

例 $I = \iint_{\Sigma} xz dy dz + 2zy dz dx + 2xy dx dy$
 $\Sigma: z = 1 - x^2 - \frac{1}{4}y^2$ ($0 \leq z \leq 1$), 下侧
 解: $\Sigma_1: \begin{cases} z=0 \\ x^2 + \frac{1}{4}y^2 \leq 1 \end{cases}$, 上侧 \sqrt{z}

$$\iint_{\Sigma+\Sigma_1} = - \iiint_{\Omega} (z + 2z) dV$$

$$= - \int_0^1 dz \iint_{x^2 + \frac{1}{4}y^2 \leq 1-z} z dxdy = - \int_0^1 z \cdot \pi \cdot 2(1-z) dz = -\pi$$

斯托克斯公式 环流量 旋度
 定理: 设光滑曲面 Σ 的边界为光滑曲线 Γ
 $P, Q, R \in C^1(\Sigma)$

$$\oint_{\Gamma} P dx + Q dy + R dz = \iint_{\Sigma} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} dxdy dz dx dy$$

$$= \iint_{\Sigma} \begin{vmatrix} \omega_1 x & \omega_1 y & \omega_1 z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} dS$$

例 $I = \oint_{\Gamma} z dx + x dy + y dz$
 $\Gamma: x+y+z=1$ 在第一卦限内
 解: Σ 为 $x+y+z=1$ 在第一卦限内

$$I = \iint_{\Sigma} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & x & y \end{vmatrix} dxdy dz = \iint_{\Sigma} (dy dz + dz dx + dx dy)$$

$$= \iint_{D_{yz}} dy dz + \iint_{D_{zx}} dz dx + \iint_{D_{xy}} dx dy = 3 \times \frac{1}{2}$$

$$= \iint_{\Sigma} \nabla \times \vec{A} \cdot d\vec{S}$$

$$= \iint_{\Sigma} \nabla \times \vec{A} \cdot \vec{n} dS$$

例 $I = \oint_{\Gamma} (4-z) dx + (z-x) dy + (x-y) dz$
 $\Gamma: \begin{cases} x^2+y^2=a^2 \\ \frac{x}{a} + \frac{z}{b} = 1 \end{cases}$ ($a>0, b>0$)
 解: $\Sigma: \pm \Gamma$
 Σ 的法向量 $\vec{n} = \left\{ \frac{b}{\sqrt{a^2+b^2}}, 0, \frac{a}{\sqrt{a^2+b^2}} \right\}$
 $(\omega_1 x, \omega_1 y, \omega_1 z) = \left\{ \frac{b}{\sqrt{a^2+b^2}}, 0, \frac{a}{\sqrt{a^2+b^2}} \right\}$

$$I = \iint_{\Sigma} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4-z & z-x & x-y \end{vmatrix} dS = \iint_{\Sigma} -2(\omega_1 x + \omega_1 y + \omega_1 z) dS$$

$$= -2 \cdot \frac{a+b}{\sqrt{a^2+b^2}} \cdot \iint_{\Sigma} dS = -2 \cdot \frac{a+b}{\sqrt{a^2+b^2}} \cdot \iint_{D_{xy}} \sqrt{1+\frac{b^2}{a^2}} dxdy$$

$$= -2 \cdot \frac{a+b}{\sqrt{a^2+b^2}} \cdot \frac{\sqrt{a^2+b^2}}{a} \cdot \pi a^2 = -2\pi a(a+b)$$

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