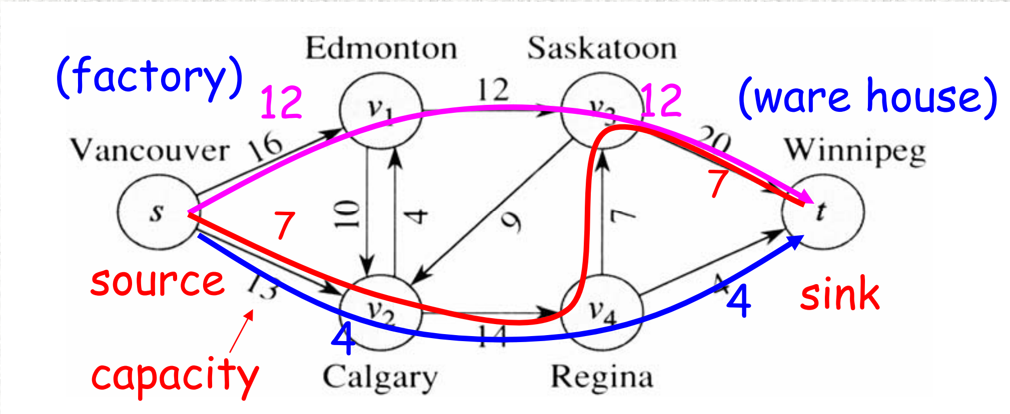
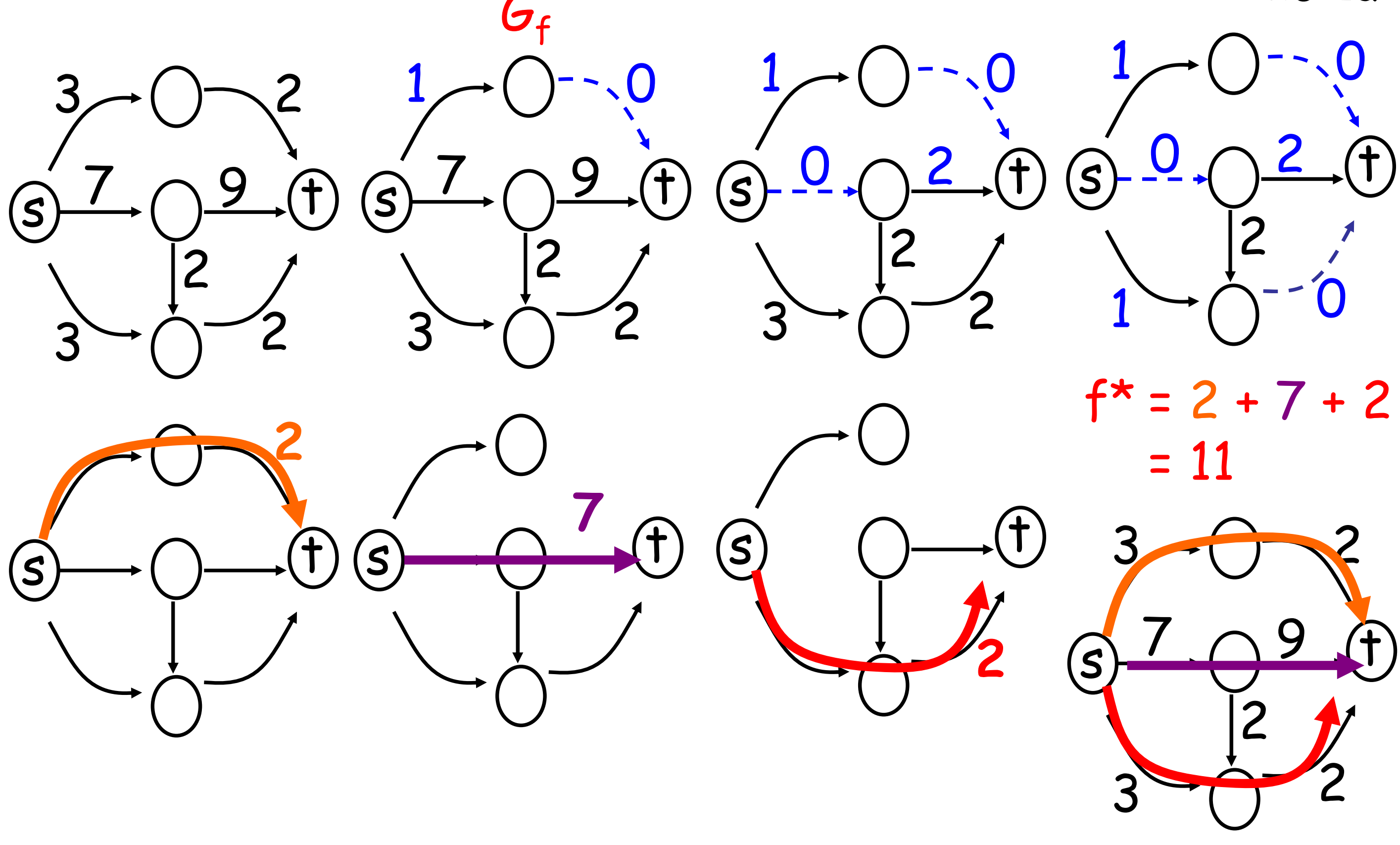
The maximum-flow problem

