Problem assignment 5

Due: Wednesday, October 9, 2019

Problem 1

Part a.

P	Q	S	T	U	KB			α
					$\neg(P\wedge\neg Q)\vee\neg(\neg S\wedge\neg T)$	$\neg(T\vee Q)$	$U\rightarrow(\neg T\rightarrow(\neg S\wedge P))$	$\neg U$
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
TRUE	TRUE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE
TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE
TRUE	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE
FALSE	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE
FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE

As showing in the table, whenever the KB is all true, the $\alpha=\neg U$ is always true. So we can prove $\neg U$ from KB.

Part b.

KB: $\neg(P\wedge\neg Q)\vee\neg(\neg S\wedge\neg T)$ $\neg(T\vee Q)$ $U\rightarrow(\neg T\rightarrow(\neg S\wedge P))$

Theorem: $\neg U$

First, use standard logical equivalences to rewrite each KB:

$\neg(P\wedge\neg Q)\vee\neg(\neg S\wedge\neg T)$ <=> $\neg P\vee Q\vee S\vee T$ --- ① (use De Morgan)

$\neg(T\vee Q)$ <=> $\neg T\wedge\neg Q$ --- ② (use De Morgan)

$U\rightarrow(\neg T\rightarrow(\neg S\wedge P))$ <=> $\neg U\vee T\vee(\neg S\wedge P)$--- ③ (use Modus ponens)

Then, make the inference:

From ② and And-elimination, we get:

$\neg T$ --- ④

$\neg Q$ --- ⑤

From ①, ④, ⑤ and Unit-resolution, we get:

$\neg P \lor S \Leftrightarrow \neg(\neg S \land P)$ --- ⑥ (use De Morgan)

From ③, ⑤, ⑥, and Unit-resolution, we get:

$\neg U$ --- ⑦

And finally we proved the Theorem: $\neg U$.

Part c.

First, convert all the KB to CNF form:

$\neg(P \land \neg Q) \lor \neg(\neg S \land \neg T) \Leftrightarrow \neg P \lor Q \lor S \lor T$

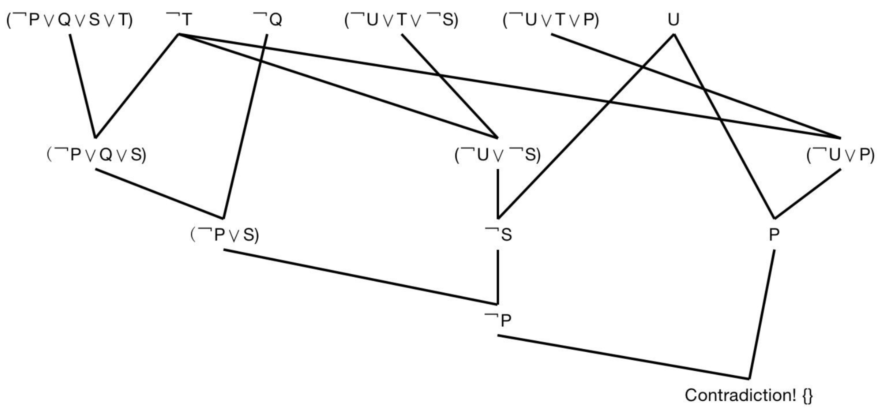
$\neg(T \lor Q) \Leftrightarrow \neg T \land \neg Q$

$U \to (\neg T \to (\neg S \land P)) \Leftrightarrow (\neg U \lor T \lor \neg S) \land (\neg U \lor T \lor P)$

Then, Negate the theorem to prove it via refutation

$\neg U$ --> U

Finally, run the resolution on the set of clauses:



So we get contradiction at $KB \land U$. In that way, we proved $\neg U$.

Problem 2.

Part a.

Let's first assign cases to each statement in the question:

A: the unicorn is mythical

B: the unicorn is immortal

C: the unicorn is a mortal mammal

D: the unicorn is horned

E: the unicorn is magical

According to the knowledge facts, we can get following propositional logic:

$A \to B$ $\quad \neg A \to C$ $\quad (B \lor C) \to D$ $\quad D \to E$

Part b.

We can't prove the unicorn is mythical.

Part c.

We can prove the unicorn is magical.

