



# COMP90014

Algorithms for Bioinformatics Week 11A: Supervised Learning

# Supervised Learning

## Recap & brief detour about GeoGuessr

Supervised learning

K-Nearest Neighbors (KNN)

Naive Bayes

Association Rule Learning

Support Vector Machines (SVM)

**Decision Trees** 

**Ensemble Methods** 

## Machine learning



Machine Learning is the field of study that gives computers the ability to learn without being explicitly programmed.

- Arthur Samuel, 1959.

Machine Learning is the study of algorithms that:

- Improve their performance P
- at some task T
- with experience E.

A well-defined learning task is given by  $\langle P, T, E \rangle$ 

- Tom Mitchell, 1998.

## Application of Unsupervised Clustering

#### GeoGuessr - Online Game

Google maps street view image is shown, player guesses the location.

Closer to the actual location = better score.

#### Difficult problem for AI

Images taken from anywhere, not uniformly sampled (cities vs rural)

Daytime / nighttime, varying weather, Illumination, season, traffic etc

#### Predicting latitudes and longitudes directly = poor performance

Latitude / longitude not really good classification metric - regression.

Modern approaches use **geocells** to divide space into categories - classification

Unsupervised clustering used to define geocells

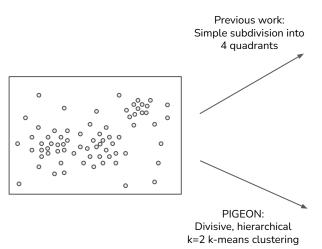


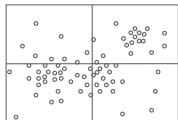
(a) Sample image in a Geoguessr location.

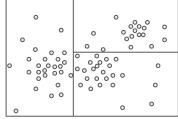


(b) Sample comparison of guesses between PIGEON and a human player.

## PIGEON: Initial idea for Geocell Creation







## PIGEON: Selected idea for Geocell Creation

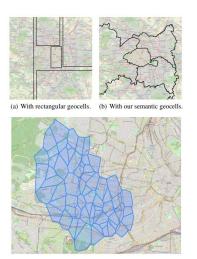


Figure 3. Voronoi tessellation applied in the process of geocell creation.

```
Algorithm 1 Semantic Geocell Division Algorithm
  Input: geocell boundaries g, training samples x,
  OPTICS parameters p, minimum cell size MINSIZE.
  Initialize j = 1.
  repeat
     Initialize C \neq \mathsf{OPTICS}(p_i).
     for q_i in q do
       Define x_i = \{x_j | x_j \in x \land x_j \in g_i\}.
        repeat
          Cluster c = C(x_i).
          c_{max} = c_k where |x_{i,k}| \ge |x_{i,l}| \forall l.
          if |c_{max}| > MINSIZE and |x \setminus x_{i,k}| > MINSIZE
          then
             New cell q_{new} = VORONOI(x_{i,k}).
             q_i = q_i \setminus q_{new}.
             Assign x_i to cells i and new.
          end if
        until convergence
     end for
     i = i + 1
  until i is |p|
```

## PIGEON: Performance & Quirks

#### Works really well

On-par or better than human world champion

#### Neural net was analysed

Which features of an image was it paying attention to?





(a) Attention attribution map for an image in Canada.

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## PIGEON: Performance & Quirks

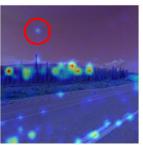
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Which features of an image was it paying attention to?





(a) Attention attribution map for an image in Canada.

#### PIGEON worked out an interesting feature to use - the car!

Has time and location data for each image.

Knows that images taken close in time & close in location were done by same car

Could use features of the car / camera to identify a new image (eg smudges on the lens)

How do we feel about this? Perhaps... **overfitting**?

