

## Assignment 21

2.

$\sigma$	s0	s1	s2	s3	s4	...
p	F	T	F	T	F	...
q	F	F	T	F	T	...
r	F	F	F	F	T	...
$pUq$	F	T	T	T	T	...
$qUr$	F	F	T	F	T	...
$pU(qUr)$	F	T	T	T	T	...
$(pUq)Ur$	F	T	T	T	T	...

I believe that the until operator is associative because the parentheses can be moved around and not change the answer, the relations are still the same

3.

$$\begin{aligned}
 & o(p \equiv q) \\
 = & \quad \langle (3.80) \rangle \\
 & o((p \Rightarrow q) \wedge (q \Rightarrow p)) \\
 = & \quad \langle (5) \rangle \\
 & o(p \Rightarrow q) \wedge o(q \Rightarrow p) \\
 = & \quad \langle (2) \text{ twice} \rangle \\
 & op \Rightarrow oq \wedge oq \Rightarrow op \\
 = & \quad \langle (3.80) \rangle \\
 & op \equiv oq \quad //
 \end{aligned}$$

4.

$$\begin{aligned}
 & ofalse \\
 = & \quad \langle (3.8) \rangle \\
 & o\neg true \\
 = & \quad \langle (1) \rangle \\
 & \neg otrue \\
 = & \quad \langle (7) \rangle \\
 & \neg true \\
 = & \quad \langle (3.8) \rangle \\
 & false \quad //
 \end{aligned}$$

5.

$$\begin{aligned}
 & p \cup \text{true} \equiv \text{true} \\
 = & \quad \langle(10)\rangle \\
 & \text{true} \vee (p \wedge o(p \cup q)) \\
 = & \quad \langle(3.24)\rangle \\
 & (p \wedge o(p \cup q)) \vee \text{true} \\
 = & \quad \langle(3.29)\rangle \\
 & \text{true} //
 \end{aligned}$$

6.

$$\begin{aligned}
 & \diamond p \\
 = & \quad \langle(29)\rangle \\
 & \text{true} \cup p \\
 = & \quad \langle(10)\rangle \\
 & p \vee (\text{true} \wedge o(\text{true} \cup p)) \\
 = & \quad \langle(29)\rangle \\
 & p \vee (\text{true} \wedge o \diamond p) \\
 = & \quad \langle(3.36)\rangle \\
 & p \vee (o \diamond p \wedge \text{true}) \\
 = & \quad \langle(3.39)\rangle \\
 & p \vee o \diamond p //
 \end{aligned}$$

7.

$$\begin{aligned}
 & \Box \Box p \\
 = & \quad \langle(43)\rangle \\
 & \neg \diamond \neg \Box p \\
 = & \quad \langle(51)\rangle \\
 & \neg \diamond \diamond \neg p \\
 = & \quad \langle(39)\rangle \\
 & \neg \diamond \neg p \\
 = & \quad \langle(43)\rangle \\
 & \Box p
 \end{aligned}$$

8.

$$\begin{aligned}
 & \Box p \\
 \Rightarrow & \quad \langle(64)\rangle \\
 & p \\
 \Rightarrow & \quad \langle(37)\rangle \\
 & \diamond p //
 \end{aligned}$$