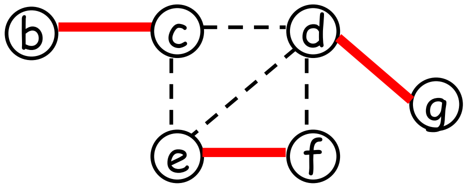
$G \supseteq$ A: three disjoint edges



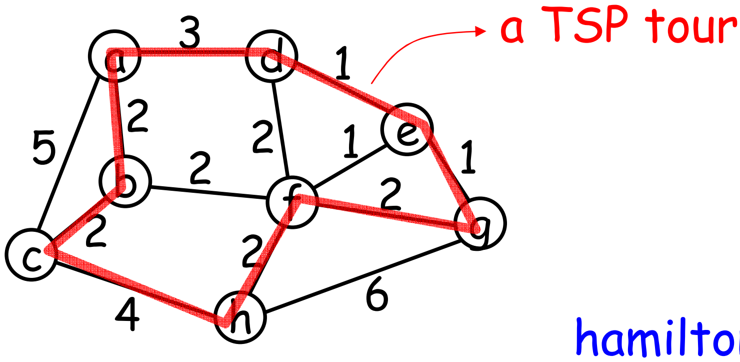

A needs at least |A| = 3 vertices

⇨ G needs at least |A| = 3 vertices

⇨ $|C^*| \geq |A|$ — ② (a lower bound on $C^*$)



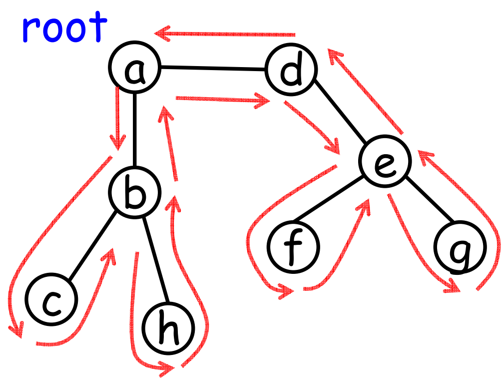
The TSP Problem




hamiltonian cycle

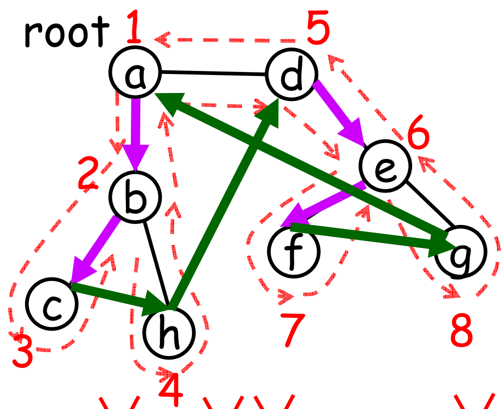
① visit each vertex exactly once

② minimum total length




full walk W=(a, b, c, b, h, b, a, d, e, f, e, g, e, d, a)





