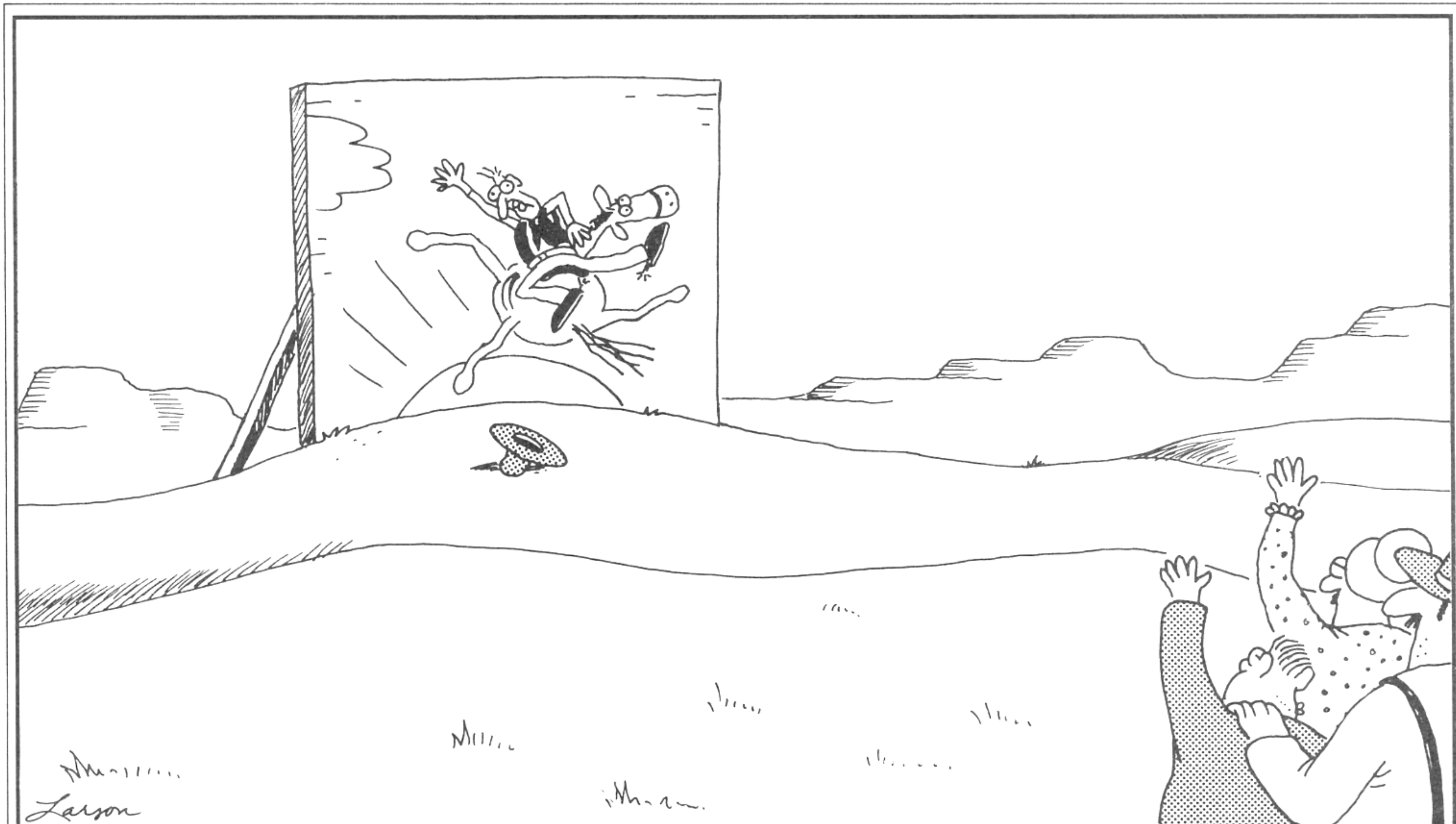
MIT 6.837 - Ray Tracing



The embarrassment of riding off into a fake sunset

Tony DeRose - *Math in the Movies*

Tony DeRose: Pixar Animation Studios - Senior Scientist

Date: **10-5-2004 (one week from today!)**

Time: 1:00 PM - 2:00 PM

Location: 32–D449 (Stata Center, Patil/Kiva)

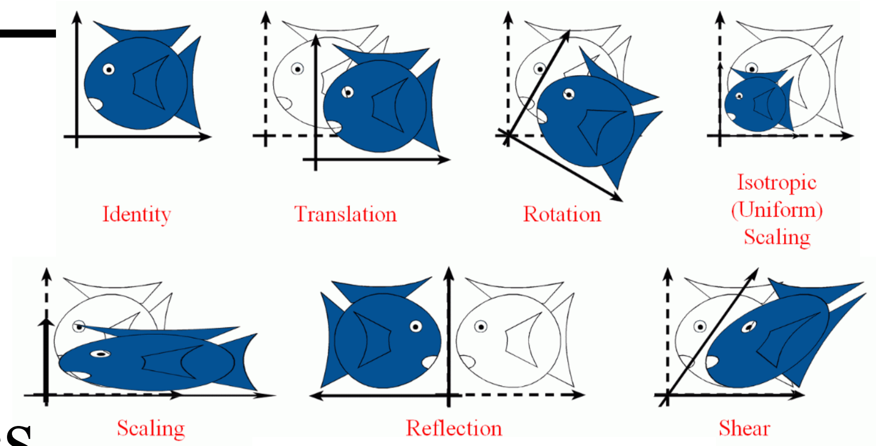
Film making is undergoing a digital revolution brought on by advances in areas such as computer technology, computational physics and computer graphics. This talk will provide a behind the scenes look at how fully digital films --- such as Pixar's "Monster's Inc" and "Finding Nemo" --- are made, with particular emphasis on the role that mathematics plays in the revolution.

Final Exam

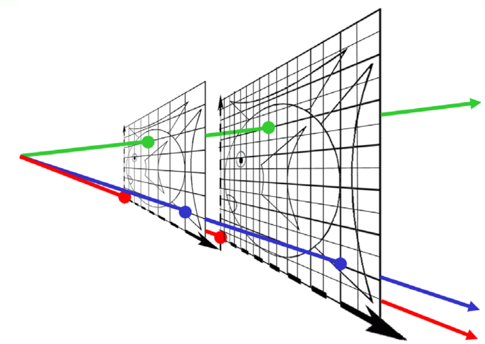
- … has been scheduled
- Thursday December 16th, 1:30-3:30pm
- DuPont
- Open Book

Last Week: Transformations

- Linear, affine and projective transforms



- Homogeneous coordinates

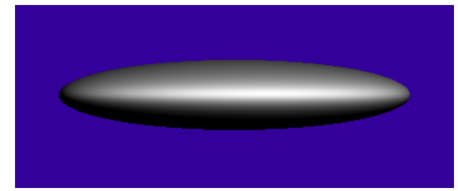


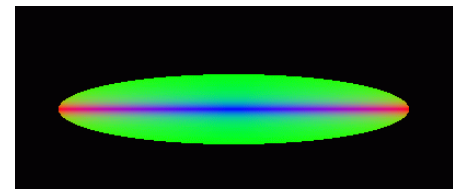
- Matrix notation
- Transformation composition is not commutative

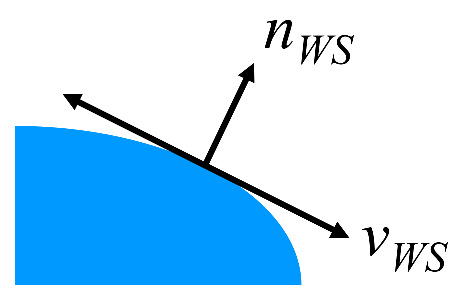
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Last Week: Transformations

- Transformations in Ray Tracing
  - Transforming the ray
    Remember: points & directions transform differently!
  - Normalizing direction & what to do with t
  - Normal transformation





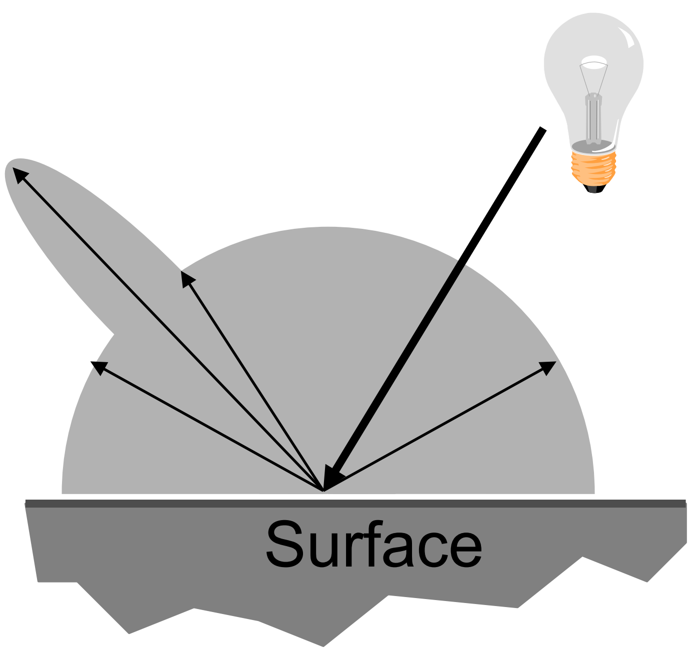


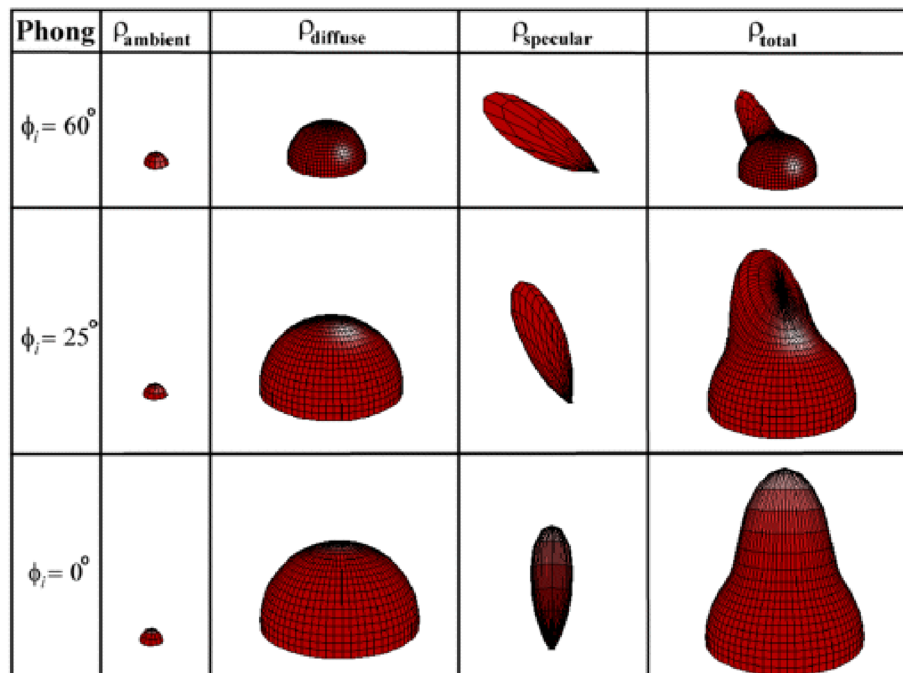
$$n_{WS}^{\mathbf{T}} = n_{OS} \ (\mathbf{M}^{-1})$$

Last Time: Local Illumination

- BRDF – Bidirectional Reflectance Distribution Function
- Phong Model - Sum of 3 components:
  - Diffuse Shading
  - Specular Highlight
  - Ambient Term

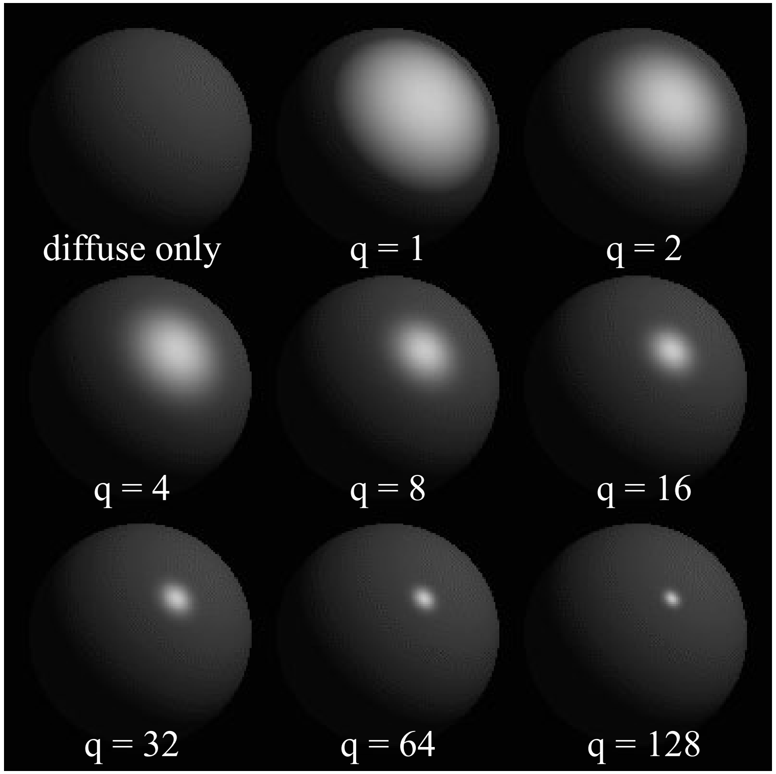
$$L_o = k_a + \left(k_d(\mathbf{n}\cdot\mathbf{l}) + k_s(\mathbf{v}\cdot\mathbf{r})^q\right)\frac{L_i}{r^2}$$



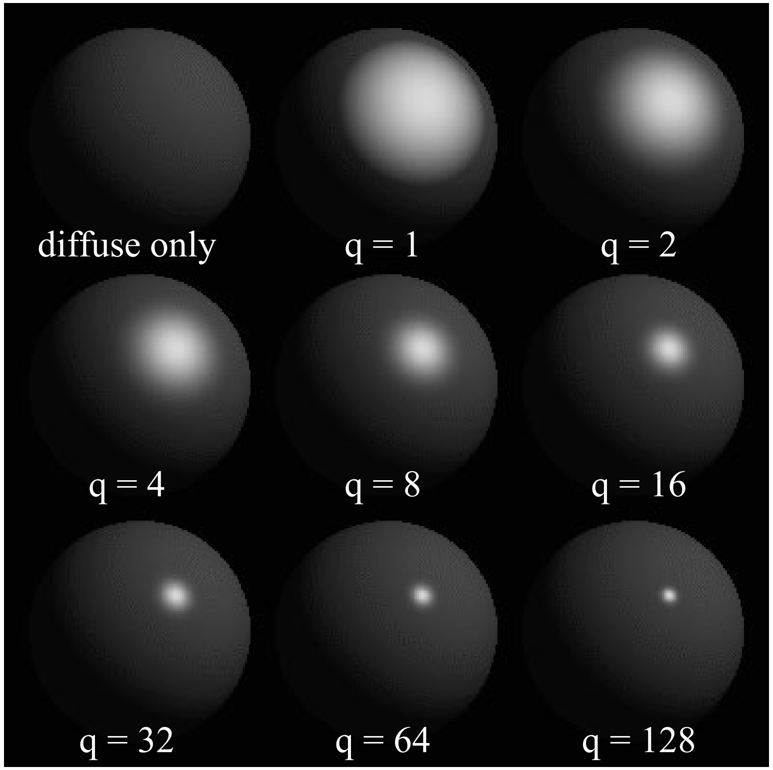


Phong Examples

- Shininess coefficient controls the "spread" of the specular highlight



Phong

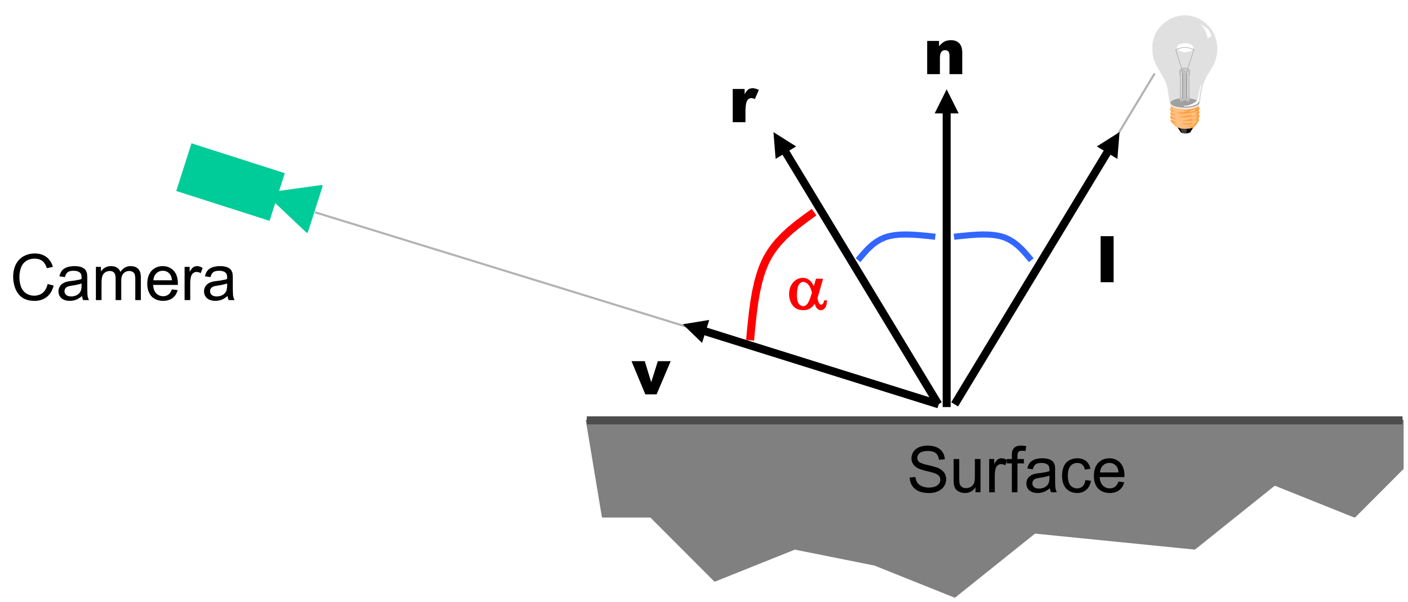


Blinn-Torrance
(scaled to approximate Phong)

