions:

1. aggregate risk follows an IID process;

2. productivity transition probabilities α^i and ρ^i are constant.

No idiosyncratic risk. As a first benchmark, we study the case where agents do not face idiosyncratic risk (i.e., $\alpha^i = 1$ for $i = 1, 2$). Due to limited participation, only type-1 agents trade the risky asset, whose constant price P^{NIR} (NIR stands for "No Idiosyncratic Risk") verifies $P^{NIR} = \beta E^{\tilde{y}}[P^{NIR} + \tilde{y}]$, where $E^{\tilde{y}}[\cdot]$ is the expectation over the next period uncertain dividend $\tilde{y}$. The gross average stock return R_s^{NIR} is therefore constant and equal to β^{-1}. The riskless bond is traded by both agents and its price is $Q^{NIR} = \beta$. The riskless gross interest rate R_f^{NIR} is identical to the stock return. The equity premium in this environment is null: $R_s^{NIR} - R_f^{NIR} = 0$. Limited participation alone does not imply a non-zero risk premium.

Zero volumes. We now assume that agents face heterogeneous but constant transition probabilities across idiosyncratic states. Aggregate risk effects dividends while the income of productive (ω^i) and unproductive (δ^i) agents are constant. Moreover, both riskless and risky securities are in zero supply. In what follows, ZV stands for "Zero Volume."

Proposition 3 (Zero volumes) *In this economy, the equilibrium features a complete asset market segmentation where productive type-1 agents trade the* *stock and productive type-2 agents trade the bond if*

$$(1-\alpha^2)(\frac{u'(\delta^2)}{\lambda^2}-1) > (1-\alpha^1)(\frac{u'(\delta^1)}{\lambda^1}-1). \tag{19}$$

The equity premium can then be expressed as:

$$R_s^{ZV} - R_f^{ZV} = \frac{(1-\alpha^2)(\frac{u'(\delta^2)}{\lambda^2}-1) - (1-\alpha^1)(\frac{u'(\delta^1)}{\lambda^1}-1)}{(1-\alpha^1)(1-\alpha^2)(\frac{u'(\delta^1)}{\lambda^1}-1)(\frac{u'(\delta^2)}{\lambda^2}-1)}, \tag{20}$$

while average consumption of a type-i agent, denoted $\bar{c}_i^{ZV}$, is:

$$\bar{c}_i^{ZV} = \frac{1-\rho^i}{2-\alpha^i-\rho^i}\omega^i + \frac{1-\alpha^i}{2-\alpha^i-\rho^i}\delta^i, \ \textit{for}\ i=1,2. \tag{21}$$

If $\tilde{\gamma}_c^{i,ZV}$ is consumption growth of a type-i agent, its variance can be expressed as:

$$V[\tilde{\gamma}_c^{i,ZV}] = \frac{(1-\rho^i)(1-\alpha^i)}{4(2-\alpha^i-\rho^i)}\left(\frac{\alpha^i(1-\rho^i)+\rho^i(1-\alpha^i)}{2-\alpha^i-\rho^i}\left(\frac{\omega^i}{\delta^i}+\frac{\delta^i}{\omega^i}-2\right)^2 + \left(\frac{\omega^i}{\delta^i}-\frac{\delta^i}{\omega^i}\right)^2\right). \tag{22}$$

In Proposition 3, the risk premium in equation (20) is strictly positive because of market segmentation, even in the absence of correlation between dividend payouts and marginal utility. In our setup, the risk premium stems from the heterogeneity in idiosyncratic risk. Indeed, for a type-i agent, the expression $(1-\alpha^i)(\frac{u'(\delta^i)}{\lambda^i}-1)$ can be interpreted as the expected magnitude of idiosyncratic (or productivity) risk since (i) $(1-\alpha^i)$ is the probability that a type-i agent faces a bad outcome due to the productivity risk, and (ii) $\frac{u'(\delta^i)}{\lambda^i}-1$ is the relative fall in marginal utility experienced by a type-i agent due to the productivity risk. This quantity drives the demand for self-insurance against the uninsurable individual risk. On the one hand, the stronger the self-insurance need of type-2 agents, the more they demand riskless bonds to hedge against the risk, causing the return on riskless bonds to decrease. On the other hand, the lower the self-insurance need of type-1 agents, the less they demand stocks and the greater the risky return they require to purchase stocks. As a result, heterogeneous demands for self-insurance –in combination with limited stock market participation– are sufficient to generate a strictly positive risk premium, even though both asset payouts are not correlated with marginal utilities.

