TUTORIAL WEEK 5

Note Title 10/02/2015

BASIC CONVERSION

base 2 to base 10

$3\,2\,1\,0$

$1011_2 = ?_{10} \rightarrow 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 8 + 2 + 1 = 11_{10}$

this works for each bases $\neq 2$ → change to base x when you multiply.

base 10 to base 2

$294_{10} \rightarrow ?_2$ → divide by 2 the # until you get ø

$294_{10} = ?_2$

294	2
147	0
73	1
36	1
18	0
9	0
4	1
2	0
1	0
0	1

$294_{10} = 100100110_2$

divide by onothe # if you're not in base 2

read this numbers from bottom to top

Floating point.

$\mathcal{F}\{\beta, t, exp_{min}, exp_{max}\}$

β = base

t = max # digits in the mantisse

exp_{min} = min exponent

exp_{max} = max exponent

How many numbers can I write in $\mathcal{F}\{10, 2, -2, 2\}$.

normalized: first digit $\neq 0$

1) list the exponents: $-2, -1, 0, 1, 2 = \{5\}$

2) all the possible normalized mantissas: $0,10;\ 0,11 \ldots 0.20, 0,21 \ldots 0,99 \rightarrow \{90\}$

possible # are: $5 \cdot 90 \cdot 2 = 900$

(2: positive and negative!)

$\mathcal{F}\{10, 2, -2, 2\}$

min poss. #? $x_{min} = \beta^{(exp_{min} - 1)} = 10^{(-2-1)} = 10^{-3}$

max poss #? $x_{max} = (1 - \beta^{-t})\beta^{e_{max}} =$

$= (1 - 10^{-2}) \cdot 10^{2}$

IEEE single precision -> 32 bits

1 sign | 8 exponent | 23 mantissa.

① $147.625_{10} \rightarrow ?_{IEEE}$

②

① Integer part.

147	2
73	1
36	1
18	0
9	0
4	1
2	0
1	0
0	1

$147_{10} = 10010011_2$

② Fractional part.

$0.625 \cdot 2 = 1.25$

$0.25 \cdot 2 = 0.5$

$0.5 \cdot 2 = 1.0$

$147.625_{10} = 10010011.101_2$

1 bit sign < 0 (+)
1 (−)

a)

0

exponent 8 → normalize the number: 1,001001101 (7)

exponent in excess 127 → $7 + 127 = 134_{10} = 10000110_2$

8 bit!

10000110

23 digits mantisse

→ rewrite the mantissa of the #

00100111010000000000000000

write zeros until 23 digits.

LU decomposition

$AX = B$ $\qquad A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$ $\qquad L \cdot U = A$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$LU = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{11}L_{21} & U_{12}L_{21} + U_{22} & L_{21}U_{13} + U_{23} \\ U_{11}L_{31} & L_{31}U_{12} + L_{32}U_{22} & U_{13}L_{31} + L_{32}U_{23} + U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

Solve the above system

$U_{11} = 1$; $U_{12} = 2$; $U_{13} = 4$

$1 \cdot L_{21} = 3 \rightarrow L_{21} = 3$; $2 \cdot 3 + U_{22} = 8 \rightarrow U_{22} = 2$; $3 \cdot 4 + U_{23} = 14 \rightarrow U_{23} = 2$

$L_{31} = 2$; $2 \cdot 2 + 2 L_{32} = 6 \rightarrow L_{32} = 1$; $4 \cdot 2 + 1 \cdot 2 + U_{33} = 13 \rightarrow U_{33} = 3$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

Newton's method : $x^{(k+1)} = x^{(k)} - \frac{f(x^{(k)})}{f'(x^{(k-1)})}$

$f(x) = x^3 - 2x + 1$

$f'(x) = 3x^2 - 2$

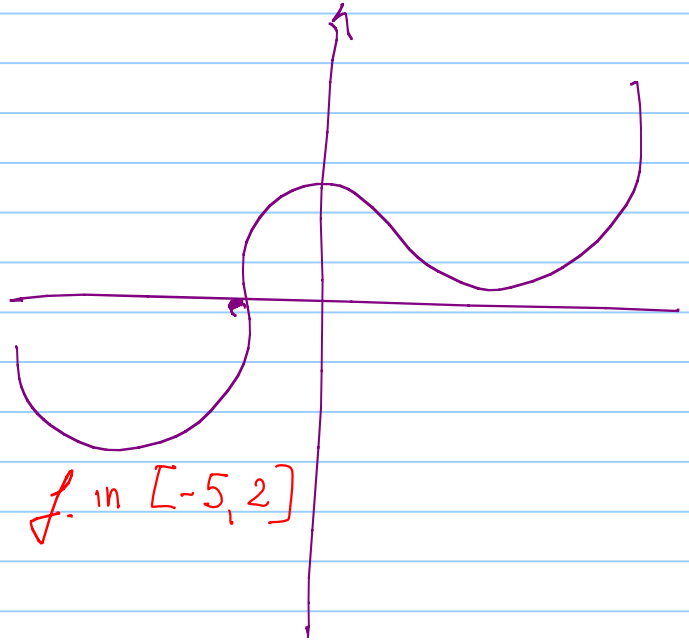
Choose and interval

$[-5, 2]$ if $f(-5) \cdot f(2) < 0 \rightarrow \exists$ zero of f in $[-5, 2]$

$f(-5) = -125 + 10 + 1 = -114$

$f(2) = 8 - 4 + 1 = 5$

it's good!



Choose x^0 in $[-5, 2]$

$x_0 = -3$

$$x^{(k+1)} = x^{(k)} - \frac{f(x^{(k)})}{f'(x^{(k)})}$$

$$x' = -3 - f(-3)/f'(-3) = -3 + \frac{20}{25} = -2{,}2$$

$$x'' = -2{,}2 - f(-2{,}2)/f'(-2{,}2) = \dots \quad \text{and so on...}$$

Another F.P. example: $\mathcal{F}\{2, 3, -3, 2\}$

possible exponents : $-3, -2, -1, 0, 1, 2 = \{6\}$

mantissas → $\overbrace{0,100 \; ; \; 0,101 \; ; \; 0,110 \; ; \; 0,111}^{4}$ they have to be normalized!

exp

$6 \cdot 4 \cdot 2 = 48$ possible #s

↓

positive and negative