The Phong Model

- Parameters
  - k_s: specular reflection coefficient
  - q : specular reflection exponent

$$L_o = k_s (\cos\alpha)^q \frac{L_i}{r^2} = k_s (\mathbf{v}\cdot\mathbf{r})^q \frac{L_i}{r^2}$$

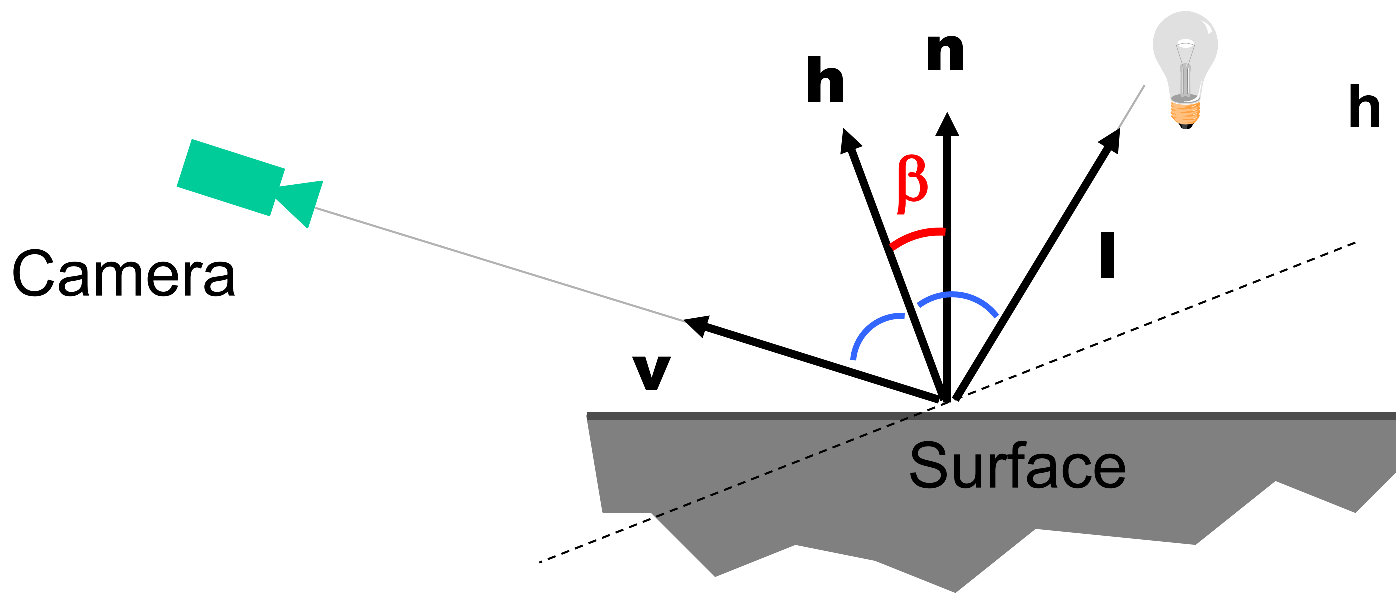


Blinn-Torrance Variation

- Parameters
  - k_s: specular reflection coefficient
  - q : specular reflection exponent

Implement this version of Phong (because it's what OpenGL uses & we want to match)

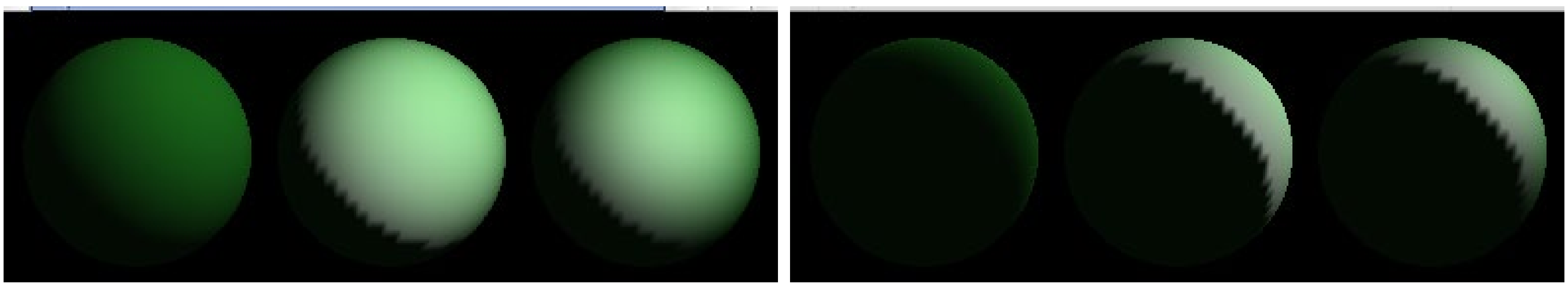
$$L_o = k_s (\cos\beta)^q \frac{L_i}{r^2} = k_s (\mathbf{n} \cdot \mathbf{h})^q \frac{L_i}{r^2}$$



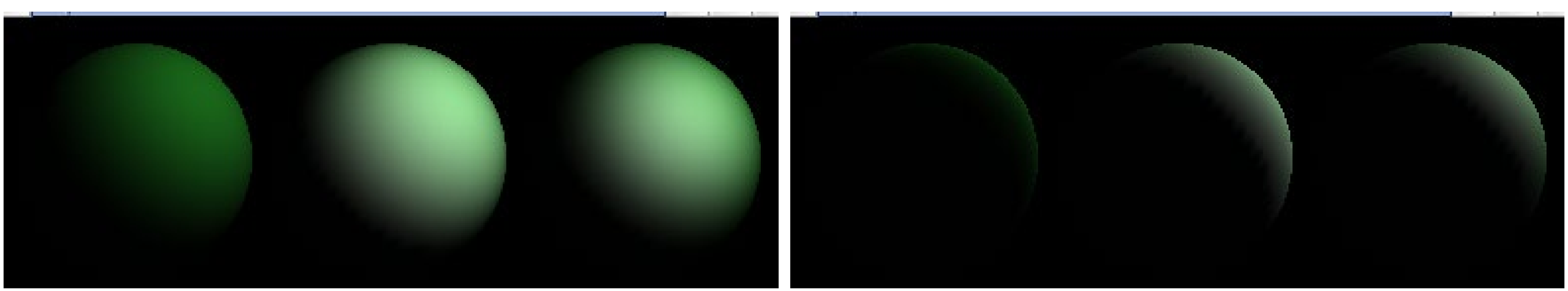
$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$

Additional Phong Clamping Term

- Surfaces facing away from the light should not be lit (if $N \cdot L < 0$)



- Scale by dot product to avoid a sharp edge at the light's grazing angle: specular *= $\max(N \cdot L, 0)$

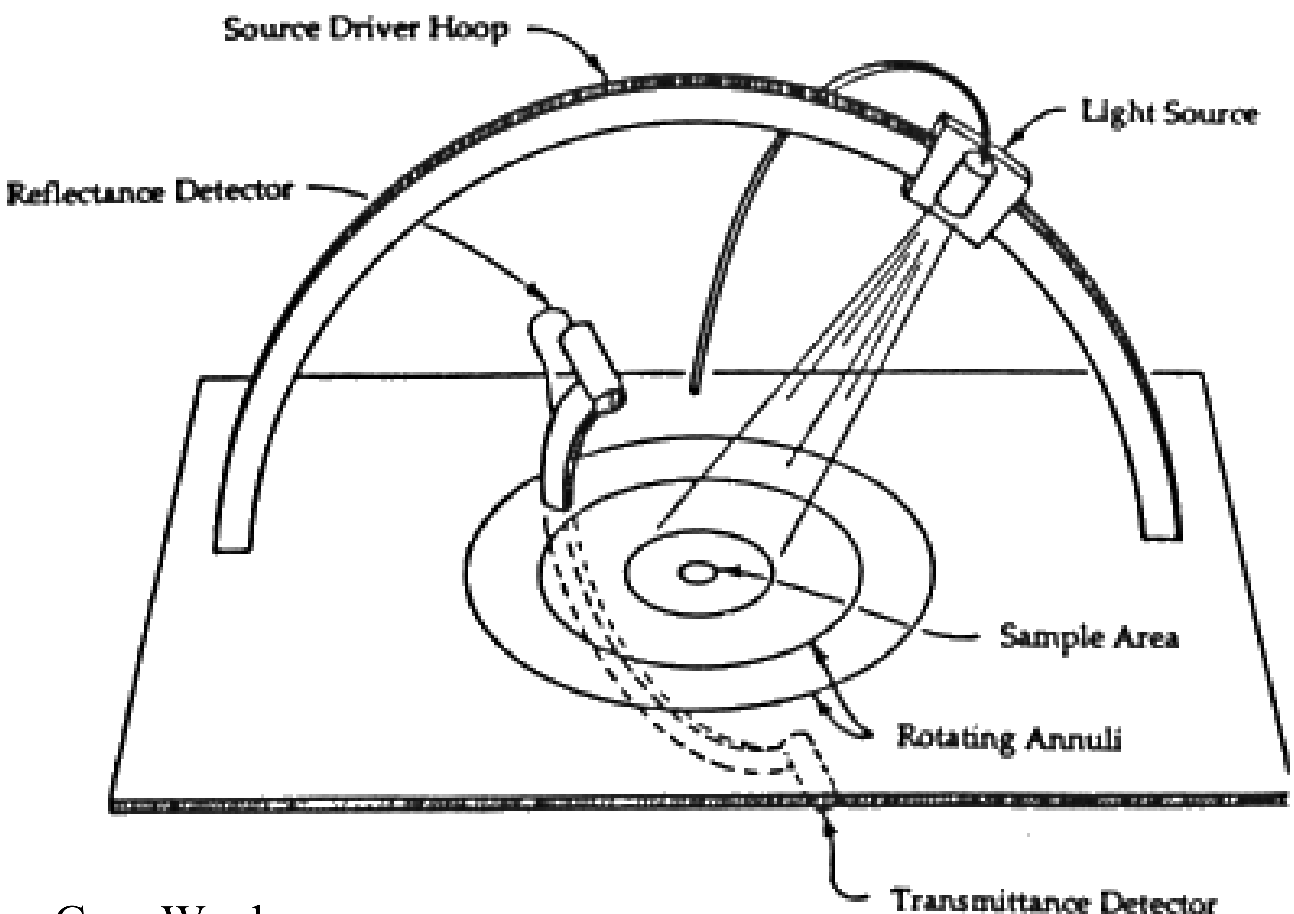


BRDFs in the Movie Industry

- http://www.virtualcinematography.org/publications/acrobat/BRDF-s2003.pdf
- For the Matrix movies
- Agent Smith clothes are CG, with measured BRDF

How Do We Obtain BRDFs?

- Gonioreflectometer
  - 4 degrees of freedom



Source: Greg Ward

BRDFs in the Movie Industry

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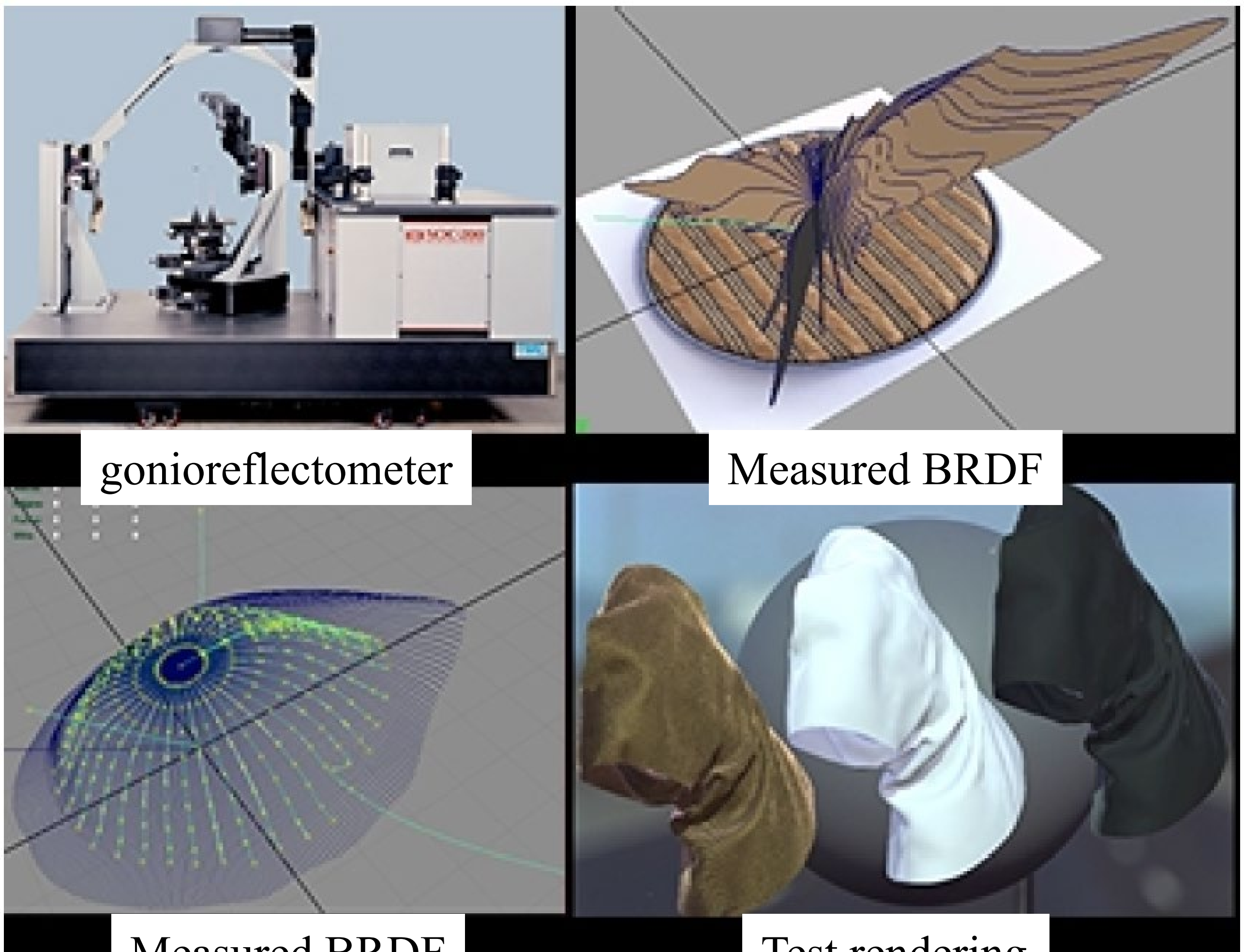