# Supervised Learning

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Naive Bayes

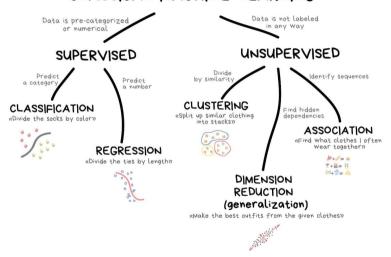
Association Rule Learning

Support Vector Machines (SVM)

**Decision Trees** 

**Ensemble Methods** 

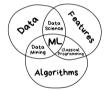
## CLASSICAL MACHINE LEARNING



## Three components of supervised learning

#### Data

- labelled (historical / experimental)
- representative and diverse
  - collection and curation is key
  - A predictive model is as good as the training data



#### **Features**

- properties to be used as evidence to train a predictive model
- requires knowledge of the problem

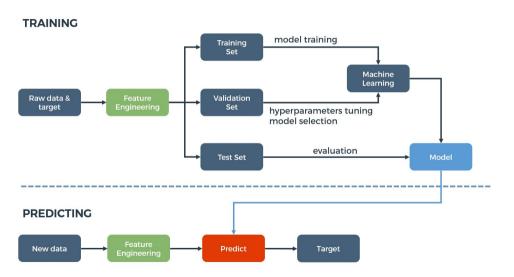
#### Learning Algorithms

which one?

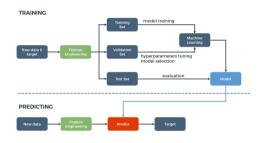
#### Infer a function *f*:

- maps input (features) to an output (target)
- experience (examples of input-output pairs)

## Supervised Learning Pipeline



## Data



#### Training set

representative set of examples used for training, where the target value is known

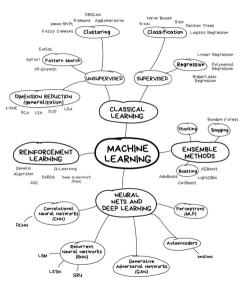
#### Validation set

representative set of examples used to tune the architecture of a learning algorithm and estimate prediction errors

#### Independent test set (blind test)

- independently assess the performance of a predictive model
- never used during the training process
- error on the blind test provides an unbiased estimate of the generalization error

## Learning algorithms



## There are tons of algorithms

- K-nearest neighbours (KNN)
- Naïve Bayes
- Support Vector Machines (SVMs)
- Decision Trees
- not an exhaustive list

# Supervised Learning

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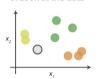
Support Vector Machines (SVM)

**Decision Trees** 

**Ensemble Methods** 

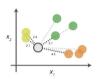
## K-nearest neighbours (KNN)

#### 0. Look at the data



Say you want to classify the grey point into a class. Here, there are three potential classes - lime green, green and orange.

#### 1. Calculate distances



Start by calculating the distances between the grey point and all other points.

#### 2. Find neighbours



Next, find the nearest neighbours by ranking points by increasing distance. The nearest neighbours (NNs) of the grey point are the ones closest in dataspace.

#### 3. Vote on labels



Vote on the predicted class labels based on the classes of the k nearest neighbours. Here, the labels were predicted based on the k=3 nearest neighbours.

- given training data,  $D = \{(x_1, y_1), ..., (x_N, y_N)\}$ and a test point  $x_u$
- prediction rule: look at the K most similar training examples to  $x_u$
- for classification: assign the majority class label (majority voting)
- for regression: assign the average response
- The algorithm requires
  - parameter K: number of nearest neighbors to look for
  - distance function: to compute similarities between points

## KNN algorithm

## Procedure KNN:

3

5

6

7

#### foreach test point do

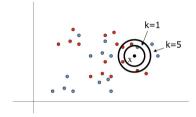
Compute the distance to each training point

Sort the distances in ascending order

Select the K-nearest neighbors

if classification then use majority rule

if regression then use averaging



- KNN is called a non-parametric method
- Unlike other supervised learning algorithms, it doesn't learn an explicit mapping function f from the training data (no explicit model)
- It simply uses the training data at the test time to make predictions

## KNN implementation

```
# Locate the most similar neighbors
     def get_neighbors(train, test_row, num_neighbors):
         distances = list()
 3
         for train row in train:
 4
             dist = euclidean distance(test row. train row)
 5
             distances.append((train_row, dist))
 6
         distances.sort(kev=lambda tup: tup[1])
        neighbors = list()
 8
         for i in range(num_neighbors):
9
             neighbors.append(distances[i][0])
10
11
         return neighbors
12
13
     # Make a prediction with neighbors
     def predict_classification(train, test_row, num_neighbors):
14
         neighbors = get_neighbors(train, test_row, num_neighbors)
15
         output_values = [row[-1] for row in neighbors]
16
         prediction = max(set(output_values), key=output_values.count)
17
         return prediction
18
```

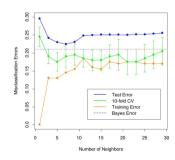
## KNN implementation

```
# kNN Algorithm
def k_nearest_neighbors(train, test, num_neighbors):
    predictions = list()
    for row in test:
        output = predict_classification(train, row, num_neighbors)
    predictions.append(output)
return(predictions)
```

- Time complexity?
- Space complexity?
- Would KNN be adequate for large-scale, real-time predictions?