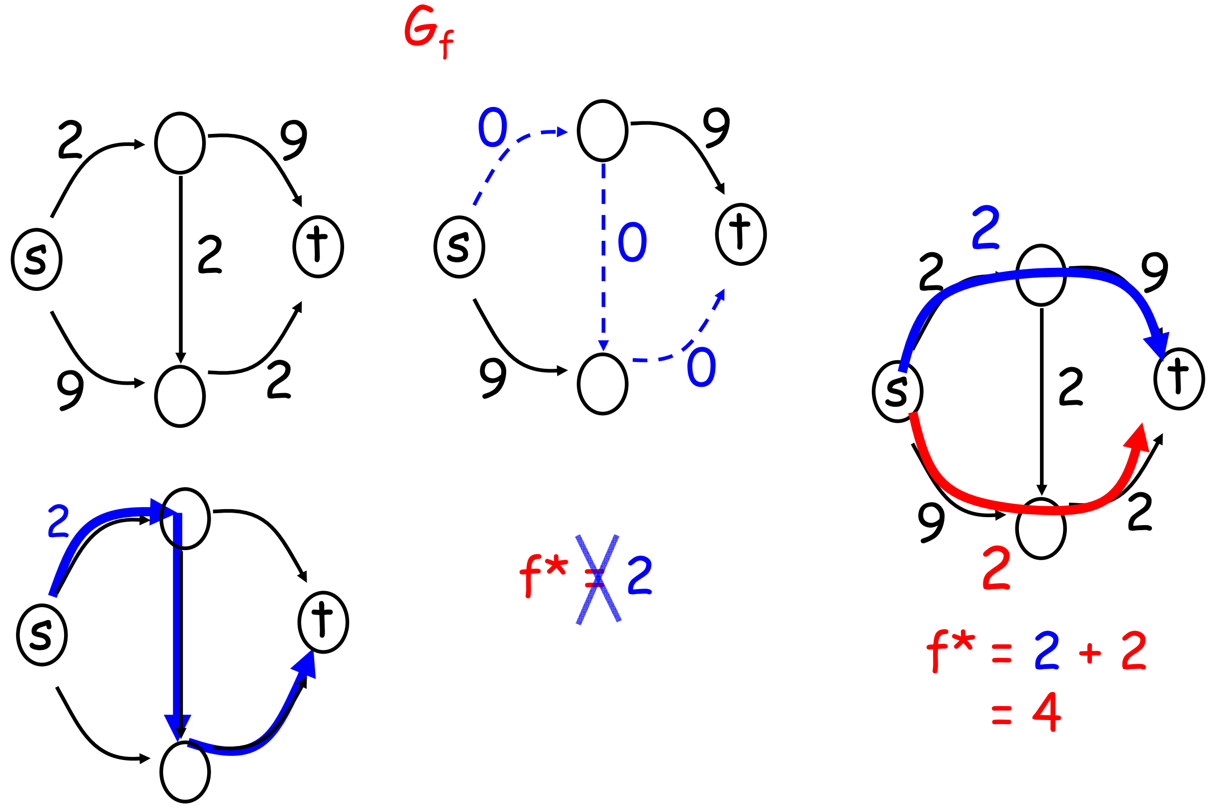
flow network



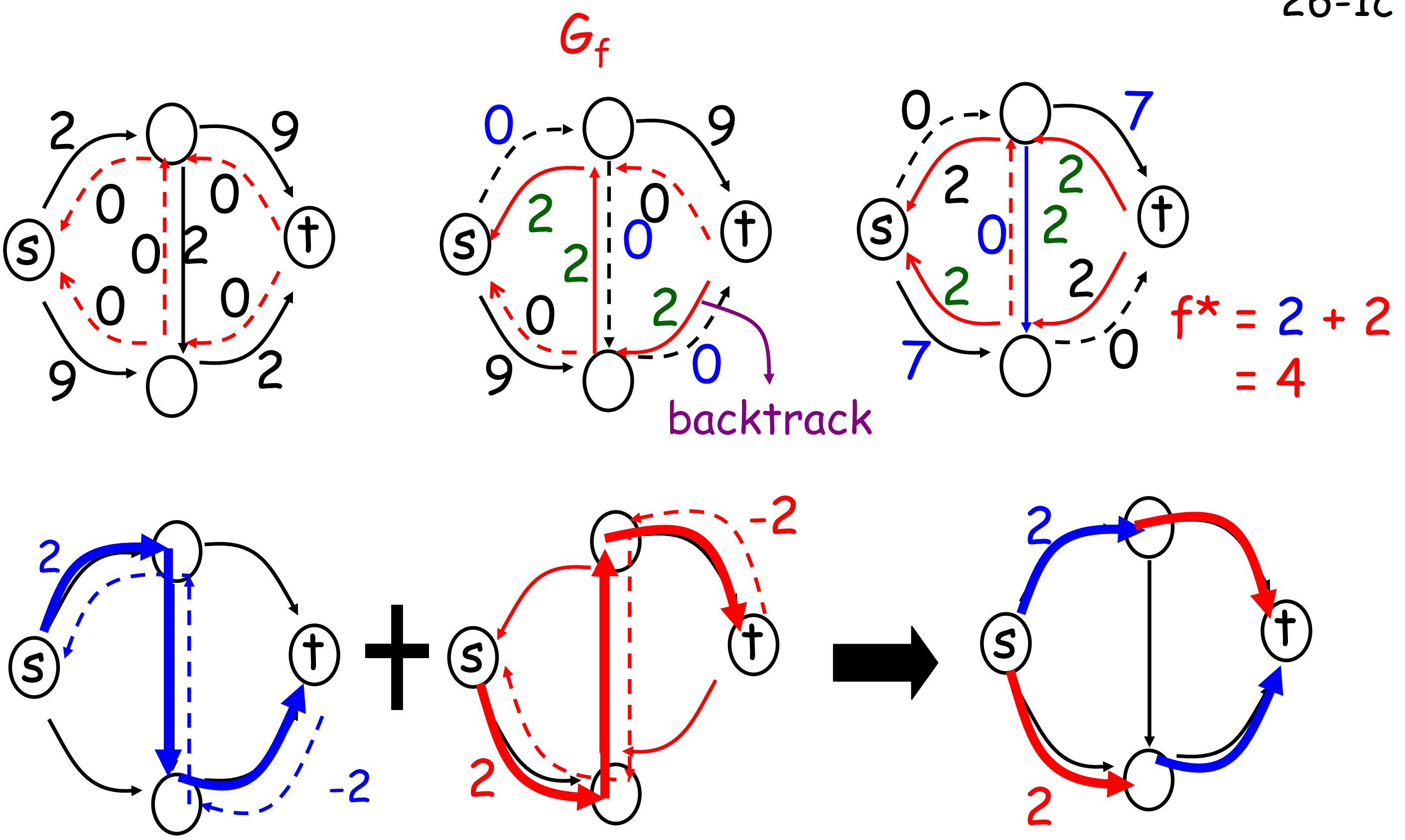
Another application: file transfer
(capacity = Kbit/s)

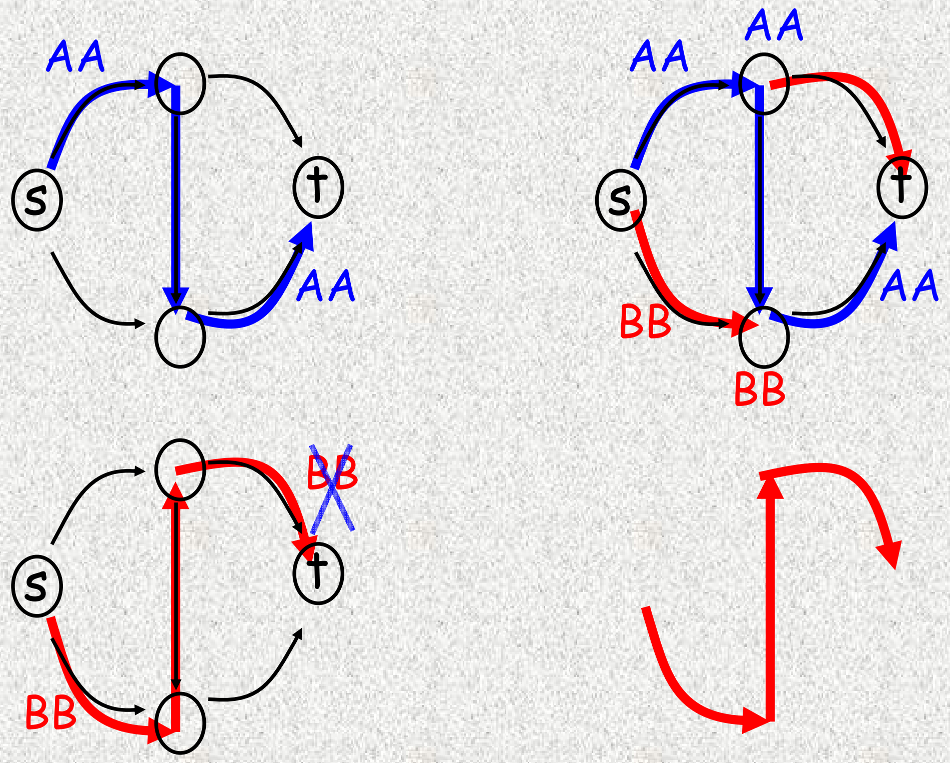


26-1b



26-1c

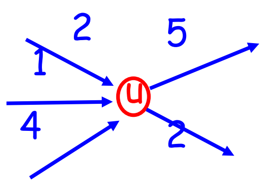




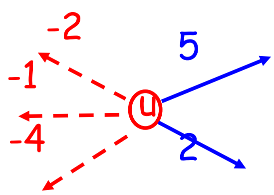
Flow Conservation: for all $u \in V - \{s, t\}$

$$\sum_{v \in V,\, f(v,u)>0} f(v, u) = \sum_{v \in V,\, f(u,v)>0} f(u, v)$$

(positive in = positive out)



$$\sum_{v \in V} f(u, v) = 0$$ (total out = 0)