Finally, expression (21) is very simple and states that consumption only depends on incomes since securities are in zero supply. The volatility of consumption growth in equation (22) also depends solely on the volatility of wage growth. This volatility increases when the difference between wages in both possible individual states rises.

Positive volumes. We now relax the assumption of zero volumes. For the sake of simplicity, we assume that both bond and stock volumes are small, so as to allow us to derive closed-form expressions –as first-order expressions– for the equity premium, consumption levels and growth rates. In addition, and to simplify expressions, we assume that incomes are not time-varying: δ^i and ω^i are constant for $i = 1, 2$. Only stock dividends are time-varying.

Proposition 4 (Small positive volumes) *If the condition (19) of Proposition 3 holds, the economy exhibits the following features:*

- *the equity premium can be expressed as:*

$$R_s^{PV} - R_f^{PV} \approx R_s^{ZV} - R_f^{ZV} + \beta(1-\alpha^1)\frac{-\frac{u''(\delta^1)}{\lambda^1}}{\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1}} \tag{23}$$

$$\times \left(\frac{E\left[P^{ZV} + y(\tilde{z})\right]}{P^{ZV}} (E_t\left[P^{ZV} + y(\tilde{z})\right]\frac{V}{\eta^1} + b^1) + \frac{V\left[P^{ZV} + y(\tilde{z})\right]}{P^{ZV}}\frac{V}{\eta^1} \right),$$

$$\textit{with: } P^{ZV} = \frac{\beta(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1})}{1 - \beta(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1})} E[y(\tilde{z})]. \tag{24}$$

- *the bond holdings of productive agents are such that*
  - *either* $b^1 = 0$ *and* $\eta^2 b^2 = V_B$ *in case of (endogenous) complete market separation;*
  - *or* $b_1 \geq 0$ *and* $b_2 \geq 0$ *verify the two following equations:*

$$\eta^2 \mu b^2 \approx (1-\alpha^2)(\frac{u'(\delta^2)}{\lambda^2} - 1) - (1-\alpha^1)(\frac{u'(\delta^1)}{\lambda^1} - 1) \tag{25}$$

$$- (1-\alpha^1)\frac{u''(\delta^1)}{\lambda^1\eta^1}(E^{\tilde{z}}[P^{ZV} + y(\tilde{z})]V_X + V_B),$$

$$\eta^1 \mu b^1 \approx (1-\alpha^1)(\frac{u'(\delta^1)}{\lambda^1} - 1) - (1-\alpha^2)(\frac{u'(\delta^2)}{\lambda^2} - 1) \tag{26}$$

$$+ (1-\alpha^1)\frac{u''(\delta^1)}{\lambda^1\eta^1}E^{\tilde{z}}[P^{ZV} + y(\tilde{z})]V_X - (1-\alpha^2)\frac{u''(\delta^2)}{\eta^2\lambda^2}V_B,$$

$$\textit{with: } \mu = -(1-\alpha^1)\frac{u''(\delta^1)}{\eta^1\lambda^1} - (1-\alpha^2)\frac{u''(\delta^2)}{\eta^2\lambda^2} > 0. \tag{27}$$

- *The average consumptions:*

$$\bar{c}_1^{PV} \approx \bar{c}_1^{ZV} + E[y(\tilde{z})]V_X + \eta^1(1 - Q^{ZV})b^1 - \eta^1\omega^1\tau, \tag{28}$$

$$\bar{c}_2^{PV} \approx \bar{c}_2^{ZV} + \eta^2(1 - Q^{ZV})b^2 - \eta^2\omega^2\tau, \tag{29}$$

$$\textit{with:} \quad Q^{ZV} = \beta(\alpha^2 + (1 - \alpha^2)\frac{u'(\delta^2)}{\lambda^2}), \tag{30}$$

