Neural Kernel Without Tangents

Convolution Kernel

Convolution. The convolution operation c_w maps an input tensor $\boldsymbol{K}_{in}$ to an output tensor $\boldsymbol{K}_{out}$ of the same shape: $N \times D_1 \times D_2 \times N \times D_1 \times D_2$. w is an integer denoting the size of the convolution (e.g. $w = 1$ denotes a 3×3 convolution).

The elements of $\boldsymbol{K}_{out} = c_w(\boldsymbol{K}_{in})$ can be written as:

$$K_{out}[i, j, k, \ell, m, n] = \sum_{dx=-w}^{w} \sum_{dy=-w}^{w} K_{in}[i, j + dx, k + dy, \ell, m + dx, n + dy]$$

For out-of-bound location indexes, we simply zero pad the $\boldsymbol{K}_{in}$ so all out-of-bound accesses return zero.

Average Pooling Kernel

Average pooling. The average pooling operation p_w downsamples the spatial dimension, mapping an input tensor $\boldsymbol{K}_{in}$ of shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$ to an output tensor $\boldsymbol{K}_{out}$ of shape $N \times (D_1/w) \times (D_2/w) \times N \times (D_1/w) \times (D_2/w)$. We assume D_1 and D_2 are divisible by w.

The elements of $\boldsymbol{K}_{out} = p_w(\boldsymbol{K}_{in})$ can be written as:

$$K_{out}[i,j,k,\ell,m,n] = \frac{1}{w^4} \sum_{a=1}^{w} \sum_{b=1}^{w} \sum_{c=1}^{w} \sum_{d=1}^{w} \left(K_{in}[i, wj+a, wk+b, \ell, wm+c, wn+d] \right)$$

ReLU Kernel

The ReLU embedding, k_{relu}, is shape preserving, mapping an input tensor $\boldsymbol{K}_{in}$ of shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$ to an output tensor $\boldsymbol{K}_{out}$ of shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$. To ease the notation, we define two auxiliary tensors: $\boldsymbol{A}$ with shape $N \times D_1 \times D_2$ and $\boldsymbol{B}$ with shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$, where the elements of each are:

$$A[i,j,k] = \sqrt{K_{in}[i,j,k,i,j,k]}$$

$$B[i,j,k,\ell,m,n] = \arccos\left(\frac{K_{in}[i,j,k,\ell,m,n]}{A[i,j,k]A[\ell,m,n]}\right)$$

ReLU Kernel

The elements of $\boldsymbol{K}_{out} = k_{relu}(\boldsymbol{K}_{in})$ can be written as:

$$K_{out}[i, j, k, \ell, m, n]$$
$$= \frac{1}{\pi}\Big(A[i, j, k]A[\ell, m, n] \sin(B[i, j, k, \ell, m, n])+$$
$$(\pi - B[i, j, k, \ell, m, n]) \cos(B[i, j, k, \ell, m, n])\Big)$$