4.图 2.4 所示电路中电容的原始储能为零，用时域分析法求解开关 K 闭合后的电容电压 $u_C(t)$ 和开关支路电流 $i_k(t)$

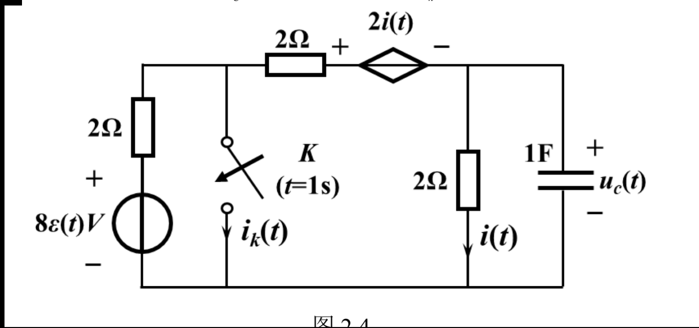
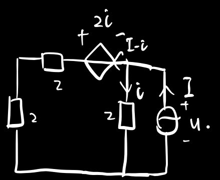


图 2.4

$t \in [0,1]s$

$U_{cf} = 8V$

$\tau = 1s$



$u = 2i = -2i + 4(I-i)$

$i = I - i$

$2i = I.$

$R_{eq} = 1$

$U_c(t) = 8(1-e^{-t})\ V \qquad U_c(1) = 8(1-e^{-1})\ V$

$t \in (1, +\infty)s$

$\tau' = \frac{2}{3}s. \quad U = 2i = -2i + 2(I-i)$

$2i = I - i \qquad i = \frac{I}{3} \qquad U = \frac{2I}{3} \qquad R = \frac{2}{3}.$

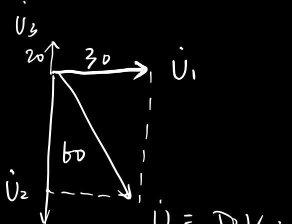
$U_c(t) = 8(1-e^{-1}) \cdot e^{-\frac{3}{2}t}$

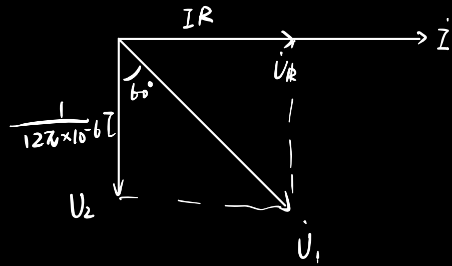
$\dot{I}(R + \frac{1}{j\omega L} + j\omega C) = \dot{U}$

令 $\dot{I} = I\angle 0°$

$\dot{U}_2 = \frac{1}{j\omega C}\dot{I} \qquad \frac{1}{j\omega C} \qquad \omega\uparrow \quad X\downarrow.$

$= \frac{1}{j\,12[illegible] \times 0.01 \times 10^{-6}}\dot{I}$





$R = \frac{\sqrt{3}}{122\times10^{-6}} = 45.9k\Omega.$

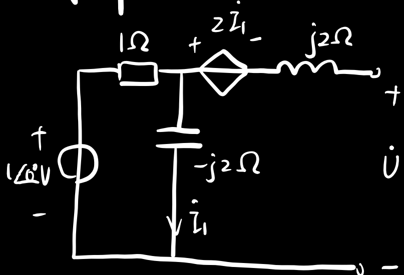
$$(R_1+R_2+j\omega L)\dot{I}_1-(j\omega L+R_1)\dot{I}_2-R_2\dot{I}_3=\dot{U}_s$$

$$-(j\omega L+R_1)\dot{I}_1+(R_1+R_3+R_4+j\omega L)\dot{I}_2-R_3\dot{I}_3=0$$

$$-R_2\dot{I}_1-R_3\dot{I}_2+\left(R_2+R_3+\frac{1}{j\omega C}\right)\dot{I}_3+\frac{1}{j\omega C}\dot{I}_4$$

$$\dot{I}_4=\dot{I}_s$$

求开口电压：

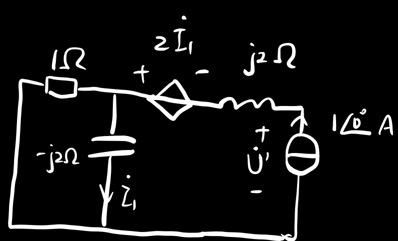


$$\begin{cases}1\angle 0^\circ=\dot{I}_1+2\dot{I}_1+\dot{U}\\ 1\angle 0^\circ=\dot{I}_1(1-j2)\end{cases}$$