$$\tau = \frac{(1 - Q^{ZV})V_B}{\omega^1\eta^1 + \omega^2\eta^2}. \tag{31}$$

- *The volatility consumption growth rates $V[\tilde{\gamma}_c^{i,PV}]$:*

$$V[\tilde{\gamma}_c^{1,PV}] \approx V[\tilde{\gamma}_c^{1,ZV}] - \nu_{10}\tau - \nu_{11}b^1 - \nu_{12}\frac{V}{\eta^1} + \nu_{13}(\frac{V}{\eta^1})^2 V[y(\tilde{z})], \tag{32}$$

$$V[\tilde{\gamma}_c^{2,PV}] \approx V[\tilde{\gamma}_c^{2,ZV}] - \nu_{20}\tau - \nu_{21}b^2, \tag{33}$$

*where ν_{1i} $(i = 0, \ldots, 3)$ and ν_{2i} $(i = 0, 1)$ are positive terms whose expressions can be found in (**??**)–(**??**).*

This proposition illustrates the role of positive asset volumes along three dimensions: the equity premium, average consumption levels, and consumption growth rates. Equation (23) describes the role of positive volumes on the equity premium. The quantity P^{ZV} in equation (24) is the stock price in the zero-volume economy. Positive volumes increase the equity premium through two channels. First, positive volumes increase the ability to self-insure for productive type-1 agents who can trade both risky and riskless securities. Type-1 agents therefore require a greater return to hold the risky asset, increasing the equity premium. Second, once they become unproductive, type-1 agents now sell a positive volume of risky assets whose liquidation value is uncertain. Type-1 agents want to be compensated for the uncertainty related to the liquidation value. This corresponds to the variance term in equation (23). Since the riskless bond pays off a certain outcome in every state of the world, there is not any liquidation premium related to short-term riskless bonds. Challe, LeGrand, and Ragot (2013) exhibit a similar mechanism for long-term bonds and the term structure of interest rates.

Equations (25) and (26) determine the bond demands b^1 and b^2, provided that there is no full market segmentation. In the event of full market segmentation, all bonds are held by productive type-2 agents –under condition (19), type-1 agents cannot hold all bonds. We deduce that the bond demand b^2 of type-2 agents is mainly driven by two factors. The first determinant is the heterogeneity in the magnitude of expected productivity risk between both agents. The greater this risk for type-2 agents –with respect to type-1 agents–, the more type-2 agents need to self-insure themselves and the more they demand bonds. The second determinant is the total quantity of securities available, of both bonds and stocks. The greater the security supply, the smaller the bond price and the more type-2 agents can purchase bonds. For type-1 agents, the intuition is similar except for the role of stock volumes. Indeed, type-1 agents can purchase either stocks or bonds, which are therefore partly substitutes. An increase in stock volumes makes stock cheaper and therefore crowds out bonds in favor of stocks for type-1 agents. These agents purchase more stocks, but need fewer bonds to achieve the same degree of self-insurance.

Equations (28) and (29) explain how positive volumes effect the average consumption of both agent types. The quantity Q^{ZV} in (30) is the price of the riskless bond in the zero-volume economy, while τ is the (first-order expansion of the) tax rate that productive agents of both types have to pay on their income ω^i. Average consumption is augmented by security payoffs, net of tax effects (due to the funding of bond issuance). The heterogeneity in average consumption levels therefore reflects (i) the heterogeneity in average incomes (i.e., the zero net supply case), and (ii) the heterogeneity in net returns of financial wealth. In consequence, the effect of an increase in risky asset volumes on consumption inequality is twofold. First, it raises the equity premium and reinforces the fact that risky stocks pay off more than riskless bonds. Second, it increases stockholder consumption while leaving that of non-stockholders unchanged. The impact of a larger public debt will imply higher taxes (see equation (31)) for both agent types and will therefore mainly benefit to agents with a greater need of bonds for self-insurance.

