

# Homework2 for EECS 340

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## 1 Give a recursive algorithm to find the average (mean) value of an array of $2^k$ decimal numbers, where $k \in \mathbb{N}$ .

*Answer:* The proposed algorithm is as follow:

**Algorithm A1:** Average( $L$ )

**Data:** A list of  $2^k$  decimal numbers  $L$ .

**Result:** The average of all the numbers in  $L$ .

**if**  $L.length() = 0$  **then**

**return**  $L[0]$

**else**

$length \leftarrow L.length()$

**return**  $0.5 \times (\text{Average}(L[0, length/2 - 1]) + \text{Average}(L[length/2, length]))$

**end if**

## 2 R-12.6

*Question:* Suppose we are given a set of telescope observation requests, specified by triples, of  $(s_i, f_i, b_i)$ , defining the start times, finish times, and benefits of each observation request as

$$L = (1, 2, 5), (1, 3, 4), (2, 4, 7), (3, 5, 2), (1, 6, 3), (4, 7, 5), (6, 8, 7), (7, 9, 4)$$

Solve the telescope scheduling problem for this set of observation requests.

*Answer:* The time of scheduling can be shown in Fig.1, the number in the bar means the value of such task.

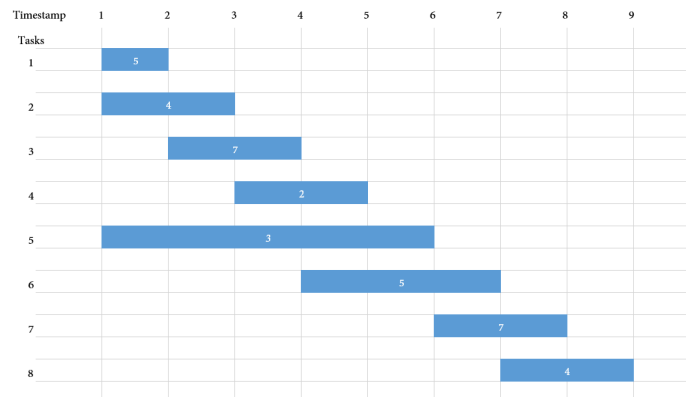


Figure 1: Time of tasks

Based on what we have discussed on class, we can have a table of  $B_i$  which stands for the maximum benefit that can be achieved with the first  $i$  requests in the task list.

To fill this table, we follow the algorithm as follow:

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 $B[0] \leftarrow 0$ 
for  $i = 1$  to  $n$  do
     $B[i] \leftarrow \max(B[i-1], B[P[i]] + b_i)$ 
end for

```

Here the  $P[i]$  stands for the array which gives the predecessor index for each request  $i$ , and  $b_i$  means the value of each single task. The table is shown as Table 1.

Table 1:  $B_i$  values

$i$	0	1	2	3	4	5	6	7	8
$B_i$	0	5	4	12	6	3	17	13	21

As we can see, the highest value is  $B_8$ , which includes task 1, 3, 6, 8 that we should select. The corresponding triples are (1, 2, 5), (2, 4, 7), (4, 7, 5), (7, 9, 4).

### 3 Implement *det-bogoSort* in pseudocode using recursion

*Answer:* This algorithm is described as follows:

**Algorithm** BogoSort( $S, L, L_{temp}$ )

**Data:** Input list  $L$ , as is described in the question, an initially empty set of lists  $S$ , and an initially empty list  $L_{temp}$

**Result:** A sorted copy of  $L$

```

if  $size(L) = 0$  then
     $flag \leftarrow true$ 
    for  $i \leftarrow 1$  to  $size(L_{temp}) - 1$  do
        if  $L_{temp}[i-1] > L_{temp}[i]$  then
             $flag \leftarrow false$ 
            break
        end if
    end for
    if  $flag = true$  then
        return  $L_{temp}$ 
    end if
else
    for  $i \leftarrow 0$  to  $size(L) - 1$  do
         $L_{temp}.append(L[i])$ 
         $L.remove(i)$ 
        BogoSort( $S, L, L_{temp}$ )
    end for
end if

```

NOTE: in the operations of lists,  $L_{temp}.append(L[i])$  means to append the element  $L[i]$  at the end of list  $L_{temp}$ . And  $L.remove(i)$  means to remove the  $i$ th element in list  $L$ .

### 4 Write pseudo-code for a new recursive function *moving-average*

*Answer:* The algorithm is shown as follows:

**Algorithm** moving-average( $S, t_{start}, t_{end}$ )

**Data:** Stock-price time series data  $S$ , a start time  $t_{start}$ , and an ending time  $t_{end}$ .

**Result:** The average stock price from  $t_{start}$  through  $t_{end}$ .

$i \leftarrow 0$

**if**  $t_{start} = t_{end}$  **then**

**return** 0

**end if**

**for**  $i \leftarrow t_{end}$  **to**  $t_{start}$  **do**

**if**  $t_{start} \% (i - t_{start}) = 0$  **and**  $((i - t_{start})$  is a power of 2) **then**

**return**  $\frac{i - t_{start}}{t_{end} - t_{start}} \times \text{pow-of-two-average}(S, t_{start}, i) + \frac{t_{end} - i}{t_{end} - t_{start}} \times \text{moving-average}(S, i, t_{end})$

**break**

**end if**

**end for**

**return** 0

## 5 Write pseudo-code for a recursive function to compute $\text{checker-sum}(A)_{i,j}$ given the triple $(A, i, j)$

*Answer:* The algorithm is shown as follows:

**Algorithm** checker-sum( $A, i, j$ )

**Data:** A matrix  $A$ , and index  $i$  and  $j$

**Result:** The checkerboard sum of matrix  $A$

**if**  $j > 1$  **then**

**return** compute-row( $A, i, j$ ) – checker-sum( $A, i, j - 1$ )

**else**

**return** compute-row( $A, i, j$ )

**end if**

**Algorithm** compute-row( $A, i, j$ )

**Data:** A matrix  $A$ , and index  $i$  and  $j$

**Result:** The checkerboard sum of row  $j$  in matrix  $A$

**if**  $i > 1$  **then**

**return**  $A(i, j)$  – compute-row( $A, i - 1, j$ )

**else**

**return**  $A(i, j)$

**end if**

## 6 Read section 17.1 in the textbook and then solve the following problems

### 6.1 R-17.7

*Question:* Show that the CLINQUE problem is in  $NP$

### 6.2 C-17.3

*Question:* Show that we can deterministically simulate in polynomial time any nondeterministic algorithm  $A$  that runs in polynomial time and makes at most  $O(\log n)$  calls to the **choose** method, where  $n$  is the size of the input to  $A$ .