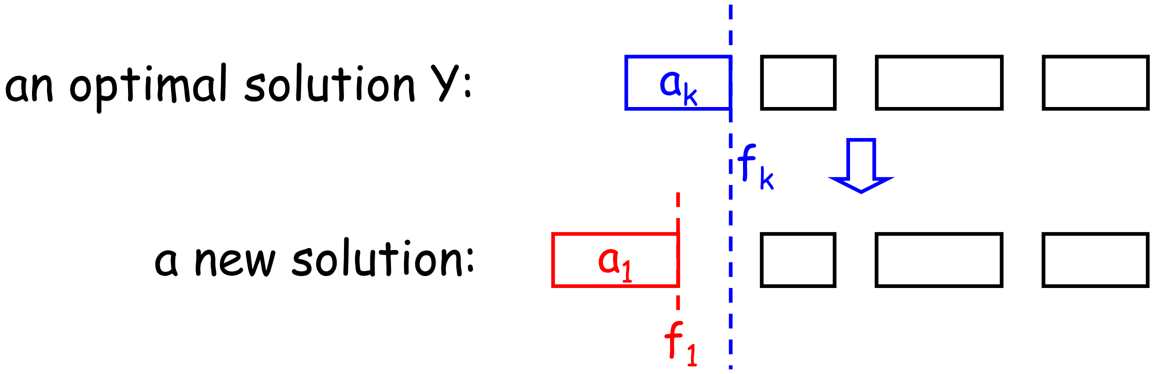
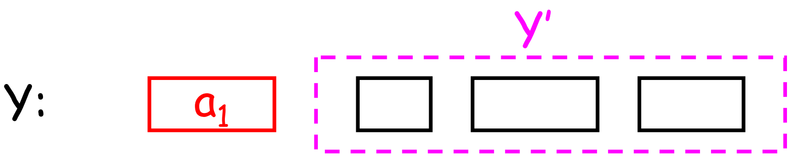
(1) Taking a_1 is correct (greedy-choice property)



(2) Optimal substructure



Let X = $\{a_i \mid s_i < f_1\}$ and A' = A - X = $\{a_i \mid s_i \geq f_1\}$.

和 a_1 衝突　　和 a_1 沒衝突

After taking a_1

(i) all a_i in X should be discarded;

(ii) the problem becomes to select a maximum set of compatible activities in A'

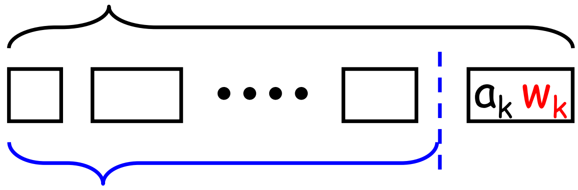
$\Rightarrow$ Y' is optimal for A'

(after a choice $\rightarrow$ same problem of smaller size)

Optimal substructure

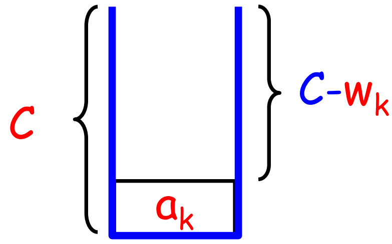
0-1:

optimal for $\begin{cases} A = \{a_1, a_2, \ldots, a_n\} \\ C \end{cases}$



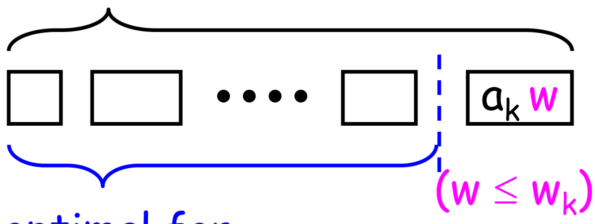
optimal for

$\begin{cases} A' = \{a_1, a_2, \ldots, a_{k-1}, \cancel{a_k}, \ldots, \cancel{a_n}\} \\ C' = C - w_k \end{cases}$



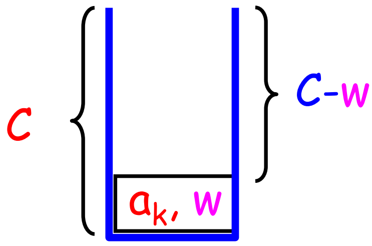
fractional:

optimal for $\begin{cases} A = \{a_1, a_2, \ldots, a_n\} \\ C \end{cases}$



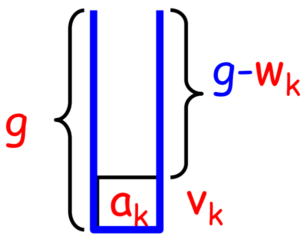
optimal for

$\begin{cases} A' = \{a_1, a_2, \ldots, a_{k-1}, \cancel{a_k}, \ldots, \cancel{a_n}\} \\ C' = C - w \end{cases}$



0-1 Knapsack problem
(integer weights, DP)

* f(g,k): optimal value for $\begin{cases} a_1\, a_2 \ldots a_k \\ \text{capacity is } g \end{cases}$



* solution: f(C,n)

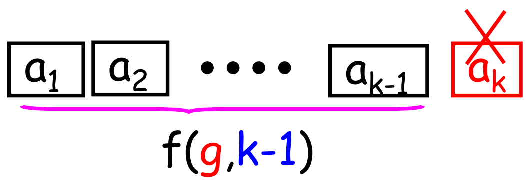
* $f(g,k) = \max \begin{Bmatrix} f(g, k-1) \\ f(g-w_k, k-1) + v_k \end{Bmatrix}$

* $f(0,k) = f(g,0) = 0$, $f(-,k) = -\infty$

* Time: O(Cn)

* optimal substructure

Case 1. a_k is not selected



Case 2. a_k is selected