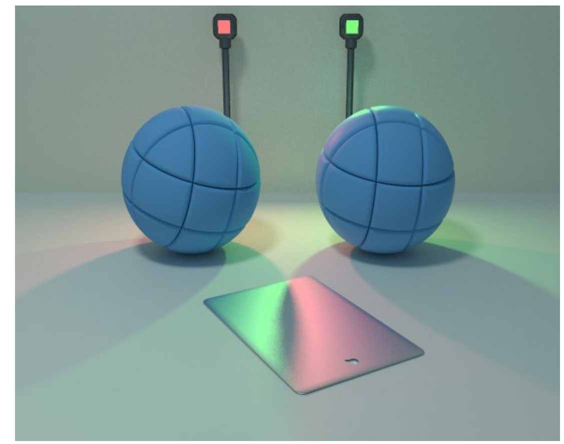
BRDFs in the Movie Industry

Photo CG Photo CG

BRDF Models

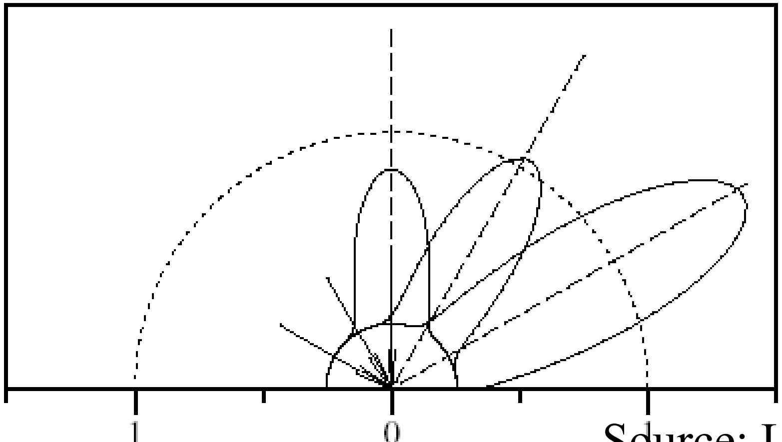
- Phenomenological
  - Phong [75]
    - Blinn [77]
  - Ward [92]
  - Lafortune et al. [97]
  - Ashikhmin et al. [00]
- Physical
  - Cook-Torrance [81]
  - He et al. [91]

Roughly increasing computation time



Fresnel Reflection

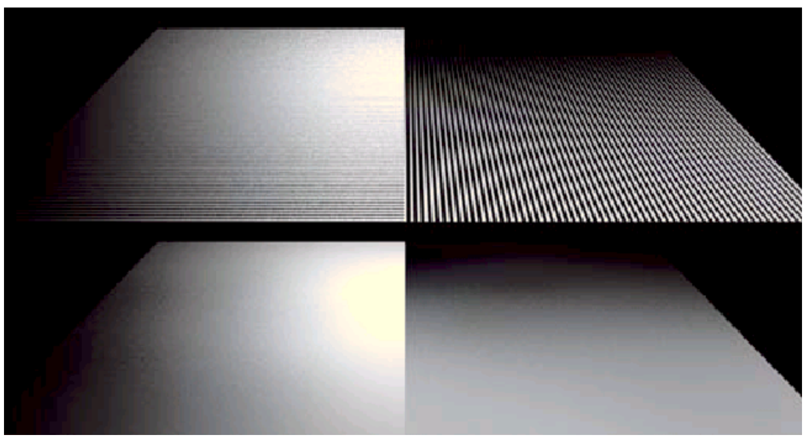
- Increasing specularity near grazing angles.



Source: Lafortune et al. 97

Anisotropic BRDFs

- Surfaces with strongly oriented microgeometry elements
- Examples:
  - brushed metals,
  - hair, fur, cloth, velvet



Source: Westin et.al 92

Questions?




Today: Ray Tracing

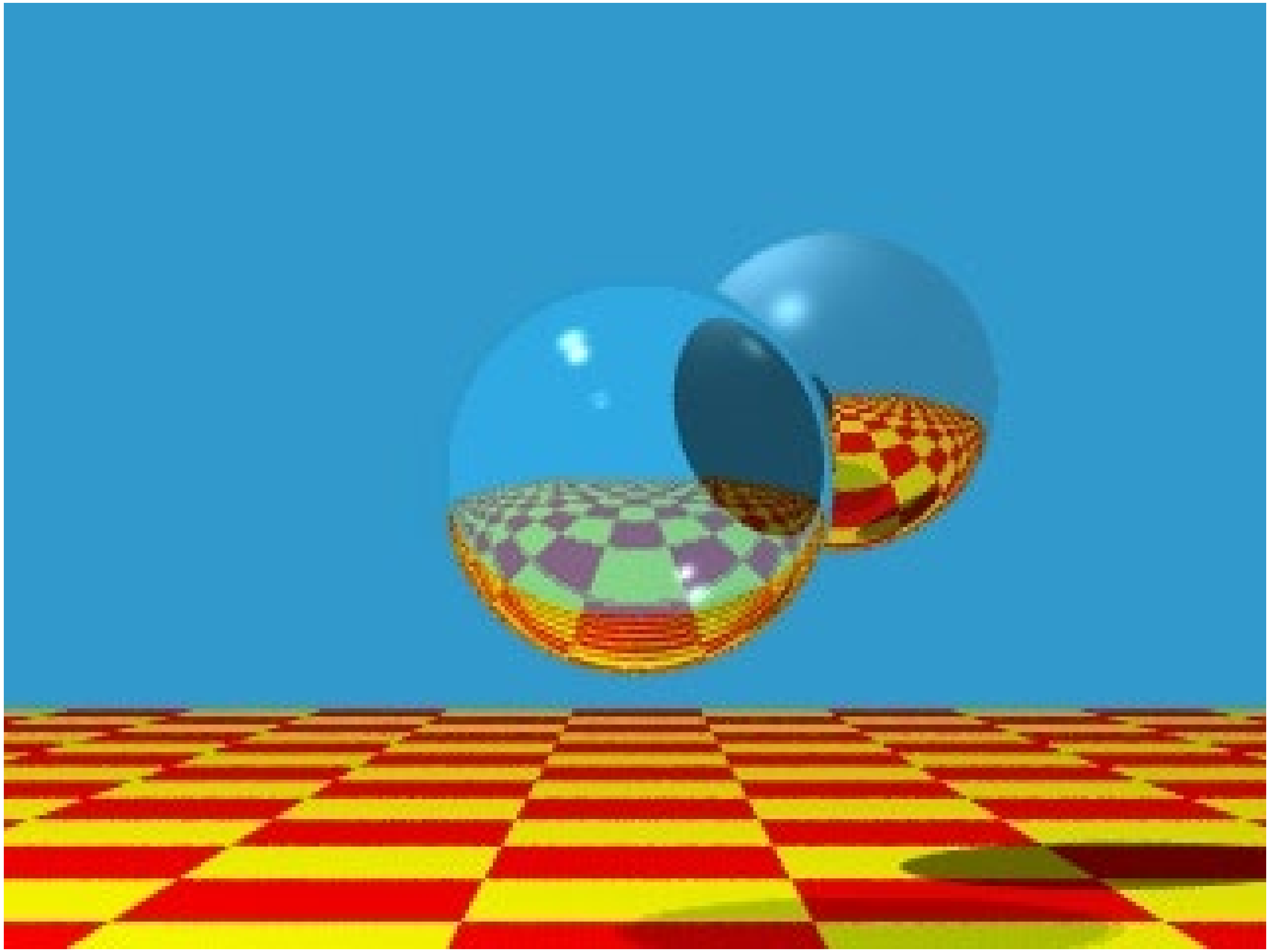
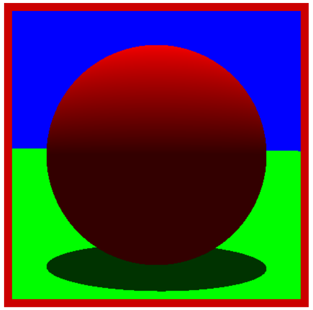
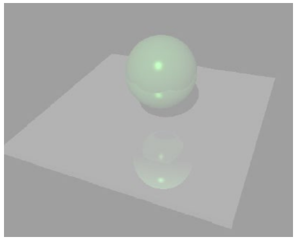


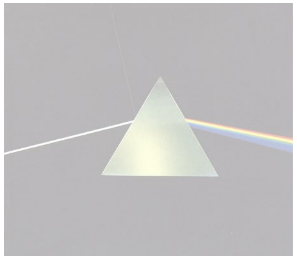
Image by Turner Whitted

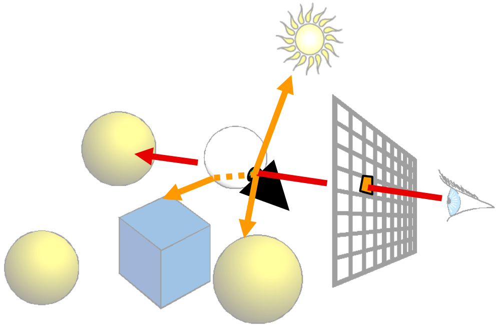
Overview of Today

- Shadows
- Reflection
- Refraction
- Recursive Ray Tracing









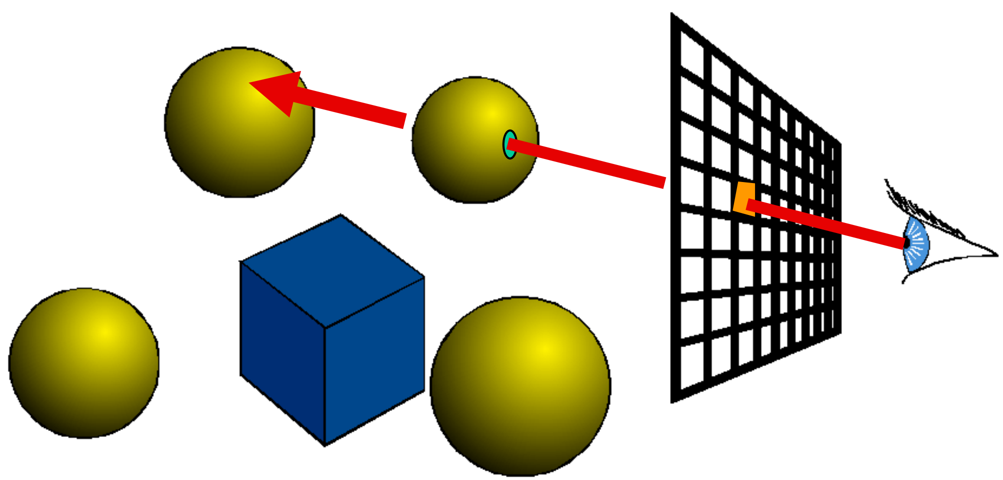
Ray Casting (a.k.a. Ray Shooting)

```
for every pixel
   construct a ray
  for every object
      intersect ray with object
```

Complexity?

O(n * m)

n = number of objects,   m = number of pixels



Ray Casting with Phong Shading

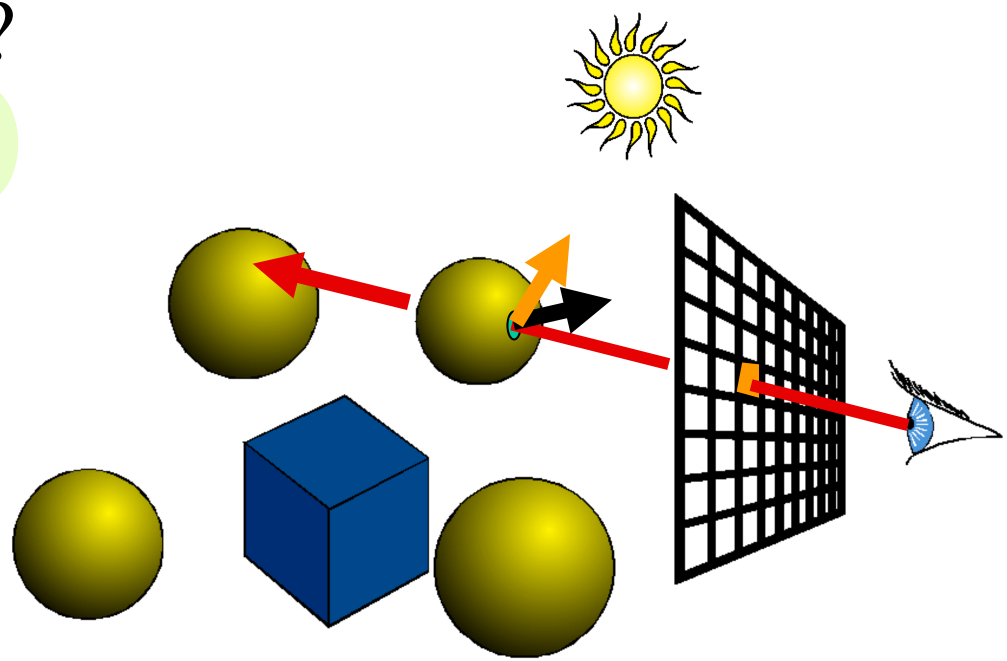
When you’ve found the closest intersection:

```
color = ambient*hit->getMaterial()->getDiffuseColor()
for every light
    color += hit->getMaterial()->Shade
                (ray, hit, directionToLight, lightColor)
return color
```

Complexity?

O(n * m * l)

l = number of lights

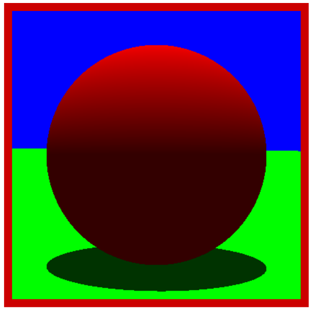


Questions?

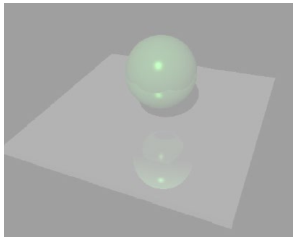
- Image computed using the RADIANCE system by Greg Ward

Overview of Today

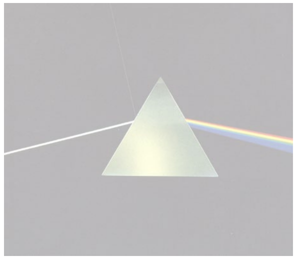
- Shadows



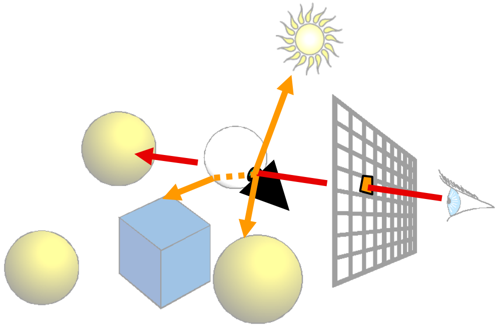
- Reflection



- Refraction

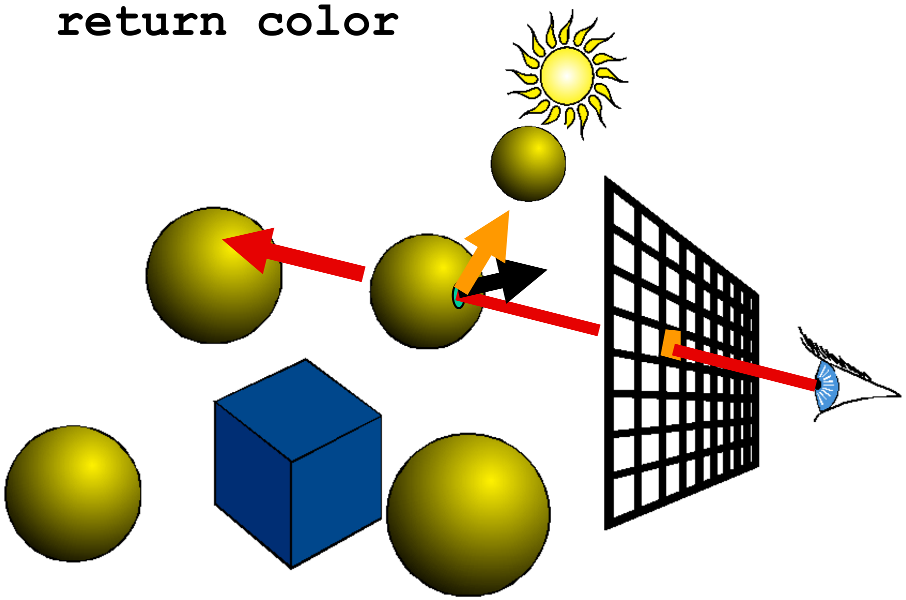


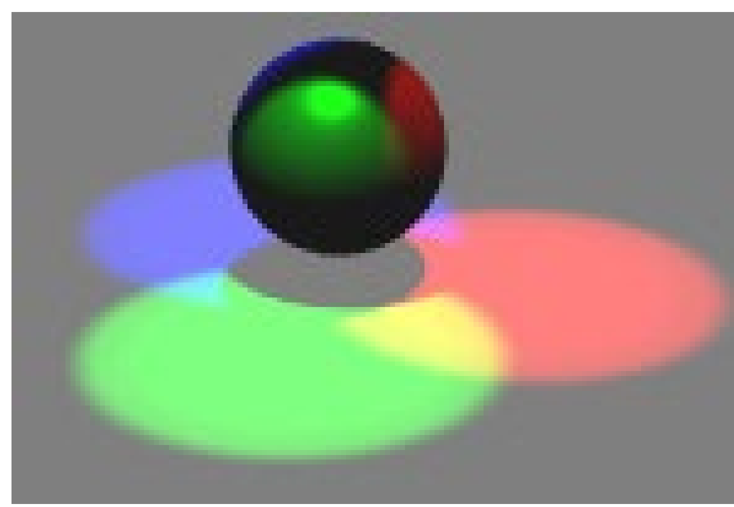
- Recursive Ray Tracing



How Can We Add Shadows?

