Randomness
and
Pseudorandomness

Review

Problem:

Integrity of message from Alice to Bob

Alice must append bits to message that only Alice (or Bob) can make

Solution:

Message Authentication Code (MAC)

Practical solution:

Hash-based MAC (HMAC) –
HMAC-SHA256$_{\mathbf{k}}$(M)

Where do these random keys **k** come from … ?

Careful: We're often sloppy about what is "random"

True Randomness

- Output of a physical process that is inherently random
- Scarce and hard to get

Pseudorandom generator (PRG)

- Takes small seed that is really random
- Generates long sequence of numbers that are “as good as random”

Definition: **PRG** is secure if it's indistinguishable from random

Similar game to PRF definition:

1. We flip a coin secretly to get a bit **b**
2. If **b**=0, let ***s*** be a truly random stream
   If **b**=1, let ***s*** be g_k for random secret **k**
3. Mallory can see as much of the output of ***s*** as he/she wants
4. Mallory guesses **b**,
   wins if guesses correctly

Say **g** is a secure PRG if there is no winning strategy for Mallory*

Here's a *simple PRG that works:*

For some random **k** and PRF ***f***,
output: $f_k(0) \,||\, f_k(1) \,||\, f_k(2) \,||\, \ldots$

Theorem: If ***f*** is a secure PRF, and ***g*** is built from ***f*** by this construction, then ***g*** is a secure PRG.

Proof: Assume ***f*** is a secure PRF, we need to show that ***g*** is a secure PRG.

Proof by contradiction:

1. Assume ***g*** is *not* secure;
   therefore Mallory can win the PRG game
2. This gives Mallory a winning strategy for the PRF game:
   a. query the PRF with inputs 0, 1, 2, …
   b. apply the PRG-distinguishing algorithm
3. Therefore, Mallory can win the PRF game, which is a contradiction
4. Therefore, g is secure

Where do we get true randomness?

Want "indistinguishable from random" which means: adversary can't guess it

Gather lots of details about the computer that the adversary will have trouble guessing [Examples?]

- Problem: Adversary can predict some of this
- Problem: How do you know when you have enough randomness?

Modern OSes typically collect randomness, give you API calls to get it

- e.g., Linux:
- `/dev/random` is a device that gives random bits, blocks until available
- `/dev/urandom` gives output of a PRG, nonblocking, seeded from `/dev/random` *eventually*

Confidentiality

Review

Problem:

Integrity of message from Alice to Bob over an untrusted channel

Alice must append bits to message that only Alice (or Bob) can make

Solution:

Random function

Practical solution:

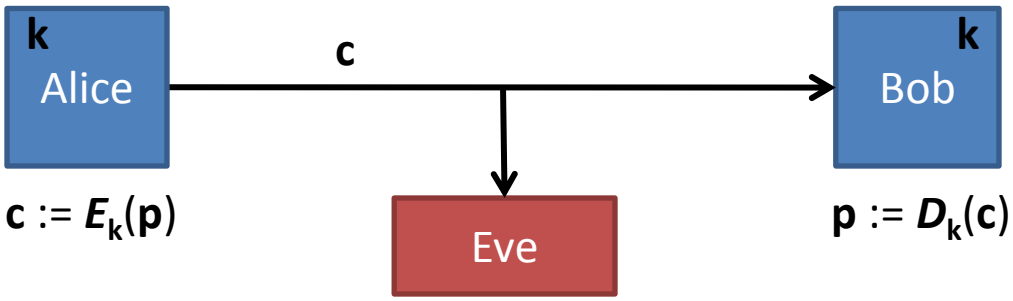
e.g. "Attack at dawn", 628369867…

(Hash-based) MAC

f_k is (we hope!) indistinguishable in practice from a random function, unless you know **k**

Confidentiality

Goal: Keep contents of message **p** secret from an *eavesdropper*



Terminology

p	plaintext
c	ciphertext
k	secret key
E	encryption function
D	decryption function

Digression: **Classical Cryptography**

Caesar Cipher

First recorded use: Julius Caesar (100-44 BC)

Replaces each plaintext letter with one a fixed number of places down the alphabet

Encryption: $\mathbf{c}_i := (\mathbf{p}_i + \mathbf{k}) \bmod 26$

Decryption: $\mathbf{p}_i := (\mathbf{c}_i - \mathbf{k}) \bmod 26$

e.g. (**k**=3):

```
   Plain: ABCDEFGHIJKLMNOPQRSTUVWXYZ
  +Shift: 33333333333333333333333333
=Cipher: DEFGHIJKLMNOPQRSTUVWXYZABC
```

```
   Plain: fox   go wolverines
   +Key: 333   33 3333333333
=Cipher: ira   jr zroyhulqhv
```

[Break the Caesar cipher?]

