Neural Kernel Without Tangents

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Motivation

- NTK, CNTK... do not match the performance of neural networks on most tasks of interest.
- The NTK constructions themselves are not only hard to compute, but their mathematical formulae are difficult to even write down.

Problem Formulation

- Are there computationally tractable/easier kernels that approach the expressive power of neural networks?
- Is there a correlation between neural architecture performance and the performance of the associated kernel?

Outline

- Main Idea
- Experiments
- Conclusion

Main Idea

- Construct CNN architecture using only 3×3 **convolutions,** 2×2 **average pooling, ReLU**.
- **Compositional Kernel**: Kernelize $1 \ldots, L$ layers as kernel functions $k_1 \ldots, k_L$ and **compute the kernel hierarchily** $k_L(k_{L-1}(\ldots k_1(x, y)))$ as the kernel of the corresponding CNN architecture.
- **5-layers compositional kernel**(in Myrtle5 architecture) can **significantly outperform(about 10% classification accuracy)** than **14-layers CNTK on CIFAR-10(**Arora et al. 2020**)** while the **training samples are less than 1000**.

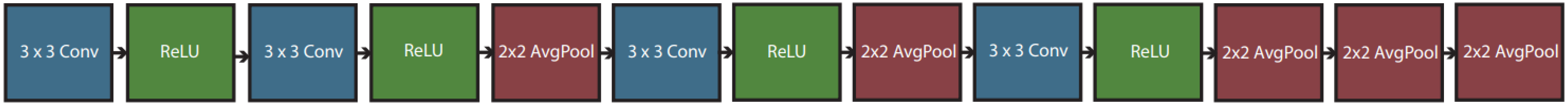


Figure 2. A 5 layer network from the "Myrtle" family (Myrtle5).

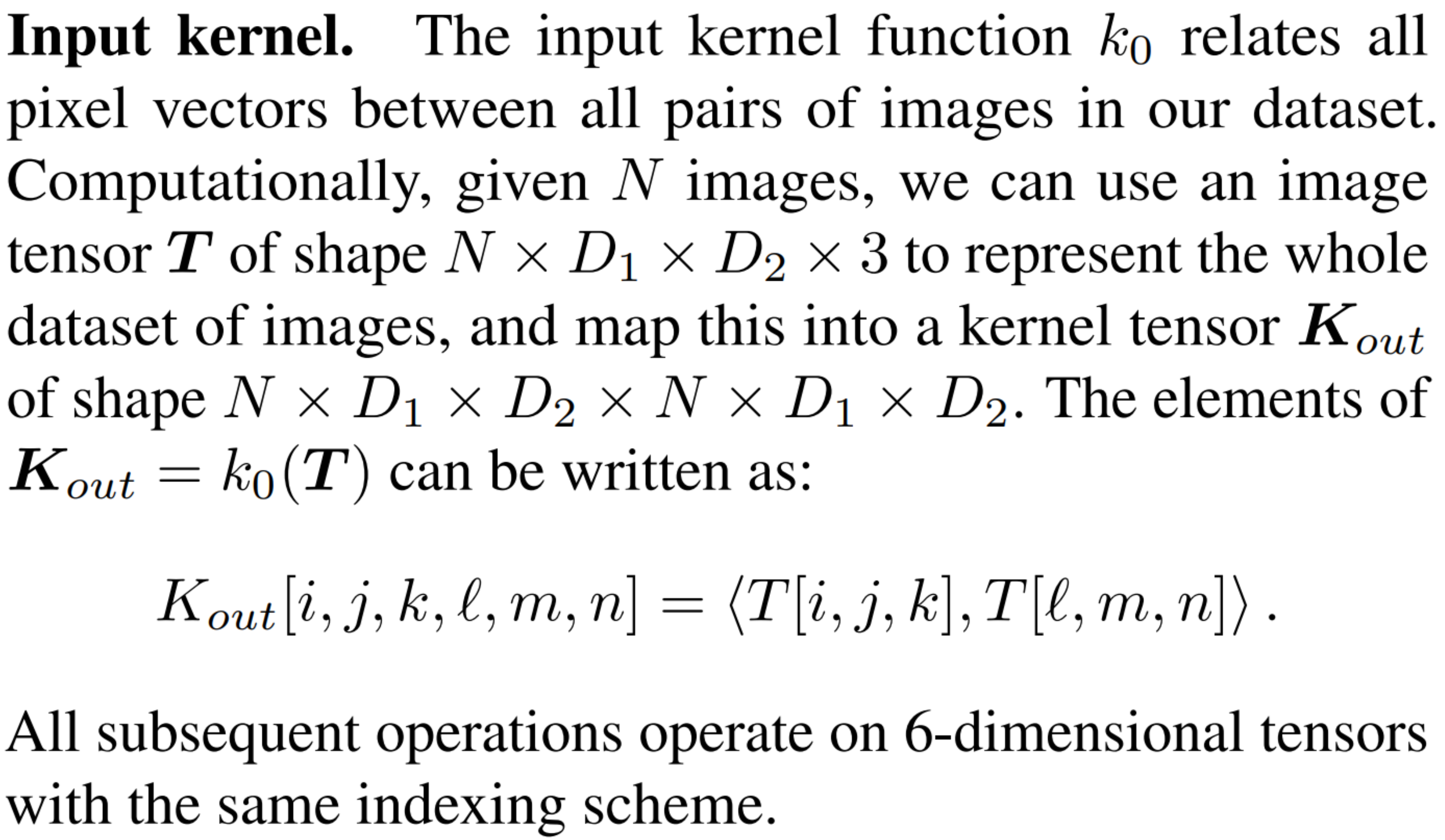
Methodology

- **Bag of features** is simply a generalization of a matrix or tensor: whereas a matrix is an indexed list of vectors, a bag of features is a collection of elements in a Hilbert space $\mathcal{H}$ with a finite, structured index set $\mathcal{B}$.
- EX: we can consider an **image** to be a bag of features where the **index set $\mathcal{B}$ is the pixel's row and column location** and $\mathcal{H}$ **is $\mathbb{R}^3$: at every pixel location, there is a corresponding vector encoding RGB in** $\mathbb{R}^3$.
- Given two bags of features with the same $(\mathcal{B}, \mathcal{H})$, we define the kernel function

$$k(\mathbf{X}, a, \mathbf{Z}, b) = \langle \mathbf{X}_a, \mathbf{Z}_b \rangle$$

It defines a **kernel matrix between two bags of features**: we compute the kernel function for each pair of indices in $\mathcal{B} \times \mathcal{B}$ to form a $|\mathcal{B}| \times |\mathcal{B}|$ **matrix**