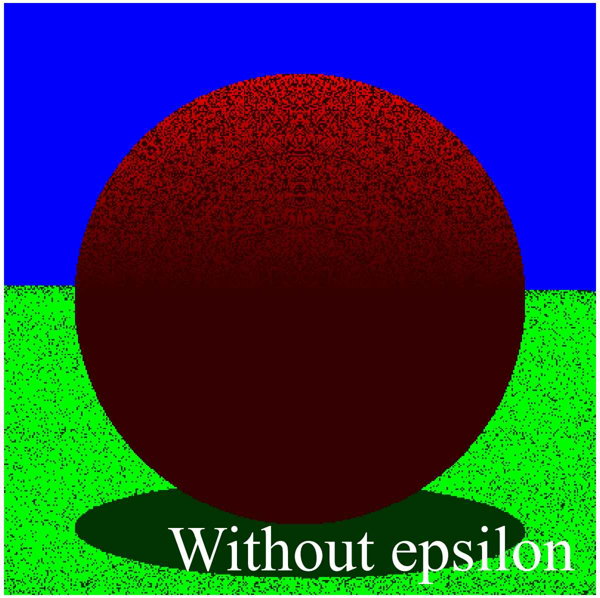
```
color = ambient*hit->getMaterial()->getDiffuseColor()
for every light
   Ray ray2(hitPoint, directionToLight)
   Hit hit2(distanceToLight, NULL, NULL)
   For every object
      object->intersect(ray2, hit2, 0)
   if (hit2->getT() = distanceToLight)
      color += hit->getMaterial()->Shade
                  (ray, hit, directionToLight, lightColor)
return color
```

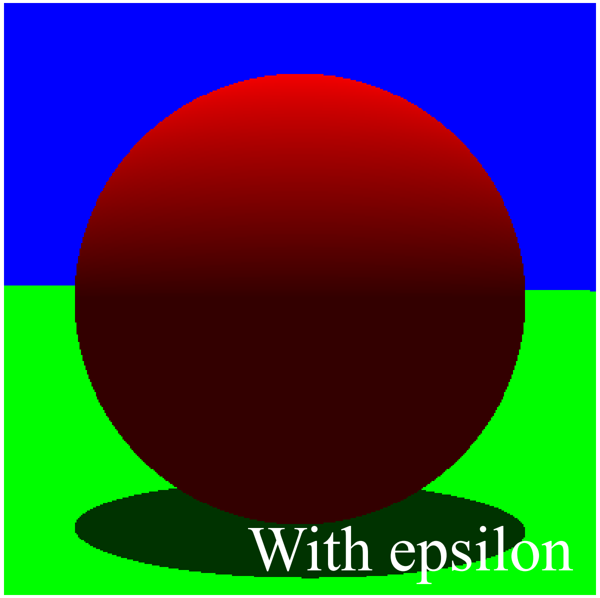




Problem: Self-Shadowing

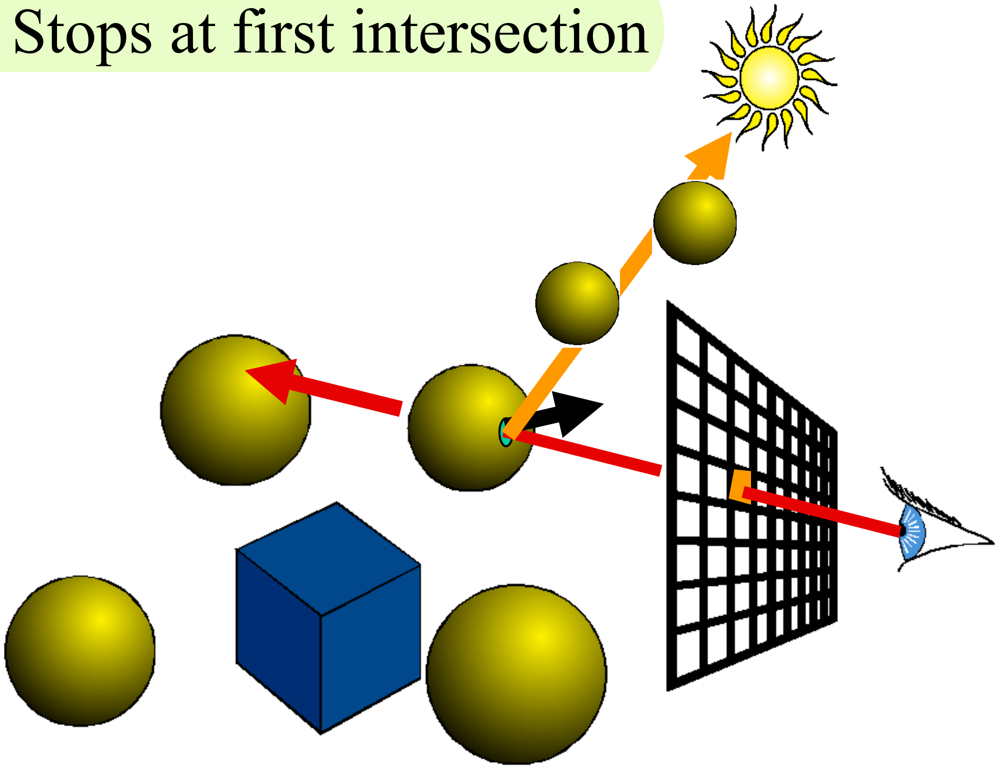
```
color = ambient*hit->getMaterial()->getDiffuseColor()
for every light
   Ray ray2(hitPoint, directionToLight)
   Hit hit2(distanceToLight, NULL, NULL)
   For every object
      object->intersect(ray2, hit2, epsilon)
   if (hit2->getT() = distanceToLight)
      color += hit->getMaterial()->Shade
                  (ray, hit, directionToLight, lightColor)
return color
```





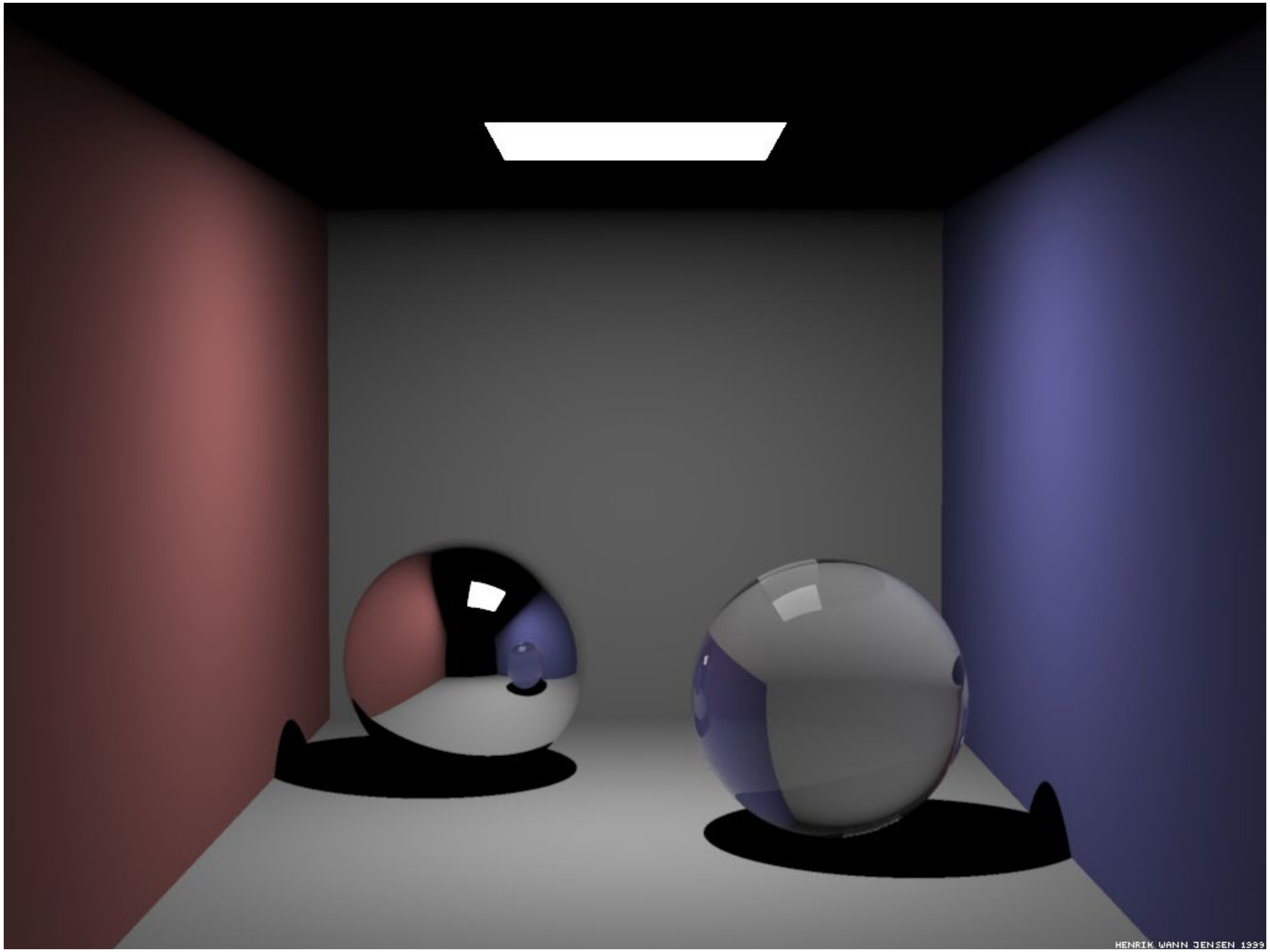
Shadow Optimization

- Shadow rays are special: Can we accelerate our code?
- We only want to know whether there is an intersection, *not* which one is closest
- Special routine `Object3D::intersectShadowRay()`
  - Stops at first intersection



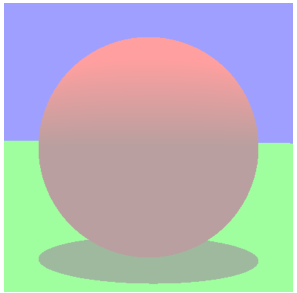
Questions?

- Image Henrik Wann Jensen

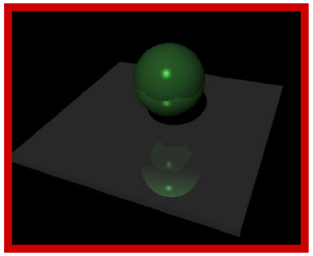


Overview of Today

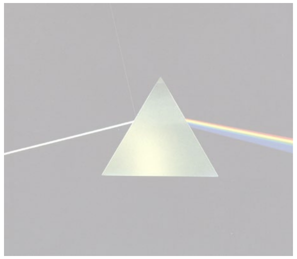
- Shadows



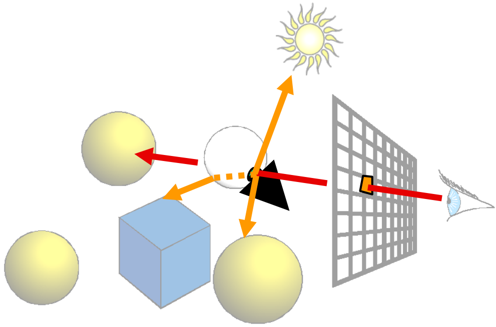
- Reflection



- Refraction

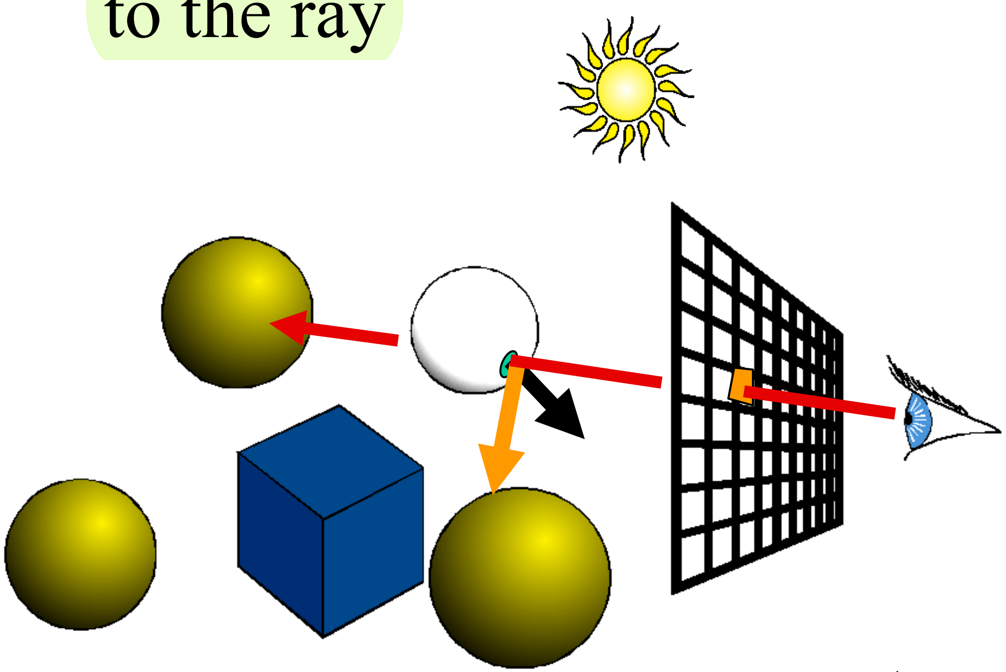


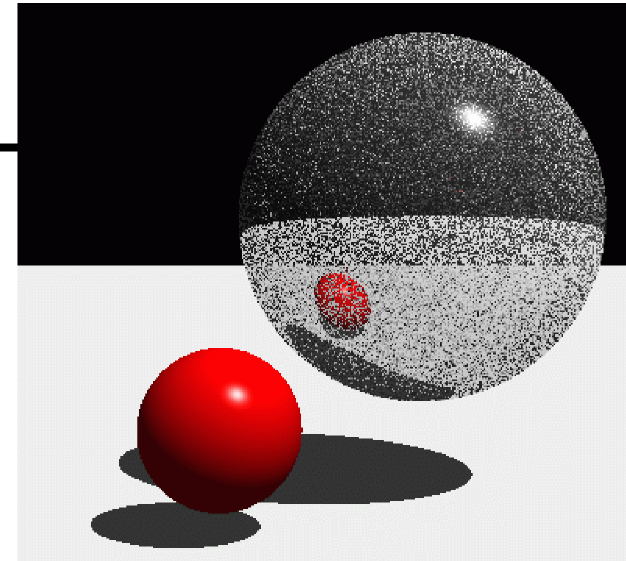
- Recursive Ray Tracing



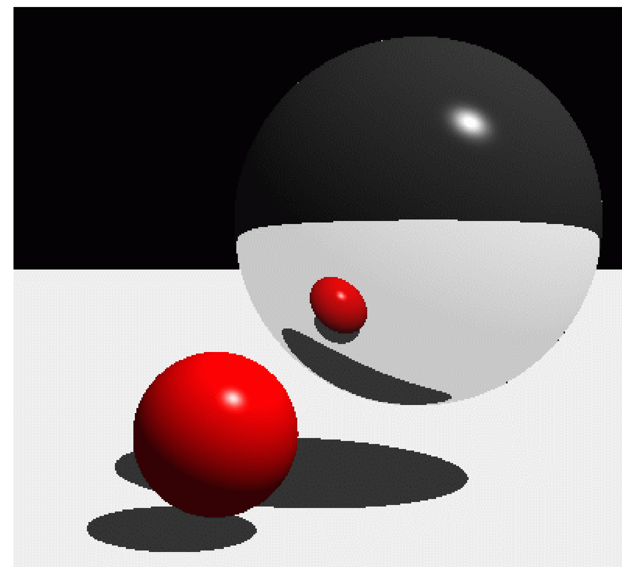
Mirror Reflection

- Cast ray symmetric with respect to the normal
- Multiply by reflection coefficient (color)
- Don't forget to add epsilon to the ray





