

$$I = \int_{\Gamma} (x^2 + y^2 + z^2) ds, \quad \Gamma: \begin{cases} x^2 + y^2 = a^2 \\ z = 1 \end{cases} \quad (a > 0)$$

$$I = \int_0^{2\pi} (a^2 + 1) ds = (a^2 + 1) \cdot 2\pi a$$

$$\therefore \frac{x^2}{4} + \frac{y^2}{3} = 1, \text{ 其同轴为 } a, \text{ 所以 } \oint_L (2xy + 3x^2 + 4y^2) ds =$$

$$= \oint_L 2xy ds + \oint_L (3x^2 + 4y^2) ds$$

$$= 0 + 12\pi a$$

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$$I = \int_0^{2\pi} (a^2 + 1) ds = (a^2 + 1) \cdot 2\pi a$$

$$\text{例 } I = \int_L [(x+2)^2 + (y-1)^2] ds$$

$$\textcircled{1} L: x^2 + y^2 + z^2 = R^2 \text{ 与 } x + y + z = 0 \text{ 的交线}$$

$$\text{解: } I = \int_L (x^2 + y^2) ds + \int_L (4x - 6y) ds + 13 \int_L ds$$

$$\int_L x^2 ds = \int_L y^2 ds = \int_L z^2 ds = \frac{1}{3} \int_L (x^2 + y^2 + z^2) ds$$

$$= \frac{1}{3} \int_L R^2 ds = \frac{1}{3} R^2 \cdot 2\pi R = \frac{2\pi R^3}{3}$$

$$I_1 = \int_L (x^2 + y^2) ds = 2 \int_L x^2 ds = \frac{4\pi}{3} R^3$$

$$\text{又 } \int_L x ds = \int_L y ds = \int_L z ds = \frac{1}{3} \int_L (x + y + z) ds = 0$$

$$\text{或 } 0 = \bar{x} = \frac{\int_L x ds}{L \text{ 长度}} \Rightarrow \int_L x ds = 0$$

$$\text{所以 } I_2 = \int_L (4x - 6y) ds = 0$$

$$I_3 = 13 \int_L ds = 13 \cdot 2\pi R$$

被积函数轮换

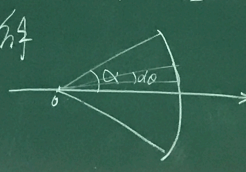


看区域函数轮换

$$I_x = \int_{\Gamma} (y^2 + z^2) f(x, y, z) ds$$

$$\text{例. 计算半圆 } (z \geq 0, \text{ 中心角为 } 2\alpha \text{ 的圆弧 } L \text{ 对 } z \text{ 轴}$$

$$\text{对称轴转动惯量 } I_x \text{ (设密度为 } \mu = 1)$$

$$\text{解}$$


$$L: \begin{cases} x = R \cos \theta \\ y = R \sin \theta \end{cases} \quad -\alpha \leq \theta \leq \alpha$$

$$I_x = \int_L y^2 ds = \int_{-\alpha}^{\alpha} R^2 \sin^2 \theta \cdot R d\theta$$

$$= R^3 \int_{-\alpha}^{\alpha} (1 - \sin 2\theta) d\theta$$

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