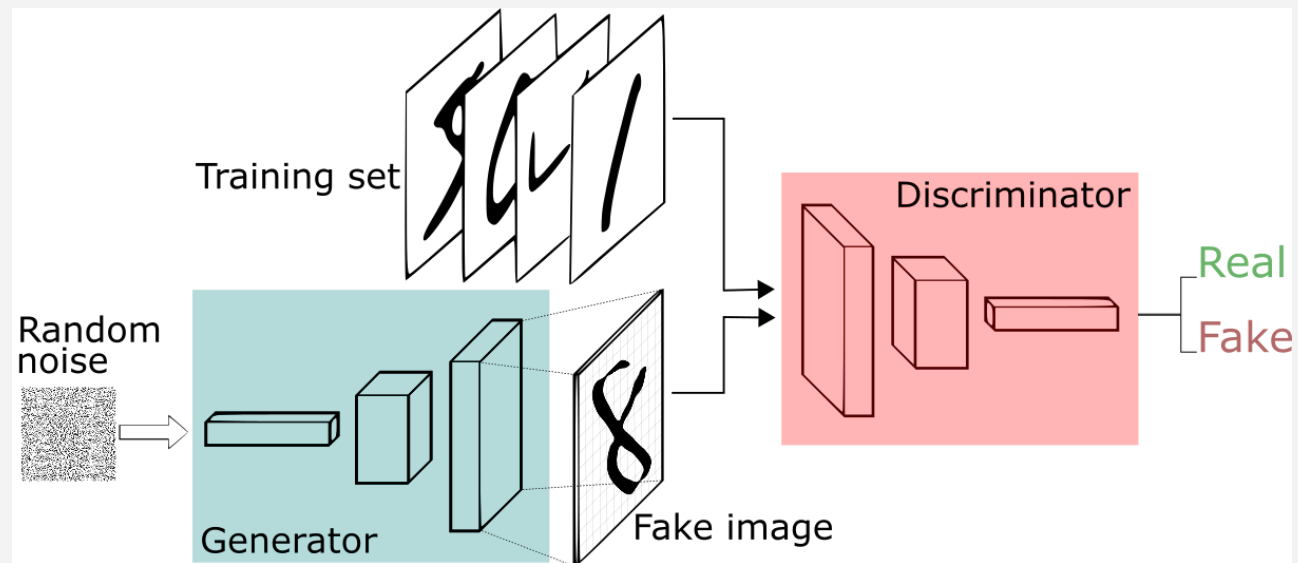


# The loss of GANs

$$\arg \min_{\theta_G} \max_{\theta_D} \mathbb{E}_{x \sim \mathcal{P}_{data}} [\log D(x)] + \mathbb{E}_{z \sim \mathcal{P}_{noise}} [\log(1 - D(G(z)))] \quad (1)$$

We denote the generator as  $G$  and the discriminator as  $D$  with training image  $x \sim \mathcal{P}_{data}$  following the distribution of training images and random noise  $z \sim \mathcal{P}_{noise}$ . The generator is parametrized by  $\theta_G$  and the discriminator is parametrized by  $\theta_D$ .



# Neural Tangent Kernel(NTK)

According to the paper [Neural Tangent Kernel: Convergence and Generalization in Neural Networks](#)(by Jacot, NIPS'18), the Neural Tangent Kernel(NTK) over the training dataset  $\mathbf{X}$  corresponding to the architecture of the neural network  $f(\cdot; \theta)$  is defined as

$$\mathbf{K} = \nabla_{\theta} f(\mathbf{X}; \theta)^{\top} \nabla_{\theta} f(\mathbf{X}; \theta) \quad (2)$$

Let  $\mathbf{K}^{n,n} \in \mathbb{R}^{n \times n}$  be the kernel matrix for  $\mathbf{X}^n$ , i.e.,  $\mathbf{K}_{i,j}^{n,n} = k(\mathbf{X}_{i,:}^n, \mathbf{X}_{j,:}^n)$ . The mean prediction of the ensemble of the neural network  $f(\cdot; \theta)$  after training  $t$  steps gradient descent can be approximated by the mean prediction of the NTK-GP with corresponding kernel. The mean prediction of the NTK-GP over  $\mathbf{X}^n$  evolves as

$$(\mathbf{I}^n - e^{-\eta \mathbf{K}^{n,n} t}) \mathbf{Y}^n \quad (3)$$

where  $\mathbf{I}^n \in \mathbb{R}^{n \times n}$  is an identity matrix and  $\eta$  is a sufficiently small learning rate.