Input Kernel

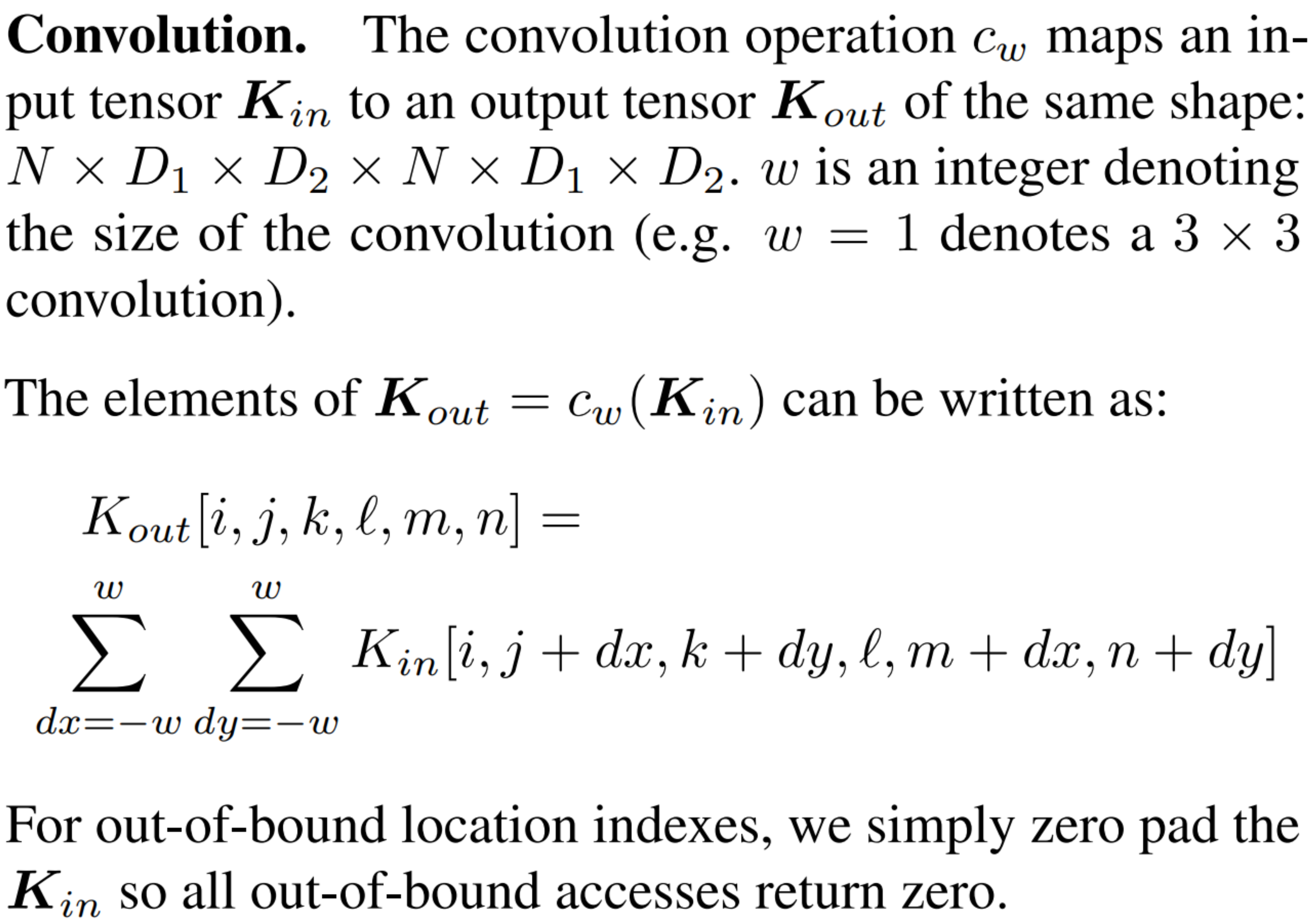


Input kernel. The input kernel function k_0 relates all pixel vectors between all pairs of images in our dataset. Computationally, given N images, we can use an image tensor $\boldsymbol{T}$ of shape $N \times D_1 \times D_2 \times 3$ to represent the whole dataset of images, and map this into a kernel tensor $\boldsymbol{K}_{out}$ of shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$. The elements of $\boldsymbol{K}_{out} = k_0(\boldsymbol{T})$ can be written as:

$$K_{out}[i, j, k, \ell, m, n] = \langle T[i, j, k], T[\ell, m, n] \rangle \,.$$

All subsequent operations operate on 6-dimensional tensors with the same indexing scheme.

Convolution Kernel



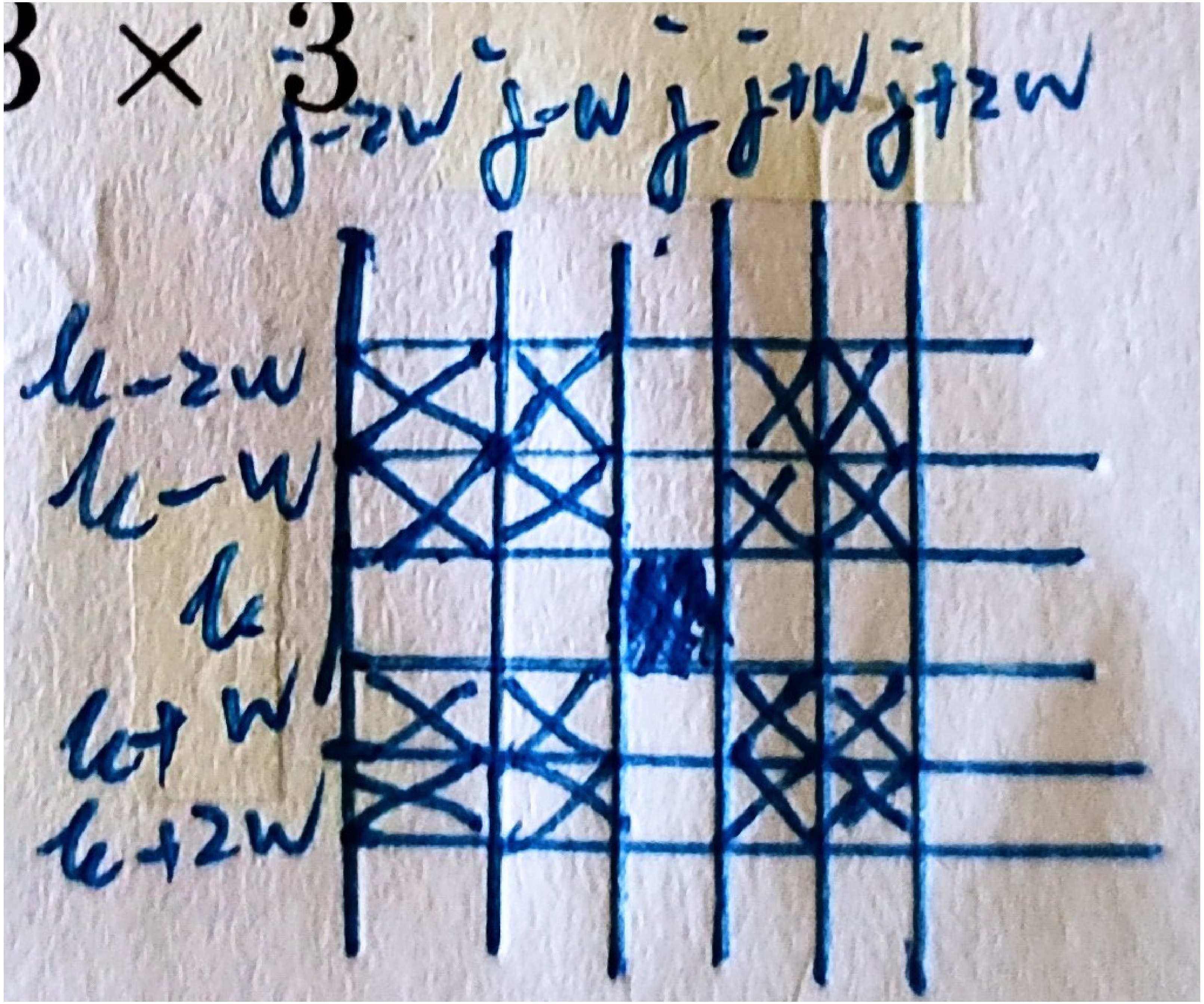
Convolution. The convolution operation c_w maps an input tensor $\boldsymbol{K}_{in}$ to an output tensor $\boldsymbol{K}_{out}$ of the same shape: $N \times D_1 \times D_2 \times N \times D_1 \times D_2$. w is an integer denoting the size of the convolution (e.g. $w = 1$ denotes a 3×3 convolution).

The elements of $\boldsymbol{K}_{out} = c_w(\boldsymbol{K}_{in})$ can be written as:

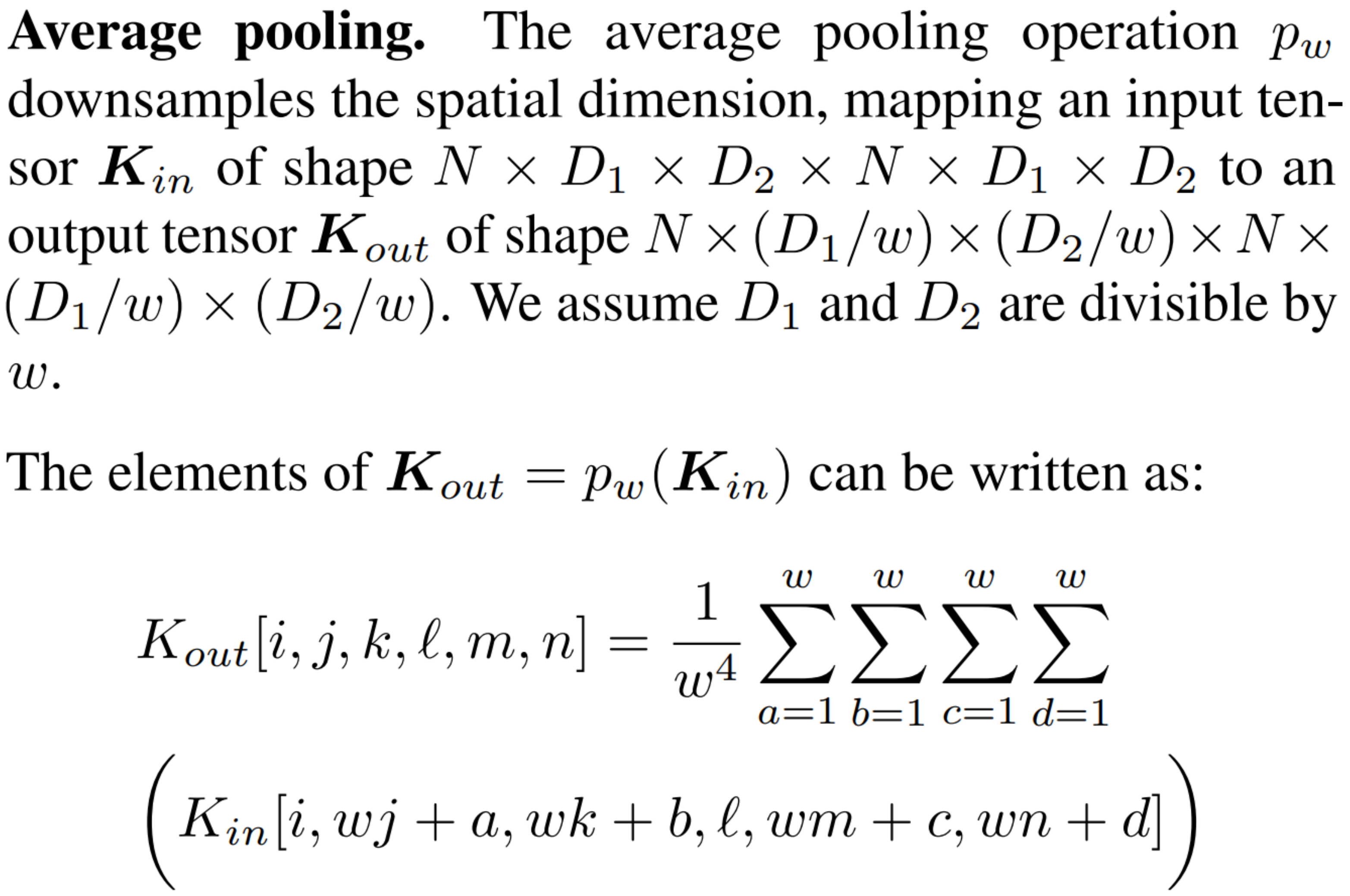
$$K_{out}[i, j, k, \ell, m, n] = \sum_{dx=-w}^{w} \sum_{dy=-w}^{w} K_{in}[i, j + dx, k + dy, \ell, m + dx, n + dy]$$

For out-of-bound location indexes, we simply zero pad the $\boldsymbol{K}_{in}$ so all out-of-bound accesses return zero.

3 × 3



Average Pooling Kernel



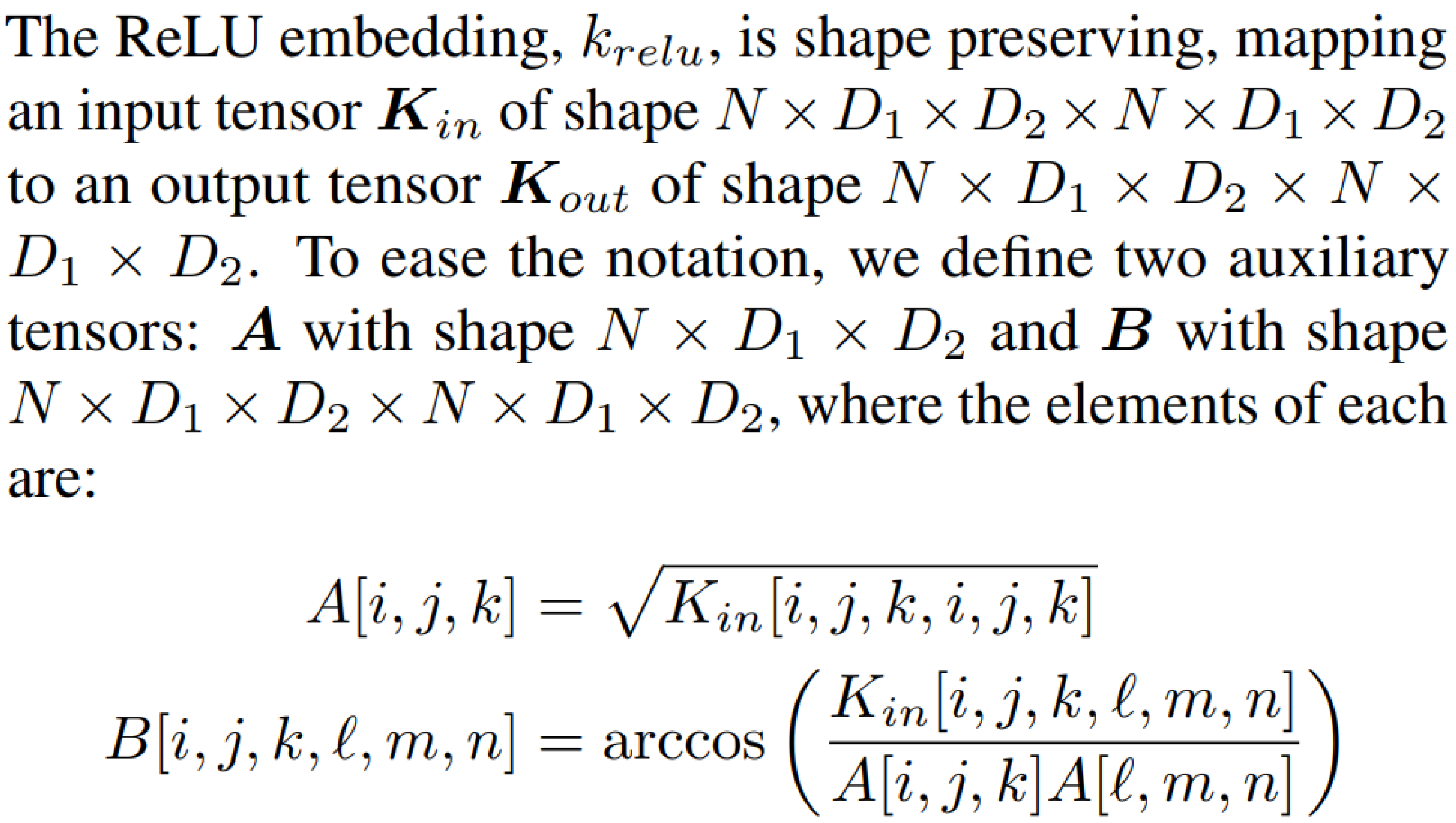
Average pooling. The average pooling operation p_w downsamples the spatial dimension, mapping an input tensor $\boldsymbol{K}_{in}$ of shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$ to an output tensor $\boldsymbol{K}_{out}$ of shape $N \times (D_1/w) \times (D_2/w) \times N \times (D_1/w) \times (D_2/w)$. We assume D_1 and D_2 are divisible by w.