Finally, equations (32) and (33) illustrate the role of positive security volumes on the volatility of consumption growth, that comes in addition to the zero volume volatility due to heterogeneity in productive and unproductive wages. A first effect (related to coefficient ν_{i0}=, $i = 1, 2$) corresponds to taxation. A greater bond volume increases tax burden and diminishes the variance of consumption growth rate. This effect is related to our taxation scheme: productive agents are taxed, while unproductive are not. A second effect (related to coefficients ν_{11} and ν_{12} for type-1 agents and ν_{21} for type-2 agents) also contributes to a reduction in the consumption growth volatility. Indeed, positive security volumes allow agents to self-insure themselves better against productivity risk. In consequence, agents are more able to smooth out their consumptions across the different idiosyncratic states, which diminishes the volatility of consumption growth. A third effect (related to coefficient ν_{13} for type-1 agents) of positive security volumes raises consumption growth volatility for stock-holders. Indeed, the consumption growth is affected by the uncertain payoff of future dividends. Therefore, the consumption growth of type-1 agents, who hold stocks, can potentially increase because of positive stock volumes.

4 Quantitative exercise

We now assess the ability of our model to match both asset prices and the allocation of risk across households. In the description of Section 2, Assumption C implies that only type-1 agents hold stocks, while type-2 do not, while the population of both types is identical. This is consistent with the empirical observation that only 50% of US households hold stocks either directly or indirectly and that stock-holders are mostly in the top 50% of the income distribution (Bricker et al., 2014). We identify type-1 participating households to the top 50% of US households in the income distribution (henceforth the *top 50%*) and type-2 nonparticipating agents to the bottom 50% (henceforth the *bottom 50%*).

The model parameters can be divided into two groups. First, we calibrate some parameters to standard values (and to ensure that our equilibrium exists). Second, we use the tractability of our framework to estimate the idiosyncratic risk parameters, by simulating the model to match several targets (described below). This empirical exercise enables us to assess to what extent our small-trade equilibrium reproduces observed risk allocations and asset prices.

4.1 Calibrated parameters and restrictions

4.1.1 Aggregate risk

The period is a quarter. The aggregate state can be either G (for good) or B (for bad). For transition probabilities, we rely on the estimation of Hamilton (1994) and choose the values $\pi_{GG} = 0.75$ and $\pi_{BB} = 0.5$. The good state is thus more persistent than the bad one.

The stock volume $V_X = 0.002$ is assumed to be small. As the model is able to reproduce the volatility of the consumption growth rate for type-1 agents (who are the only agents holding stocks), this small value is key for the results. The volume of bonds is $V_B = 0.01$. These low values ensure that our equilibrium exists. We further discuss in Section 5 the realism of this assumption.

4.1.2 Preference parameters

The shape of the periodic utility function $\tilde{u}$ is defined by three parameters –see equation (3). First, we set $\sigma = 1$, implying that we have $\tilde{u}(c) = \log(c)$ in the non-linear part. Second, to avoid arbitrariness in the choice of the slopes $\lambda^1 < \lambda^2$ of the two linear parts, we require them to be equal to the derivatives –computed at the relevant point– of the utility function $\log(c)$:

$$\lambda^i = \frac{1}{c_{GG}^{i,pp}},\ i = 1, 2. \tag{34}$$

Our choice for λ^1 and λ^2 is consistent with our interpretation of the linear parts in the utility function as an approximation of a general utility function. Obviously, the values of $c_{GG}^{i,pp}$, $i = 1, 2$ in turn depend on λ^1 and λ^2, implying that equation (34) defining λ^i involves solving a fixed-point problem.

4.1.3 Parameter restrictions

To bring discipline to the calibration strategy, we impose some constraints on the model parameters, which are consistent with the mechanisms identified in Section 3. We first set $\omega_B^1 = 1.0$ to scale the income process of type-1 agents. Second, we set $\eta_k^1 = \eta_k^2 = \eta$ $(k = G, B)$ such that a constant fraction of the population is productive in every period. Third, we assume that the income risk faced by type-2 agents is not time-varying, such that $\omega^2 \equiv \omega^2_G = \omega^2_B$. The model could easily accommodate these parameters being time-varying, but this is not necessary to match key moments of the data. Finally, we assume that the average value of dividends is $\overline{y} = 1$, such that only the ratio y_G/y_B is used in the estimation strategy.

