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作业.周一.P48.1.3.8

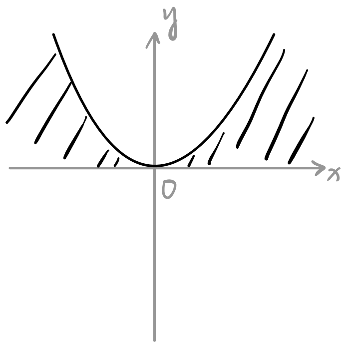
1. 写出函数 $z=\sqrt{x-\sqrt{y}}$ 的定义域，并画出草图.

1° $y \geq 0$

2° $x-\sqrt{y} \geq 0$

$\sqrt{y} \leq x$

$y \leq x^2$

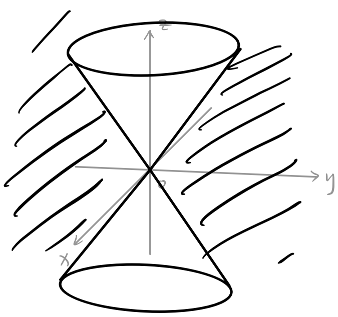


3. 3. 求函数 $u=\arcsin\left(\frac{\sqrt{x^2+y^2}}{z}\right)$ 的定义域，并画出草图.

$\frac{\sqrt{x^2+y^2}}{z} \in [-1,1]$

$\begin{cases} z>0 \\ \sqrt{x^2+y^2} \leq z \end{cases} \Rightarrow x^2+y^2-z^2 \leq 0.$

$\begin{cases} z<0. \\ \sqrt{x^2+y^2} \geq -z \end{cases} \quad x^2+y^2-z^2 \geq 0.$



8. 8. 设 $z=xf\left(\frac{y}{x}\right)$，其中 $x \neq 0$，如果当 $x=1$ 时，$z=\sqrt{1+y^2}$，试确定 $f(x)$ 及 z.

令 $\frac{y}{x}=t \quad x=\frac{y}{t}$

$z=\frac{y}{t}f(t)$

$x=1$时，$y=t$，则 $z=f(t)=\sqrt{1+t^2}$

$\therefore f(x)=\sqrt{1+x^2}$

$z=x\sqrt{1+\frac{y^2}{x^2}}=\frac{x}{|x|}\sqrt{x^2+y^2}$

周三.P48.9.奇.12.13.15.

9. 求下列二元函数的极限:

(1) $\lim\limits_{\substack{x\to0\\y\to0}}\frac{3y^3+2yx^2}{x^2-xy+y^2}$;

$0 \leq \left|\frac{3y^3+2yx^2}{x^2-xy+y^2}\right| \leq \left|\frac{3y^3+2yx^2}{\frac{x^2+y^2}{2}}\right| = 2|y|\left|\frac{3y^2+2x^2}{x^2+y^2}\right| = 2|y|\cdot\left|\frac{3+\frac{2x^2}{y^2}}{1+\frac{x^2}{y^2}}\right|$

$\lim\limits_{\substack{x\to0\\y\to0}} 2|y|\cdot\left|\frac{3+\frac{2x^2}{y^2}}{1+\frac{x^2}{y^2}}\right|=0$

$\therefore \lim\limits_{\substack{x\to0\\y\to0}}\left|\frac{3y^3+2yx^2}{x^2-xy+y^2}\right|=0 \Rightarrow$ 原极限 $=0$.

(3) (3) $\lim\limits_{\substack{x\to0\\y\to0}}\frac{x^2y^{7/3}}{x^4+y^4}$;

$=\lim\limits_{\substack{x\to0\\y\to0}} y^{\frac{1}{3}}\frac{\frac{x^2}{y^2}}{\frac{x^4}{y^4}+1} \overset{\frac{x}{y}=u}{=} \lim\limits_{y\to0} y^{\frac{1}{3}}\cdot\lim\limits_{u\to0}\frac{u^2}{u^4+1}=0.$

(5). (5) $\lim\limits_{\substack{x\to0\\y\to0}}\frac{x^2+y^2}{|x|+|y|}$;

$\begin{cases} |x|+|y| \geq |x+y| \\ x^2+y^2 \leq (x+y)^2 \end{cases}$

$\therefore 0 \leq \frac{x^2+y^2}{|x|+|y|} \leq |x+y|$

而 $\lim\limits_{\substack{x\to0\\y\to0}}|x+y|=0.$

$\therefore \lim\limits_{\substack{x\to0\\y\to0}}\frac{x^2+y^2}{|x|+|y|}=0.$

(7) (7) $\lim\limits_{\substack{x\to0\\y\to1}}(1+xe^y)^{\frac{2y+x}{x}}$;

$=\lim\limits_{\substack{x\to0\\y\to1}}(1+xe^y)^{\frac{1}{xe^y}\cdot\frac{2y+x}{x}\cdot xe^y}$

$=e^{\lim\limits_{\substack{x\to0\\y\to1}}(2y+x)e^y}$

$=e^{2e}$

(9) $\lim\limits_{\substack{x\to0\\y\to0}}\frac{1-\sqrt{x^2y+1}}{x^3y^2}\sin(xy)$.