The elements of $\boldsymbol{K}_{out} = p_w(\boldsymbol{K}_{in})$ can be written as:

$$K_{out}[i, j, k, \ell, m, n] = \frac{1}{w^4} \sum_{a=1}^{w} \sum_{b=1}^{w} \sum_{c=1}^{w} \sum_{d=1}^{w}$$

$$\left(K_{in}[i, wj + a, wk + b, \ell, wm + c, wn + d] \right)$$

ReLU Kernel

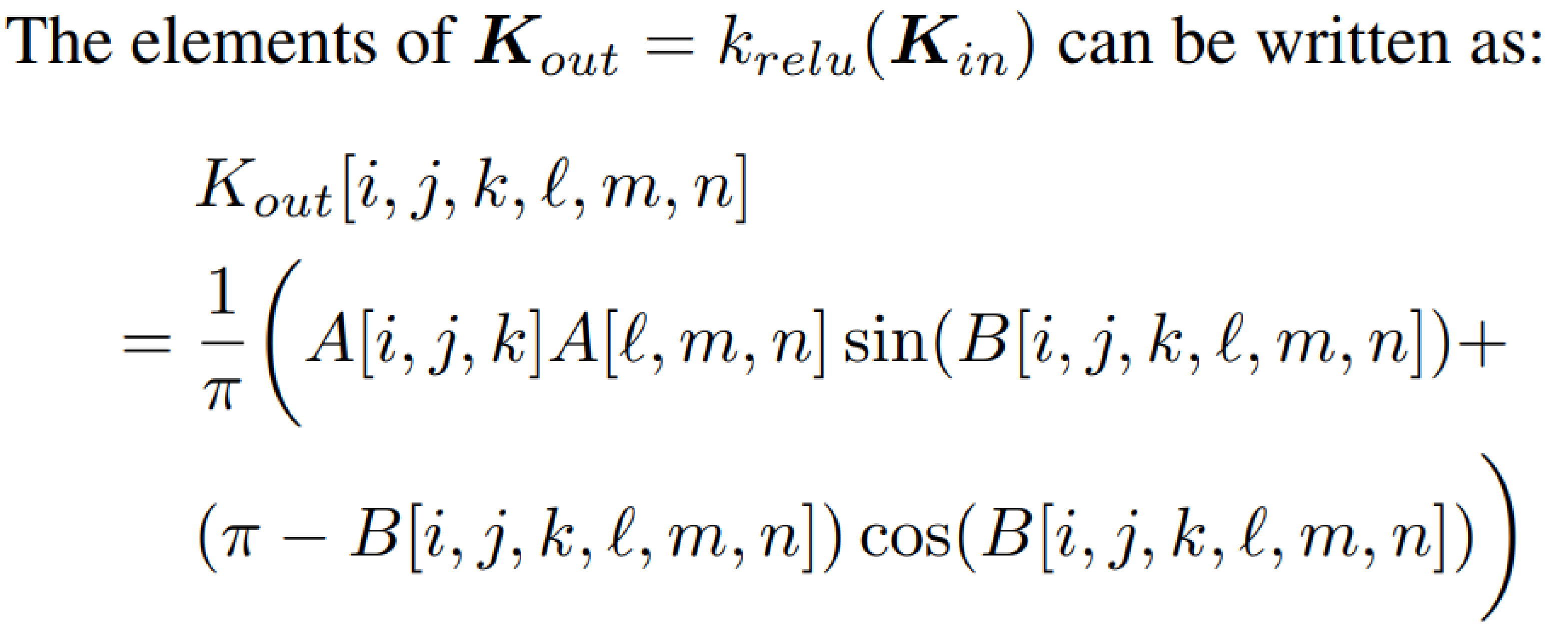


The ReLU embedding, k_{relu}, is shape preserving, mapping an input tensor $\boldsymbol{K}_{in}$ of shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$ to an output tensor $\boldsymbol{K}_{out}$ of shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$. To ease the notation, we define two auxiliary tensors: $\boldsymbol{A}$ with shape $N \times D_1 \times D_2$ and $\boldsymbol{B}$ with shape $N \times D_1 \times D_2 \times N \times D_1 \times D_2$, where the elements of each are:

$$A[i,j,k] = \sqrt{K_{in}[i,j,k,i,j,k]}$$

$$B[i,j,k,\ell,m,n] = \arccos\left(\frac{K_{in}[i,j,k,\ell,m,n]}{A[i,j,k]A[\ell,m,n]}\right)$$

ReLU Kernel

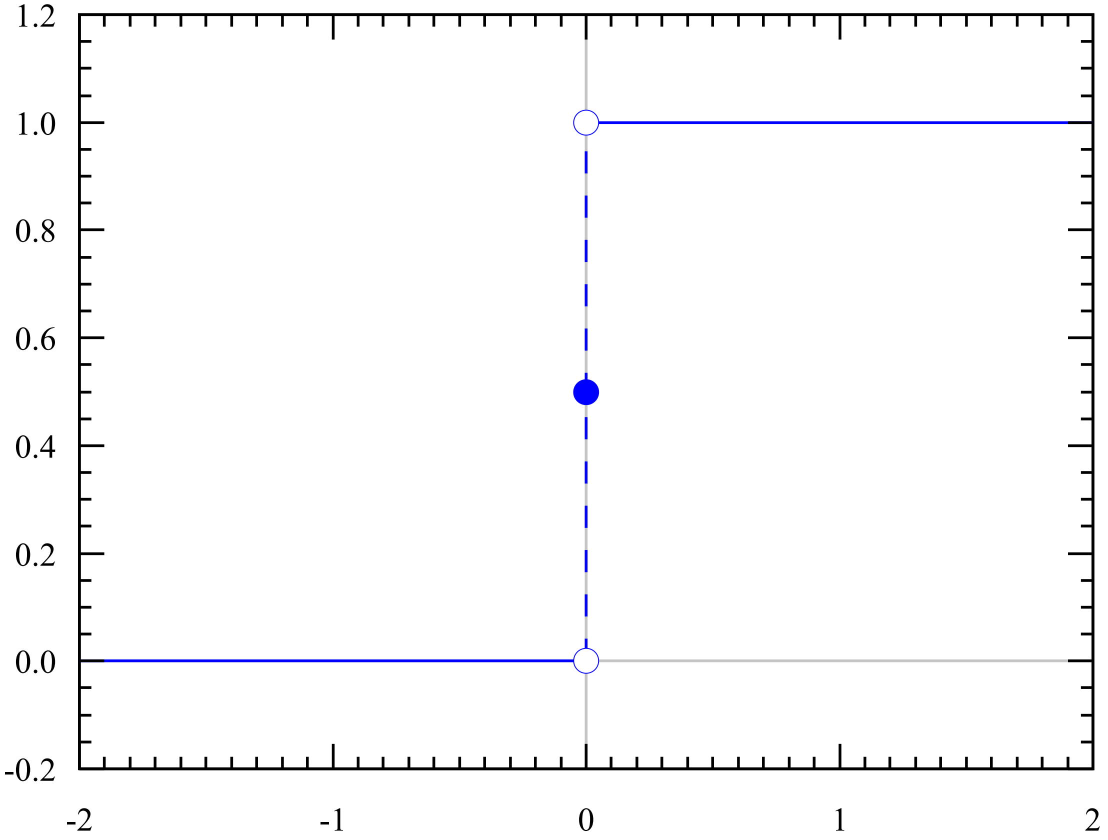


The elements of $\boldsymbol{K}_{out} = k_{relu}(\boldsymbol{K}_{in})$ can be written as:

$$K_{out}[i,j,k,\ell,m,n]$$
$$= \frac{1}{\pi}\Big(A[i,j,k]A[\ell,m,n]\sin(B[i,j,k,\ell,m,n]) +$$
$$(\pi - B[i,j,k,\ell,m,n])\cos(B[i,j,k,\ell,m,n])\Big)$$

