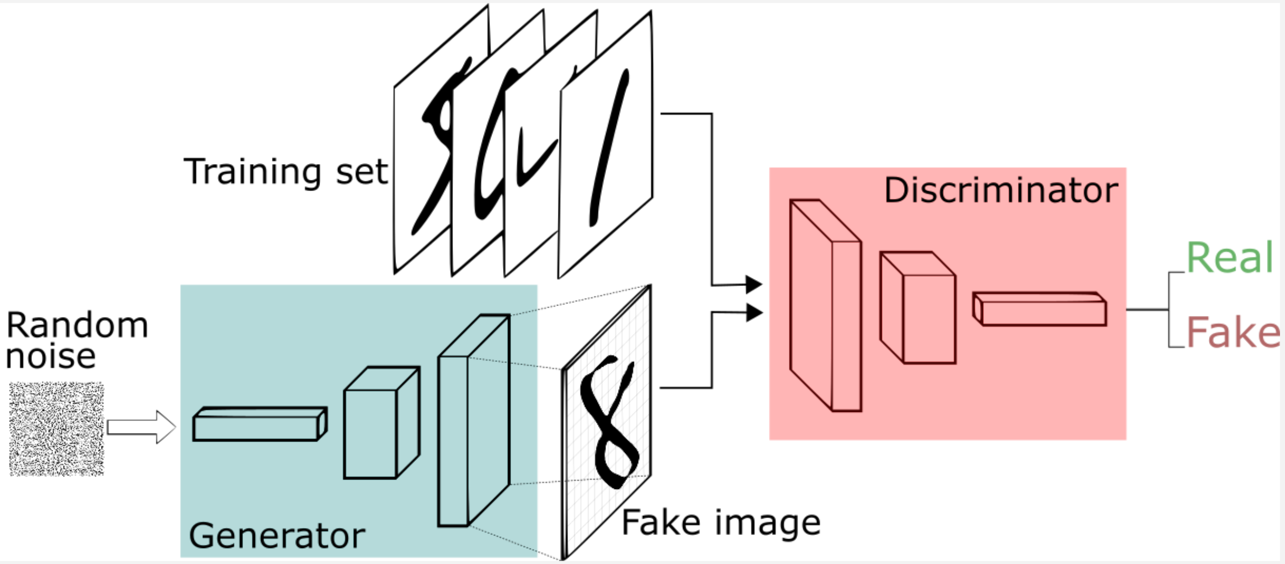
The loss of GANs

$$\arg\min_{\theta_G}\max_{\theta_D}\mathbb{E}_{x\sim\mathcal{P}_{data}}[\log D(x)]+\mathbb{E}_{z\sim\mathcal{P}_{noise}}[\log D(1-G(x))] \qquad (1)$$

We denote the generator as G and the discriminator as D with training image $x \sim \mathcal{P}_{data}$ following the distribution of training images and random noise $z \sim \mathcal{P}_{noise}$. The generator is parametrized by θ_G and the discriminator is parametrized by θ_D.



Neural Tangent Kernel(NTK)

According to the paper Neural Tangent Kernel: Convergence and Generalization in Neural Networks(by Jacot, NIPS'18), the Neural Tangent Kernel(NTK) over the training dataset $\mathbf{X}$ corresponding to the architecture of the neural network $f(\cdot;\theta)$ is defined as

$$\mathbf{K} = \nabla_\theta f(\mathbf{X};\theta)^\top \nabla_\theta f(\mathbf{X};\theta) \qquad (2)$$

Let $\mathbf{K}^{n,n} \in \mathbb{R}^{n\times n}$ be the kernel matrix for $\mathbf{X}^n$, i.e.,$\mathbf{K}^{n,n}_{i,j} = k(\mathbf{X}^n_{i,:}, \mathbf{X}^n_{j,:})$. **The mean prediction of the ensemble of the neural network $f(\cdot;\theta)$ after training t steps gradient descent can be approximated by the mean prediction of the NTK-GP with corresponding kernel.** The mean prediction of the NTK-GP over $\mathbf{X}^n$ evolves as

$$(\mathbf{I}^n - e^{-\eta\mathbf{K}^{n,n}t})\mathbf{Y}^n \qquad (3)$$

where $\mathbf{I}^n \in \mathbb{R}^{n\times n}$ is an identity matrix and η is a sufficiently small learning rate.