$$\Downarrow$$

$$\dot{U}=\frac{\sqrt{5}}{3}\angle -63.43^\circ\ V$$

求 Z_{eq}



外加电流源

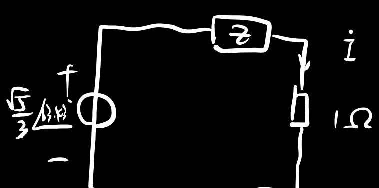
$$\begin{cases}(1\angle 0^\circ-\dot{I}_1)\times 1=\dot{I}_1\cdot(-j2)\\ \dot{U}'=1\angle 0^\circ\cdot j2-2\dot{I}_1+\dot{I}_1\cdot(-j2)\end{cases}$$

$$\Downarrow$$

$$\dot{U}'=\frac{2\sqrt{5}}{5}\angle 63.43^\circ\ V$$

$$\therefore Z_{eq}=\frac{\dot{U}'}{1\angle 0^\circ}=\frac{2\sqrt{5}}{5}\angle 63.43^\circ\ \Omega$$

等效电路：



$$\dot{I}=\frac{\dot{U}}{1+Z_{eq}}=0.462\angle 33.19^\circ\ A$$

于是 $\int_{OB}$ ...

故 $\int_L (x+y)\,ds = \frac{1}{2}+\frac{3}{2}+\sqrt{2} = 2+\sqrt{2}$.

例 3 计算 $\int_L \sqrt{R^2-x^2-y^2}\,ds$，其中 L 为上半圆弧 $x^2+y^2=Rx$，$y\geqslant 0$.

解 如图 10－5，采用半圆弧的参数方程

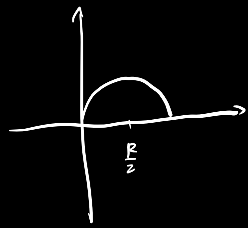
$$\begin{cases} x = R\cos^2\theta, \\ y = R\cos\theta\sin\theta \end{cases} \quad \left(0\leqslant\theta\leqslant\frac{\pi}{2}\right),$$

则 $$\int_L \sqrt{R^2-x^2-y^2}\,ds = \int_0^{\frac{\pi}{2}} \sqrt{R^2\sin^2\theta}\sqrt{(-R\sin 2\theta)^2+(R\cos 2\theta)^2}\,d\theta$$

$\int\sqrt{x^2+y^2}=\sqrt{Rx}$ $$= \int_0^{\frac{\pi}{2}} |R\sin\theta|\sqrt{R^2}\,d\theta = R^2\int_0^{\frac{\pi}{2}}\sin\theta\,d\theta = R^2.$$

例 4 计算 $\int_\Gamma (x^2+y^2+z^2)\,ds$，其中 Γ 为螺旋线 $x=\cos t$，$y=\sin t$，$z=$ ...

$(x-\frac{R}{2})^2+y^2=\frac{R^2}{4}$



$x-\frac{R}{2}=\frac{R}{2}\cos 2\theta$

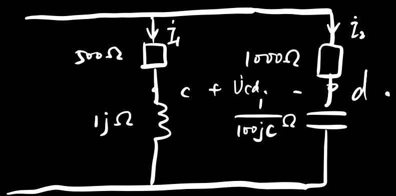
$y=\frac{R}{2}\sin 2\theta$

$\Downarrow$

$x=\frac{R}{2}(1+\cos 2\theta)=R\cos^2\theta$

$y=\frac{R}{2}\cdot 2\cos\theta\sin\theta = R\cos\theta\sin\theta$

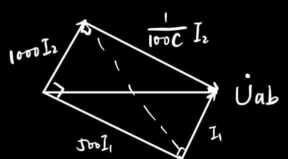
$2\theta\in(0,\pi) \Rightarrow \theta\in(0,\frac{\pi}{2})$



$\dot{I}_1(500+j1)=\dot{I}_2(1000+\frac{1}{100jc})=\dot{U}_{ab}$

$\dot{U}_{cd}=\dot{I}_1\cdot j-\frac{1}{100jc}\dot{I}_2=\dot{U}_{ab}=\dot{I}_1(500+j1)$

$\dot{I}_1\cdot 500+\frac{1}{100jc}\dot{I}_2=0$



$1000\dot{I}_2=\dot{I}_1$

$500\dot{I}_1=\frac{1}{100C}\dot{I}_2$

$5\times10^5=\frac{1}{100C}$

$C=\frac{1}{5\times10^7}=0.2\times10^{-7}=0.02\mu F$.