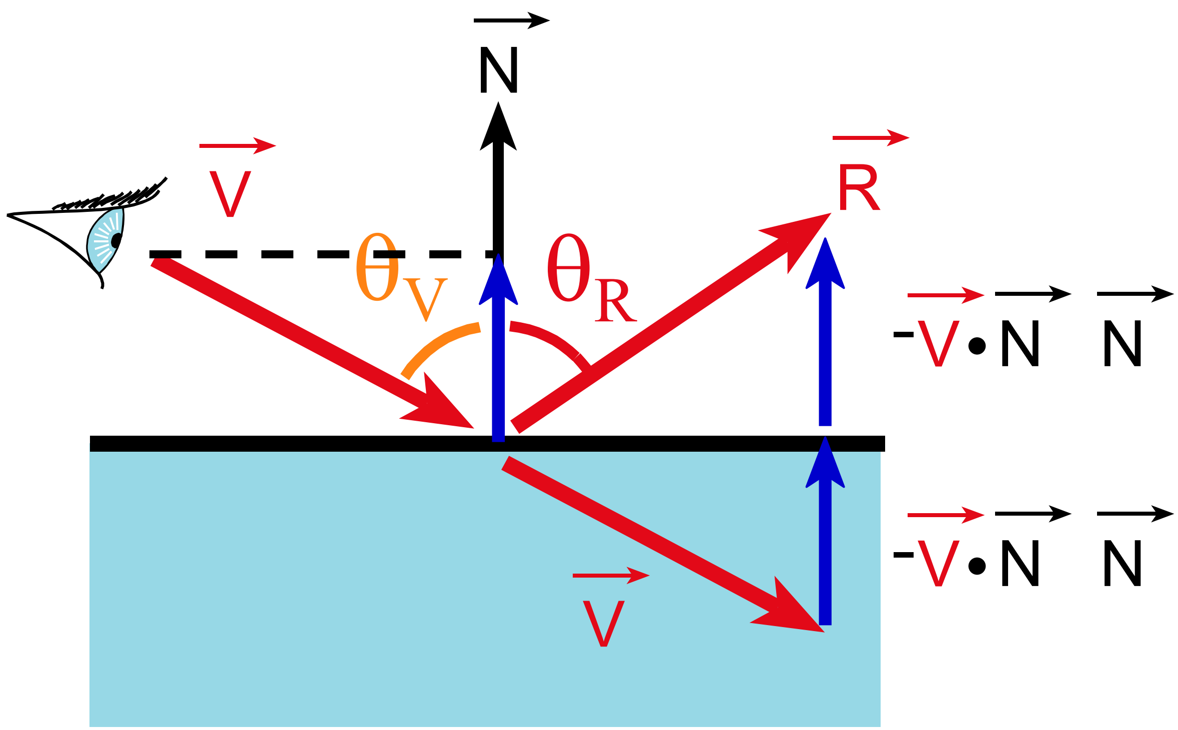
Without epsilon



With epsilon

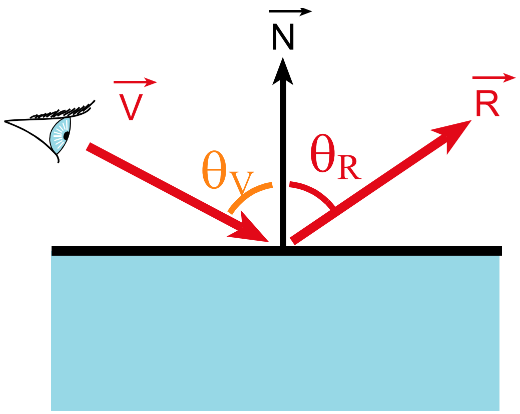
Reflection

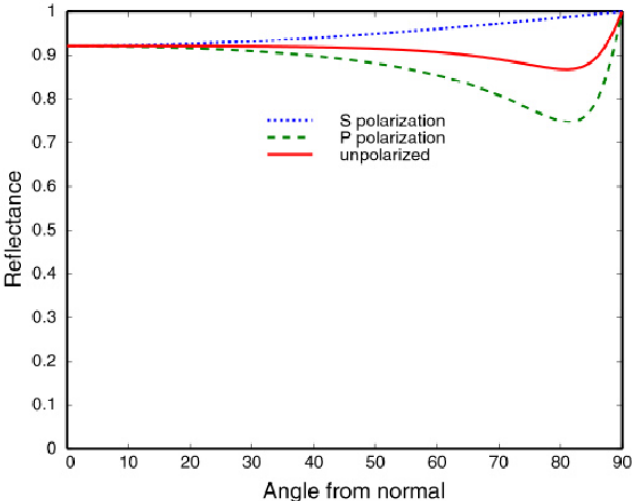
- Reflection angle = view angle
- $\mathbf{R} = \mathbf{V} - 2\,(\mathbf{V} \cdot \mathbf{N})\,\mathbf{N}$



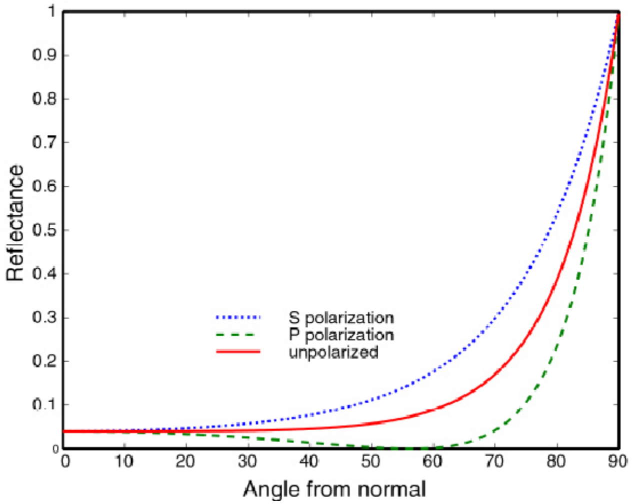
Amount of Reflection

- Traditional ray tracing (hack)
  - Constant **`reflectionColor`**
- More realistic:
  - Fresnel reflection term (more reflection at grazing angle)
  - Schlick's approximation: $R(\theta)=R_0+(1-R_0)(1-\cos\theta)^5$





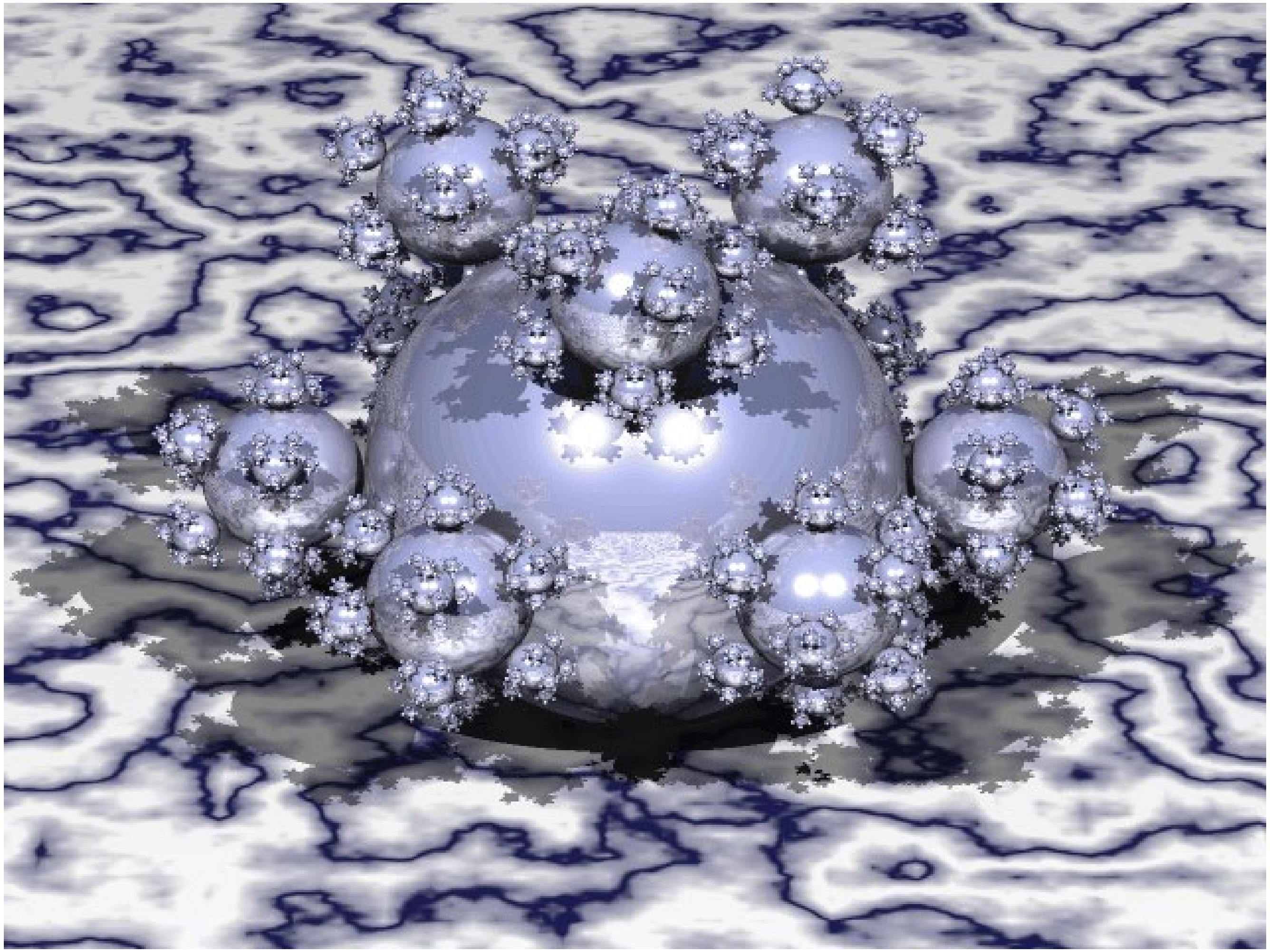
metal



Dielectric (glass)

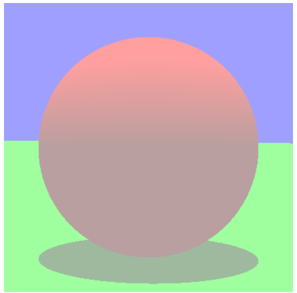
Questions?

- Image by Henrik Wann Jensen

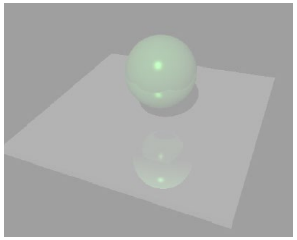


Overview of Today

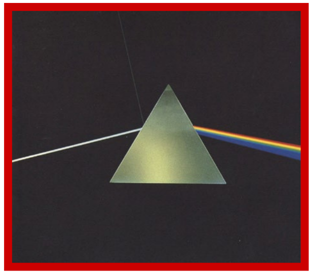
- Shadows



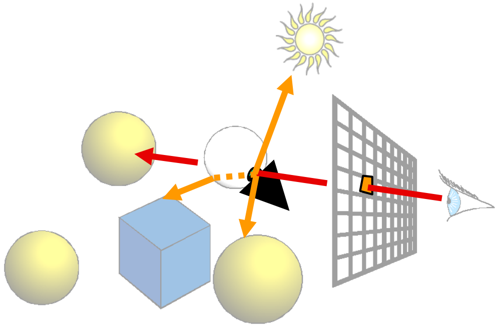
- Reflection



- Refraction

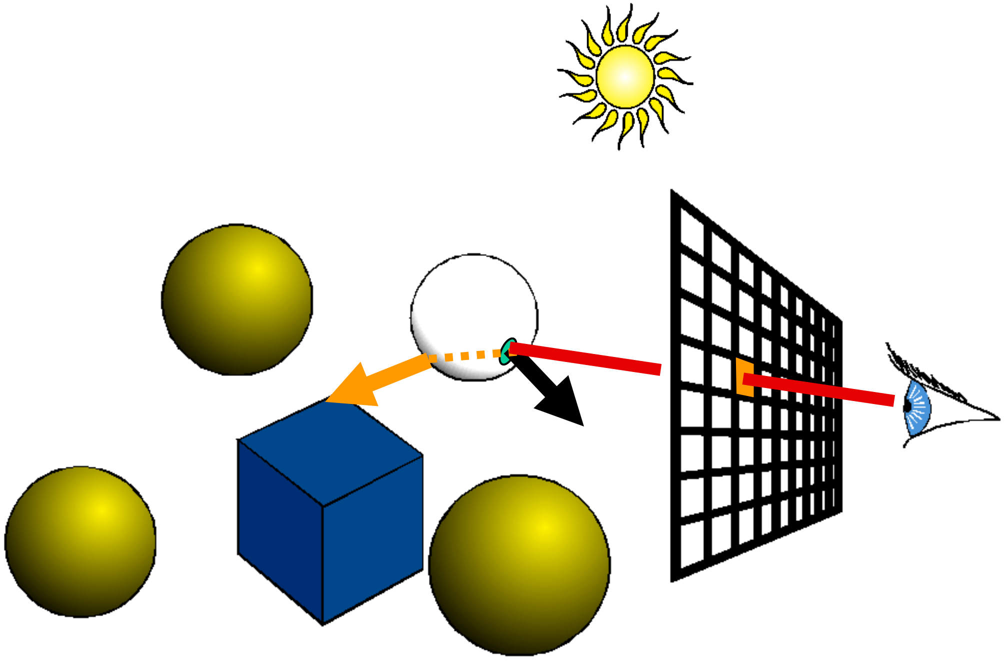


- Recursive Ray Tracing

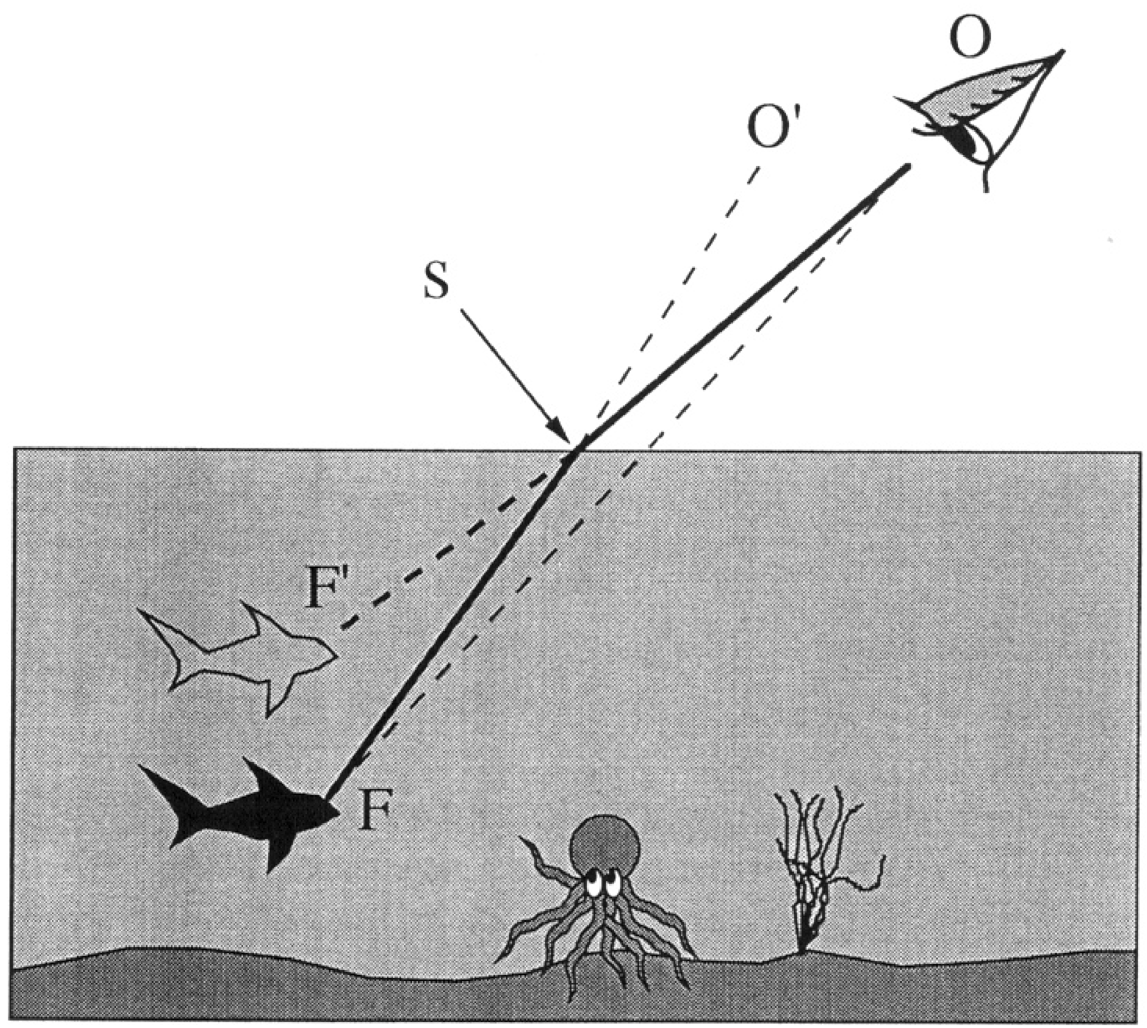


Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)