## Properties of KNN

$$d(x_i, x_j) = \sum_{m=1}^{D} (x_{im} - x_{jm})^2$$



#### David Sontag, New York University

#### KNN requires computing distances

several different distance metrics (e.g. Euclidean)

#### Features should be on the same scale

e.g. kcal and kJ (we need to normalize them)

## Choosing K

#### Pros:

simple and intuitive, easily to implement/interpret

#### Cons:

- memory usage (store training data in memory)
- may perform badly in high dimensions
- sensitive to noisy features
- categorical attributes?

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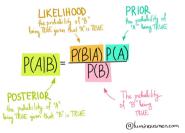
**Decision Trees** 

**Ensemble Methods** 

## Naïve Bayes

#### Conditional probability:

# $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ Probability of A given B Probability of B Probability of B



#### Maximum a posteriori (MAP):

$$rg \max P(A \mid B)$$
 for each class  $A$   $rg \max P(A \mid B) = rg \max P(B \mid A) imes rac{P(A)}{P(B)}$   $= rg \max P(B \mid A) imes P(A)$ 

- uses Bayes's rule
- P(B) will be the same for all classes
- for a set of features  $x_i$  we calculate the joint probability:  $B = (x_1, x_2, ..., x_n)$

$$\begin{array}{c} ^{\tiny \textcircled{\tiny \textbf{arg}}} \ \operatorname{arg} \max P(x_1, x_2, ..., x_n \mid A) \times P(A) \\ P(x_1, x_2, ..., x_n \mid A) = \prod_i P(x_i \mid A) \end{array}$$

- simply the product of individual probabilities
- assumption: features are independent given a class

Sneezing	Fever	Flu
Yes	No	No
No	Yes	Yes
No	No	No
Yes	Yes	Yes
	Yes No No	Yes No No Yes No No

- Given this training data, if we have a new patient with only a cough, should we diagnose them with the flu?
- → P(? | Cough, No Sneezing, No Fever)
- $P(Cough, No Sneezing, No Fever | Flu) \times P(Flu)?$
- $P(Cough, No Sneezing, No Fever | No Flu) \times (No Flu)?$

Procedure Naïve Bayes Learn(examples):
 foreach target value v<sub>j</sub> do
 | P̂(V<sub>j</sub>) ← estimate P(V<sub>j</sub>)
 foreach attribute value a<sub>i</sub> of each attribute a
 do

 $\hat{P}(a_i \mid v_i) \leftarrow \text{estimate } P(a_i \mid v_i)$ 

- 1 Procedure Classify New Instance(x):
- $\mathbf{2} \quad \middle| \ \mathbf{v}_{NB} = \operatorname*{arg\,max}_{\mathbf{v}_j \in V} \hat{P}(\mathbf{v}_j) \prod_{a_i \in \mathbf{x}} \hat{P}(a_i \mid \mathbf{v}_j)$

1 Procedure Naïve Bayes Learn(examples):
2 | foreach target value 
$$v_j$$
 do
3 |  $\hat{P}(V_j) \leftarrow$  estimate  $P(V_j)$ 
4 | foreach attribute value  $a_i$  of each attribute  $a$  do
5 |  $|\hat{P}(a_i \mid v_i) \leftarrow$  estimate  $P(a_i \mid v_i)$ 

1 Procedure Classify New Instance(x):

$$2 \quad \left| \begin{array}{l} v_{NB} = \arg \max_{v_j \in V} \hat{P}(v_j) \prod_{a_i \in x} \hat{P}(a_i \mid v_j) \end{array} \right.$$

Туре	Long	Not long	Sweet	Not sweet	Yellow	Not yellow	Total
Banana	400	100	350	150	450	50	500
Orange	0	300	150	150	300	0	300
Other	100	100	150	50	50	150	200
Total	500	500	650	350	800	200	1000

Step 1: Compute the 'Prior' probabilities for each of the classes.