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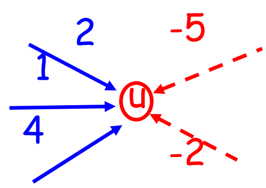
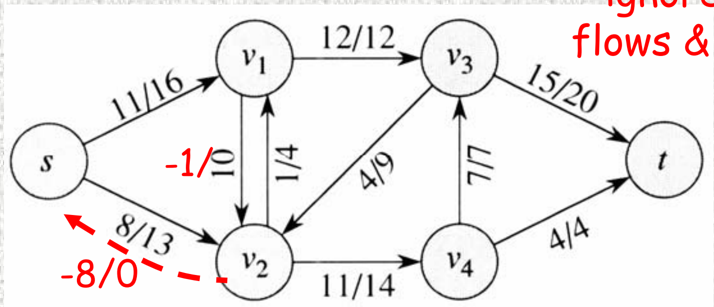


Example:

* ignore negative flows & zero edges

G and f



only non-zero edges

Residual network G_f with an augmenting path p

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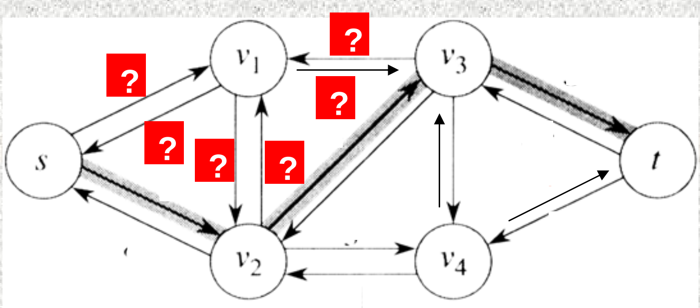

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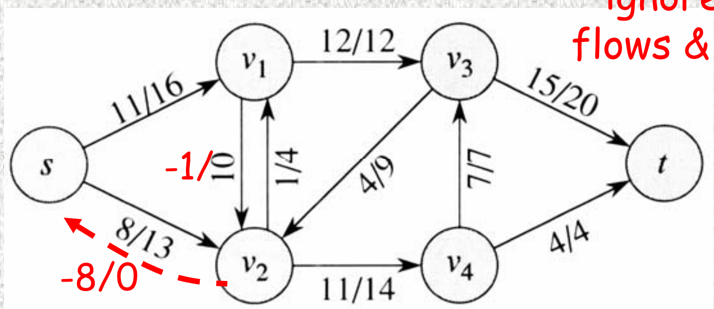

only non-zero edges

Residual network G_f with an augmenting path p

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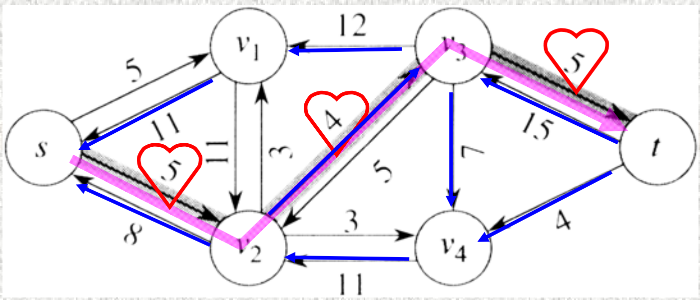
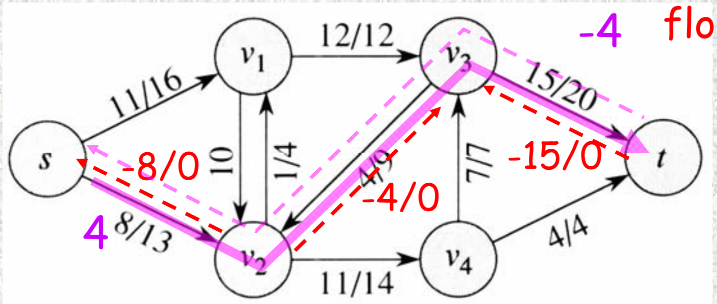
backtrack



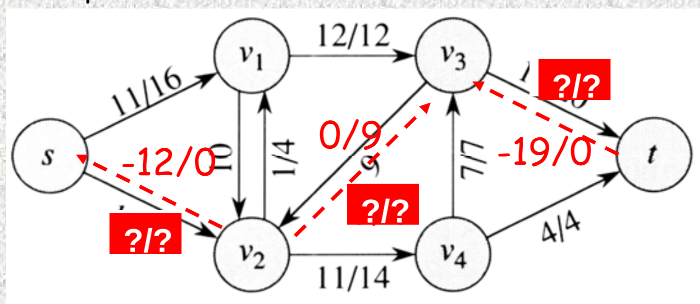
Example:

G and *f*

* ignore negative flows & zero edges



New f ← f + f_p



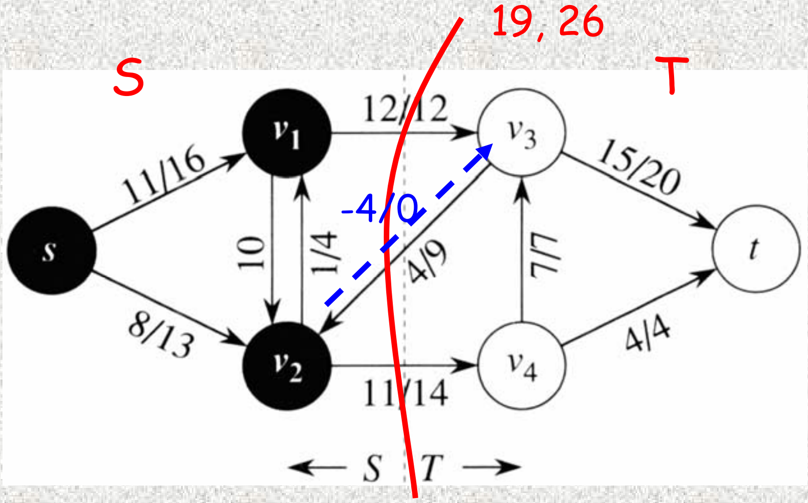
Net flow across a cut: ***f*(*S*, *T*).** = 12 + (-4) + 11

19

Capacity of a cut: ***c*(*S*, *T*).** = 12 + 0 + 14

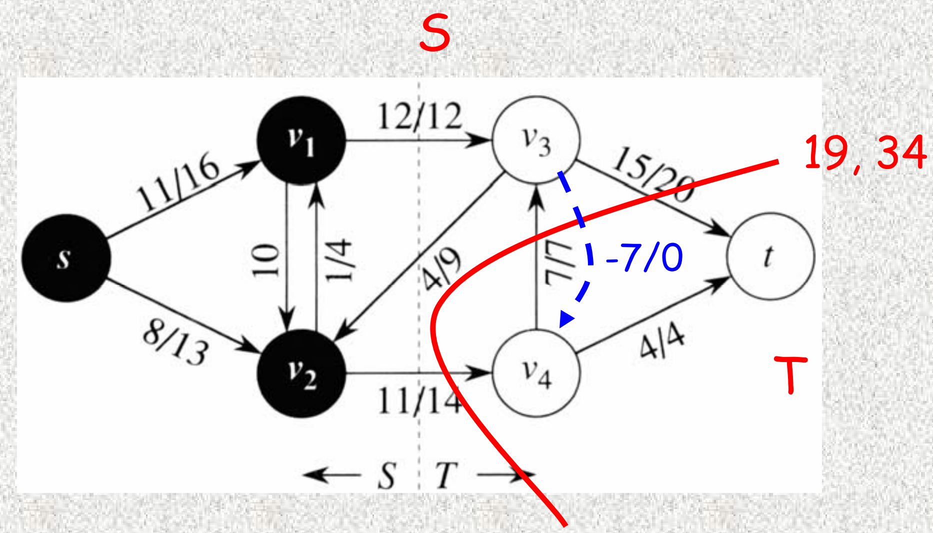
26

Example: |*f*|=19, *f*(*S*, *T*)=19, and *c*(*S*, *T*)=26.



Net flow across a cut: $f(S, T)$. = 15 + (-7) + 11 = 19

Capacity of a cut: $c(S, T)$. = 20 + 0 + 14 = 34

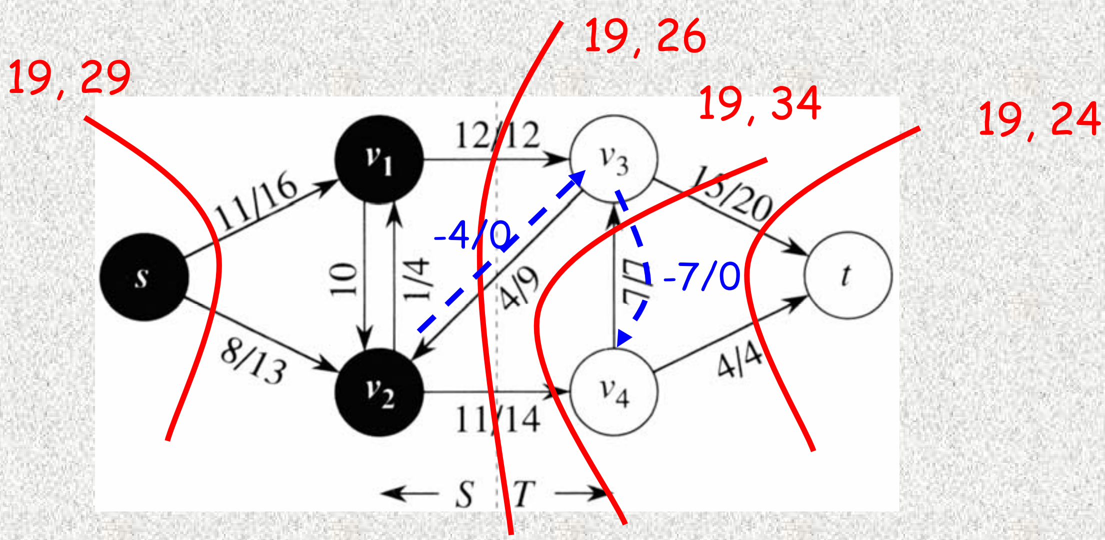


Lemma 26.5: $f(S, T) = |f|$

(flow conservation)