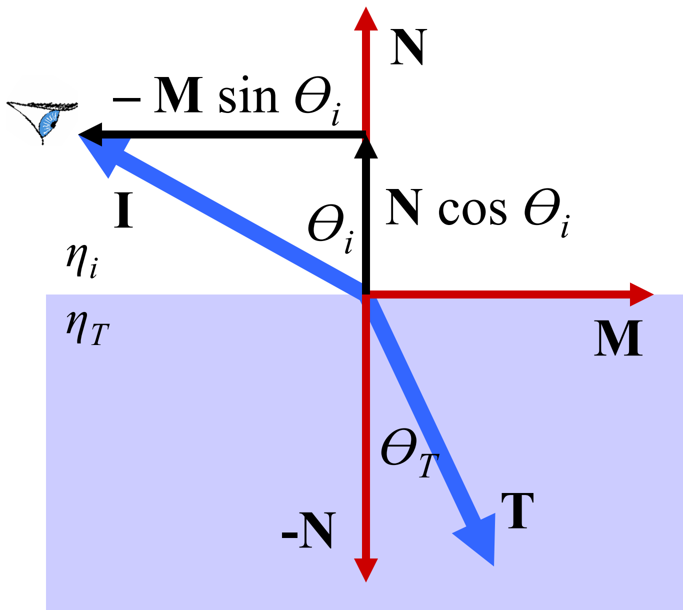


Qualitative Refraction



From “Color and Light in Nature” by Lynch and Livingston

Refraction



$$\mathbf{I} = \mathbf{N}\cos\Theta_i - \mathbf{M}\sin\Theta_i$$

$$\mathbf{M} = (\mathbf{N}\cos\Theta_i - \mathbf{I}) / \sin\Theta_i$$

$$\begin{aligned}
\mathbf{T} &= -\mathbf{N}\cos\Theta_T + \mathbf{M}\sin\Theta_T \\
&= -\mathbf{N}\cos\Theta_T + (\mathbf{N}\cos\Theta_i - \mathbf{I})\sin\Theta_T / \sin\Theta_i \\
&= -\mathbf{N}\cos\Theta_T + (\mathbf{N}\cos\Theta_i - \mathbf{I})\,\eta_r \\
&= [\,\eta_r\cos\Theta_i - \cos\Theta_T\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r\cos\Theta_i - \sqrt{1 - \sin^2\Theta_T}\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r\cos\Theta_i - \sqrt{1 - \eta_r^2\sin^2\Theta_i}\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r\cos\Theta_i - \sqrt{1 - \eta_r^2(1 - \cos^2\Theta_i)}\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r(\mathbf{N}\cdot\mathbf{I}) - \sqrt{1 - \eta_r^2(1 - (\mathbf{N}\cdot\mathbf{I})^2)}\,]\,\mathbf{N} - \eta_r\,\mathbf{I}
\end{aligned}$$

Snell-Descartes Law:

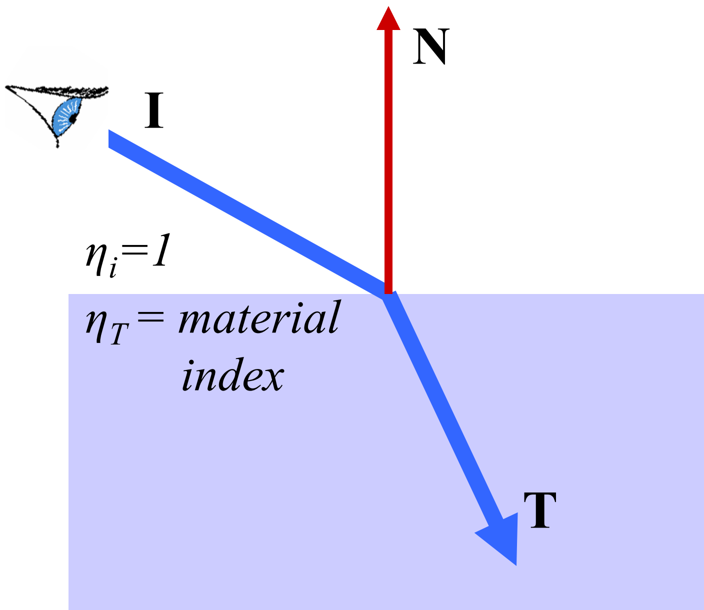
$$\eta_i \sin\Theta_i = \eta_T \sin\Theta_T$$

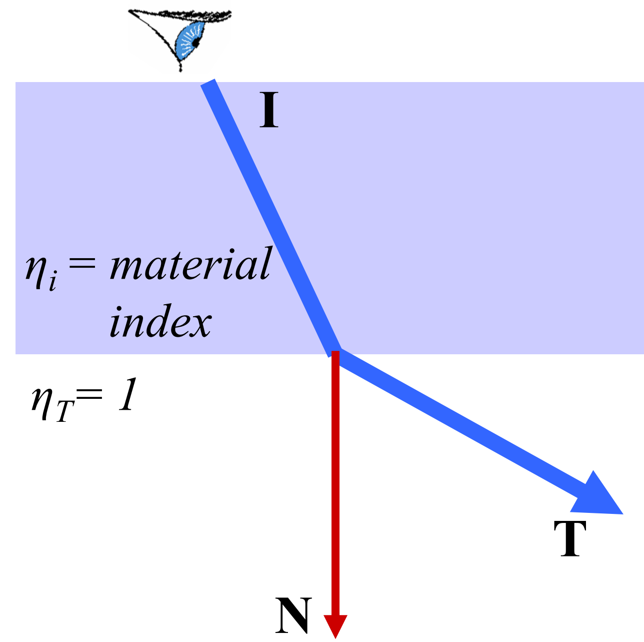
$$\frac{\sin\Theta_T}{\sin\Theta_i} = \frac{\eta_i}{\eta_T} = \eta_r$$

- **Total internal reflection when the square root is imaginary**
- **Don't forget to normalize!**

Refraction & the Sidedness of Objects

- Make sure you know whether you're entering or leaving the transmissive material:





- Note: We won't ask you to trace rays through intersecting transparent objects

Total Internal Reflection

Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air, Section 1.9. (Photo by D. Granger)

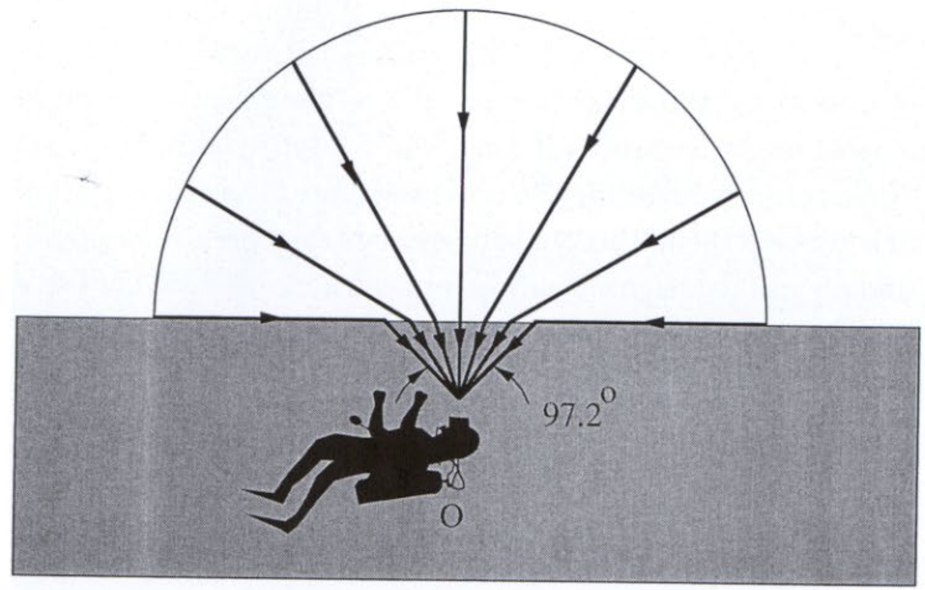


Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

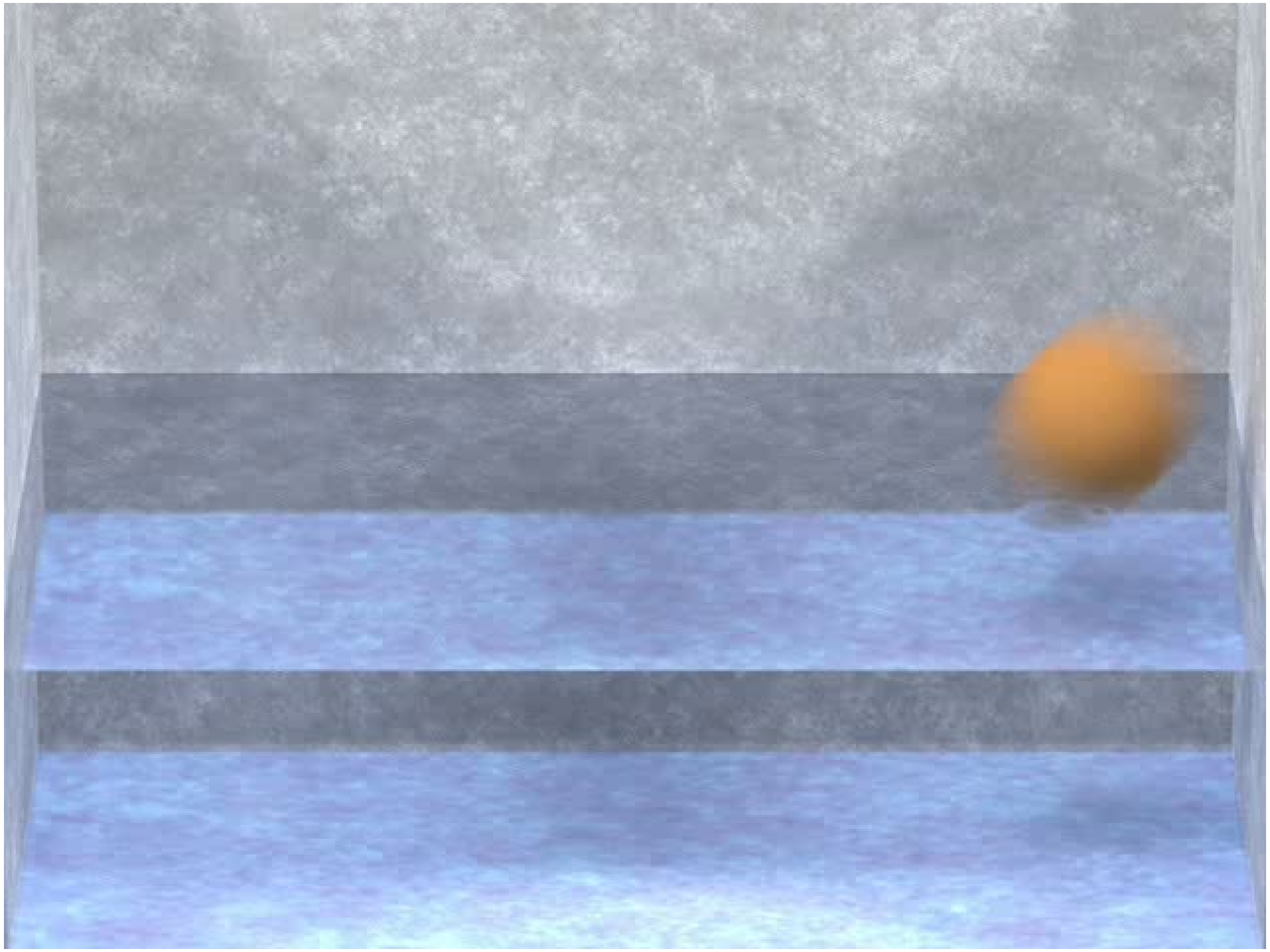
From "Color and Light in Nature" by Lynch and Livingston

Cool Refraction Demo

- Enright, D., Marschner, S. and Fedkiw, R.,

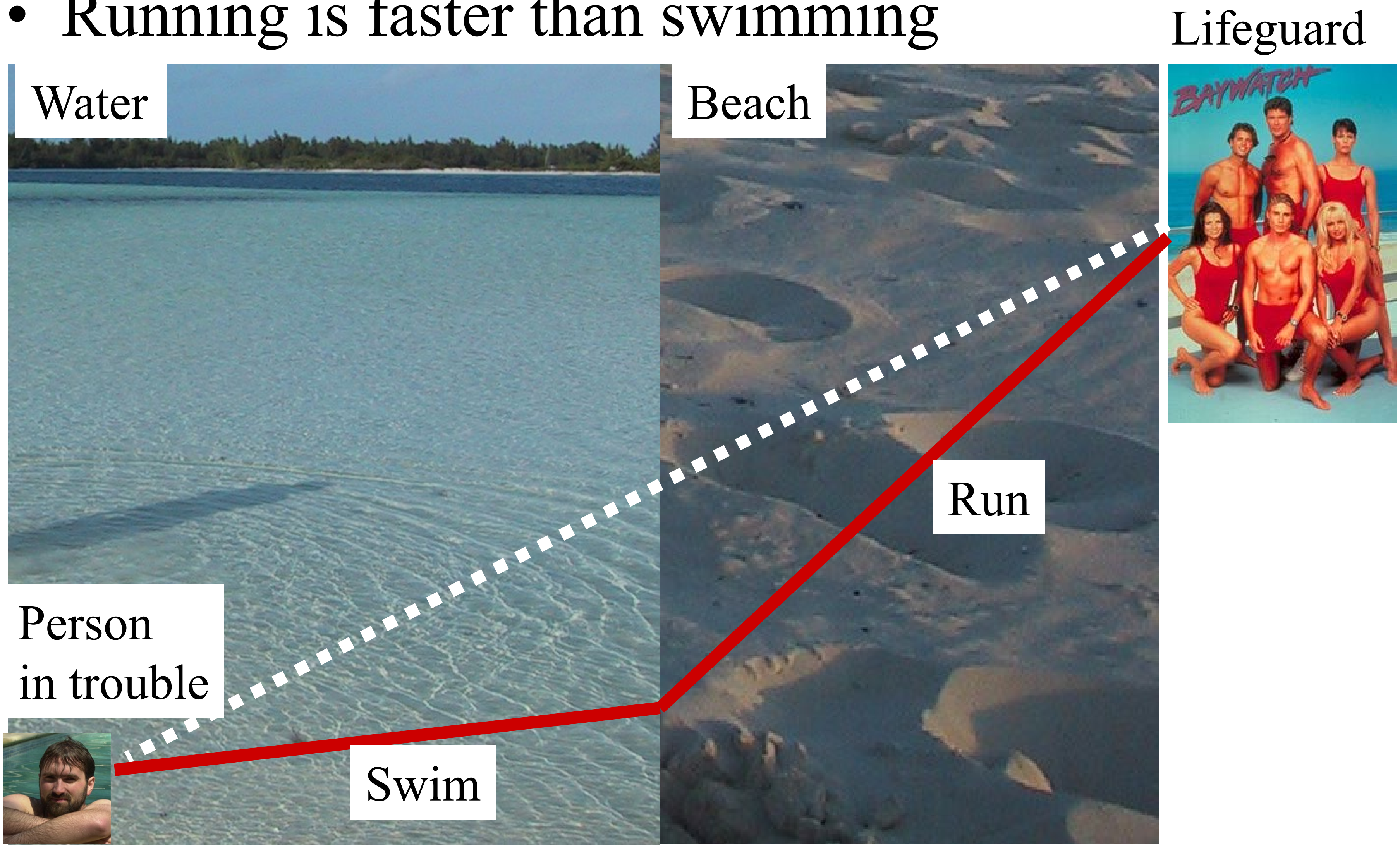
Cool Refraction Demo

- Enright, D., Marschner, S. and Fedkiw, R.,



Refraction and the Lifeguard Problem

- Running is faster than swimming

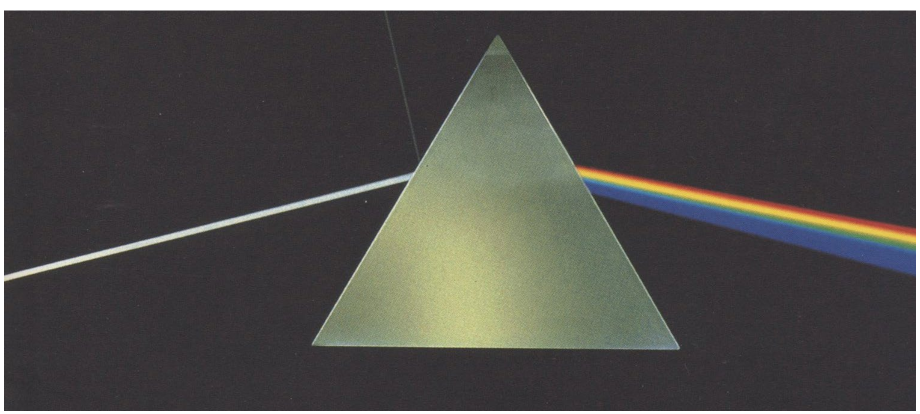


How does a Rainbow Work?

