Homework1 for EECS 340

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1 Warm-up: Big-Oh and Counting Primitive Operations

Show your work on the following questions. Use the limit-based definitions of asymptotic notation on the "Big-Oh Cheat Sheet" on Canvas wherever applicable.

1.1 Solve R-1.20, R-1.22, and R-1.23 in the text

1.1.1 R-1.20

Show that $(n+1)^5$ is $O(n^5)$.

Proof: Let $f(n) = (n+1)^5$ and $g(n) = n^5$ so that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{(n+1)^5}{n^5} = \lim_{n\to\infty} \frac{n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1}{n^5}$$
$$= 1 + \lim_{n\to\infty} \frac{5}{n} + \frac{10}{n^2} + \frac{10}{n^3} + \frac{5}{n^4} + \frac{1}{n^5} = 1$$

Since $0 \leq 1 < \infty$, $(n+1)^5$ is $O(n^5)$.

1.1.2 R-1.22

Show that n is $o(n \log n)$.

Proof: Let $f(n) = n$ and $g(n) = n \log n$ so that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n}{n \log n} = \lim_{n\to\infty} \frac{1}{\log n} = 0$$

Since $0 = 0$, n is $o(n \log n)$.

1.1.3 R-1.23

Show that n^2 is $\omega(n)$.

Proof: Let $f(n) = n^2$ and $g(n) = \omega(n)$ so that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^2}{n} = \lim_{n\to\infty} n = \infty$$

Since $\infty = \infty$, n^2 is $\omega(n)$.