## P(Banana) = 
$$\frac{500}{1000}$$
 = 0.50  
## P(Orange) =  $\frac{300}{1000}$  = 0.30

$$P(\text{Orange}) = \frac{300}{1000} = 0.30$$

$$P(\text{Other}) = \frac{200}{1000} = 0.20$$

		Not		Not		Not	
Type	Long	long	Sweet	sweet	Yellow	yellow	Total
Banana	400	100	350	150	450	50	500
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1 Procedure Classify New Instance(x):

$$\begin{array}{c|c} 2 & v_{NB} = \displaystyle \argmax_{v_j \in V} \hat{P}(v_j) \prod_{a_i \in x} \hat{P}(a_i \mid v_j) \end{array}$$

Step 2: Compute the 'Likelihood' probabilities for each feature/class combination.

P(Long | Banana) = 
$$\frac{400}{500}$$
 = 0.80

P(Sweet | Banana) =  $\frac{350}{500}$  = 0.70

P(Yellow | Banana) =  $\frac{450}{500}$  = 0.90

P(A | B) =  $\frac{P(A \cap B)}{P(B)}$ 

Probability of A given B Probability of B

Step 3: Compute the 'Posterior' probabilities for each class given the features.

$$P(Y \mid Long, Sweet, Yellow)$$
?

$$P(\text{Banana} \mid \text{Long, Sweet, Yellow})$$

$$= (0.8 \times 0.7 \times 0.9) \times 0.5 = 0.252$$

$$P(\text{Orange} \mid \text{Long, Sweet, Yellow})$$

$$= ... = 0.0 \rightarrow P(\text{Long} \mid \text{Orange}) = 0$$

$$P(\text{Other} \mid \text{Long, Sweet, Yellow})$$

$$= ... = 0.019$$

$$\begin{vmatrix}
\hat{P}(V_j) \leftarrow \text{ estimate } P(V_j) \\
\text{foreach } attribute \ value \ a_i \ of each \ attribute \ a
\end{vmatrix}$$

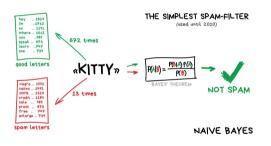
$$|\hat{P}(a_i \mid v_j) \leftarrow \text{estimate } P(a_i \mid v_j)$$

do

$$v_{NB} = \underset{v_j \in V}{\operatorname{arg max}} \hat{P}(v_j) \prod_{a_i \in x} \hat{P}(a_i \mid v_j)$$



## Naïve Bayes



#### Pros

- Easy to implement
- Very efficient

## Cons

- Independence assumption doesn't always hold
- Loss of accuracy

# Supervised Learning

Recap & brief detour about GeoGuessr

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Support Vector Machines (SVM)

**Decision Trees** 

**Ensemble Methods** 

Tries to find "interesting" **dependent** associations between variables.

Cough	Sneezing	Fever	Flu
Yes	Yes	No	No
Yes	No	Yes	Yes
No	No	No	No
No	Yes	Yes	Yes

Tries to find "interesting" **dependent** associations between variables.

Transaction:  $X \Rightarrow Y$  (if X then Y)

- $\{Sneezing\}$   $\Rightarrow \{Flu\}$
- {Cough}  $\Rightarrow$  {Flu}
- {Sneezing, Cough} ⇒ {Flu}

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Apply metrics known as **rules** on transactions to identify interesting relationships.

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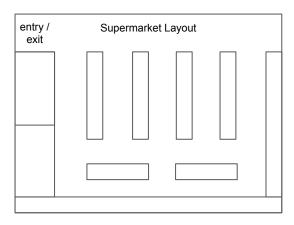
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Apply metrics known as **rules** on transactions to identify interesting relationships.