- From “Color and Light in Nature” by Lynch and Livingstone

Wavelength

- Refraction is wavelength-dependent
  - Refraction increases as the wavelength of light decreases
  - violet and blue experience more bending than orange and red
- Newton's experiment
- Usually ignored in graphics

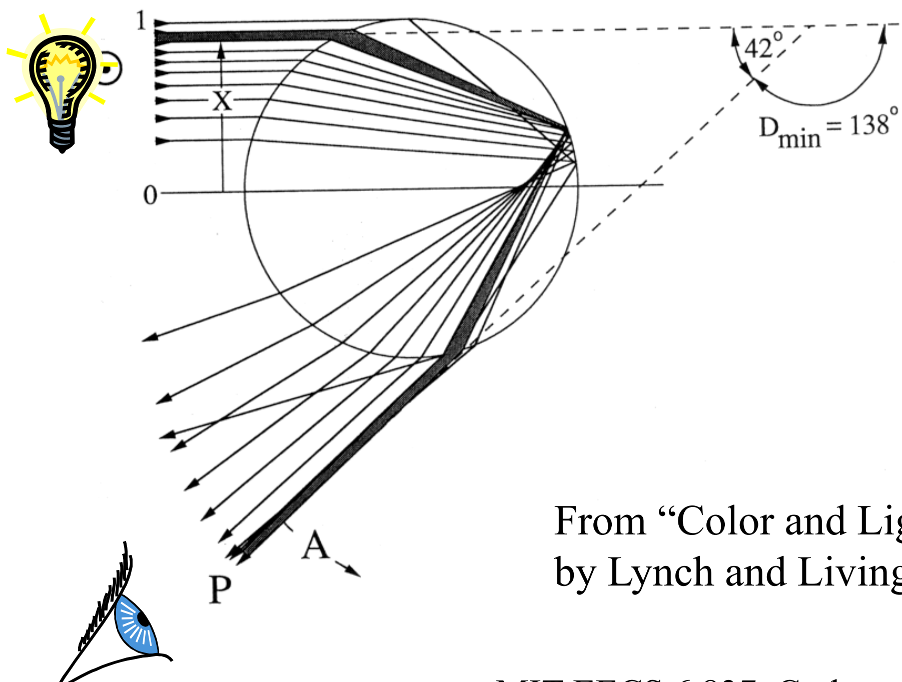


Pink Floyd, *The Dark Side of the Moon*

Pittoni, 1725, Allegory to Newton

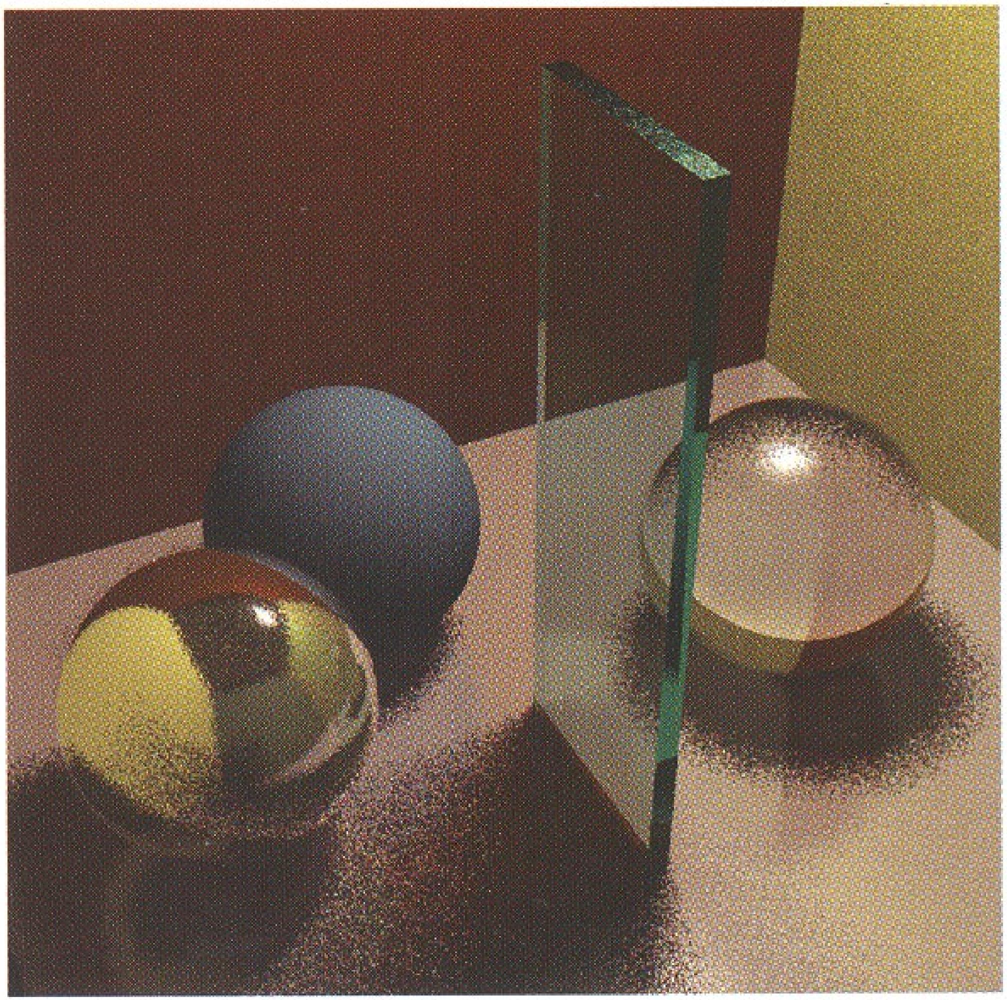
Rainbow

- Refraction depends on wavelength
- Rainbow is caused by refraction + internal reflection + refraction
- Maximum for angle around 42 degrees



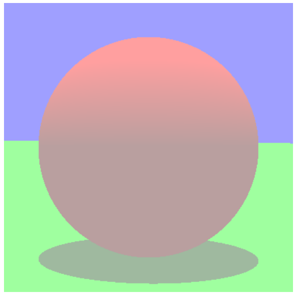
From "Color and Light in Nature" by Lynch and Livingstone

Questions?

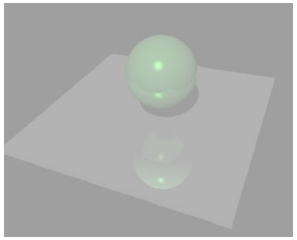


Overview of Today

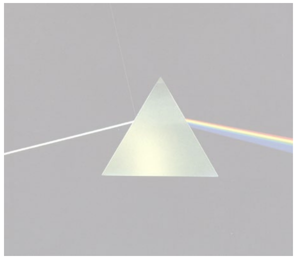
- Shadows



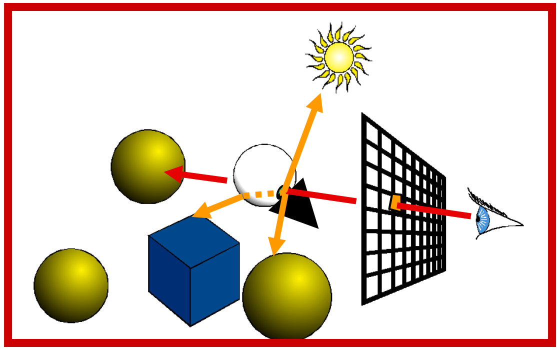
- Reflection



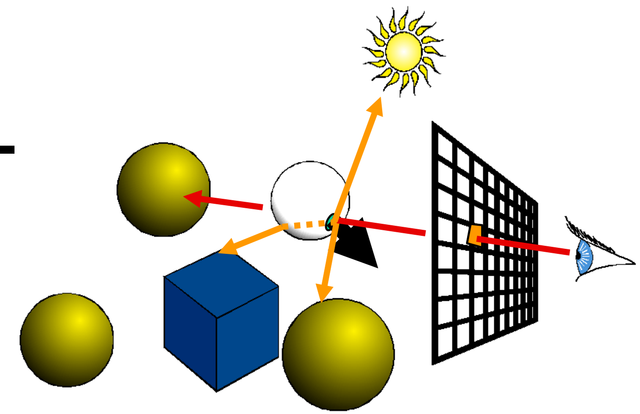
- Refraction



- Recursive Ray Tracing



Recap: Ray Tracing



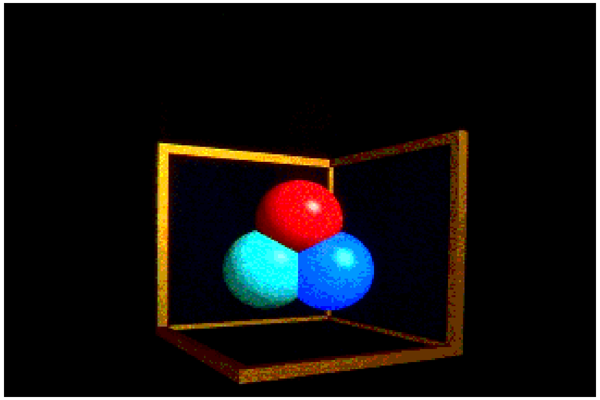
```
trace ray
    Intersect all objects
    color = ambient term
    For every light
        cast shadow ray
        color += local shading term
    If mirror
        color += color_refl *
            trace reflected ray
    If transparent
        color += color_trans *
            trace transmitted ray
```

- ***Does it ever end?***

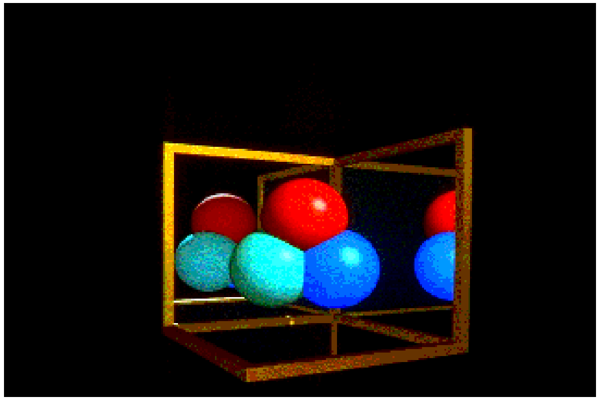
Stopping criteria:

- Recursion depth
  - Stop after a number of bounces
- Ray contribution
  - Stop if reflected / transmitted contribution becomes too small