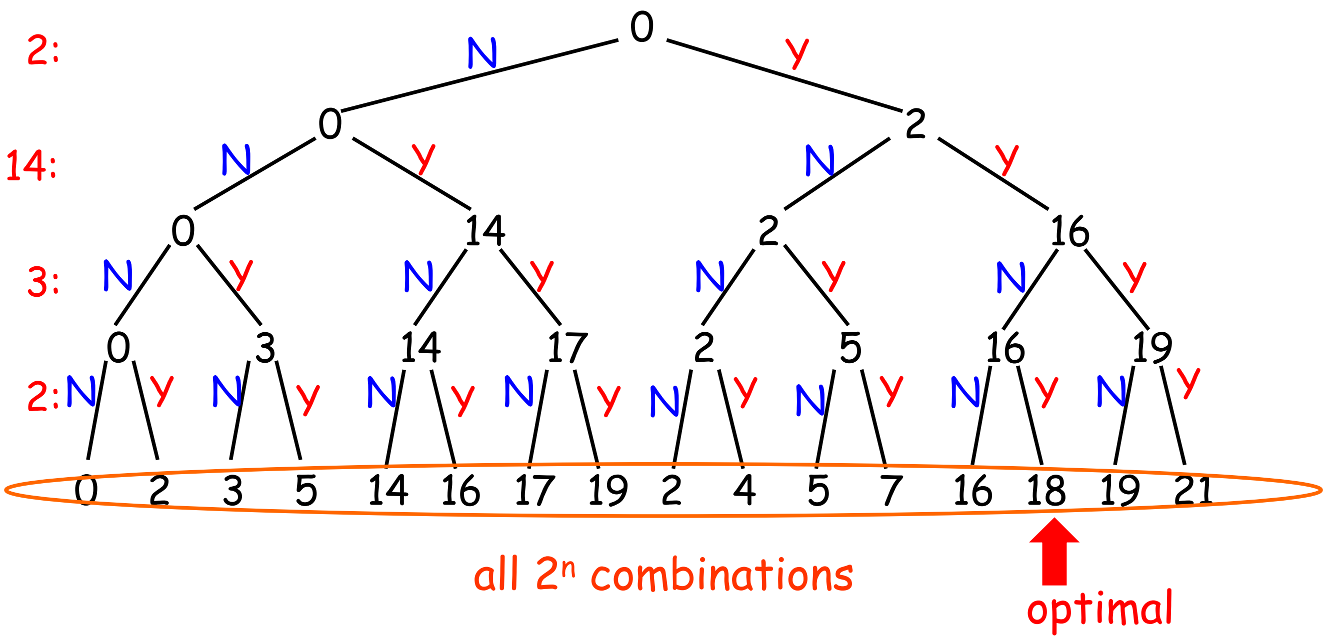

full walk W=(a, b, c, ~~b~~, h, ~~b~~, ~~a~~, d, e, f, ~~e~~, g, ~~e~~, ~~d~~, a)

1 2 3 4 5 6 7 8 1

H=(a, b, c, h, d, e, f, g, a)

pre-order traversal on T

t = 18
S = (2, 14, 3, 2)

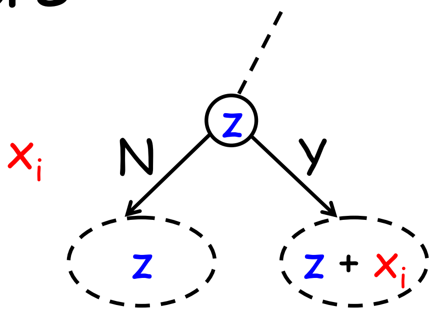




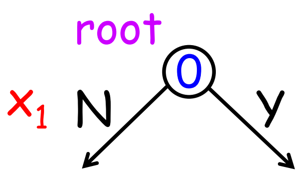

## Brute-Force-DFS

```
DFS(i, z)
begin
   if i ≤ n then
       DFS(i + 1, z)
       DFS(i + 1, z + x_i)
   else /* z is a leaf-candidate
       z* = better(z*, z)
end
```