$Z_{eq}=5\,//\,5\,//\,(j\cdot2\cdot2.5)$

$=\frac{2.5\times5j}{2.5+5j}=2+i$

$\lambda+\frac{1}{j\cdot2\cdot C_0}=\lambda+j$

$C_0=\frac{1}{2j^2}=-0.5$

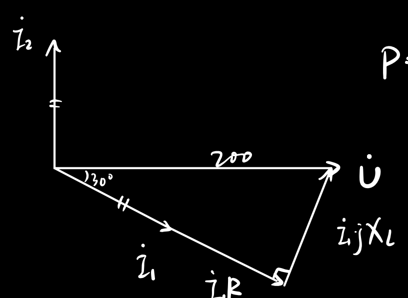
$\frac{U}{I} = R + wL$

$w = 2\pi f$.

$R = 30$.

$30 + 314L = 50$

$L = \frac{20}{314} = 0.0637$



$\left.\begin{aligned} I_1R &= 100\sqrt{3} \\ P = I_1^2R &= 866 \end{aligned}\right\}$

$I_1 = 5$.

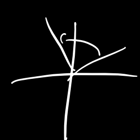
$R = 20\sqrt{3}$.

$X_L = \frac{100}{5} = 20\,\Omega$

$X_C = 40\,\Omega$.

$\dot U_s = 12\angle 90^\circ\ V$.

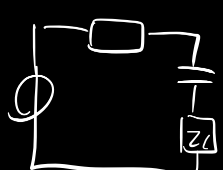
$\dot I_s = 4\sqrt{2}\angle 0^\circ\ A$



$\dot U = -4\angle 0^\circ \cdot 16 + 12\angle 90^\circ$

$= 65.115\angle 169.38\ V$

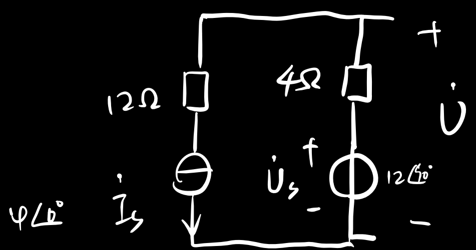
$X_{eq} = 16\angle 0^\circ\,\Omega$



$Z + \frac{1}{j1000\times 250\times 10^{-6}} = 16\angle 0^\circ$

$Z = \cancel{16.49\angle 14.04^\circ}\ 16+4j$

$P_{max} = \frac{|U|^2}{X_{eq}} = 521\ W$



$\dot U = -4\angle 0^\circ \times 4 + 12\angle 90^\circ = 20\angle 143.13^\circ$ V

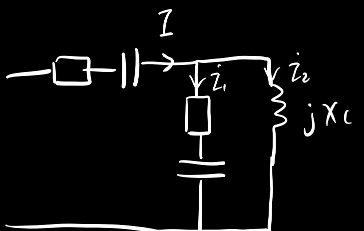
$X_{eq} = 4\Omega$.

$\therefore Z_L - 4j = 4$

$Z_L = 4 + 4j \ \Omega$

$$P_{max} = \frac{20^2}{4\times 4} = 25W$$

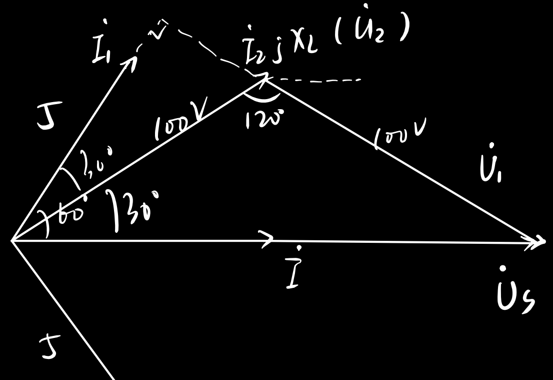
$$\frac{\frac{L_1}{C}}{j\omega L_1 - \frac{j}{\omega C}}$$



$U_s = 100\sqrt{3}$ V.

$5X_L = 100$

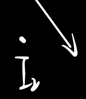
$X_L = 20\Omega$

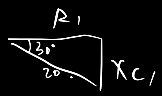


$\frac{\dot U_1}{\dot I} = 20\angle -30^\circ = R_1 - jX_{C1}$.

$\Rightarrow R_1 = 10\sqrt{3}\,\Omega$.

$X_{C1} = 10\Omega$





$$\frac{\dot{U}_2}{\dot{I}_1} = 20\angle -30^\circ = R_2 - jX_{C2}$$

$$R_2 = 10\sqrt{3}\,\Omega$$

$$X_{C2} = 10\,\Omega$$