# Our Method: GA-NTK

Let  $\mathbf{K}^{2n,2n} \in \mathbb{R}^{2n \times 2n}$  be the kernel matrix for  $\mathbf{X}^n \oplus \mathbf{Z}^n$ , where the value each element  $\mathbf{K}_{i,j}^{2n,2n} = k((\mathbf{X}^n \oplus \mathbf{Z}^n)_{i,:}, (\mathbf{X}^n \oplus \mathbf{Z}^n)_{j,:})$ . Let  $\lambda = \eta \cdot t$  The discriminator can be written as:

$$D(\mathbf{X}^n, \mathbf{Z}^n; k, \lambda) = \underbrace{(\mathbf{I}^{2n} - e^{-\lambda \mathbf{K}^{2n,2n}})}_{NTK-GP} (\mathbf{1}^n \oplus \mathbf{0}^n) \in \mathbb{R}^{2n} \quad (4)$$

where  $\mathbf{I}^{2n} \in \mathbb{R}^{2n \times 2n}$  is an identity matrix. We formulate the objective of GA-NTK as follows:

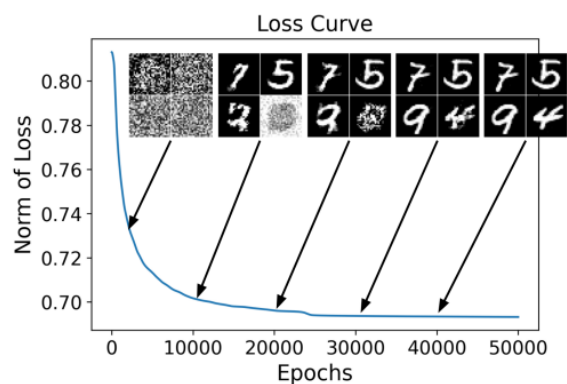
$$\arg \min_{\mathbf{Z}^n} \|\mathbf{1}^{2n} - D(\mathbf{X}^n, \mathbf{Z}^n; k, \lambda)\|_2 \quad (5)$$

where  $\mathbf{1}^{2n} \in \mathbb{R}^{2n}$  is a vector of ones. Then, we update the fake image at the last time step

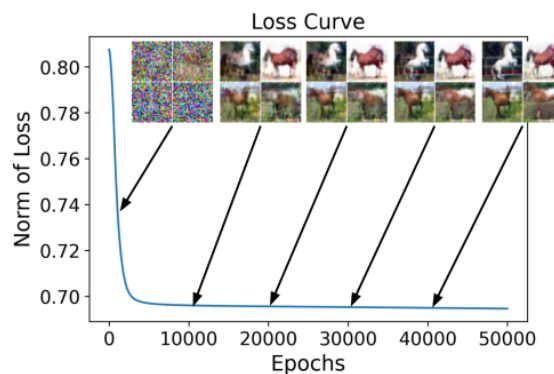
$$\mathbf{Z}^{n+1} = \mathbf{Z}^n + \alpha \nabla_{\mathbf{Z}^n} \|\mathbf{1}^{2n} - D(\mathbf{X}^n, \mathbf{Z}^n; k, \lambda)\|_2 \quad (6)$$

# Stable Convergence

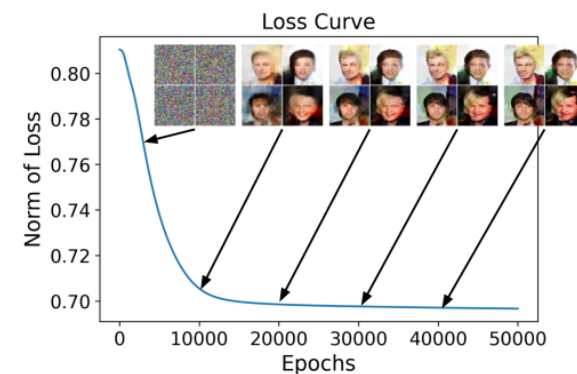
The image quality is consistent with the loss. As the **loss decreases**, the **image quality is improved**.