also check valid or not



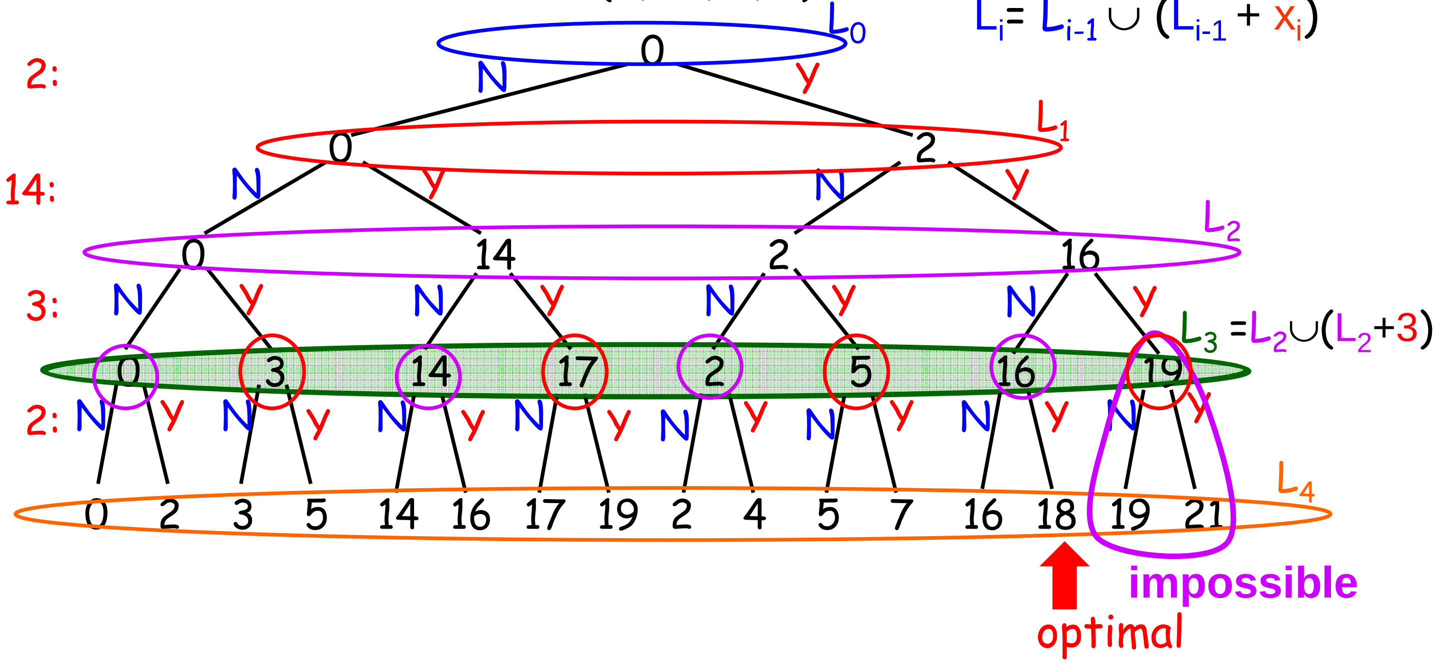

```
SubsetSum(S = (x_1, x_2, ..., x_n), t)
begin
   z* = 0
   DFS(1, 0)
   Output z*
end
```



t = 18

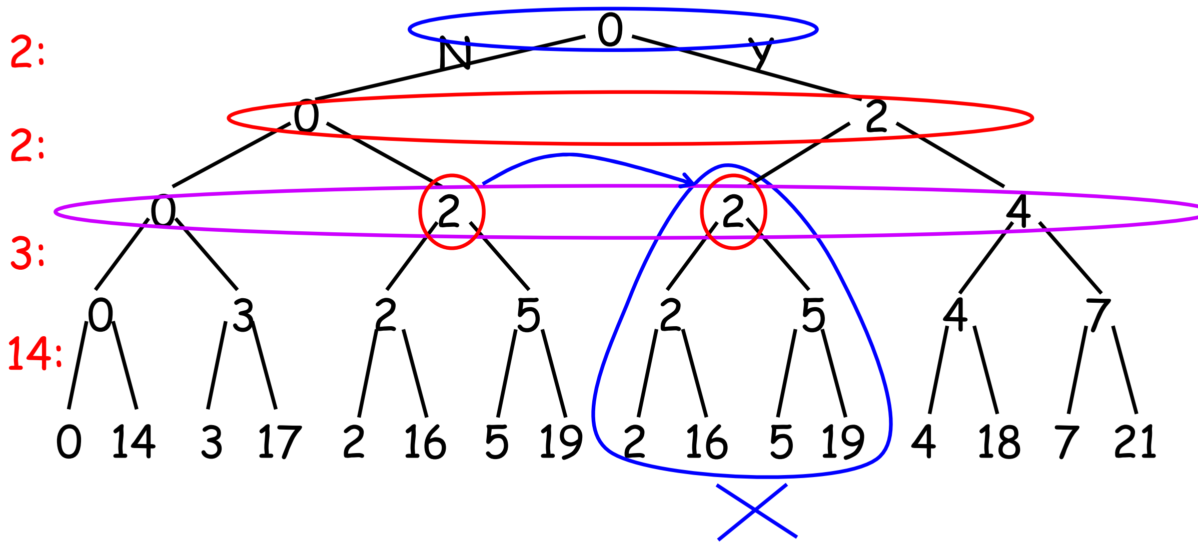
S = (2, 14, 3, 2)