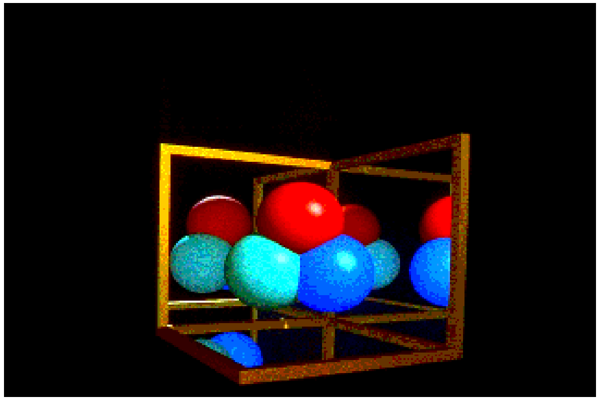
Recursion For Reflection



0 recursion

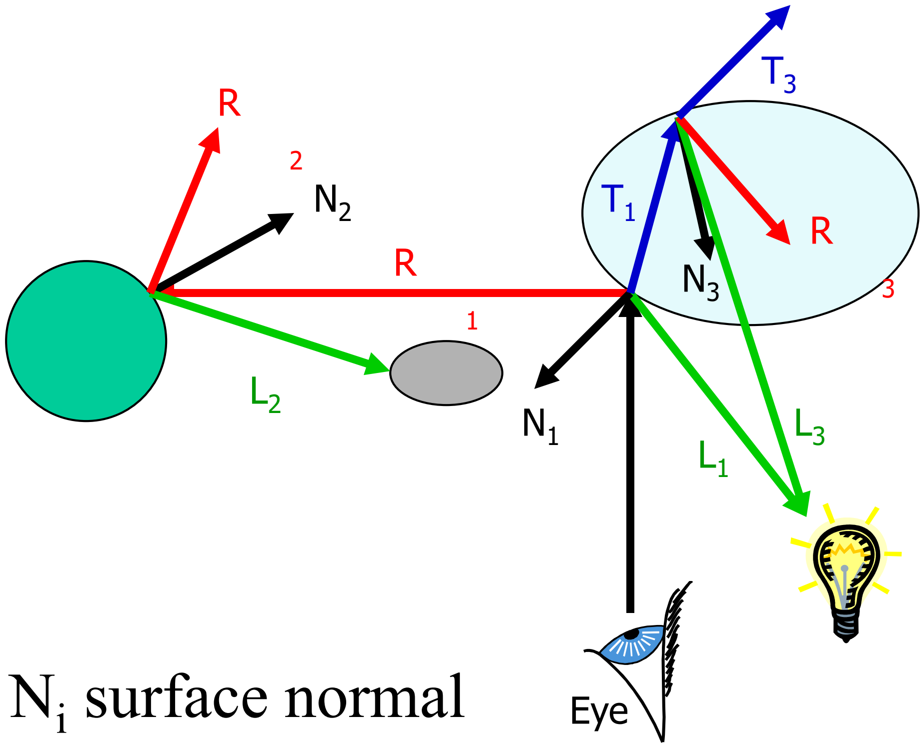


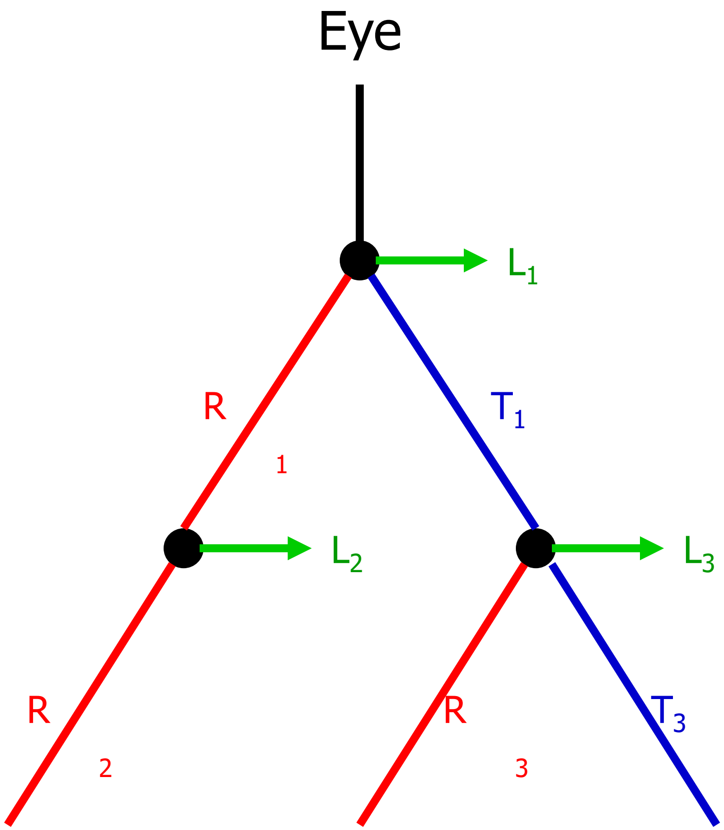
1 recursion



2 recursions

The Ray Tree





N_i surface normal

R_i reflected ray

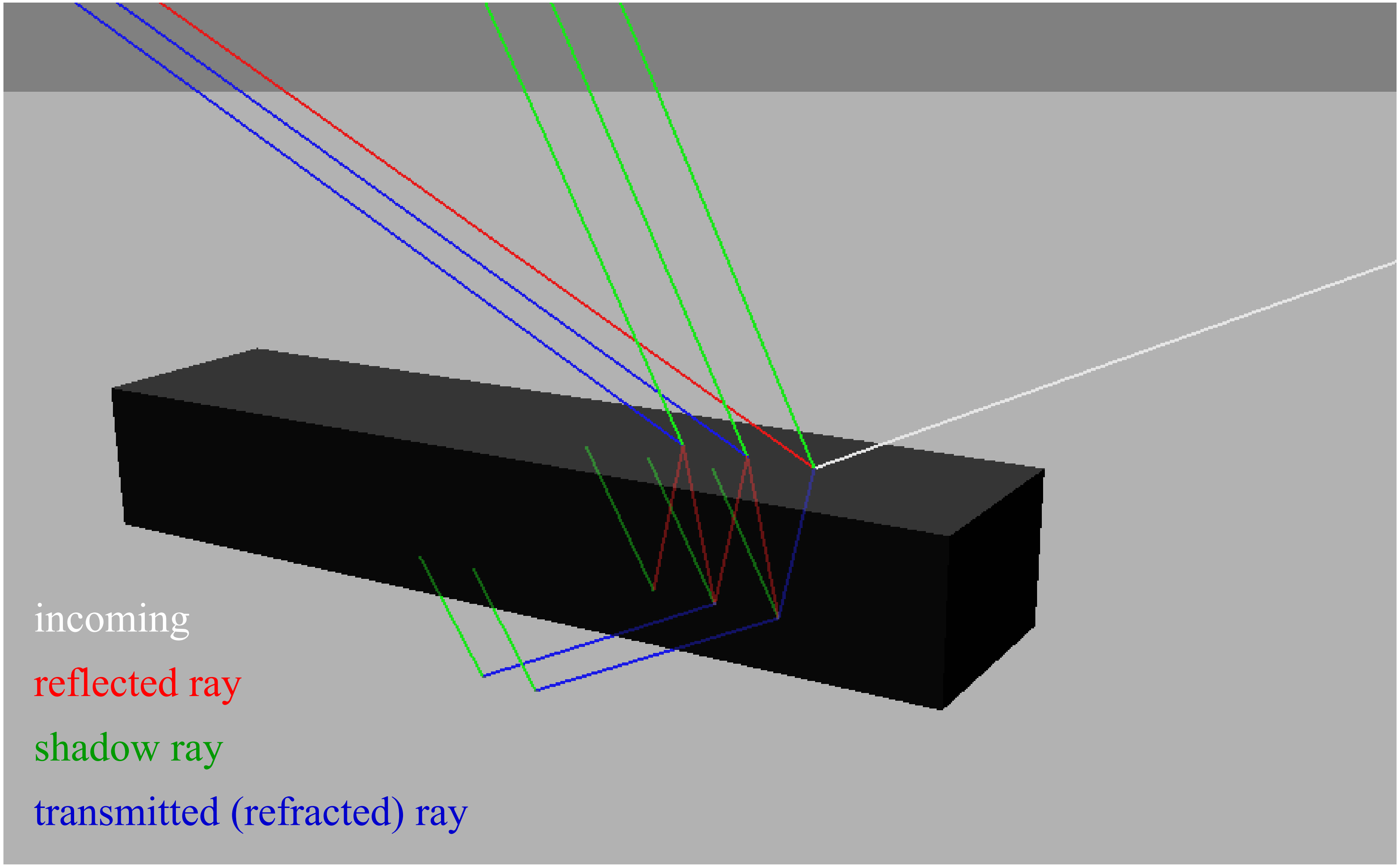
L_i shadow ray

T_i transmitted (refracted) ray

Complexity?

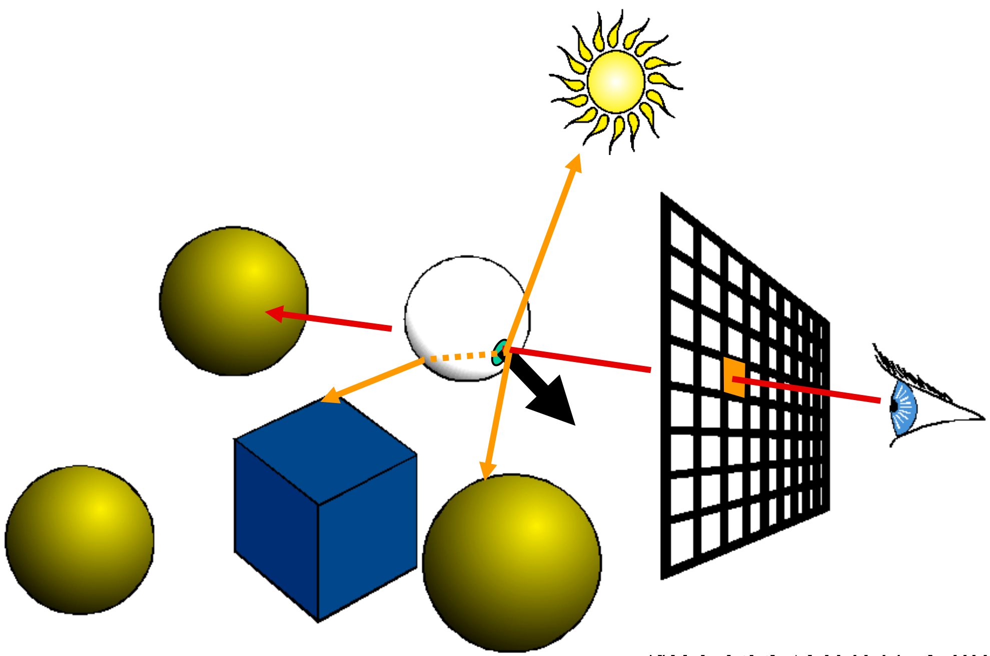
Ray Debugging (Assignment 4)

- Visualize the ray tree for single image pixel



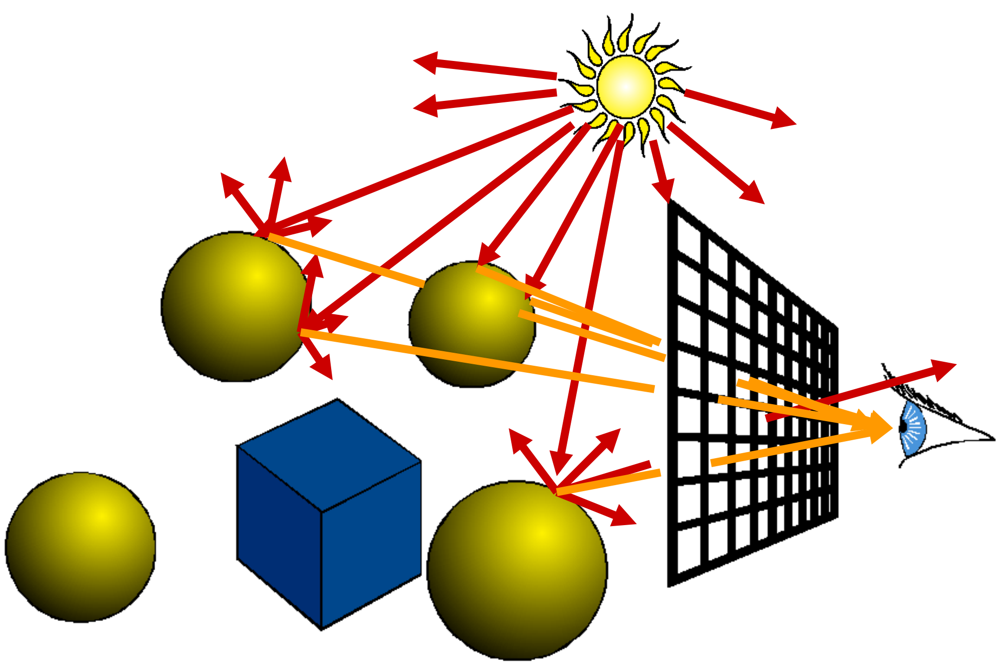
Does Ray Tracing Simulate Physics?

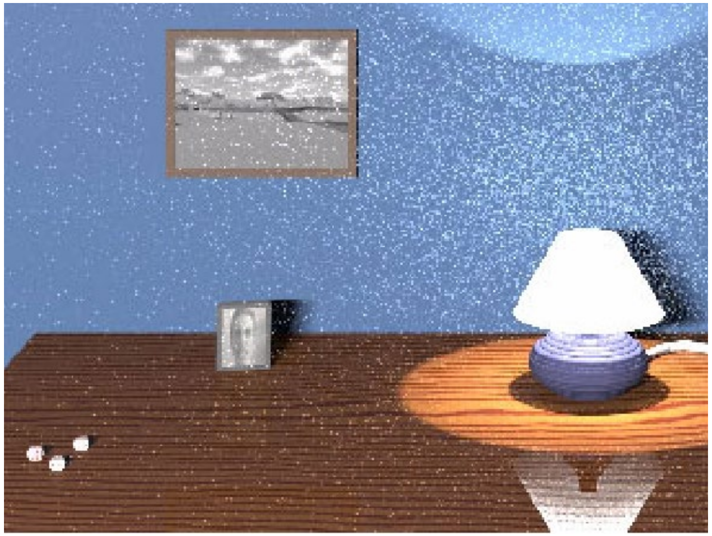
- Photons go from the light to the eye, not the other way
- What we do is *backward ray tracing*



Forward Ray Tracing

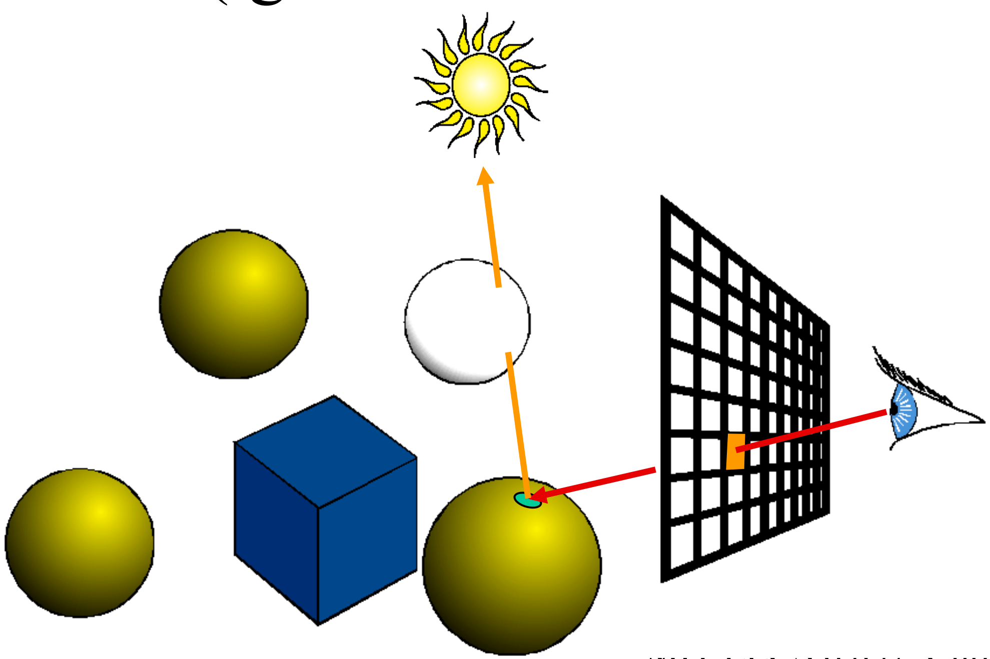
- Start from the light source
  - But low probability to reach the eye
- What can we do about it?
  - Always send a ray to the eye…. still not efficient





Does Ray Tracing Simulate Physics?

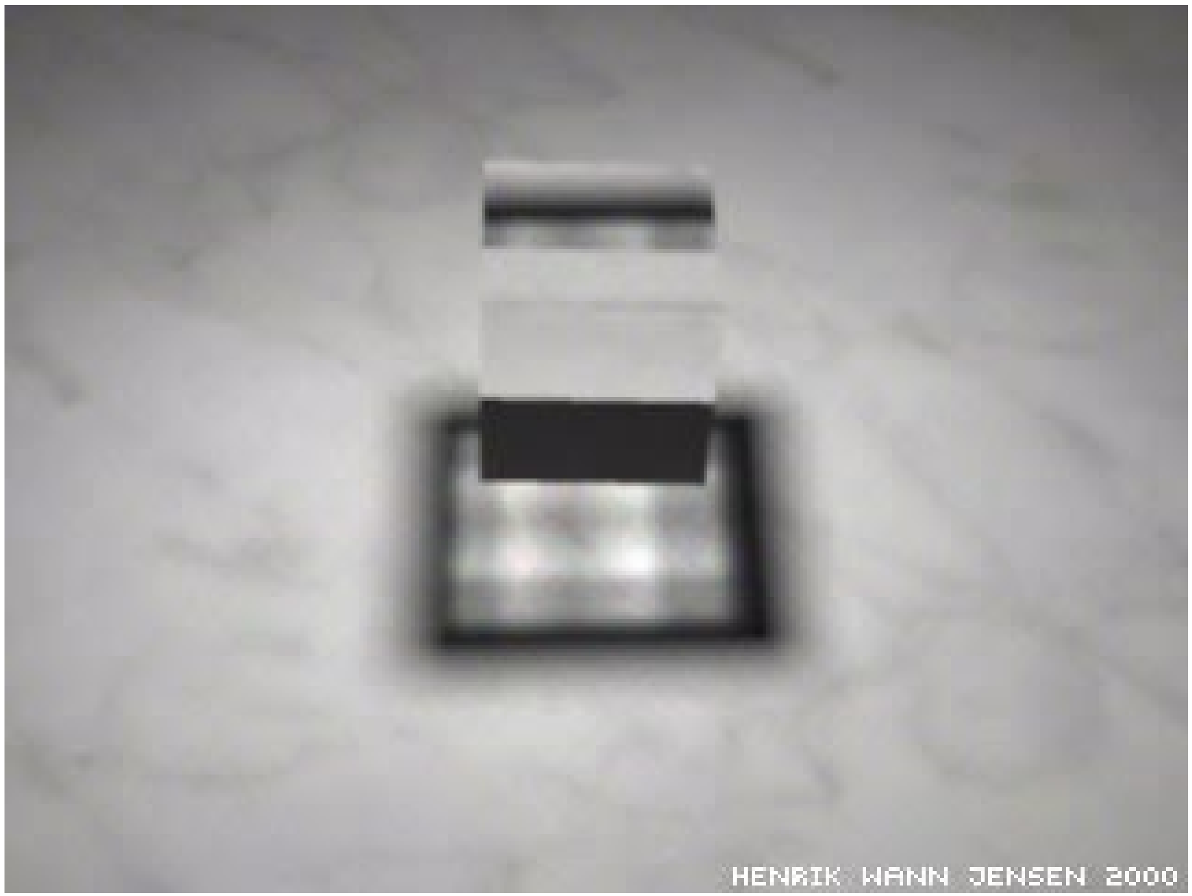
- Ray Tracing is full of dirty tricks
- For example, shadows of transparent objects:
  - opaque?
  - multiply by transparency color?
    (ignores refraction & does not produce caustics)



Correct Transparent Shadow

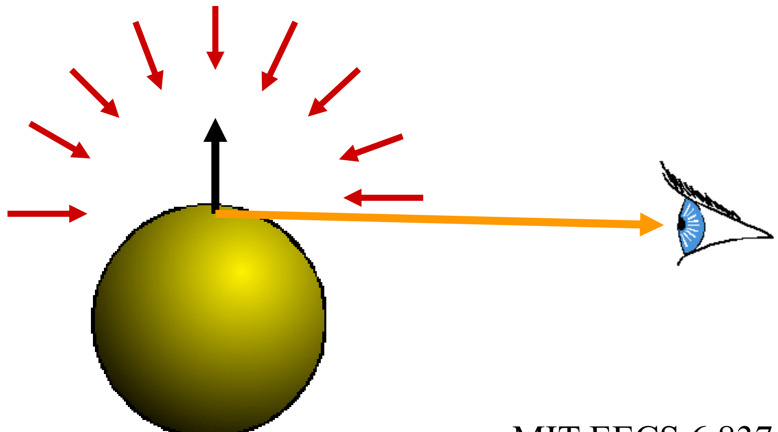
Animation by Henrik Wann Jensen

Using advanced refraction technique
(refraction for illumination is usually not handled that well)



The Rendering Equation

- Clean mathematical framework for light-transport simulation
- We'll see this later
- At each point, outgoing light in one direction is the integral of incoming light in all directions multiplied by reflectance property



A Look Ahead

- Assignment 2
  - Transformations & More Primitives
- Assignment 3
  - OpenGL Pre-Visualization & Phong Shading
- Assignment 4
  - Ray Tracing (Shadows, Reflections, Refractions)

Next Time

- Modeling Transformations
- Illumination (Shading)
- Viewing Transformation (Perspective / Orthographic)
- Clipping
- Projection (to Screen Space)
- Scan Conversion (Rasterization)
- Visibility / Display

Introduction to the Graphics Pipeline