Problem assignment 3

Due: Wednesday, September 25, 2019

Problem 1

Variables inferred by forward checking

vars	x	y	z	w	u	t	v
domain	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9
x = 2	2	0,1,2...9	2,5,8	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9
y = 0	2	0	5	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9
t = 0	2	0	5	0,1,2...9	0,1,2...9	0	0
Infer w	2	0	5	5,9	0,1,2...9	0	0
Infer u	2	0	5	9	0,3,6,9	0	0

For the infer of w, since u isn't assigned, we only consider the arc between w and z. Then, the constraints would be:

w mod 4 = z mod 4 = 1

so w = {5,9}

For the infer of u, the constraints would be:

u mod 3 = v mod 3 = 0

u mod 3 = w mod 3 = 5 mod 3 or 9 mod 3 = {0,2}

For the first equation, we get u = {0,3,6,9}, for the second equation, we get u = {0,2,3,5,6,8,9}, then u = {0,3,6,9} ∩ {0,2,3,5,6,8,9} = {0,3,6,9}

Then since u is inferred, we should infer w again. Now the constraint for w is:

W mod 4 = z mod 4 = 1 -> w = {5,9}

W mod 3 = u mod 3 = 0 -> w = {0,3,6,9}

So the w = {5,9} ∩ {0,3,6,9} = 9.

So, the reasonable variable groups inferred by forward checking is:

x = 2

y = 0

z = 5

w = 9

u = {0,3,6,9}

t = 0

v = 0

Variables inferred by arc consistency

vars	x	y	z	w	u	t	v
domain	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9
x = 2	2	0,1,2...9	2,5,8	0,1,2...9	0,1,2...9	0,1,2...9	0,1,2...9
y = 0 (check arc	2	0	5	0,1,2,4,5,6,8,9	0,1,2...9	0,1,2...9	0,1,2...9

z->w)							
t = 0 (check arc t->v, w->u, z->w)	2	0	5	5,9	0,1,2...9	0	0
(check arc v->u, w->u)	2	0	5	5,9	0,3,6,9	0	0
(check u->w)	2	0	5	9	0,3,6,9	0	0

So, the reasonable variable groups inferred by forward checking is:

x = 2

y = 0

z = 5

w = 9

u = {0,3,6,9}

t = 0

v = 0

Problem 2

Part a.

Initial tour: [0, 58, 36, 53, 10, 18, 17, 2, 6, 42, 41, 19, 13, 11, 29, 4, 51, 44, 30, 15, 27, 40, 37, 22, 7, 31, 3, 54, 50, 48, 21, 23, 38, 9, 47, 33, 26, 16, 56, 43, 28, 59, 34, 52, 45, 24, 20, 8, 14, 35, 55, 46, 1, 32, 12, 5, 39, 49, 25, 57]

Initial tour distance: 306.43717

Initial temperature: 100

Number of tours tried: 100000

Number of tours accepted: 92435

The best tour found: [0, 5, 53, 32, 34, 15, 18, 33, 25, 36, 17, 3, 44, 40, 55, 20, 9, 14, 46, 2, 4, 59, 48, 10, 21, 38, 24, 26, 19, 35, 6, 52, 12, 37, 22, 8, 49, 41, 28, 13, 58, 31, 43, 57, 11, 51, 47, 23, 54, 56, 29, 7, 50, 1, 27, 45, 39, 30, 42, 16]

The best tour found distance: 103.65166

Part b.

Parameters		Initial Tour Distance	Number of Tours tried	Number of Tours Accepted	Best tour Distance
Steps: 100000	Temperature: 10000	~~311.539122~~	~~100000~~	~~99838~~	~~242.586178~~
	Temperature: 1000	~~300.557104~~	~~100000~~	~~98869~~	~~187.442838~~
	Temperature: 100	281.221808	100000	92424	102.865312