Corollary 26.6: $|f| \le c(S, T)$

(capacity constraint)



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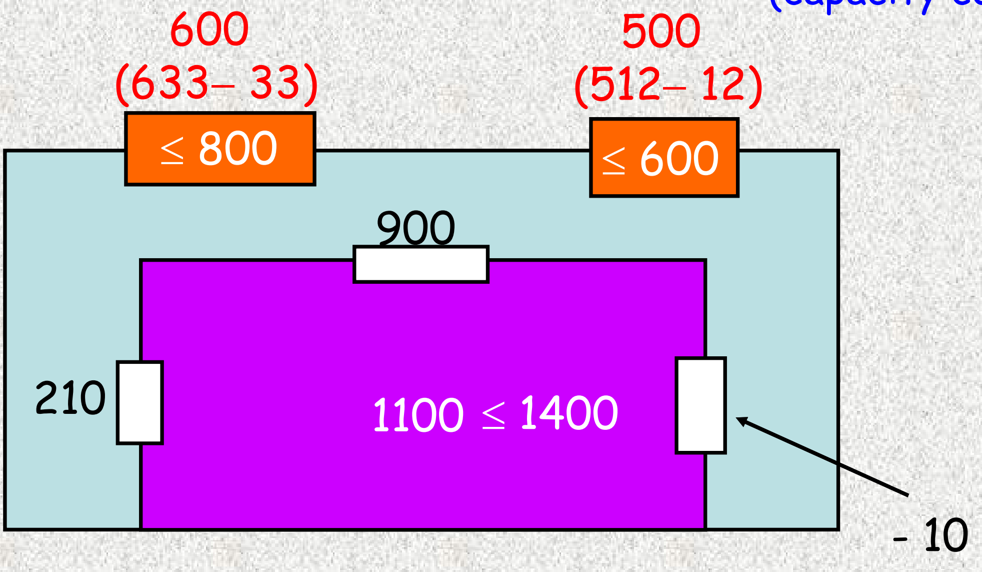
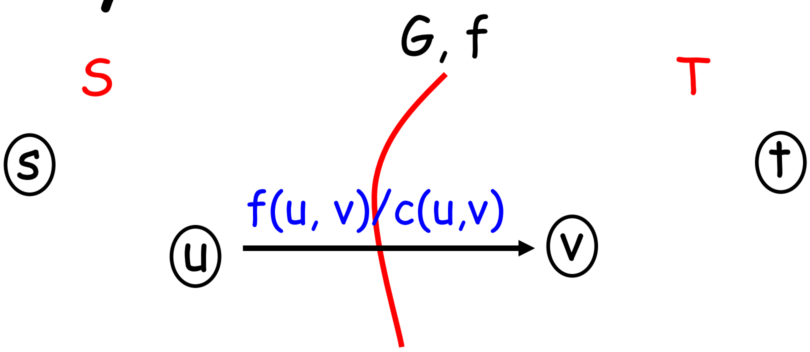


26-6s

Corollary 26.6

26-6a



$$f(u, v) \le c(u, v)$$

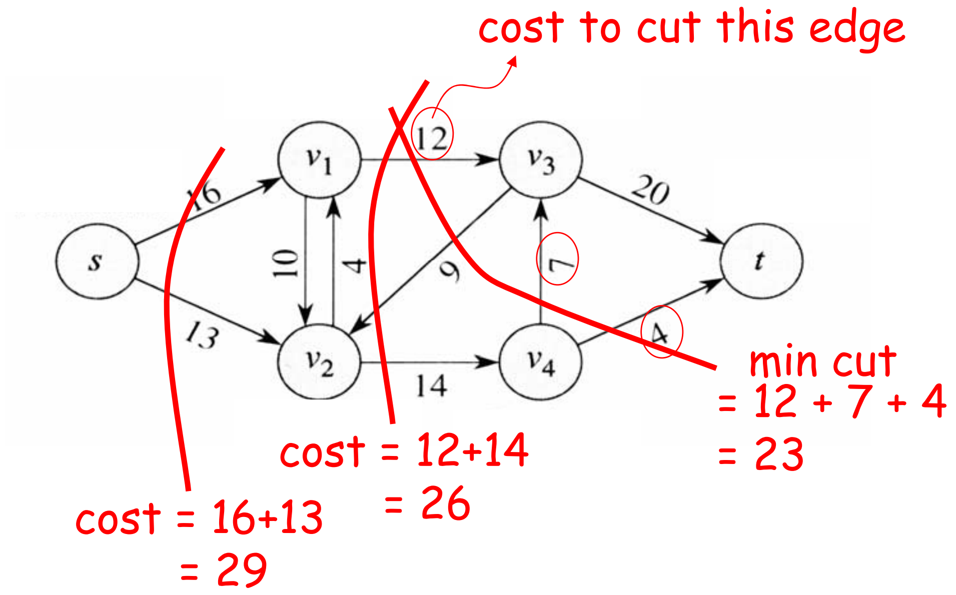
$$\Rightarrow \sum_{u \in S, v \in T} f(u, v) \le \sum_{u \in S, v \in T} c(u, v)$$