4.2 Estimated parameters

We are left with 11 parameters to estimate: the discount factor β, the dividend process y_G/y_B and 9 parameters driving the income process: 4 probabilities α^i_k ($i = 1, 2$ and $k = G, B$), 4 levels ω^1_G, ω^2, δ^1, δ^2 and the share η. Using equation (2), the values of $\rho^i_{k_1 k_2}$ ($i = 1, 2$ and $k_1, k_2 = G, B$) are uniquely determined by the values of η and α^i_k for $i = 1, 2$ and $k = G, B$. To estimate these 11 parameters, we match 12 empirical targets, that we denote $T = [T_1, \ldots, T_{12}]$.

4.2.1 Consumption and the risk exposure of households

We first target parameters concerning the consumption risk. Following the literature (Parker and Vissing-Jorgensen, 2009, among others), the risk faced by each category of households is proxied by the volatility of the consumption growth rate for non-durable goods and services. Consumption is measured by quarterly expenditures on non-durable goods and on a subset of services deflated with the relevant price index. We use data of the Consumer Expenditure Survey (CEX) from 1980 to 2007.[18] A detailed discussion can be

[18] In what follows, we apply the methodology of Parker and Vissing-Jorgensen (2009) to a different subset of households, so as to be consistent with our model. Moreover, it is known that consumption data are not as accurate as data on household income (see Aguiar and Bils, 2011, among others, for a discussion). Nevertheless, as our results are consistent with those derived using different datasets, we are confident that the facts presented here are robust.

found in Appendix **??**. Our first two targets are the standard deviations of the consumption growth rate for the top 50%, equal to $T_1 \equiv \sigma(\Delta \log C_1) = 0.14$, and for the bottom 50%, equal to $T_2 \equiv \sigma(\Delta \log C_2) = 0.19$. The bottom 50% face higher total risk than the top 50%, as is standard in the literature. Our third target is the standard deviation of aggregate consumption which is $T_3 \equiv \sigma(\Delta \log C^{tot}) = 1\%$. This last value is not implied by the first two targets, because of agent heterogeneity.

We also target the exposure of both groups to aggregate shocks. Following Parker and Vissing-Jorgensen (2009), we compute, for each group, the coefficient equal to: (Change in real group consumption per household)∗(Group share of population)/(Lagged aggregate real consumption per household).[19] The coefficients, that sum to one for both groups, can be interpreted as the fraction of aggregate risk born by each group. According to this metric, the top 50% bear $T_4 \equiv S_1 = 84\%$ of aggregate risk, whereas the bottom 50% bears the remaining 16%. Finally, the consumption share of the top 50% amounts to $T_5 \equiv C_1/C_{tot} = 72.1\%$, which drives consumption inequalities in our economy.

4.2.2 Asset returns and dividend process

We target 4 moments of stock and bond returns, and 3 moments for the dividend process, using Campbell (1999)'s dataset. The stock returns are computed from the S&P 500 index, while the bond returns are computed from the six-month commercial paper rate. These are real returns and correspond to historical US data from 1890–1991. The average bond interest rate is $T_6 \equiv E(R_f) = 0.9\%$, while the standard deviation of the bond interest rate is $T_7 \equiv \sigma(R_f) = 1.7\%$. The average average stock return is equal $T_8 \equiv E(R_s) =$

[19]This measure does not depend on the share of the type-1 agent consumption in total consumption, as explained by Parker and Vissing-Jorgensen (2009).

8.1%, whereas its standard deviation is $T_9 \equiv \sigma(R_s) = 15.6\%$. The last three targets are the average price dividend ratio $T_{10} \equiv E(P_s/D) = 21$, the standard deviation of the log of the price dividend ratio $T_{11} \equiv \sigma(\log(P_s/D)) = 30\%$, and the standard deviation of the log of dividend growth $T_{12} \equiv \sigma(\Delta\log(D)) = 28.3\%$.

