

Application of Cointegration to Index Tracking, Index Arbitrage, and Related Market-neutral Strategies

Introduction

These are my expository notes on a paper titled "[Cointegration portfolios of European equities for index tracking and market neutral strategies](#)" by C. Dunis and R. Ho.

In addition to my summary and exposition of their main ideas, I have added some musings of my own: how to apply their portfolio construction method to a version of index arbitrage that can be profitable even in an environment where every investment bank is practising index arbitrage already.

The original paper by Dunis and Ho contains 2 applications of cointegration:

- a) Construction of a portfolio of stocks which closely tracks a market index, but which involves lower transactions cost due to less frequent rebalancing; and
- b) Construction of a long-short market neutral portfolio, where the long side tracks the market index plus some fixed return δ , and the short side tracks the same market index minus the fixed return δ , so that the market-neutral portfolio is likely to return 2δ over the long term.

The market index in question is the Dow Jones EUROStoxx50 index, whose components are available from www.finance.yahoo.co.uk

How do we select stocks for the portfolio?

1. Decide on how many stocks you want: 5, 10, 15, 20 are acceptable numbers.
2. Pick one stock at a time from the index, test for cointegration.
3. Add to the portfolios the 5, 10, 15 or 20 stocks with the highest probabilities of cointegration between the log price of the stock, and the log index.
4. Run a multivariate linear regression with the log index as the dependent variable, and the log prices of the stocks in the portfolio as the independent variables. The regression coefficients give the capital weightings of the stocks. (The weights can be negative: i.e. short positions are allowed.)
5. Re-estimate cointegration and regression coefficients once in a while.

For the long (short) portfolio in application b), one only need to add (subtract) δ from the log index for the above steps 1-5.

One may ask: can we use raw prices rather than log prices in these steps? My belief is that it doesn't make any difference. But then, you would have to add a compounded return component to the index prices if you want to add (subtract) the δ for the long (short) portfolio. Also, the regression coefficients are no longer capital weightings, but the number of shares you need.

Index arbitrage

There is another way to generate returns from this portfolio construction method. Many of you have heard of index arbitrage: when the value of a basket of the index component stocks becomes higher than the futures representing that index (taking into account present value, interest costs, etc.), one can short the basket, and buy the futures, and earn riskless profits. And of course, vice versa.

This kind of arbitrage has been practised by investment banks the world over, and is therefore almost impossible to eke out a profit these days due to the infinitesimal deviation from fair value of the basket. However, when we construct our portfolio not with all the component stocks in the index, but with just a subset using the method described above, the deviation between the basket value and the futures become much more significant -- and therefore profitable.

One caveat though: if you use just 5 stocks to construct the portfolio, the significant deviation of value from the futures may be due to an irreversible change in economic value of one of the companies. In this case, the specific risk may be too overwhelming. I would recommend that one choose at least 20 stocks to form the portfolio.

Other applications

This paper is not only interesting in its own right, but it references a number of different papers that relates to arbitrage trading. Here are a few:

- Kasa (1992) Relationship among developed markets (US/Japan/UK/Germany/Canada) indices.
- Choudhury (1997) Relationship between 6 Latin American stock indices to US index.
- Cheung and Mak (1992) Relationship between developed markets and other Asian Pacific markets.
- Ng (2002) Relationship among various South-East Asian markets.