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P149. 3. 4

3. 设一薄板由 $y=e^x$，$y=0$，$x=0$，$x=2$ 所围成，其面密度 $\mu(x,y)=xy$. 求薄板对两个坐标轴的转动惯量 I_x 和 I_y.

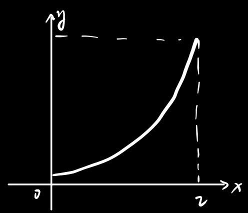
$$I_x=\int_0^2 dx\int_0^{e^x} y^2\cdot xy\,dy$$

$$=\int_0^2 \frac{1}{4}x\cdot e^{4x}dx$$

$$=\frac{1}{27}\int_0^2 3x\, e^{3x}d3x$$

$$=\frac{1}{27}\left[e^{3x}(3x-1)\right]_0^2$$

$$=\frac{5e^6+1}{27}$$



$$I_y=\int_0^2 dx\int_0^{e^x} x^2 y\,dy$$

$$=\int_0^2 \frac{1}{2}x^2\cdot e^{2x}dx$$

$$=\frac{1}{4}\int_0^2 x^2 e^{2x}d2x$$

$$=\frac{1}{4}\int_0^2 x^2 de^{2x}$$

$$=\frac{1}{4}\left[\left(x^2e^{2x}\right]_0^2-\frac{1}{2}\int_0^2 e^{2x}\cdot 2x\,d2x\right]$$

$$=\frac{1}{4}\left(4e^4-\frac{1}{2}e^{2x}(2x-1)\Big|_0^2\right)$$

$$=e^4-\frac{1}{8}\left(e^4\times 3+1\right)$$

$$=e^4-\frac{3e^4+1}{8}$$

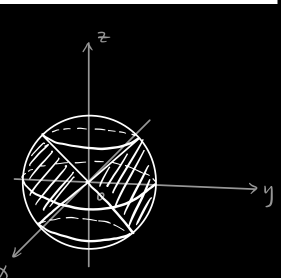
$$=\frac{5e^4-1}{8}$$

4. 求均匀物体：$x^2+y^2+z^2\leqslant 2$，$x^2+y^2\geqslant z^2$ 对 z 轴的转动惯量.

$$I=\iiint_\Omega (x^2+y^2)\,\rho\,dv$$

$$=\rho\int_0^{2\pi}d\theta\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}d\varphi\int_0^{\sqrt{2}}(r^4\sin^3\varphi)\,dr$$

$$=-\rho\cdot 2\pi\cdot\left(\cos\varphi-\frac{1}{3}\cos^3\varphi\right)\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\cdot\frac{1}{5}r^5\Big|_0^{\sqrt{2}}$$



$$=-2\pi\rho\cdot\left[-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}-\frac{1}{3}\cdot\left(-\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4}\right)\right]\cdot\frac{4\sqrt{2}}{5}$$

$$=-2\pi\rho\left(-\sqrt{2}+\frac{1}{3}\cdot\frac{\sqrt{2}}{2}\right)\cdot\frac{4\sqrt{2}}{5}$$

$$=2\pi\rho\cdot\frac{5\sqrt{2}}{6}\cdot\frac{4\sqrt{2}}{5}$$

$$=\frac{8\pi\rho}{3}$$