$L_i = L_{i-1} \cup (L_{i-1} + x_i)$






S = (2, 2, 3, 14)



$T(n) = \sum_{i=0}^{n-1} 2|L_i|$ $\quad L_i = (l_1, l_2, \ldots, \underset{\underset{100}{\wedge|}}{l_k})$ $\quad$ all $l_j$ are distinct and $l_k \le t = 100$ $\Rightarrow |L_i| \le \underset{\underset{101}{\wedge|}}{l_k + 1}$

① $|L_i| \le 2^i$

$T(n) = 2^0 + 2^1 + 2^2 + \ldots + 2^{n-1}$
$= O(2^n)$

② $l_k \le t \Rightarrow |L_i| \le t+1$

$T(n) = O(nt)$

③ $l_k \le \overset{W}{\overset{=}{\text{sum}(S)}} \Rightarrow |L_i| \le W+1$

$T(n) = O(nW)$

④ let $m = \max(S) \Rightarrow W \le nm$
$\Rightarrow |L_i| \le nm+1$

$T(n) = O(n \times nm) = O(n^2 m)$

T(n) is polynomial if one of t, W, m is polynomial !

⇨ T(n) is pseudo-polynomial! (t, W, m may be $\infty$)



## Pseudo-Polynomial:

polynomial in the numeric value of an integer
(exponential in the length (# of bits) of the integer

The subset sum problem ($S = \{x_1, \ldots, x_n\}$, t)

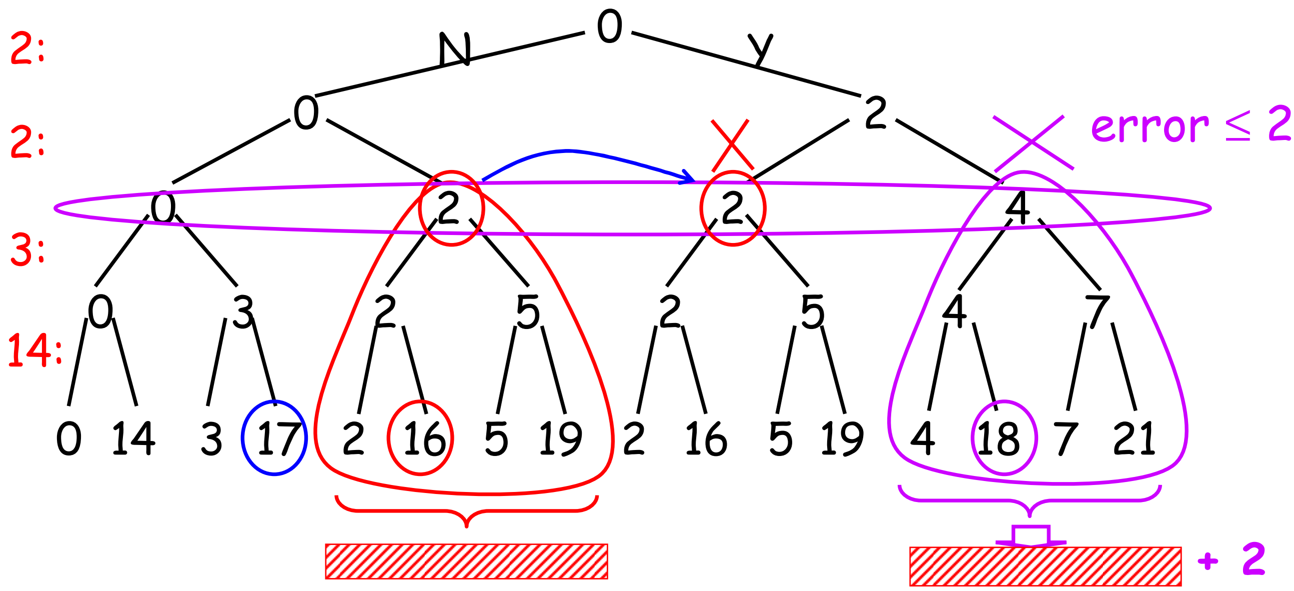
* Consider $s = \lg t$ as the "input size" of t. (t is an s-bit integer)
  e.g. t = 60000, s = lg t = 16 bits
* $T(n) = O(nt) = O(n2^s)$ is exponential in s (pseudo-polynomial)
* $A(n) = O(n^2 \log t) = O(n^2 s)$ is polynomial (in n and s)