	Temperature: 1	304.905496	100000	8224	63.707482
	Temperature: 0.1	293.5911	100000	3672	70.27154
Steps: 200000	Temperature: 10000	~~322.237354~~	~~200000~~	~~199742~~	~~214.168836~~
	Temperature: 1000	~~315.761656~~	~~200000~~	~~197847~~	~~161.578628~~
	Temperature: 100	325.083958	200000	184680	85.275408
	Temperature: 1	334.881072	200000	15452	63.81284
	Temperature: 0.1	278.214292	200000	7408	68.78746
Steps: 400000	Temperature: 10000	~~290.435064~~	~~400000~~	~~399416~~	~~207.303222~~
	Temperature: 1000	~~301.722132~~	~~400000~~	~~395590~~	~~123.624858~~
	Temperature: 100	290.907394	400000	369320	77.615786
	Temperature: 1	327.964436	400000	31233	63.666698
	Temperature: 0.1	289.172244	400000	14092	67.207378
Steps: 800000	Temperature: 10000	~~333.904144~~	~~800000~~	~~798804~~	~~165.260582~~
	Temperature: 1000	307.826264	800000	791020	103.25354
	Temperature: 100	307.21158	800000	738602	72.957344
	Temperature: 1	299.86413	800000	62739	65.110992
	Temperature: 0.1	312.247572	800000	28704	67.02021
Steps: 1600000	Temperature: 10000	~~283.527504~~	~~1600000~~	~~1597645~~	~~156.056872~~
	Temperature: 1000	278.29668	1600000	1582088	85.168188
	Temperature: 100	319.700394	1600000	1477070	71.719568
	Temperature: 1	287.510608	1600000	124901	64.41229
	Temperature: 0.1	305.709702	1600000	58653	63.677714

With the same steps, as initial temperature decreased, the best tour distance found would decrease in general.

With the same temperature, as the steps increased, the best tour distance found would decrease in general.

The best solution appears with the parameter: steps=400000, initial_ temperature=1.

The best solution is 63.666698.

The red solutions are those beat the best solution in problem (72.892862).

Part c.

My cooling method for the temperature is:

$T_i = T_{init} - (T_{half} / (0.3K)^2) * i^2$ $\quad$ $0<i<0.3K$

$T_i = T_{half}/(0.7K)^2*(i-K)^2$ $\quad$ $0.3K \le i<K$

T_{init} is the initial temperature.

T_{half} is half of the initial temperature.

K is the number of steps.

Generally, it's two part of quadratic function.

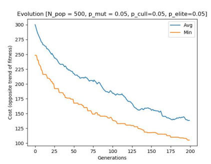
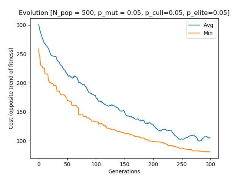
10 tries with tours accepted over 20000:

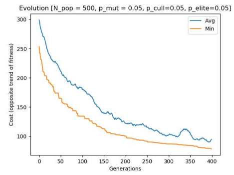
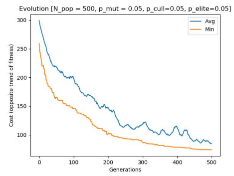
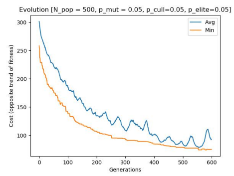
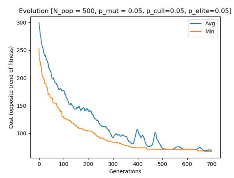
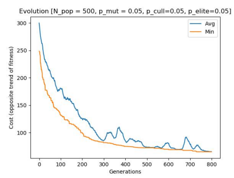
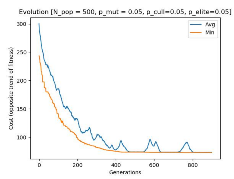
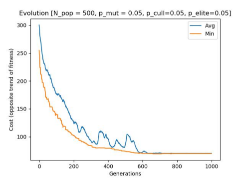
Initial distance	322.813260	320.257628	309.813396	292.906218	281.33309	283.037410	314.996344	316.949372	288.46531	296.115862
Best tour distance	81.042122	82.351288	78.570942	79.272502	79.434684	79.663748	78.559722	83.073736	81.429768	82.892166
Tours accepted	20180	20245	20204	20169	20145	20131	20090	20115	20263	20329

Then, the average energy for the above 10 best distance are: 80.6290678

Problem 3.