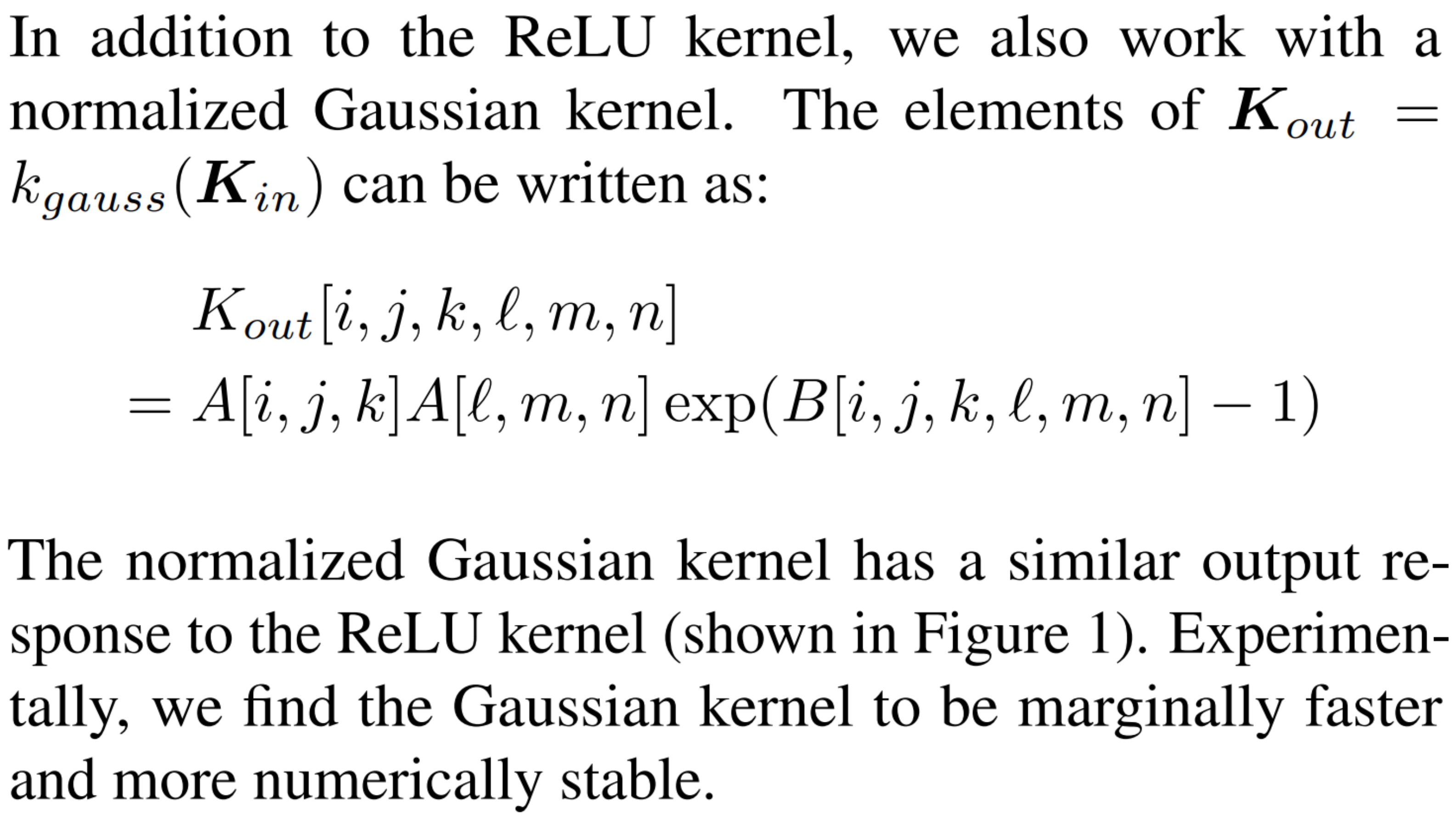
It's the same as the **arccosine kernel** used in NTK. Refers to NIPS'09 Kernel Methods for Deep Learning

Heaviside Function



The derivative of the ReLU function is Heaviside function. First we can compute the kernel of the Heaviside function and then integrate it, we can get the kernel of the ReLU function.

Gaussian Kernel



In addition to the ReLU kernel, we also work with a normalized Gaussian kernel. The elements of $\boldsymbol{K}_{out} = k_{gauss}(\boldsymbol{K}_{in})$ can be written as:

$$\begin{aligned} & K_{out}[i,j,k,\ell,m,n] \\ = {} & A[i,j,k]A[\ell,m,n]\exp(B[i,j,k,\ell,m,n]-1) \end{aligned}$$

The normalized Gaussian kernel has a similar output response to the ReLU kernel (shown in Figure 1). Experimentally, we find the Gaussian kernel to be marginally faster and more numerically stable.

Related to Neural Network

Let a simple convolution layer as

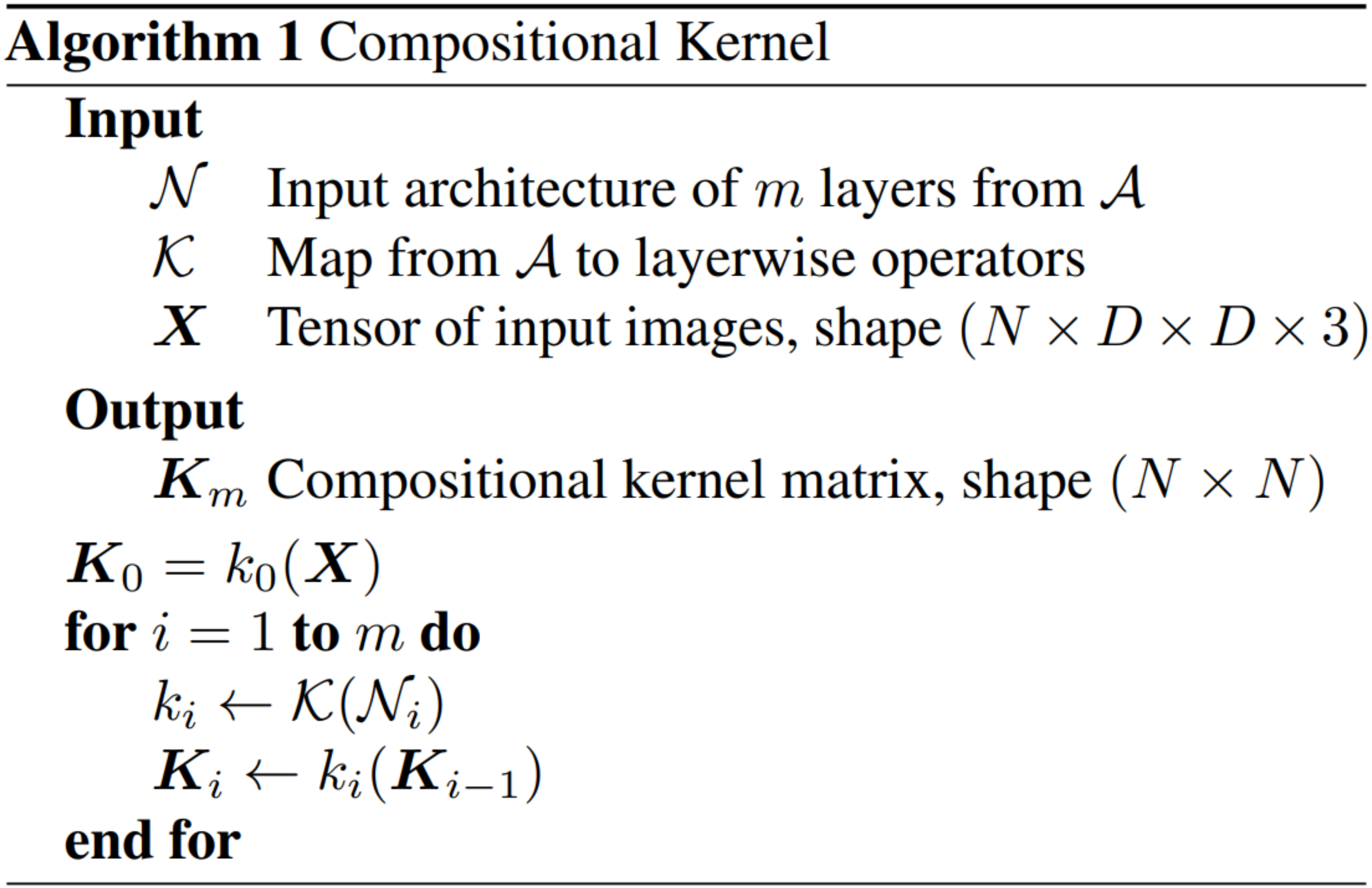
$$\Psi(U) = relu(W * U)$$

Where $W \in \mathbb{R}^{(2w+1)\times(2w+1)\times D_3 \times D_4}$ is a weight tensor and $U \in \mathbb{R}^{N \times D_1 \times D_2 \times D_3}$ is the input, which can be N images. Suppose the entries of W are appropriately **scaled random Gaussian variables**. We can evaluate the following **expectation according to the calculation**

$$\mathbb{E}[\sum_{c=1}^{D_4} \Psi(U)[i,j,k,c]\Psi(U)[l,m,n,c]] = k_{relu}(c_w(k_0(U)))[i,j,k,l,m,n]$$

Refers to NIPS'16 Toward Deeper Understanding of Neural Networks: The Power of Initialization and a Dual View on Expressivity