Examples:

| pseudo-polynomial | polynomial |
|---|---|
| Counting sort - $O(n + k)$ | GCD- $O(\lg b)$ |
| Knapsack - $O(nC)$ | $X^a$- $O(\lg a)$ |
| GCD- $O(b)$ | |
| $X^a$- $O(a)$ | |

35-9c

$S = (2, 2, 3, 14)$




e.g., t = 18 $\Rightarrow$ error $\leq 2$ $\Rightarrow$ error = 1

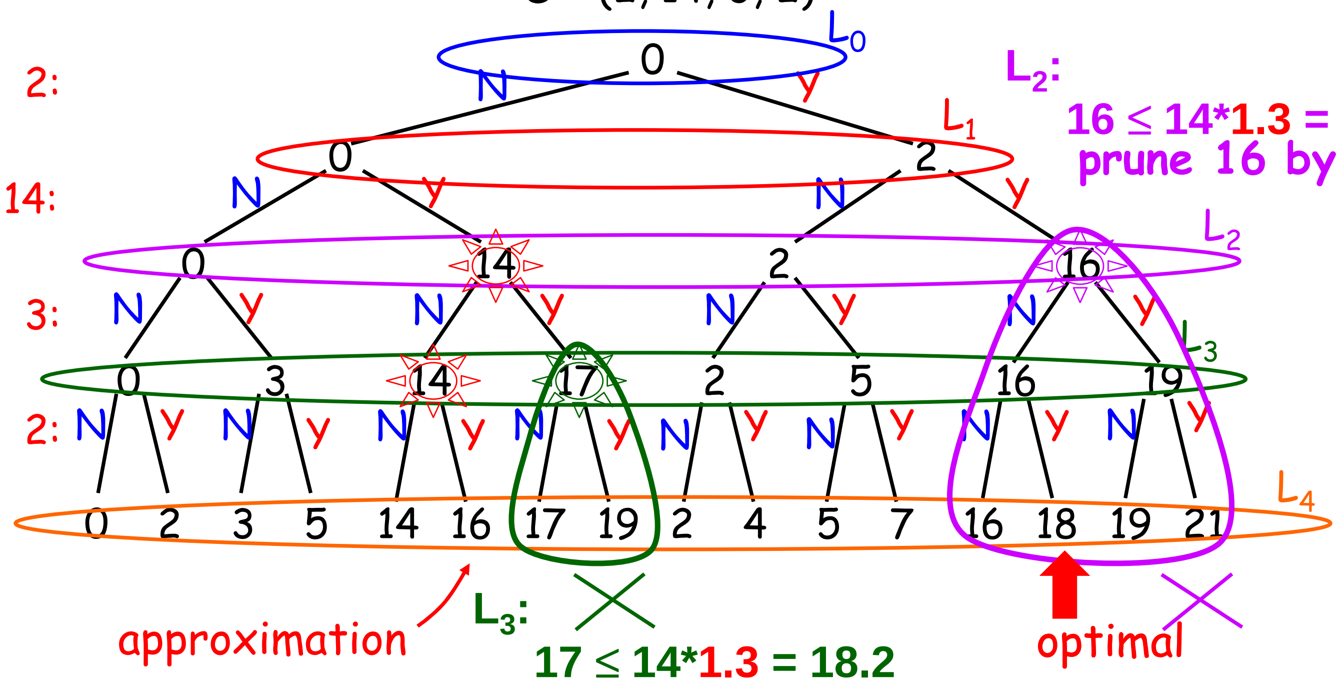
$\Rightarrow$ ~~error~~ = 2

35-9d

t = 18

$S = (2, 14, 3, 2)$

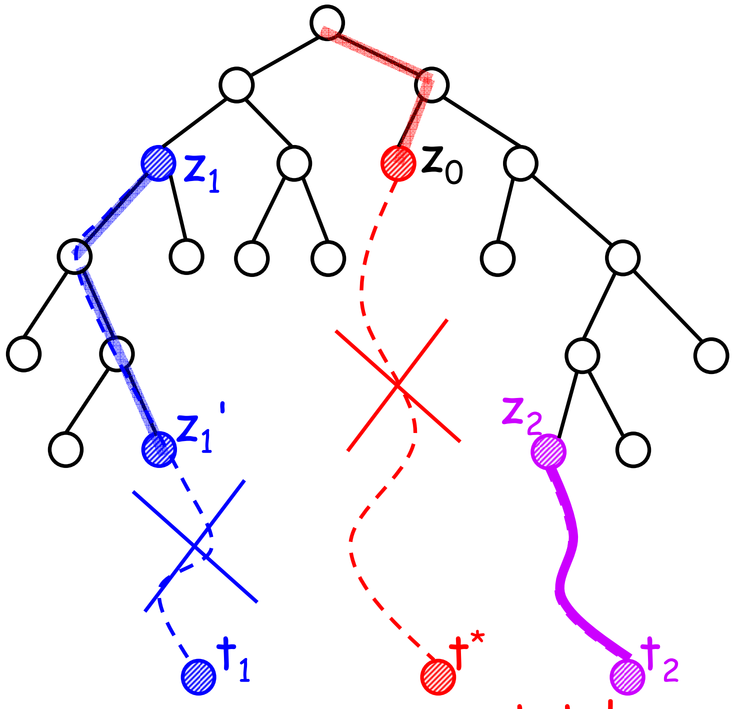
$\delta$ = 0.3




$L_2$:
$16 \leq 14*1.3 = 18.2$
prune 16 by 14

$L_3$:
$17 \leq 14*1.3 = 18.2$
prune 17 by 14

**Question: Can we prune 14 by 16 at $L_2$?**




$z_0 \le z_1(1+\delta)$
error $\le \delta \times z_1$

$z_1' \le z_2(1+\delta)$
error $\le \delta \times z_2$

total error $\le \delta \times (z_1+z_2)$

Note: $z_1$ and $z_2$ are valid ($\le t$)



Step i: seed for $t^*$

$L_i = \{\_, \_, \ldots, \square, \square, \ldots\}$

trim

$L_i = \{\_, \_, \ldots, \square, \ldots\}$

error $\le \delta \times \square$

Step j:

$L_j = \{\_, \_, \ldots, \square, \blacksquare, \ldots\}$

trim

$L_j = \{\_, \_, \ldots, \square, \ldots\}$

error $\le \delta \times \square$

Note: all $z_i$ are valid ($\le t$)

$z_k \leftarrow \cdots \leftarrow z_3 \leftarrow z_2 \leftarrow z_1 \leftarrow z_0$

total error

$$\le \sum z'_{i-1} - z_i$$

$$\le \sum_{i=1}^{k} \delta \times z_i$$

$$\le \sum_{i=1}^{k} \delta \times t^* \quad (z_i \le t^*)$$

$$\le k\,\delta\, t^*$$

$$\le n\,\delta\, t^* \quad (k \le n)$$

$$\le t^*\,\varepsilon \quad (\delta = \varepsilon / n)$$