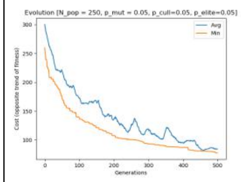
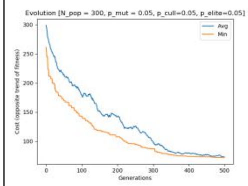
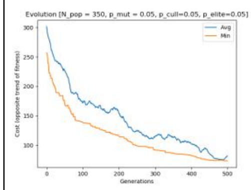
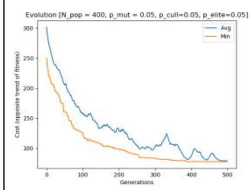
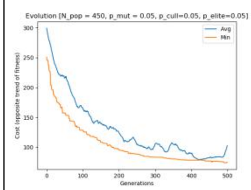
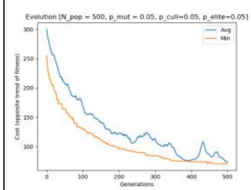
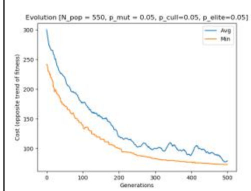
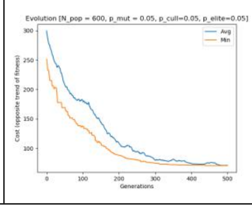
number of generations range between 200-1000, population=500, mutation probability=0.05, culling=0.05, elite=0.05

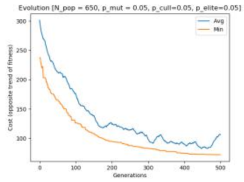
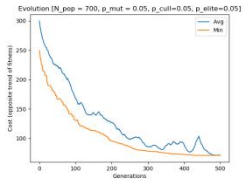
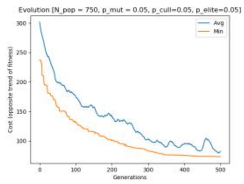
	Start state distance	Best distance found	pics
N_generations=200	248.48	105.17	
N_generations=300	258.48	80.85	

N_generations=400	253.62	78.43	
N_generations=500	259.32	74.04	
N_generations=600	258.34	75.14	
N_generations=700	252.7	67.2	
N_generations=800	248.47	64.99	
N_generations=900	243.34	72.84	
N_generations=1000	254.68	69.8	

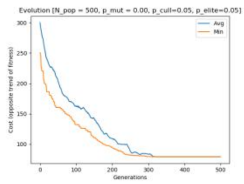
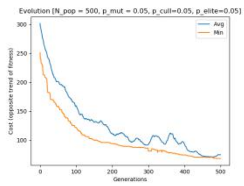
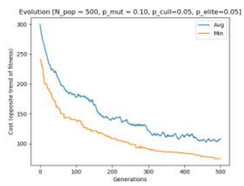
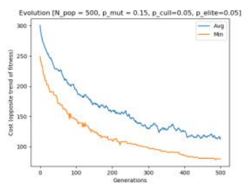
number of generations=500, population range between 250-750, mutation probability=0.05, culling=0.05, elite=0.05

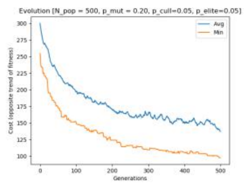
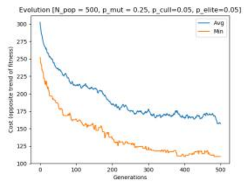
	Start state distance	Best distance found	pics

N_pop=250	259.92	77.48	
N_pop=300	260.95	72.07	
N_pop=350	257.11	73.94	
N_pop=400	250.02	77.72	
N_pop=450	250.79	74.75	
N_pop=500	255.39	70.79	
N_pop=550	241.84	72.85	
N_pop=600	251.63	70.52	

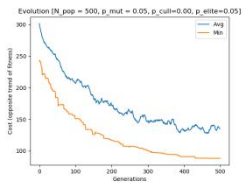
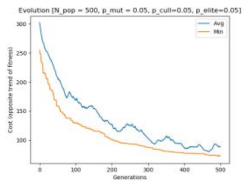
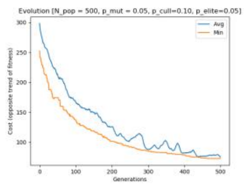
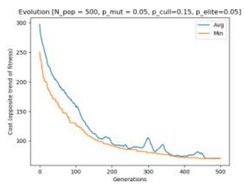
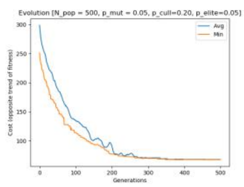
N_pop=650	237.17	72.11	
N_pop=700	249.25	70.76	
N_pop=750	237.13	73.22	