Algorithm



Algorithm 1 Compositional Kernel

```
Input
    $\mathcal{N}$   Input architecture of $m$ layers from $\mathcal{A}$
    $\mathcal{K}$   Map from $\mathcal{A}$ to layerwise operators
    $\boldsymbol{X}$   Tensor of input images, shape $(N \times D \times D \times 3)$
Output
    $\boldsymbol{K}_m$ Compositional kernel matrix, shape $(N \times N)$
$\boldsymbol{K}_0 = k_0(\boldsymbol{X})$
for $i = 1$ to $m$ do
    $k_i \leftarrow \mathcal{K}(\mathcal{N}_i)$
    $\boldsymbol{K}_i \leftarrow k_i(\boldsymbol{K}_{i-1})$
end for
```

Experiment Setup

MNIST, CIFAR-10, CIFAR-10.1, CIFAR-100 Dataset

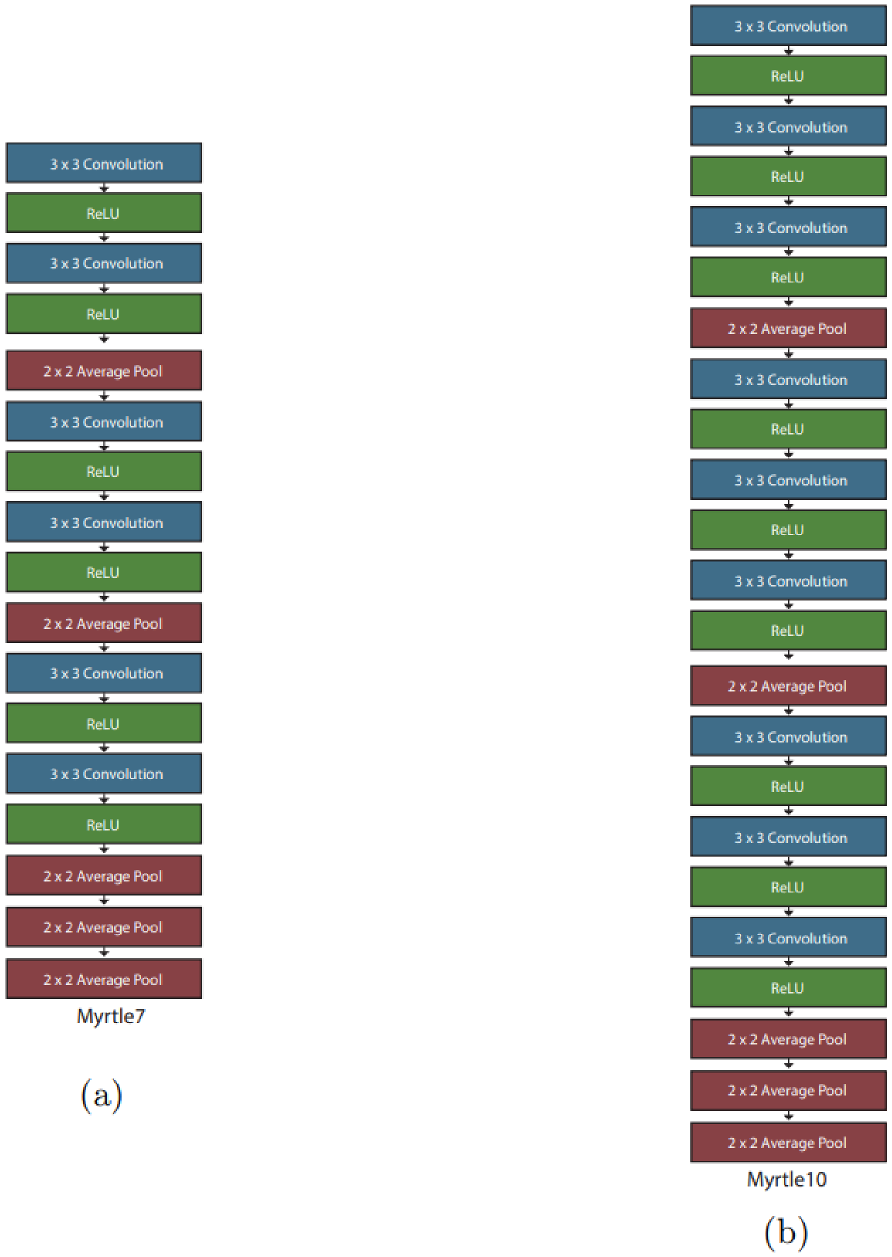
- Myrtle5, 7, 10 with ReLU kernel
- ZCA whitening preprocessing
- Flip data augmentation to our kernel method by flipping every example in the training set across the vertical axis
- Kernel ridge regression with respect to one-hot labels

90 UCI Dataset

- Myrtle5, 7, 10 with Gaussian kernel
- Hinge loss with libSVM

Architecture

All architectures that can be represented as a list of operations from the set **{conv3, pool2, relu}** as the **"Myrtle" family**. The right one is **Myrtle7** and the left on is **Myrtle10**



(a)

(b)

Figure 1: a) 7 layer b) 10 layer variants of the Myrtle architectures

MNIST

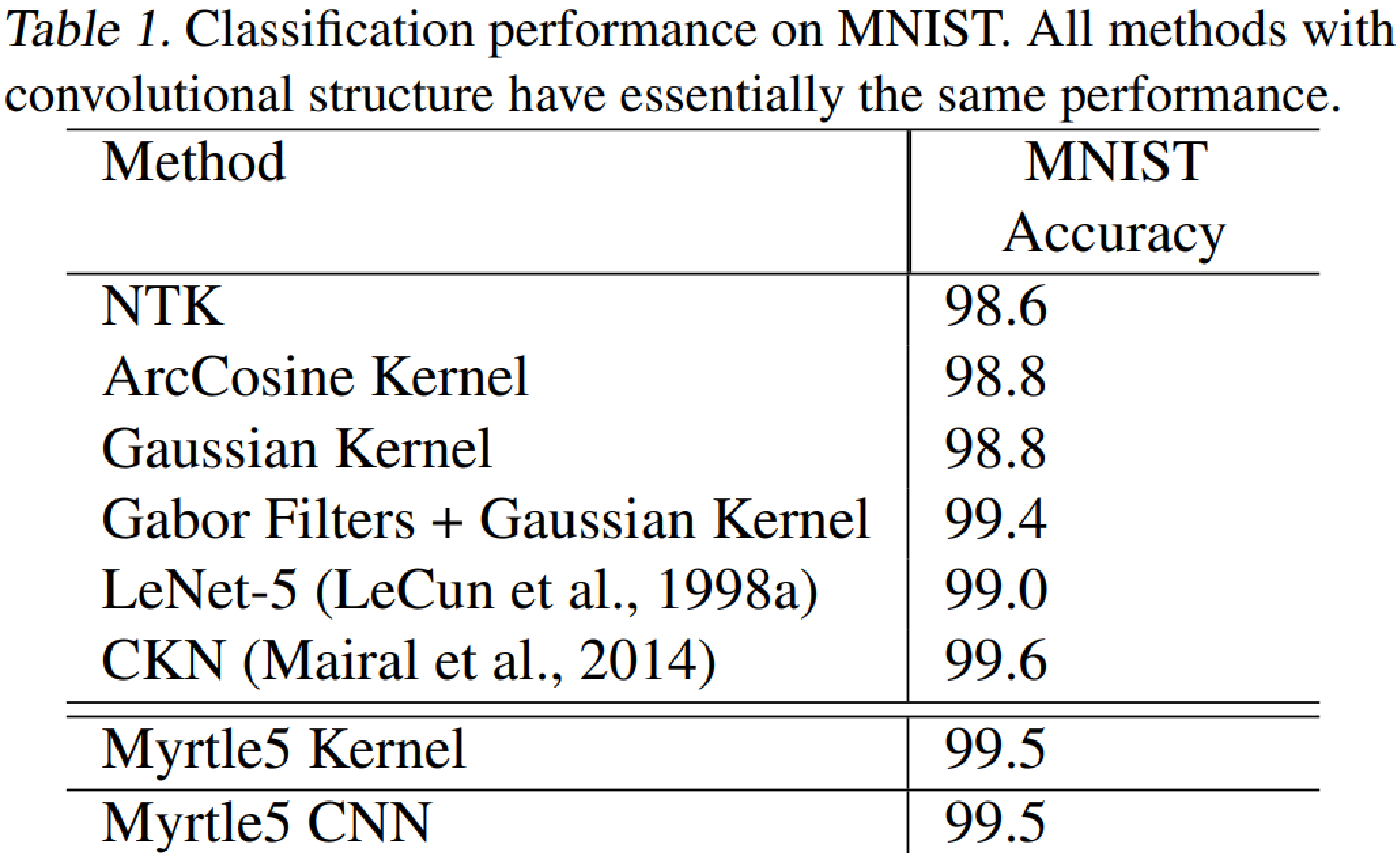


Table 1. Classification performance on MNIST. All methods with convolutional structure have essentially the same performance.