$=\lim\limits_{\substack{x\to0\\y\to0}}\frac{\frac{1}{2}x^2y\cdot xy}{x^3y^2}$

$=\frac{1}{2}.$

12.

12. 讨论函数 $f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+2y^4}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0)\end{cases}$ 在点 $(0,0)$ 处的连续性.

$$\lim_{\substack{x\to0\\ y=\sqrt{x}}}\frac{x^2}{x^2+2x^2}=\frac{1}{3}\neq0.$$

∴ $f(x,y)$ 在 $(0,0)$ 处不连续.

13.

13. 求函数 $f(x,y)=\begin{cases}x\sin\dfrac{1}{y}, & y\neq0,\\ 0, & y=0\end{cases}$ 的间断点.

$$\lim_{\substack{x\to0\\ y\to0}}x\sin\frac{1}{y}=0=f(0,0)$$

$\lim\limits_{\substack{y\to0\\ x=k\neq0}}x\sin\dfrac{1}{y}$ 不存在，故 $f(x,y)$ 在 $(k,0)$ $(k\neq0)$ 处不连续.

图五. P56. 2. 3. 4.奇. 5. 6. 12. 14.奇 (8-2)

2. 设 $f(x,y)=\sqrt{x^2+y^4}$，问 $f_x(0,0)$ 与 $f_y(0,0)$ 是否存在？若存在，求其值.

$f'_x(0,0)=\lim\limits_{\Delta x\to0}\dfrac{f(\Delta x,0)-f(0,0)}{\Delta x}=\lim\limits_{\Delta x\to0}\dfrac{|\Delta x|}{\Delta x}$ 不存在

$$f'_y(0,0)=\lim_{\Delta y\to0}\frac{f(0,\Delta y)-f(0,0)}{\Delta y}=\lim_{\Delta y\to0}\frac{\Delta y^2}{\Delta y}=0$$

3. 证明：$f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0)\end{cases}$ 在点 $(0,0)$ 不连续，但存在一阶偏导数.

$$\lim_{\substack{x\to0\\ y=\sqrt{x}}}f(x,y)=\lim_{x\to0}\frac{x^2}{2x^2}=\frac{1}{2}\neq f(0,0)=0$$

故 $f(x,y)$ 在 $(0,0)$ 不连续.

$$f'_x(0,0)=\lim_{\Delta x\to0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x}=\lim_{\Delta x\to0}\frac{0}{\Delta x^2}=0$$

$$f'_y(0,0)=\lim_{\Delta y\to0}\frac{f(0,\Delta y)-f(0,0)}{\Delta y}=0.$$

4. 求下列函数的偏导数：

(1) $u=(xy)^z$;

$$\frac{\partial u}{\partial x}=z(xy)^{z-1}\cdot y=(zy)(xy)^{z-1}$$

$$\frac{\partial u}{\partial y}=z(xy)^{z-1}\cdot x=(zx)(xy)^{z-1}$$

$$\frac{\partial u}{\partial z}=(xy)^z\ln(xy)$$

(3) $z=\int_x^y e^{t^2}\mathrm{d}t$;

$$\frac{\partial z}{\partial x}=-e^{x^2}$$

$$\frac{\partial z}{\partial y}=e^{y^2}$$

$$\frac{\partial z}{\partial t}=0$$

(5) 设 $z=(y\sin x)^y$，求 $\dfrac{\partial z}{\partial x}$;

$$\frac{\partial z}{\partial x}=y(y\sin x)^{y-1}\cdot y\cos x=y^2\cos x\cdot(y\sin x)^{y-1}$$

5. 求下列函数在指定点处的一阶偏导数：

(1) $z=x+(y-1)\arcsin\sqrt{\dfrac{x}{y}}$，点 $(0,1)$;

$$z'_x(0,1)=\lim_{\Delta x\to0}\frac{z(0+\Delta x,1)-z(0,1)}{\Delta x}=\lim_{\Delta x\to0}\frac{\Delta x}{\Delta x}=1$$

$$z'_y(0,1)=\lim_{\Delta y\to0}\frac{z(0,1+\Delta y)-z(0,1)}{\Delta y}=\lim_{\Delta y\to0}0=0$$