Our Method: GA-NTK

Let $\mathbf{K}^{2n,2n} \in \mathbb{R}^{2n \times 2n}$ be the kernel matrix for $\mathbf{X}^n \oplus \mathbf{Z}^n$, where the value each element $\mathbf{K}^{2n,2n}_{i,j} = k((\mathbf{X}^n \oplus \mathbf{Z}^n)_{i,:}, (\mathbf{X}^n \oplus \mathbf{Z}^n)_{j,:})$. Let $\lambda = \eta \cdot t$ The discriminator can be written as:

$$D(\mathbf{X}^n, \mathbf{Z}^n; k, \lambda) = \underbrace{(\mathbf{I}^{2n} - e^{-\lambda \mathbf{K}^{2n,2n}})}_{NTK-GP}(\mathbf{1}^n \oplus \mathbf{0}^n) \in \mathbb{R}^{2n} \qquad (4)$$

where $\mathbf{I}^{2n} \in \mathbb{R}^{2n \times 2n}$ is an identity matrix. We formulate the objective of GA-NTK as follows:

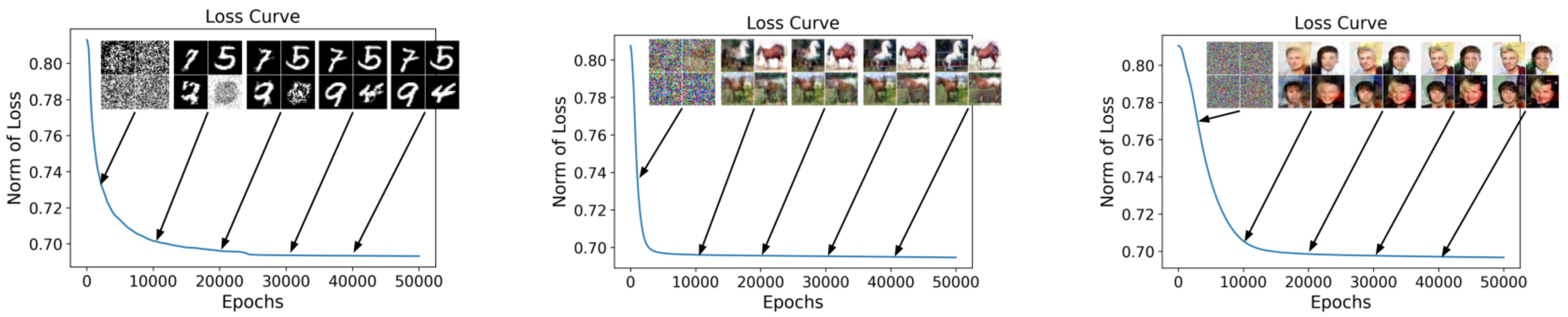
$$\arg\min_{\mathbf{Z}^n} ||\mathbf{1}^{2n} - D(\mathbf{X}^n, \mathbf{Z}^n; k, \lambda)||_2 \qquad (5)$$

where $\mathbf{1}^{2n} \in \mathbb{R}^{2n}$ is a vector of ones. Then, we update the fake image at the last time step

$$\mathbf{Z}^{n+1} = \mathbf{Z}^n + \alpha \nabla_{\mathbf{Z}^n} ||\mathbf{1}^{2n} - D(\mathbf{X}^n, \mathbf{Z}^n; k, \lambda)||_2 \qquad (6)$$

Stable Convergence

The image quality is consistent with the loss. As the **loss decreases, the image quality is improved**.