(a) MNIST



(b) CIFAR-10

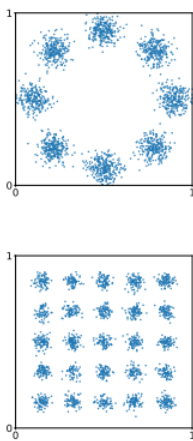


(c) CelebA

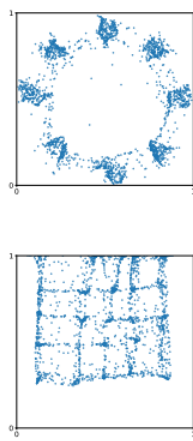
Figure 3. Loss curve and image quality at different iterations on different dataset. We can see that image quality issues correlated with loss curve.

# Mode Collapse

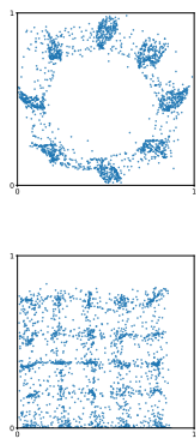
Our method(**GA-FNTK**) fits to **multi-modal distribution perfectly**, rather than other baseline GANs.



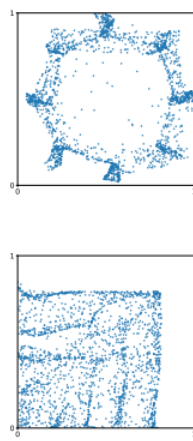
(a) Ground truth



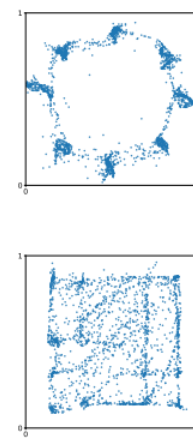
(b) Vanilla GAN



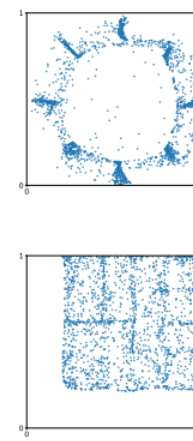
(c) LSGAN



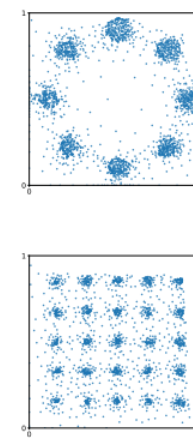
(d) WGAN



(e) WGAN-GP



(f) SN-GAN

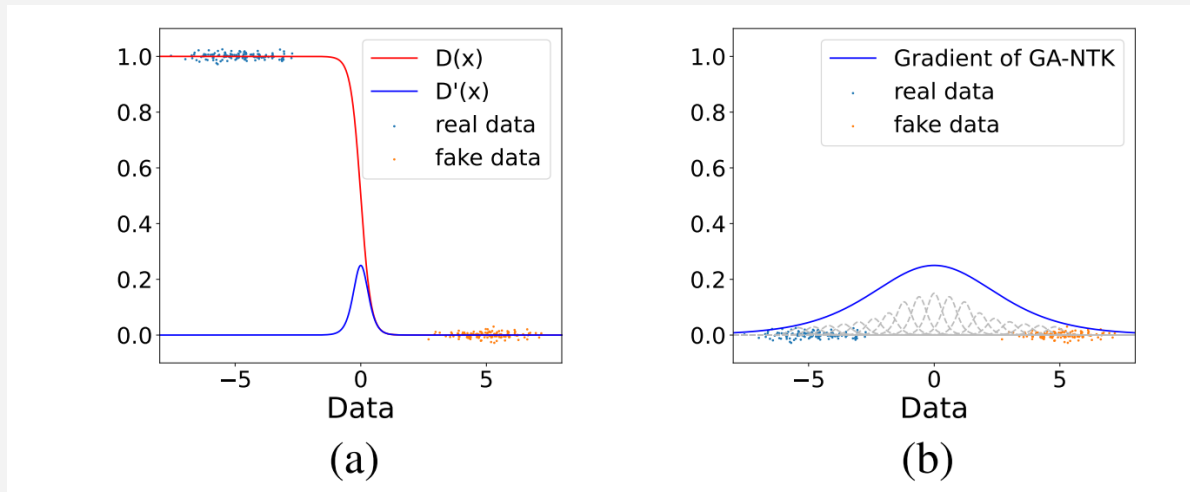


(g) GA-FNTK

# Gradient Vanish

Decision boundary of the discriminator and the corresponding gradients for the generator  $\mathbf{Z}^n$  in (a) GANs and (b) GA-NTK on 1D toy dataset. The blue points are real data points and the red ones are fake. The discriminator output 1 for the real data points and 0 for the fake ones (fig (a) red line). The gradients (fig (a) blue line) vanishes while the data points are far away from the decision boundary.