(2) $z=x^2e^y+(x-1)\arctan\dfrac{y}{x}$，点 $(1,0)$.

$$z'_x(1,0)=\lim_{\Delta x\to0}\frac{z(1+\Delta x,0)-z(1,0)}{\Delta x}=\lim_{\Delta x\to0}\frac{(1+\Delta x)^2-1}{\Delta x}=2$$

$$z'_y(1,0)=\lim_{\Delta y\to0}\frac{z(1,\Delta y)-z(1,0)}{\Delta y}=\lim_{\Delta y\to0}\frac{e^{\Delta y}-1}{\Delta y}=1$$

6. 设 $f(x,y)=\begin{cases} xy-\dfrac{x^3+y^3}{x^2+y^2}, & (x,y)\neq(0,0), \\ 0, & (x,y)=(0,0), \end{cases}$ 根据偏导数定义求 $f_x'(0,0)$，$f_y'(0,0)$.

$$f_x'(0,0)=\lim_{\Delta x\to 0}\frac{f(\Delta x,0)-0}{\Delta x}=-1$$

$$f_y'(0,0)=\lim_{\Delta y\to 0}\frac{f(0,\Delta y)-0}{\Delta y}=-1$$

12. 设 $f(x,y)=\begin{cases} e^{\frac{-1}{x^2+y^2}}, & x^2+y^2\neq 0, \\ 0, & x^2+y^2=0, \end{cases}$ 求 $f_{xx}(0,0)$.

$$f_x'(0,0)=\lim_{\Delta x\to 0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{e^{\frac{-1}{\Delta x^2}}}{\Delta x}$$

$$\overset{\text{令}u=\frac{1}{\Delta x}}{=}\lim_{u\to\infty}\frac{u}{e^{u^2}}$$

$$\overset{\text{洛必达}}{=}\lim_{u\to\infty}\frac{1}{2ue^{u^2}}$$

$$=0$$

$x^2+y^2\neq 0$ 时 $\dfrac{\partial f}{\partial x}=\dfrac{2x}{(x^2+y^2)^2}e^{\frac{-1}{x^2+y^2}}$

$$\text{则 } f_{xx}'(0,0)=\lim_{\Delta x\to 0}\frac{f_x'(\Delta x,0)-f_x'(0,0)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\frac{2}{\Delta x^3}e^{-\frac{1}{\Delta x^2}}}{\Delta x}$$

$$\overset{\frac{1}{\Delta x}=t}{=}\lim_{t\to\infty}\frac{2t^4}{e^{t^2}}$$

$$=\lim_{t\to\infty}\frac{4t^2}{e^{t^2}}$$

$$=0.$$

14. 验证：

（1）函数 $r=\sqrt{x^2+y^2+z^2}$ 满足方程 $\dfrac{\partial^2 r}{\partial x^2}+\dfrac{\partial^2 r}{\partial y^2}+\dfrac{\partial^2 r}{\partial z^2}=\dfrac{2}{r}$ $(r\neq 0)$；

$$\frac{\partial r}{\partial x}=\frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial^2 r}{\partial x^2}=\frac{\sqrt{x^2+y^2+z^2}-\frac{x^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$\text{同理 }\frac{\partial^2 r}{\partial y^2}=\frac{\sqrt{x^2+y^2+z^2}-\frac{y^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$\frac{\partial^2 r}{\partial z^2}=\frac{\sqrt{x^2+y^2+z^2}-\frac{z^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$\text{则 }\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}+\frac{\partial^2 r}{\partial z^2}=\frac{3\sqrt{x^2+y^2+z^2}-\sqrt{x^2+y^2+z^2}}{x^2+y^2+z^2}=\frac{2}{r}$$

（3）函数 $u=z\arctan\dfrac{y}{x}$ 满足拉普拉斯方程 $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}=0$；

$$\frac{\partial u}{\partial x}=z\cdot\frac{1}{1+\frac{y^2}{x^2}}\cdot\left(-\frac{y}{x^2}\right)=-\frac{zy}{x^2+y^2}\qquad \frac{\partial^2 u}{\partial x^2}=\frac{2xyz}{(x^2+y^2)^2}$$

$$\frac{\partial u}{\partial y}=\frac{z}{x}\cdot\frac{1}{1+\frac{y^2}{x^2}}=\frac{zx}{x^2+y^2}\qquad \frac{\partial^2 u}{\partial y^2}=\frac{-2xyz}{(x^2+y^2)^2}$$

$$\frac{\partial u}{\partial z}=\arctan\frac{y}{x}\qquad \frac{\partial^2 u}{\partial z^2}=0$$

$$\text{则 }\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{zxy(2-2)}{(x^2+y^2)^2}=0.$$