Method	MNIST Accuracy
NTK	98.6
ArcCosine Kernel	98.8
Gaussian Kernel	98.8
Gabor Filters + Gaussian Kernel	99.4
LeNet-5 (LeCun et al., 1998a)	99.0
CKN (Mairal et al., 2014)	99.6
Myrtle5 Kernel	99.5
Myrtle5 CNN	99.5

CIFAR-100

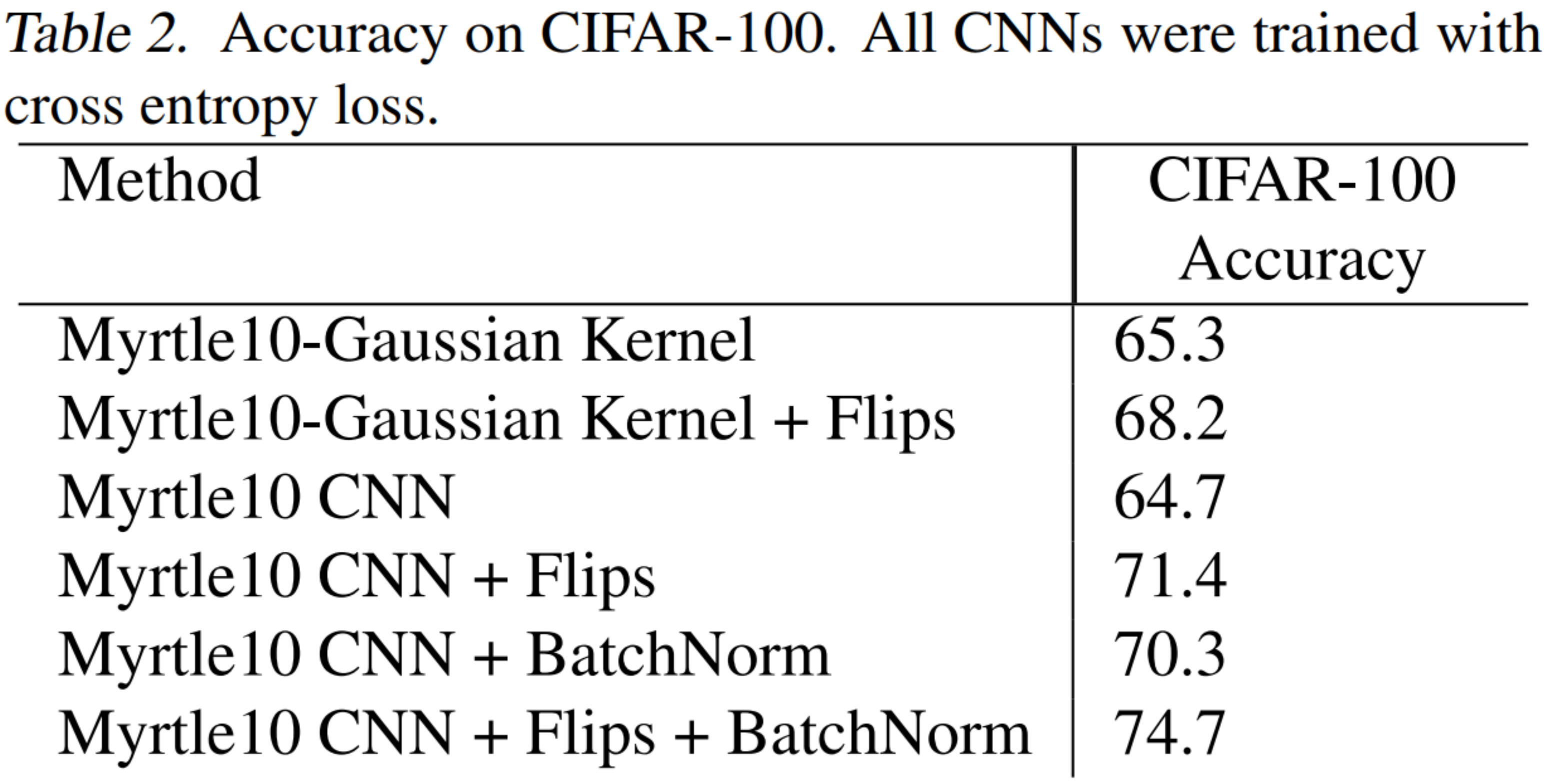


Table 2. Accuracy on CIFAR-100. All CNNs were trained with cross entropy loss.

Method	CIFAR-100 Accuracy
Myrtle10-Gaussian Kernel	65.3
Myrtle10-Gaussian Kernel + Flips	68.2
Myrtle10 CNN	64.7
Myrtle10 CNN + Flips	71.4
Myrtle10 CNN + BatchNorm	70.3
Myrtle10 CNN + Flips + BatchNorm	74.7

90 UCI

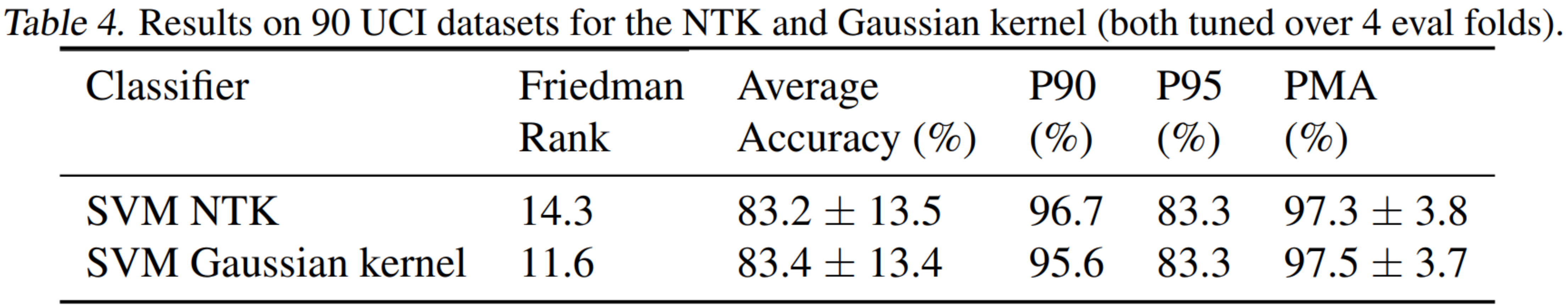


Table 4. Results on 90 UCI datasets for the NTK and Gaussian kernel (both tuned over 4 eval folds).

Classifier	Friedman Rank	Average Accuracy (%)	P90 (%)	P95 (%)	PMA (%)
SVM NTK	14.3	83.2 ± 13.5	96.7	83.3	97.3 ± 3.8
SVM Gaussian kernel	11.6	83.4 ± 13.4	95.6	83.3	97.5 ± 3.7

- **Friedman rank**: The ranking metric reports the average ranking of a given classifier compared to all other classifiers on datasets. The lower, the better.
- **P90/P95**: The percentage of datasets on which the classifier achieves more than 90%/95% of the maximum achievable accuracy. The higher, the better.
- **PMA**: The average percentage of the maximum accuracy of the classifier for datasets. The higher, the better.

CIFAR-10

- Evaluate on 10,000 test images from CIFAR-10 and the additional 2,000 "harder" test images from CIFAR-10.1
- For all kernel results on CIFAR-10, we gained an improvement of roughly 0.5% with **Leave-One-Out tilting** and **ZCA augmentation** techniques.
- A substantial drop in accuracy for the compositional kernel without ZCA preprocessing.