4.2.3 Parameter values

Table 1 presents the model parameter values. First, we find that the income of type-1 agents barely moves, as the income in the good state $\omega_G^1 = 1.01$ is very close to the income in the bad state $\omega_B^1 = 1$. As a consequence, time variation in the income process is mostly driven by time-varying probabilities. The probabilities α_G^1, α_B^1, α_G^2 and α_B^2 oscillate around the value of 0.9, which is close but slightly lower than the value obtained when identifying the idiosyncratic risk with the employment risk. Indeed, on US data from 1948Q1-2007Q4, the quarterly probability of remaining employed equals 0.953, using Shimer (2005) methodology (see Challe and Ragot, 2014). The share of productive agents is $\eta = 0.89$. The discount factor β amounts to 0.86, which ensures equilibrium existence (see condition (9)) and a realistic price dividend ratio. The dividend process $y_G/y_B = 1.12$ enables to match the standard deviation of the dividend growth rate. Finally, the two preference parameters λ^i $(i = 1, 2)$ are pinned down by equation (34).

4.3 Results

Table 2 summarizes the targets and provides the model outcome. This simple model does a surprisingly good job in reproducing empirical moments. The volatility of consumption growth for each group is close to its empirical coun-

Calibrated parameters									***"Equilibrium" parameters***	
π_{GG}	π_{BB}	σ	ω_B^1	V_X	V_B				λ^1	λ^2
0.75	0.50	1	1	0.002	0.1				0.99	2.25
Estimated parameters										
α_G^1	α_B^1	α_G^2	α_B^2	ω_G^1	ω^2	δ^1	δ^2	η	β	y_G/y_B
0.91	0.94	0.90	0.88	1.01	0.45	0.32	0.07	0.89	0.86	1.12

Table 1: Parameter values

terpart and the returns are in line with their empirical values. We find a low return for the safe asset and a high equity premium, which amounts to 7%.

	US Data	**Model**	**Description and Remarks**
Consumption Growth			
$\sigma(\Delta\log C_1)$(in %)	14	15	std. dev. of agg. type 1 cons. growth
$\sigma(\Delta\log C_2)$(in %)	19	22	std. dev. of type 2 cons. growth
$\sigma(\Delta\log C^{tot})$(in %)	1.0	1.0	std. dev. of agg. cons. growth
S_1(in %)	84	85	share of agg. risk born by type 1
C_1/C_{tot}(in %)	72	70	cons. share of type 1 in agg. cons.
Asset Returns			
$E(R_f)$(in %)	0.9	0.9	average safe return
$\sigma(R_f)$(in %)	1.7	1.2	std. dev. of the safe return
$E(R_s)$(in %)	8.1	7.9	average risky return
$\sigma(R_s)$(in %)	15.6	6.8	std. dev. of the risky return
Price-Dividend (P/D) Ratio			
$E(P_s/D)$	21	13	average P/D ratio
$\sigma(\log(P_s/D))$(in %)	30	17	std. dev. of log of P/D ratio
$\sigma(\Delta\log(D))$(in %)	28.3	27.7	std. dev. of log dividend growth

Note: See the text for the description of the statistics.

Table 2: Targets and model outcomes

The model closely reproduces average moments of asset returns, a low volatility of the safe return and a high volatility of the risky return although the latter is lower than the data, which may not be surprising for such a simple framework. We find that the average income of type-1 agents is roughly twice as high as the one for type-2 agents. These values are consistent with empirical estimates of the skill premium, which is between 1.4 and 1.7 (Murphy and Welch, 1992) and with the fact that populations of skilled and unskilled workers are roughly of the same size over the last 20 years (Mukoyama and Sahin, 2006).

Implicit valuation of the risky asset by type-2 agents. In Assumption C, we set a high participation cost of type-2 agent to ensure that these agents will not trade any stock. We provide in equation (16) a value of $\overline{\chi}_2$ which ensures that holding stocks is a dominated choice. Using our calibration and as the median annual income of US households was $52,250 in 2014, we find an annual participation cost of $33. This is relatively small compared to other estimates of the participation cost (Vissing-Jorgensen, 2002, among others).[20]

Role of participation costs. The ability of the previous model to reproduce asset prices crucially relies on the participation cost, which generates limited stock market participation. To see this, we perform the same quantitative analysis as in the previous section but we relax Assumption C and set all participation costs to zero: $\chi_1 = \chi_2 = 0$. As explained in Proposition 2, both agent types may trade stocks and bonds. We provide the full model in Appendix **??**. We re-estimate the model without participation costs and we find that the safe and risky returns are almost identical and equal to 7.0%. The model without participation costs thus fails to reproduce realistic asset prices. The reason is the following. A higher idiosyncratic risk for type-2 agents is necessary to match the difference in the consumption growth rate volatilities. Such a difference generates a higher desire to self-insure for type-2

