

ARTIFICIAL VISCOSITY IN STARSMAHER

JAMES C. LOMBARDI, JR.

Department of Physics, Allegheny College, Meadville, PA 16335, USA

Subject headings:

Our SPH evolution equations are described by Gaburov et al. (2010), but with a different AV implementation. In particular, the AV contribution to the acceleration of particle i is calculated as

$$\dot{\mathbf{v}}_{\text{AV},i} = - \sum_j m_j [\Pi_{ij} \nabla_i W_{ij}(h_i) + \Pi_{ji} \nabla_j W_{ij}(h_j)] / 2. \quad (1)$$

We use the AV form

$$\Pi_{ij} = 2 \frac{P_i}{\rho_i^2} (-\alpha \mu_{ij} + \beta \mu_{ij}^2), \quad (2)$$

where

$$\mu_{ij} = \begin{cases} \frac{(\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)}{c_i |\mathbf{r}_i - \mathbf{r}_j|} f_i, & \text{if } (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) < 0; \\ 0, & \text{if } (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) \geq 0. \end{cases} \quad (3)$$

Here c_i is the sound speed at the location \mathbf{r}_i of particle i . The Balsara switch f_i for particle i is defined by

$$f_i = \frac{|\nabla \cdot \mathbf{v}|_i}{|\nabla \cdot \mathbf{v}|_i + |\nabla \times \mathbf{v}|_i + \eta' c_i / h_i}, \quad (4)$$

with $\eta' = 10^{-5}$ preventing numerical divergences. The function f_i approaches unity in regions of strong compression ($|\nabla \cdot \mathbf{v}|_i \gg |\nabla \times \mathbf{v}|_i$) and vanishes in regions of large vorticity ($|\nabla \times \mathbf{v}|_i \gg |\nabla \cdot \mathbf{v}|_i$). Consequently, our evolution equations have the advantage that the artificial viscosity (AV) is suppressed in shear layers. We note that the AV term is not symmetric under interchange of the indices i and j (that is, $\Pi_{ij} \neq \Pi_{ji}$). Such an approach reduces the number of arrays shared among parallel processes. As the term in square brackets in equation (1) is antisymmetric under the interchange of particles i and j , momentum conservation is maintained.

The rate of change of the specific internal energy due to AV is

$$\frac{du_i}{dt}_{\text{AV}} = \sum_j \frac{\Pi_{ij}}{2} m_j (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla_i W_{ij}(h_i), \quad (5)$$

which guarantees conservation of entropy in the absence of shocks. It is straightforward to show total energy is conserved by our AV treatment: $\sum_i m_i (\mathbf{v}_i \cdot \dot{\mathbf{v}}_{\text{AV},i} + du_i/dt_{\text{AV}}) = 0$.