$$\Rightarrow f(S, T) \le c(S, T)$$

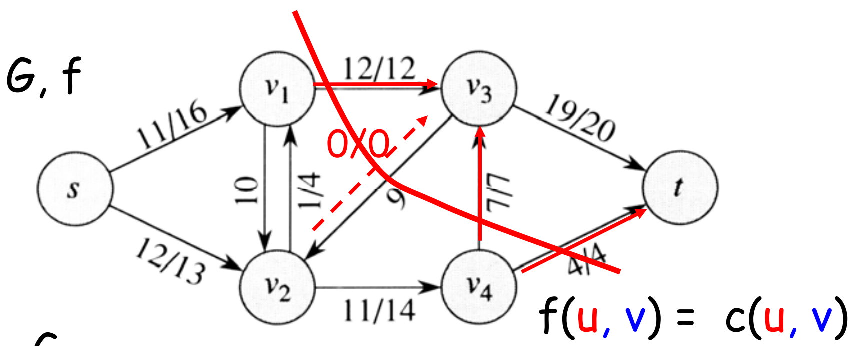
$$\Rightarrow |f| \le c(S, T)$$

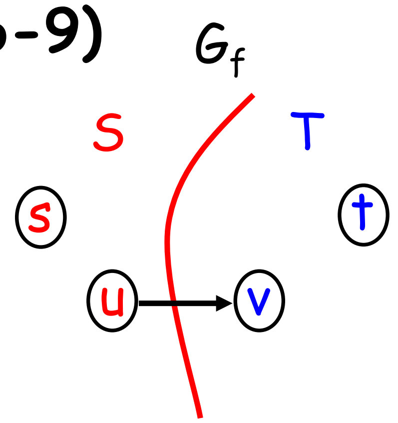
(**by Lemma 26.5:** $f(S, T) = |f|$)

The min cut problem

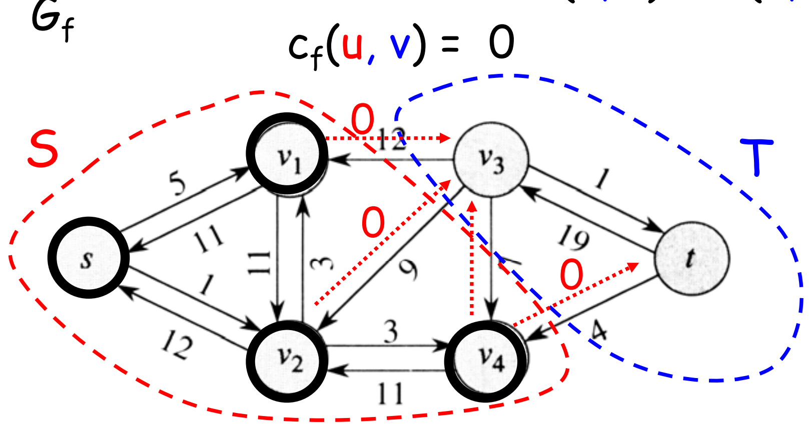


Thm. 26.7 (2)-->(3) (See 26-9)





no augmenting path



(no argumenting path)