[20]As this estimation may depend on the small volume of debt, we also compute the implicit valuation of the risky asset by type-2 agents (i.e., their valuation with their own pricing kernel). We find that type-2 agents will never participate in the stock market, if they face a proportional participation cost as low as 1.1%.

agents and thus a higher valuation of all assets. In consequence, only one type of agents prices all the assets at the equilibrium and type-2 agents hold all stocks and bonds. Krusell, Mukoyama and Smith (2011) have shown that in such an economy, it is not possible to reproduce empirical asset prices with realistic idiosyncratic risks. Participation cost is thus a key ingredient for the ability of our model to match empirical data.[21]

5 Discussion of our assumptions

As explained in the discussion of Proposition 1, our equilibrium relies on two assumptions: (i) the linear parts in the utility function and (ii) the upper bound on security volumes. The concave-linear utility function generalizes Fishburn (1977)'s. The shape of the periodic utility function implies that agents with a low consumption level are sensitive to small variations in consumption levels, while agents consuming a higher amount (i.e., those in a linear part) have a marginal utility which is invariant to small variations in consumption. However, these agents can experience a sensible increase in marginal utility if they are hit by a negative idiosyncratic shock that would force them to consume a low amount (that would be valued by the strictly concave part of the utility function). The concave-linear utility function accounts for extensive variations in consumption due to individual shocks but neglects the impact of small intensive variations. We believe it to be a simplified but relevant representation of consumption smoothing and of the behavior with respect to idiosyncratic risk. Consumption variations matter much more when consumption levels are low than when they are high. The quantitative exercise

[21]The role for participation costs in asset prices confirms findings in Guvenen (2009) in a model with heterogeneous preferences.

has shown that the linear part can be chosen for the marginal utility to be consistent with a regular and globally concave utility function. Finally, productive agents are *not* risk neutral as they always have a positive probability of valuing next period consumption with a strictly concave utility function.

The upper bound on security volumes is the second important assumption. Indeed, for our limited-heterogeneity equilibrium to exist, we have to limit the amount of self-insurance, such that unproductive agents remain credit-constrained. Considering the bottom 50% of US households (in the consumption distribution), these households hold a small amount of liquid wealth – using the SCF, this amounts to less than one thousand dollars. The assumption of small asset volumes is not unrealistic for this fraction of the population. The top 50% of US households obviously hold a much higher amount of assets. For them, we justify our assumption by the result of our estimation exercise. Indeed, the estimation shows that this simple model reproduces quite well the aggregate risk exposure and asset price properties. The availability of assets for participating agents to self-insure can thus be captured by parameter values in our small-trade equilibrium.

Other assumptions are much less critical with respect to our equilibrium existence. For instance, Assumption A could replaced by a less strict assumption, at the cost of a greater number of agent classes in the equilibrium. As discussed in Footnote 9, we could also allow agents to hold a negative wealth, as long as the borrowing limit is not too loose. Our equilibrium would still exist, provided that the set $\mathcal{V}$ of admissible volumes in Proposition 1 is changed accordingly.[22]

[22] There is a kind of substitution between the negative wealth constraint and the maximal bounds (in $\mathcal{V}$) allowing for equilibrium existence.

6 Conclusion

We have constructed an analytically tractable incomplete insurance market model with participation cost, heterogeneity in risk exposure, and aggregate shocks. Our small-trade equilibrium relies on not-too-large security volumes and a concave-linear utility function introduced by Fishburn (1977). Though simple, this model reproduces asset price properties and household risk exposure quite well. This parsimonious setup could be used to study other forms of heterogeneity with aggregate shocks and for instance the heterogeneity of agents according to both sides of their balance sheet (i.e., asset and liability sides). This would allow us to study financial intermediation in an incomplete market setting with aggregate shocks. Such an environment could improve our understanding of the functioning of financial markets.

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