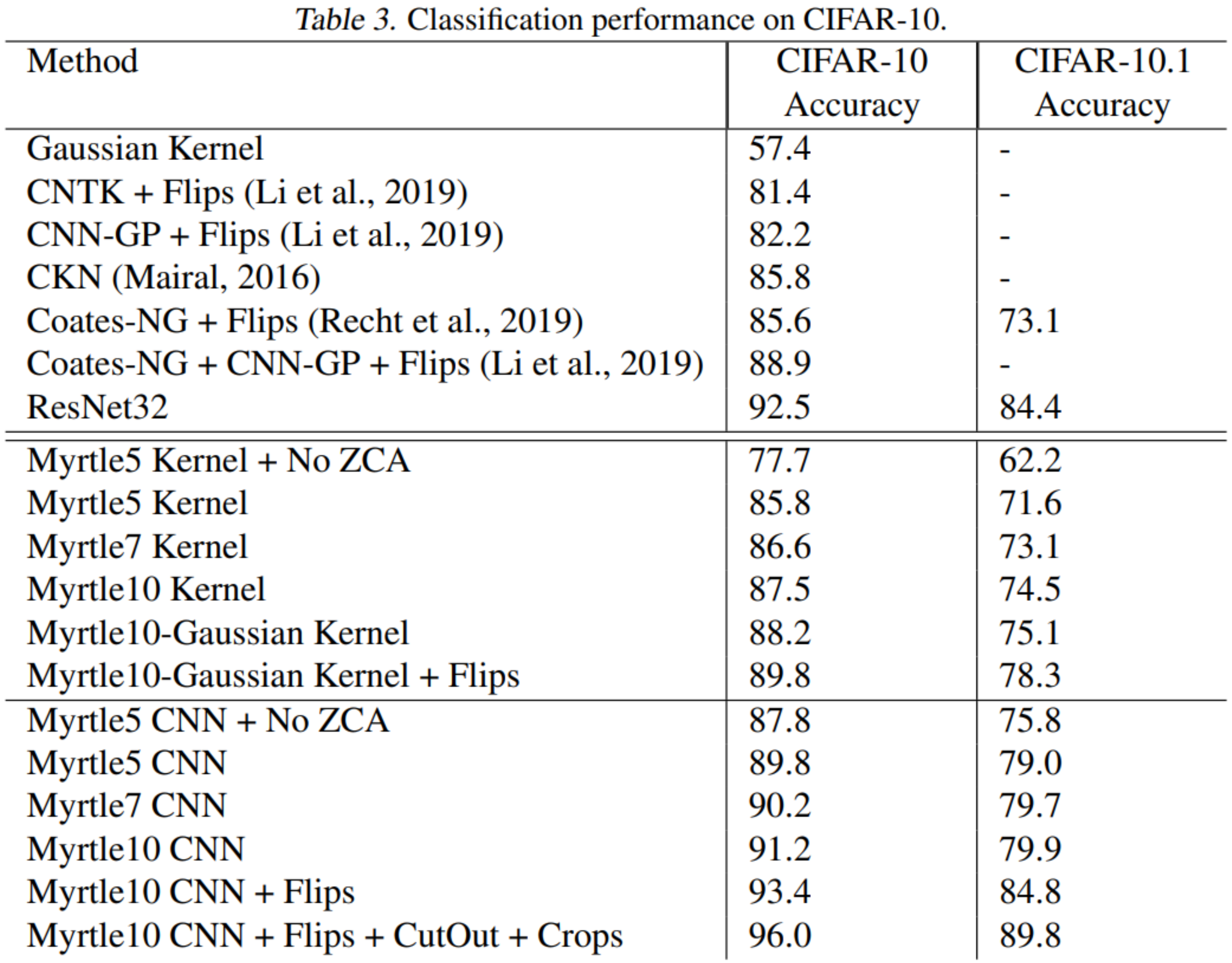


Table 3. Classification performance on CIFAR-10.

Method	CIFAR-10 Accuracy	CIFAR-10.1 Accuracy
Gaussian Kernel	57.4	-
CNTK + Flips (Li et al., 2019)	81.4	-
CNN-GP + Flips (Li et al., 2019)	82.2	-
CKN (Mairal, 2016)	85.8	-
Coates-NG + Flips (Recht et al., 2019)	85.6	73.1
Coates-NG + CNN-GP + Flips (Li et al., 2019)	88.9	-
ResNet32	92.5	84.4
Myrtle5 Kernel + No ZCA	77.7	62.2
Myrtle5 Kernel	85.8	71.6
Myrtle7 Kernel	86.6	73.1
Myrtle10 Kernel	87.5	74.5
Myrtle10-Gaussian Kernel	88.2	75.1
Myrtle10-Gaussian Kernel + Flips	89.8	78.3
Myrtle5 CNN + No ZCA	87.8	75.8
Myrtle5 CNN	89.8	79.0
Myrtle7 CNN	90.2	79.7
Myrtle10 CNN	91.2	79.9
Myrtle10 CNN + Flips	93.4	84.8
Myrtle10 CNN + Flips + CutOut + Crops	96.0	89.8

Subsampled CIFAR-10

- Subsampled datasets are class balanced
- **Compositional kernel and NTK in the low data regime**
- **Network** with the same architecture as compositional kernel severely **underperforms both the compositional kernel and NTK in the low data regime**
- After adding batch normalization, the network outperforms both compositional kernel and the NTK

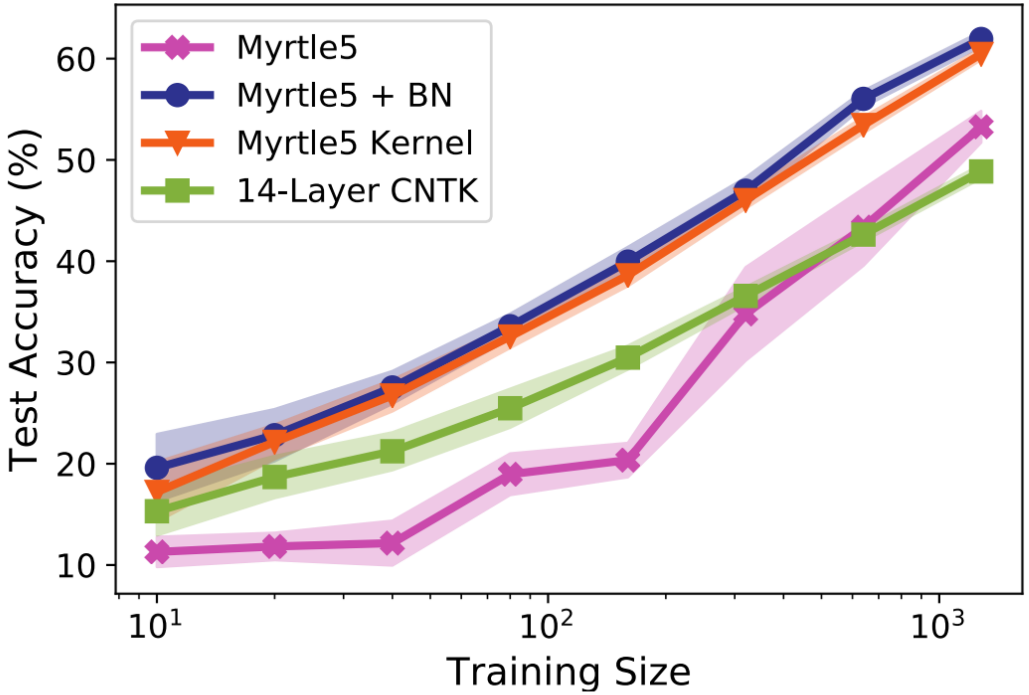


Figure 3. Accuracy results on random subsets of CIFAR-10, with standard deviations over 20 trials. The 14-layer CNTK results are from Arora et al. (2020).

Conclusion

- Provide a promising starting point for designing practical, high performance, domain specific kernel functions
- Some notion of **compositionality and hierarchy** may be necessary to build kernel predictors that match the performance of neural networks
- **NTKs** themselves may **not actually provide particularly useful guides** to the practice of kernel methods.
- We may underscores the importance of proper preprocessing for kernel methods
- There **still performance gaps between kernel methods and neural networks** and the reasons remain unknown.

Reference

- Preprocessing for deep learning: from covariance matrix to image whitening
- 知乎 - CNN数值——ZCA
- NIPS'09 Kernel Methods for Deep Learning
- ICLR'20 Harnessing the Power of Infinitely Wide Deep Nets on Small-data Tasks