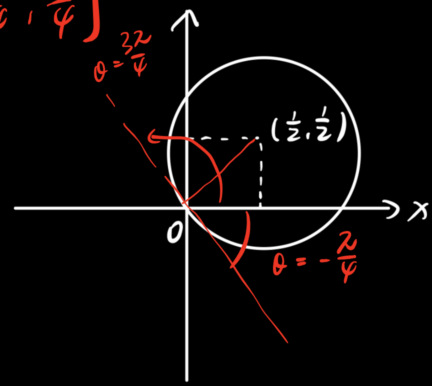
16. (2) $\iint\limits_D (x+y)\,d\sigma$，其中 $D=\{(x,y)\mid x^2+y^2\le x+y\}$；

$D=\{(r,\theta)\mid r\le \cos\theta+\sin\theta\}$ $\theta\in[-\frac{\pi}{4},\frac{3\pi}{4}]$



$I=\int_{\cancel{0}\ -\frac{\pi}{4}}^{\cancel{2\pi}\ \frac{3\pi}{4}} d\theta \int_0^{\cos\theta+\sin\theta} r^2(\cos\theta+\sin\theta)\,dr$

$=\frac{1}{3}\int_{\cancel{0}\ -\frac{\pi}{4}}^{\cancel{2\pi}\ \frac{3\pi}{4}} \underbrace{(\cos\theta+\sin\theta)^4}\, d\theta$

$=\cancel{0}$

$(c^2+s^2+2cs)^2$

$=(1+2cs)^2$

$=1+4c^2s^2+4cs$

$=1+\sin^2 2\theta+2\sin 2\theta$

$s^2=\frac{1-c2\theta}{2}$

$I=\frac{1}{3}\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}(1+\sin^2 2\theta+2\sin 2\theta)\,d\theta$

$=\frac{1}{3}\left(\pi+\underbrace{\frac{1}{4}\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}(1-c2\theta)\,d2\theta}+\underbrace{\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}\sin 2\theta\,d2\theta}\right)$

$\underbrace{\frac{1}{4}\left(2\pi-\underbrace{s2\theta\Big|_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}}_{-1-(-1)=0}\right)}_{\frac{\pi}{2}} - \underbrace{c2\theta\Big|_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}}_{0}$

$=\frac{1}{3}\cdot\frac{3\pi}{2}=\frac{\pi}{2}$

13.证明：$\frac{12\pi}{5} < \iiint_{\Omega} \sqrt[3]{2x+4y-4z+21}dv < 4\pi$，其中$\Omega: x^2+y^2+z^2 \le 1$.

提示:求被积函数在Ω上的最值,再用三重积分的估值定理.

解:

$$F = 2x+4y-4z+\lambda(x^2+y^2+z^2-1)$$

$$\left.\begin{aligned} \frac{\partial F}{\partial x} &= 2+2\lambda x=0 \\ \frac{\partial F}{\partial y} &= 4+2\lambda y=0 \\ \frac{\partial F}{\partial z} &= -4+2\lambda z=0 \\ \frac{\partial F}{\partial x} &= x^2+y^2+z^2-1=0 \end{aligned}\right\}$$

$$\Rightarrow \begin{cases} x=-\frac{1}{3} \\ y=-\frac{2}{3} \\ z=\frac{2}{3} \\ \lambda=3 \end{cases} \Rightarrow \sqrt[3]{2x+4y-4z+21} = \sqrt[3]{15}$$

$$\Rightarrow \begin{cases} x=\frac{1}{3} \\ y=\frac{2}{3} \\ z=-\frac{2}{3} \\ \lambda=-3 \end{cases} \Rightarrow \sqrt[3]{2x+4y-4z+21} = \sqrt[3]{27} = 3$$

$$\frac{12\pi}{5} < \frac{4}{3}\pi \cdot \sqrt[3]{15} < \iiint_{\Omega} \sqrt[3]{2x+4y-4z+21}dv < \quad 3 \times \frac{4}{3}\pi = 4\pi$$