The KB are: $A \to B$ $\neg A \to C$ $(B \lor C) \to D$ $D \to E$

Theorem: E

First, use standard logical equivalences to rewrite each KB

$A \to B \Leftrightarrow \neg A \lor B$ --- ①

$\neg A \to C \Leftrightarrow A \lor C$ --- ②

$(B \lor C) \to D \Leftrightarrow \neg(B \lor C) \lor D$ --- ③

$D \to E \Leftrightarrow \neg D \lor E$ --- ④

Then, make the inference:

From ① and ②, using Resolution, we get:

$B \lor C$ --- ⑤

From ③ and ⑤, using Unit Resolution, we get:

D --- ⑥

From ④ and ⑥, using Unit Resolution, we get:

E --- ⑦

Finally, we proved the Theorem: E. which means the unicorn is magical.

Part d.

We can prove the unicorn is horned.

The KB are: $A \to B$ $\neg A \to C$ $(B \lor C) \to D$ $D \to E$

Theorem: D

First, use standard logical equivalences to rewrite each KB

$A \to B \Leftrightarrow \neg A \lor B$ --- ①

$\neg A \to C \Leftrightarrow A \lor C$ --- ②

$(B \lor C) \to D \Leftrightarrow \neg(B \lor C) \lor D$ --- ③

$D \to E \Leftrightarrow \neg D \lor E$ --- ④

Then, make the inference:

From ① and ②, using Resolution, we get:

$B \lor C$ --- ⑤

From ③ and ⑤, using Unit Resolution, we get:

D --- ⑥

Finally, we proved the Theorem: D. which means the unicorn is horned.

Problem 3

First, assign case to the initial facts, the FB:

A: the animal gives milk

B: it chews cud

C: it has long legs

D: it has long neck

E: it has tawny color

F: it has dark spots

Theorem 1.

Suppose that:

G: the animal is a mammal

H: the animal is an ungulate

And our theorem is:

I: the animal is a giraffe

From rule 2, we can get:

$A \to G$ using modus ponens, $A \to G, A \Rightarrow G$

From rule 8, we can get:

$(G \wedge B) \to H$ using modus ponens, $(G \wedge B) \to H, G, B \Rightarrow H$

From rule 12, we can get:

$(H \wedge C \wedge D \wedge E \wedge F) \to I$ using modus ponens, $(H \wedge C \wedge D \wedge E \wedge F) \to I, H, C, D, E, F \Rightarrow I$

So we can prove theorem 1 is true. We use rule 2, 8 and 12 to derive the conclusion.

Theorem 2.

Suppose that:

J: the animal is a bird

K: the animal does not fly

L: the animal swims

M: it is black and white

And our theorem is:

N: the animal is penguin

From rule 15, we can get:

$(J \wedge K \wedge L \wedge M) \to N$

However, for the part $J \wedge K \wedge L \wedge M$, we can not use any other rules listed above to prove their truth. So we can not prove the theorem 2 is true.

Theorem 3.

From rule 2, we can get:

$A \to G$ using modus ponens, $A \to G, A \Rightarrow G$

So we can prove theorem 3 is true. We just use rule 2.

Problem 4.