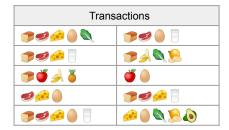
Rule	Description
Support	How frequently does "X" appear in the dataset
Confidence	How many itemsets with "X" also contain "Y"
Lift	Ratio of observed support, to expected support if "X" and "Y" were independent
Conviction	How likely an association is genuine, not just random chance

Application: Product placement in supermarkets

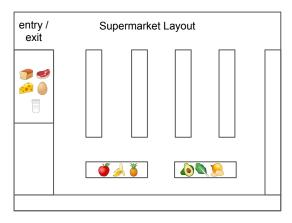
Transactions		
<b>₽ 3 3</b>	<b>5</b>	



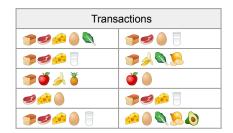
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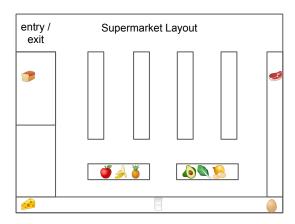


#### Easiest for customer

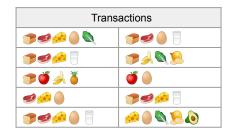


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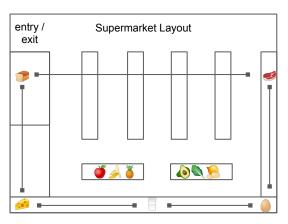




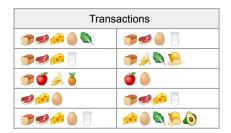
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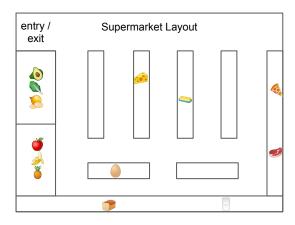


### Max distance

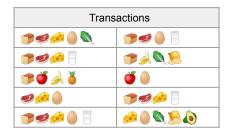


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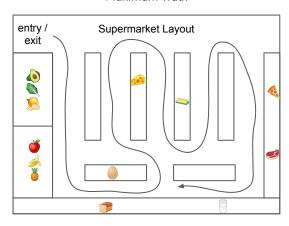




Application: Product placement in supermarkets



#### Maximum walk



# **Cutoff Criteria**Min Support = 3

Example: Apriori algorithm

Bottom up approach where subsets extended by 1 item at a time

Uses BFS to explore item sets efficiently

Identifies branches which will not meet minimum cutoff criteria

П	ransactio	ns
	Itemsets	
	{1,2,3,4}	
	{1,2,4}	
	{1,2}	
	{2,3,4}	
	{2,3}	
	{3,4}	
	{2,4}	

1 item		2 items		3 items	
Item	Support	Item	Support	Item	Suppor
{1}	3	{1,2}	3	{2,3,4}	2
{2}	6	<del>{1,3}</del>	1		
{3}	4	<del>{1,4}</del>	2		
{4}	5	{2,3}	3		
		{2,4}	4		
		{3,4}	3		

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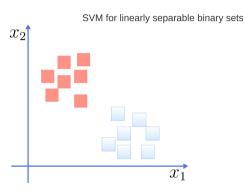
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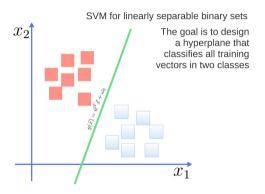
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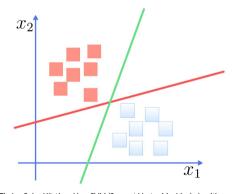


- consider linearly separable points
- for given training data, find decision boundaries separating classes (hyperplane)
- maximises the perpendicular distance to the nearest points
- maximum-margin hyperplane
- gives a linear separator
- optimisation problem



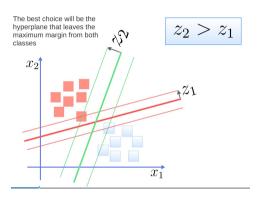
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The best choice will be the hyperplane that leaves the maximum margin from both classes

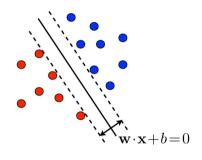


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Thales Sehn Körting, How SVM (Support Vector Machine) algorithm works on  $\underline{youtube}$ 

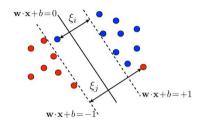


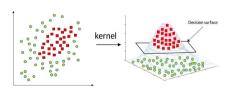
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- optimisation problem



- finding maximum-margin hyperplane is a quadratic optimisation problem to find vector weights w and b
- w can be found as a linear combination of a subset of the training points
  - Points along the margins the support vectors
- computationally straightforward
- minimising a convex function will find global optimum solution
- gives a linear boundary with maximum distance to points