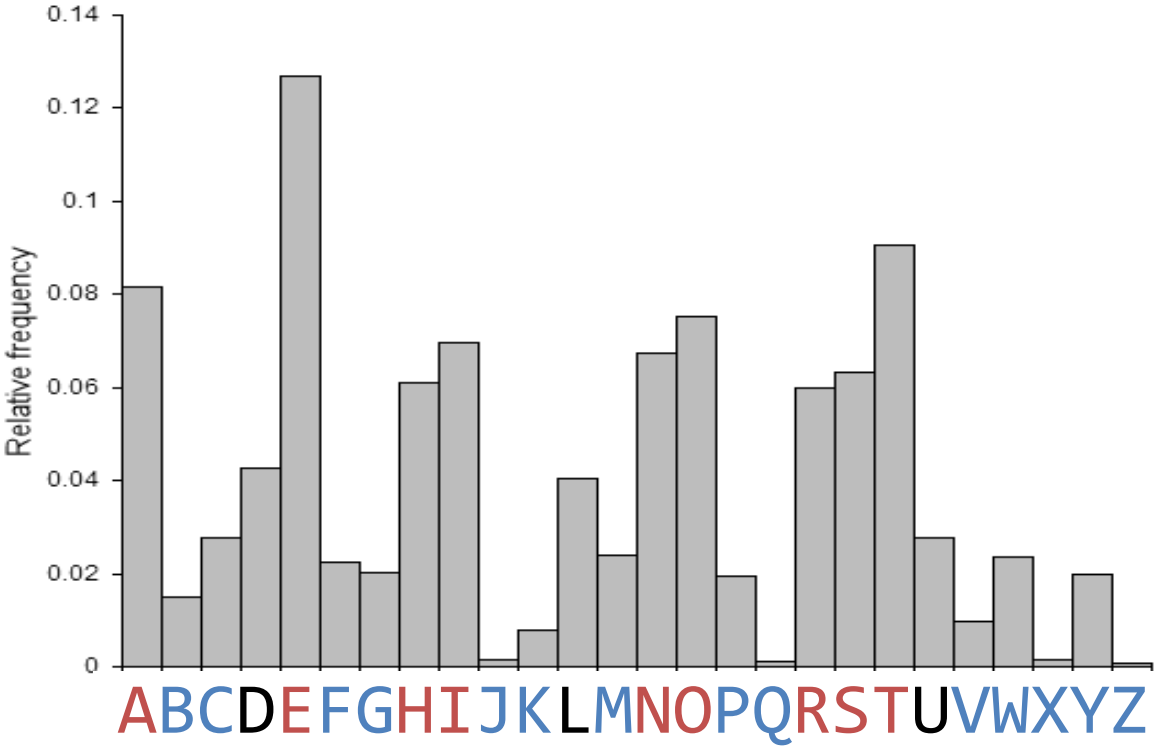
Cryptanalysis of the Caesar Cipher

Only 26 possible keys:

Try every possible **k** by "*brute force*"

Can a computer recognize the right one?

Use *frequency analysis*: English text has distinctive letter frequency distribution



Recognize with (e.g.) chi-square test

Later advance: Vigènere Cipher

First described by Bellaso in 1553, later misattributed to Vigenère

Called « le chiffre indéchiffrable » (“the indecipherable cipher”)

Encrypts successive letters using a sequence of Caesar ciphers determined by the letters of a keyword

For an **n**-letter keyword **k**,

Encryption: $\mathbf{c}_i := (\mathbf{p}_i + \mathbf{k}_{i \bmod n}) \bmod 26$

Decryption: $\mathbf{p}_i := (\mathbf{c}_i - \mathbf{k}_{i \bmod n}) \bmod 26$

Example: **k**=ABC (i.e. $\mathbf{k}_0$=0, $\mathbf{k}_1$=1, $\mathbf{k}_2$=2)

```
    Plain:  bbbbbb  amazon
    +Key:   012012  012012
=Cipher:    bcdbcd  anczpp
```

[Break *le chiffre indéchiffrable*?]

Cryptanalysis of the Vigènere Cipher

Simple, if we know the keyword length, **n**:

1. Break ciphertext into **n** slices
2. Solve each slice as a Caesar cipher

How to find **n**? One way: **Kasiski method**

Published 1863 by Kasiski (earlier known to Babbage?)

Repeated strings in long plaintext
will sometimes, by coincidence,
be encrypted with same key letters

Plain: CRYPTOISSHORTFORCRYPTOGRAPHY
+Key: ABCDABCDABCDABCDABCDABCDABCD
=Cipher: CSASTPKVSIQUTGQUCSASTPIUAQJB

Distance between repeated strings in the ciphertext is likely a multiple of key length

e.g., distance 16 implies **n** is 16, 8, 4, 2, or 1

Find multiple repeats to narrow down

[What if key is as long as the plaintext?]

Back to the present:

One-time Pad (OTP)

Alice and Bob jointly generate a secret, very long, string of random bits (the one-time pad, **k**)

To encrypt: $\mathbf{c}_i = \mathbf{p}_i$ xor $\mathbf{k}_i$

To decrypt: $\mathbf{p}_i = \mathbf{c}_i$ xor $\mathbf{k}_i$

a	**b**	**a** xor **b**
0	0	0
0	1	1
1	0	1
1	1	0

a xor **b** xor **b** = **a**

a xor **b** xor **a** = **b**

"one-time" means you should never reuse any part of the pad.

If you do:

Let $\mathbf{k}_i$ be pad bit

Adversary learns (**a** xor $\mathbf{k}_i$) and (**b** xor $\mathbf{k}_i$)

Adversary xors those to get (**a** xor **b**), which is useful to him [How?]

Provably secure [Why?]

Usually impractical [Why? Exceptions?]

Obvious idea: Use a **pseudorandom generator** instead of a truly random pad

(Recall: Secure **PRG** inputs a seed **k**, outputs a stream that is practically indistinguishable from true randomness unless you know **k**)

Called a **stream cipher**:

1. Start with shared secret key **k**
2. Alice & Bob each use **k** to seed the PRG
3. To encrypt, Alice XORs next bit of her generator's output with next bit of plaintext
4. To decrypt, Bob XORs next bit of his generator's output with next bit of ciphertext

Works nicely, but: don't *ever* reuse the key, or the generator output bits

So Far

The Security Mindset

Message Integrity

Randomness /Pseudorandomness

Confidentiality: Stream Ciphers

Thursday…

Block Ciphers and Cipher Modes