The gray dashed lines indicate the gradients for  $\mathbf{Z}^n$  from different element networks of the ensemble discriminator in GA-NTK.



# Compare GA-CNTK & GANs

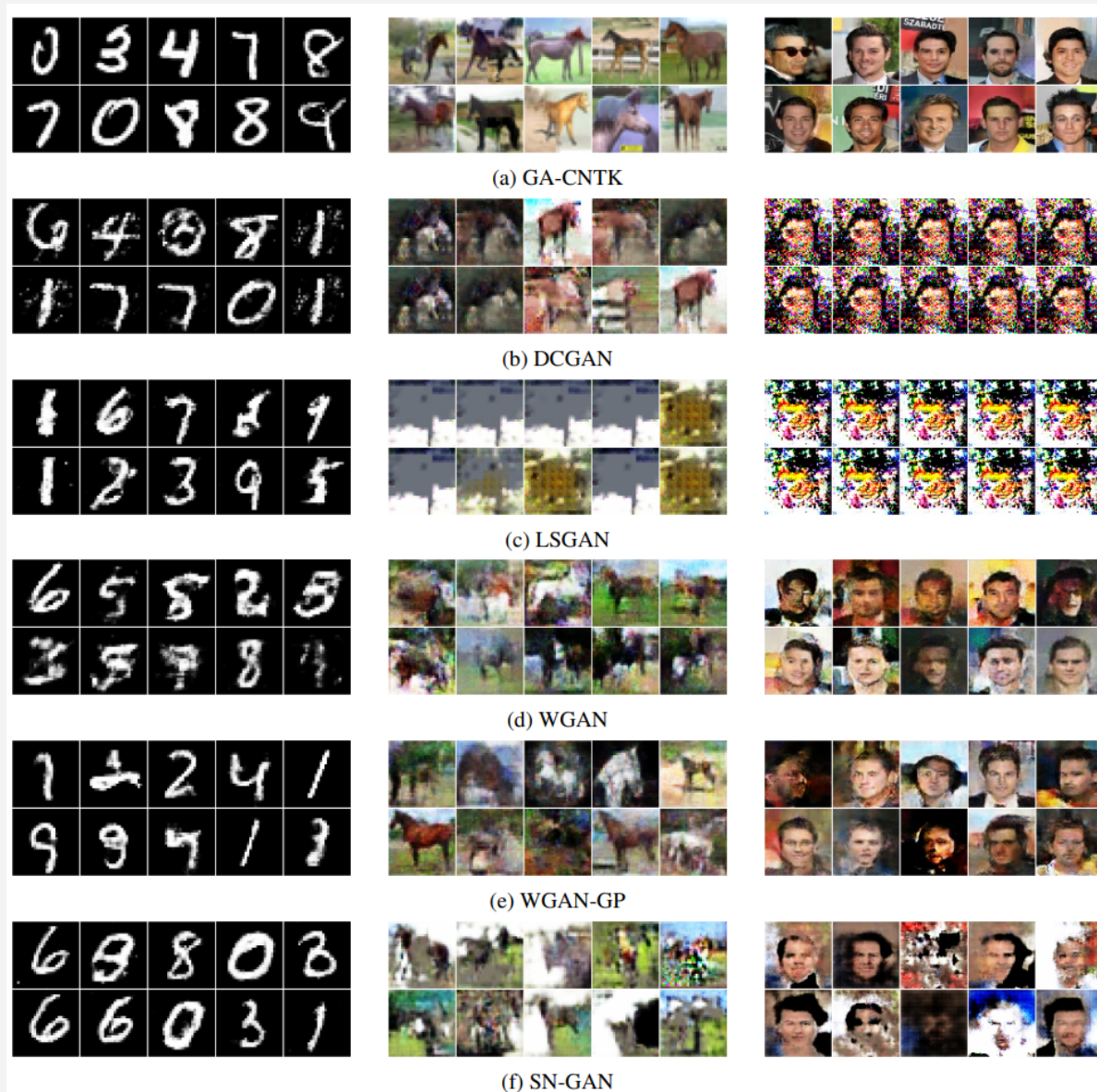


Figure 2. Generated samples from MNIST, CIFAR10, and CelebA. The sizes of the training datasets are 256.