Part b.

```
Initial theorem to prove: is_a_giraffe
The new added rule is R2. The antecedent part is gives_milk. The consequent part is is_a_mammal.
The new added rule is R8. The antecedent part is is_a_mammal,chews_cud. The consequent part is is_an_ungulate.
The new added rule is R9. The antecedent part is is_a_mammal,chews_cud. The consequent part is is_even-toed.
The new added rule is R12. The antecedent part is is_an_ungulate,has_long_legs,has_a_long_neck,has_a_tawny_color,has_dark_spots.
 The consequent part is is_a_giraffe.
Theorem is_a_giraffe is successfully proved!
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
is_a_mammal
is_an_ungulate
is_even-toed
is_a_giraffe
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Initial theorem to prove: is_a_penguin
The new added rule is R2. The antecedent part is gives_milk. The consequent part is is_a_mammal.
The new added rule is R8. The antecedent part is is_a_mammal,chews_cud. The consequent part is is_an_ungulate.
The new added rule is R9. The antecedent part is is_a_mammal,chews_cud. The consequent part is is_even-toed.
The new added rule is R12. The antecedent part is is_an_ungulate,has_long_legs,has_a_long_neck,has_a_tawny_color,has_dark_spots.
 The consequent part is is_a_giraffe.
Theorem is_a_penguin prove failed!
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
is_a_mammal
is_an_ungulate
is_even-toed
is_a_giraffe
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```
Initial theorem to prove: is_a_mammal
The new added rule is R2. The antecedent part is gives_milk. The consequent part is is_a_mammal.
Theorem is_a_mammal is successfully proved!
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
is_a_mammal
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Initial theorem to prove: has_a_tawny_color
Theorem has_a_tawny_color is successfully proved!
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
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```
Initial theorem to prove: is_a_bird
The new added rule is R2. The antecedent part is gives_milk. The consequent part is is_a_mammal.
The new added rule is R8. The antecedent part is is_a_mammal,chews_cud. The consequent part is is_an_ungulate.
The new added rule is R9. The antecedent part is is_a_mammal,chews_cud. The consequent part is is_even-toed.
The new added rule is R12. The antecedent part is is_an_ungulate,has_long_legs,has_a_long_neck,has_a_tawny_color,has_dark_spots.
 The consequent part is is_a_giraffe.
Theorem is_a_bird prove failed!
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
is_a_mammal
is_an_ungulate
is_even-toed
is_a_giraffe
```

Part c.

```
Initial theorem to prove: is_a_giraffe
Using rule R7 prove has_hoofs: failed!
Using rule R2 prove is_a_mammal: success!
Theorem is_a_mammal is Proved!
Using rule R8 prove is_an_ungulate: success!
Theorem is_an_ungulate is Proved!
Using rule R12 prove is_a_giraffe: success!
Theorem is_a_giraffe is Proved!

The initial theorem: is_a_giraffe is proved!

All the theorem trying to prove:
is_a_giraffe
is_an_ungulate
has_hoofs
is_a_mammal
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
is_a_mammal
is_an_ungulate
is_a_giraffe
Press Enter to continue.
```

```
Initial theorem to prove: is_a_penguin
Using rule R3 prove has_feathers: failed!
Using rule R4 prove flies: failed!
Using rule R4 prove lays_eggs: failed!
Using rule R3 prove has_feathers: failed!
Using rule R15 prove swims: failed!
Using rule R14 prove is_black_and_white: failed!
Using rule R15 prove is_black_and_white: failed!
Using rule R14 prove does_not_fly: failed!
Using rule R15 prove does_not_fly: failed!

We can't prove the initial theorem: is_a_penguin!

All the theorem trying to prove:
is_a_penguin
is_a_bird
has_feathers
flies
lays_eggs
swims
is_black_and_white
does_not_fly
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
Press Enter to continue.
```

```
Initial theorem to prove: is_a_mammal
Using rule R2 prove is_a_mammal: success!
Theorem is_a_mammal is Proved!
Using rule R3 prove has_feathers: failed!
Using rule R4 prove flies: failed!
Using rule R4 prove lays_eggs: failed!
Using rule R3 prove has_feathers: failed!

The initial theorem: is_a_mammal proved!

All the theorem trying to prove:
is_a_mammal
is_a_bird
has_feathers
flies
lays_eggs
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
is_a_mammal
Press Enter to continue.
```

```
Initial theorem to prove: has_a_tawny_color
The theorem is in the FB!

The initial theorem: has_a_tawny_color proved!

All the theorem trying to prove:
has_a_tawny_color
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots


Press Enter to continue.
```

```
Initial theorem to prove: is_a_bird
Using rule R3 prove has_feathers: failed!
Using rule R4 prove flies: failed!
Using rule R4 prove lays_eggs: failed!
Using rule R3 prove has_feathers: failed!

We can't prove the initial theorem: is_a_bird
!
All the theorem trying to prove:
is_a_bird
has_feathers
flies
lays_eggs
The fact base after the procedure:
gives_milk
chews_cud
has_long_legs
has_a_long_neck
has_a_tawny_color
has_dark_spots
```

As shown above, part b is the result of forward chain and part c is backward chain. In terms of the fact base of both methods, if the initial theorem is proved, the fact base has no difference on two methods. However, if the initial theorem is not proved, like theorem2 and theorem4, the forward chain method would still add proved theorem into fact base, the backward chain won’t add.