$\Rightarrow$ $c_f(u, v) = 0$

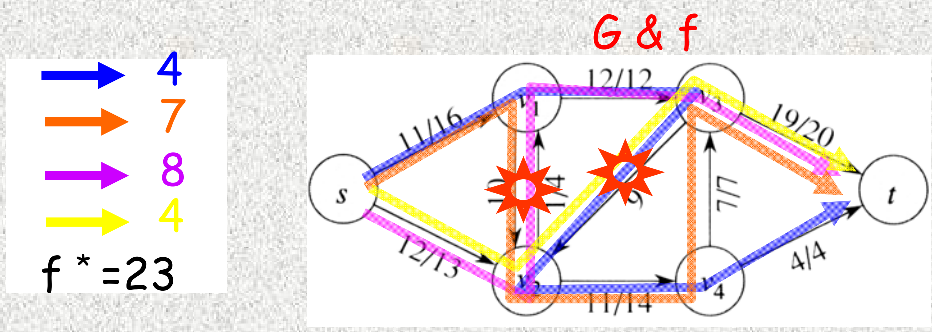
$\Rightarrow$ $f(u, v) = c(u, v)$

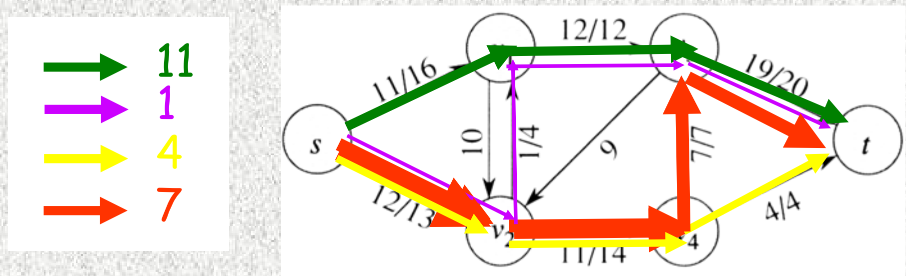
$\Rightarrow$ $\sum_{u \in S, v \in T} f(u, v) = \sum_{u \in S, v \in T} c(u, v)$

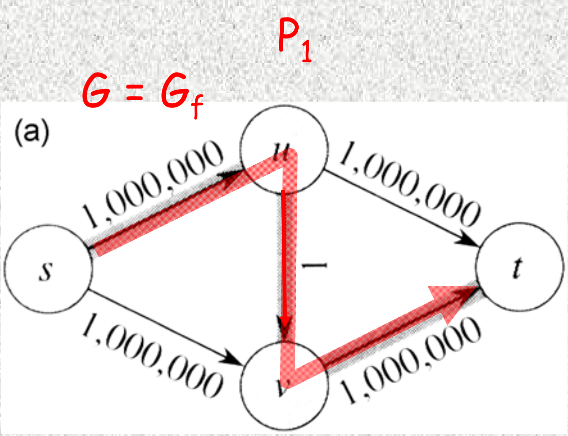
$\Rightarrow$ $f(S, T) = c(S, T)$

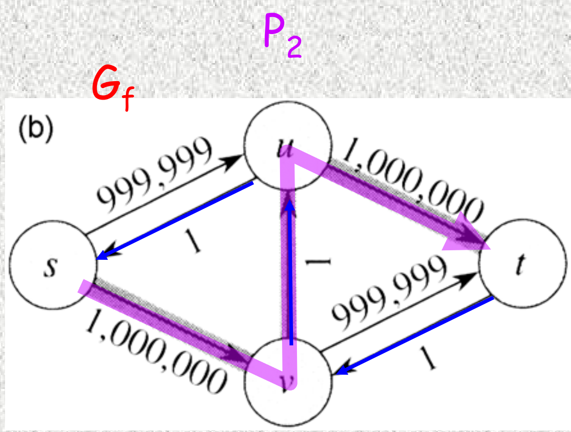
Constructing flow paths

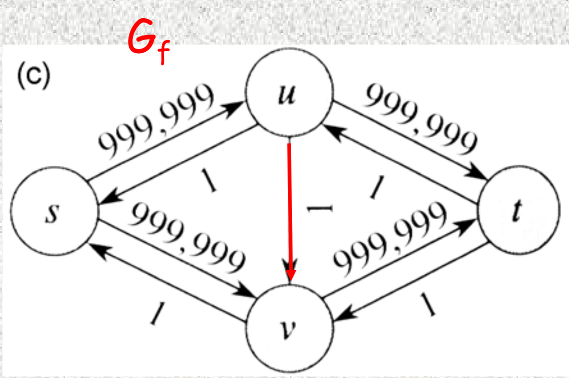
G & f



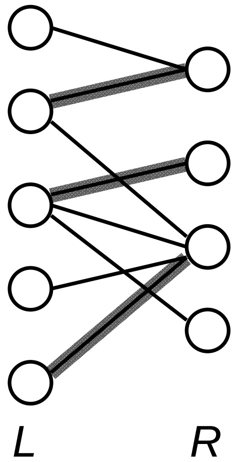


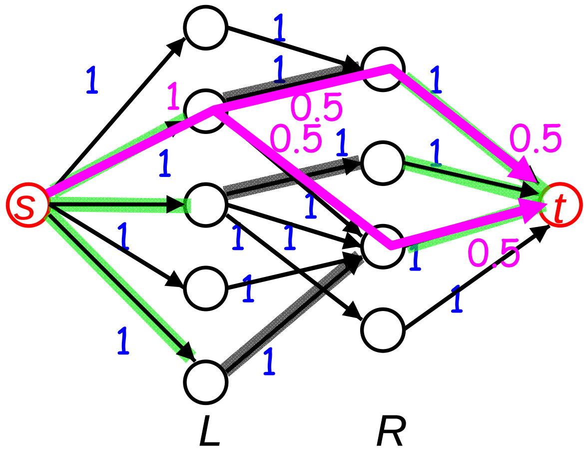






2×10^6 iterations





* matching ---> flow

* flow ~~--~~-> matching ? * integer flow ----> matching

* integer flow <----> matching

* max integer flow <----> max matching

Max flow on undirected G