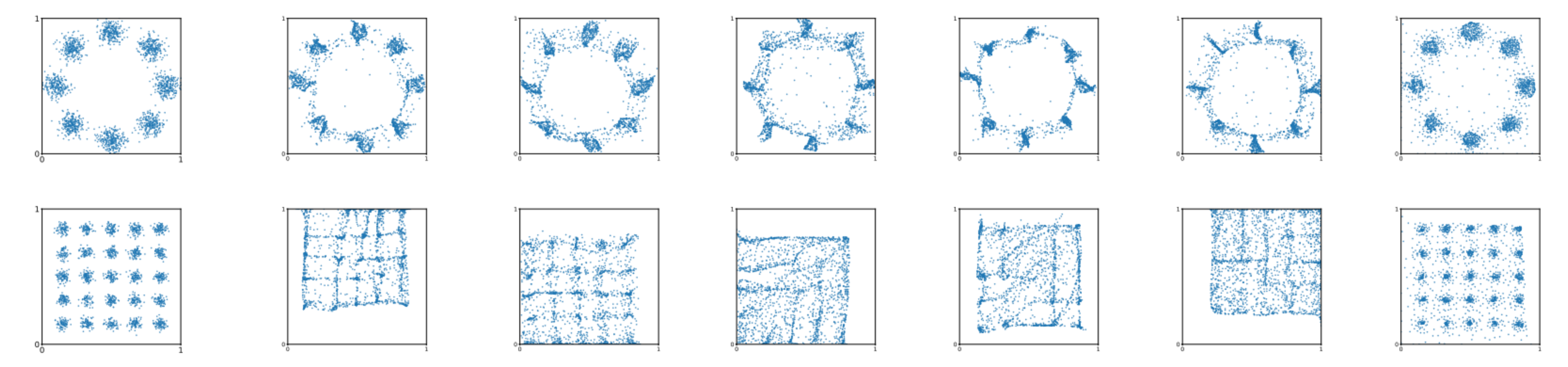


(a) MNIST (b) CIFAR-10 (c) CelebA

Figure 3. Loss curve and image quality at different iterations on different dataset. We can see that image quality issues correlated with loss curve.

Mode Collapse

Our method(**GA-FNTK**) **fits to multi-modal distribution perfectly**, rather than other baseline GANs.

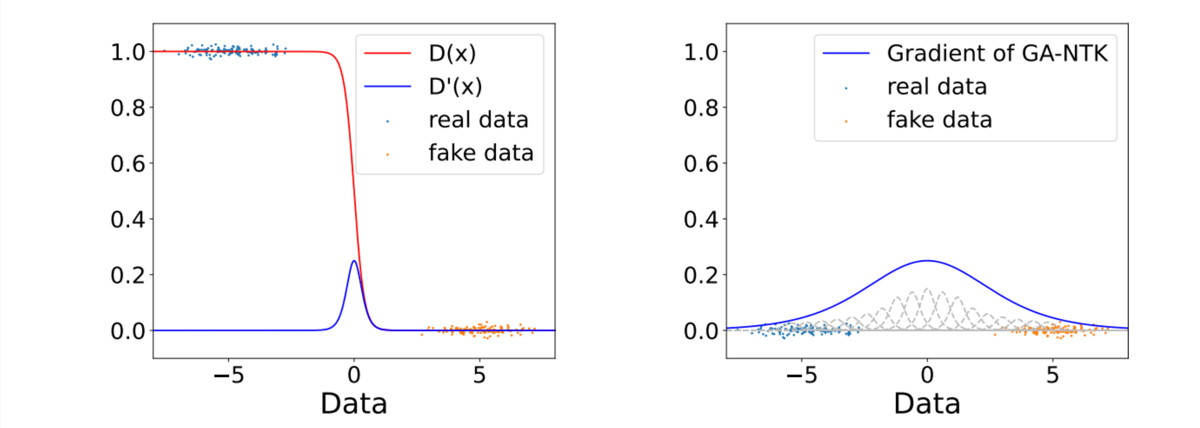


(a) Ground truth (b) Vanilla GAN (c) LSGAN (d) WGAN (e) WGAN-GP (f) SN-GAN (g) GA-FNTK

Gradient Vanish

Decision boundary of the discriminator and the corresponding gradients for the generator $\mathbf{Z}^n$ in (a) GANs and (b) GA-NTK on 1D toy dataset. The blue points are real data points and the red ones are fake. The discriminator output 1 for the real data points and 0 for the fake ones(fig (a) red line). The **gradients(fig (a) blue line) vanishes while the the data points are far away from the decision boundary**.

The **gray dashed lines indicate the gradients for $\mathbf{Z}^n$ from different element networks of the ensemble discriminator in GA-NTK**.



(a) (b)

Compare GA-CNTK & GANs

(a) GA-CNTK

(b) DCGAN

(c) LSGAN

(d) WGAN

(e) WGAN-GP

(f) SN-GAN

Figure 2. Generated samples from MNIST, CIFAR10, and CelebA. The sizes of the training datasets are 256.