## What if our data is not linearly separable?





- Use a soft SVM
  - Add terms to objective function to penalise points on the "wrong side" of boundary, but don't forbid them
- Use a kernel function
  - Transform input data into a separable space
  - Kernel SVM allows non-linear boundaries in the original space and is a popular approach

Polynomials of degree exactly d:  $K(u,v) = (u \cdot v)^d$ Polynomials of degree up to d:  $K(u,v) = (u \cdot v + 1)^d$ Gaussian kernels:  $K(\vec{u},\vec{v}) = \exp\left(-\frac{\|\vec{u}-\vec{v}\|_2^2}{2\sigma^2}\right)$ Sigmoid:  $K(u,v) = \tanh(nu \cdot v + v)$ 

# Supervised Learning

Recap & brief detour about GeoGuessr

Supervised learning

K-Nearest Neighbors (KNN)

Naive Bayes

Association Rule Learning

Support Vector Machines (SVM)

**Decision Trees** 

**Ensemble Methods** 

### **Decision Trees**

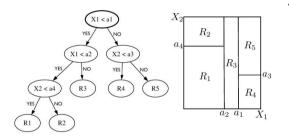
#### GIVE A LOAN?



DECISION TREE

- a decision tree consists of a hierarchy of rules for prediction
- we can visualise this as a tree where every node is either:
  - an internal node, with a decision criterion, with two children (binary)
  - a leaf node, which predicts an outcome
- trees can be built for classification or for regression

### How to build a decision tree from data?



- 1 Choose a decision point yielding best purity
- 2 Partition data into corresponding subsets
- 3 Reiterate with resulting subsets
- 4 Stop when regions are approximately pure

### Impurity in classification

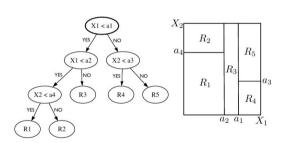
- misclassification
- Gini impurity: probability of incorrectly classifying a randomly chosen data point

### Impurity in regression

mean squared error

$$F(R) = \sum_{x_i \in R} (y_i - \langle y \rangle)^2$$

## Recursive binary splitting is a greedy heuristic



#### Prediction

a path from root to leaf based on the features

#### Pros

- decision trees facilitate explaining complex data
  - interpretable
- results can be easy to analyze and understand
- different types of variables: categorical, numerical

#### Cons

- large trees can easily overfit
- use a pruning criteria
  - minimize (impurity + tree size)

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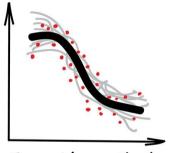
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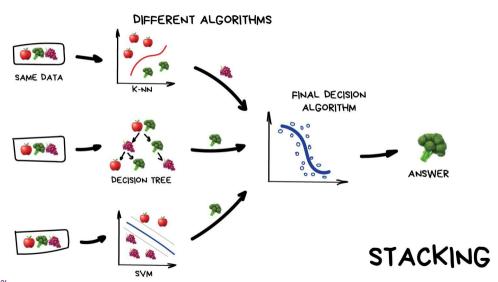
## Ensemble methods



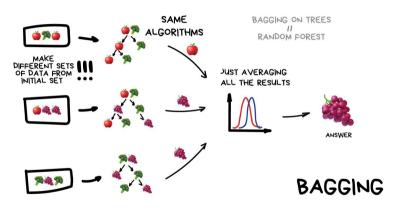
Ensemble Methods

- rather than just one, combine multiple learning algorithms (or instances) into one predictive model
- ensemble methods are meta-algorithms aiming to:
  - decrease variance;
  - decrease bias;
  - improve predictions
- Stacking
- Bagging
- Boosting

## Stacking



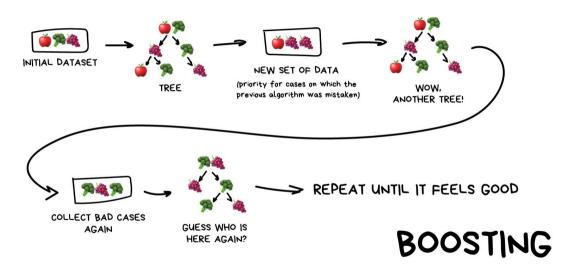
## Bagging



# Random Forest (Breiman *et al.*, 2001)

- Very popular and effective
- Classification & regression
- Robust to outliers & feature importance

## Boosting



vas3k





# Thank you!

Today: Supervised Learning

Next time: Model selection, tuning, validation