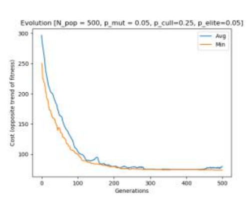
number of generations=500, population=500, mutation probability range between 0.00-0.25, culling=0.05, elite=0.05

	Start state distance	Best distance found	pics
mutation_prob=0.00	250.23	79.17	
mutation_prob=0.05	251.27	68.19	
mutation_prob=0.10	240.41	74.95	
mutation_prob=0.15	247.96	79.87	

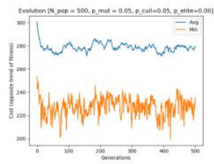
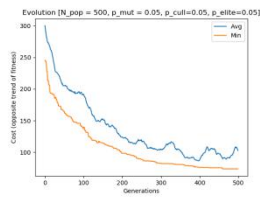
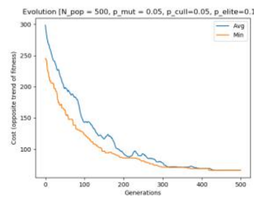
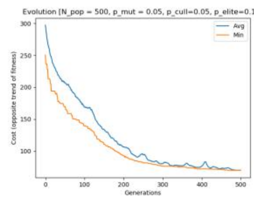
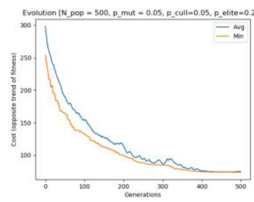
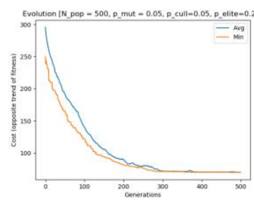
mutation_prob=0.20	254.75	97.73	
mutation_prob=0.25	251.86	110.12	

number of generations=500, population=500, mutation probability=0.05, culling range between 0.00-0.25, elite=0.05

	Start state distance	Best distance found	pics
culling_perc=0.00	242.99	88.26	
culling_perc=0.05	254.31	73.32	
culling_perc=0.10	252.39	72.67	
culling_perc=0.15	250.24	70.62	
culling_perc=0.20	251.24	67.32	

culling_perc=0.25	250.04	73.8	

number of generations=500, population=500, mutation probability=0.05, culling=0.05, elite range between 0.00-0.25

	Start state distance	Best distance found	pics
elite_perc =0.00	253.74	235.69	
elite_perc =0.05	245.17	73.7	
elite_perc =0.10	245.18	65.81	
elite_perc =0.15	250.26	69.83	
elite_perc =0.20	253.78	73.47	
elite_perc =0.25	249.95	69.85	

Analysis and conclusion:

As the number of generations increase, the best distance result is getting smaller till n_generations=800, then the result would fluctuate around 70, which might mean the evolution of the population is meeting the bottle-neck. However, as the number of generations increase, the average distance tend to become more closer to the best distance, which means every single “person” in the whole population tend to become the best “person”.

About the population, it seems that there isn’t any specific relations between the population and the best distance found. That still make sense for GA. The population only presents a sample space for the whole process. Its size somehow doesn’t really matter.

The mutation probability is very interesting. Reasonable percentage of the mutation would increase the performance of the system. For out test, the mutation_prob=0.05 seems to meet the best distance, then the results become worse. Also, as the mutation probability increases, the average distance would get far away from the best distance, since more mutations means instability of the “genotype” in the population.

About the culling percentage, the best distance result appears when culling_perc=0.20. Thus, same to the mutation probability, the reasonable percentage of culling would make it closer to best distance.

And for elite percentage, something very funny happened when elite_perc=0. The whole result would fluctuate severely, which means elite is necessary in genetic algorithm. Then, the best distance result is when elite_perc=0.10. Also we need reasonable elite percentage.

Finally, for both culling and elite percentage, the higher the rate, the closer the average distances and best results are. That’s easy to understand. These two parameters both means to preserve the “good gene”, so as generation increases, the “genotype” would generally become as good as the best.