---------------------------- 1 & 2 ----------------------------

Introduction & Getting Started

1.1 Algorithms

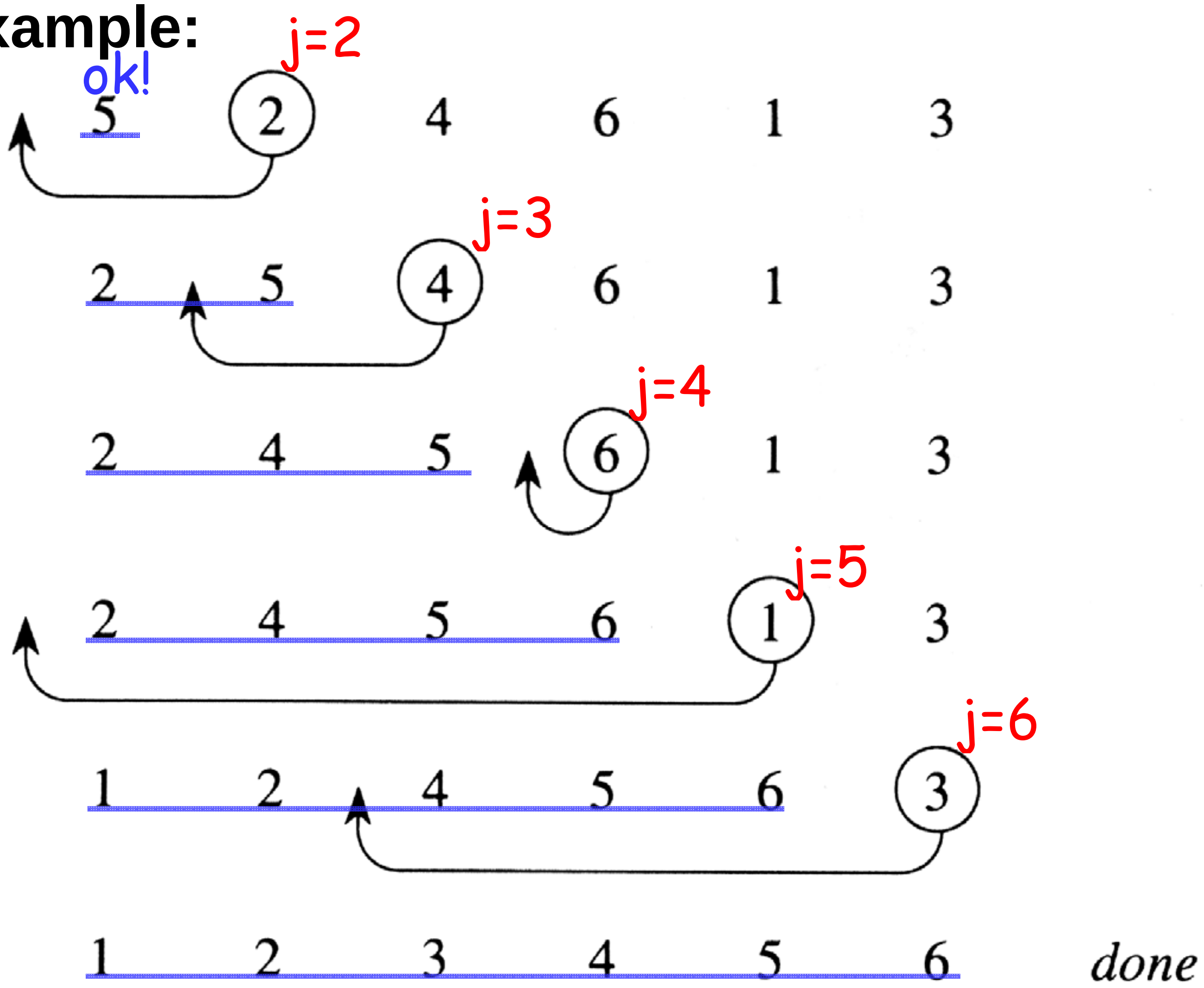
電腦可以執行

Algorithm: A sequence of computational steps that transform the ***input*** of a ***computational problem*** into the ***output***.

2.1 Insertion Sort: An efficient algorithm for sorting a small number of elements.

(Sort a hand of cards)

Example:



2.2 Analyzing Algorithms

Model

RAM: *Random-access machine*, in which each memory access takes unit time and instructions are executed one by one.

Running time: number of steps, which is a function of the input size.

Primitive operations (memory access, +, -, *, /, ...)

a := b + c; ⇨ 4 steps

Example: Insertion Sort

INSERTION-SORT(A)	*cost*	*times*
for $j \leftarrow 2$ **to** $length[A]$ n	c_1	n
do $key \leftarrow A[j]$	c_2	$n-1$
▷ Insert $A[j]$ into the sorted sequence $A[1\,..\,j-1]$.	0	$n-1$
$i \leftarrow j-1$	c_4	$n-1$
while $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2}^{n} t_j$
do $A[i+1] \leftarrow A[i]$	c_6	$\sum_{j=2}^{n} (t_j - 1)$
$i \leftarrow i-1$	c_7	$\sum_{j=2}^{n} (t_j - 1)$
$A[i+1] \leftarrow key$	c_8	$n-1$

1&2-2a

← i   j=6   A[6]

1   4   8   9   11   7

⇨ 1   4   7   8   9   11

$t_6 = 4$

$1 \le t_j \le j$

$$T(n) = c_1 n + (c_2 + c_4 + c_8)(n-1) + c_5 \sum_{j=2}^{n} t_j +$$

$$(c_6 + c_7) \sum_{j=2}^{n} (t_j - 1)$$

***Best-case*:**

Each t_j=1. (The input *A* is sorted.)

$$\begin{aligned} T(n) &= (c_1+c_2+c_4+c_5+c_8)n - (c_2+c_4+c_5+c_8) \\ &= \Theta(n) \end{aligned}$$

(***rate of growth***, ***order of growth***)

***Worst-case*: (upper bound)**

Each t_j=j.

$$\begin{aligned} T(n) &= k_1 n^2 + k_2 n + k_3 \\ &= \Theta(n^2) \end{aligned}$$

***Average-case*: (Expected running time)**

Each t_j=j/2.

$$\begin{aligned} T(n) &= t_1 n^2 + t_2 n + t_3 \\ &= \Theta(n^2) \end{aligned}$$

2.3 Designing Algorithms

Divide-and-Conquer: 1&2-4a

*not similar

Divide: (into the same problems of smaller size)

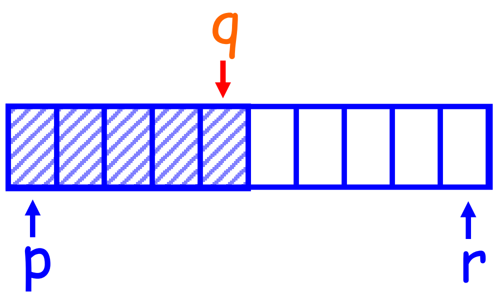
Conquer:

Combine:

*A simple example: Finding maximum 1&2-4b

1&2-4c

Example: Merge Sort



$p \geq r$ ($n \leq 1$)
"termination condition"

MERGE-SORT(A, p, r)

if $\boxed{p < r}$ → $n \geq 2$

  then $q \leftarrow \lfloor (p + r)/2 \rfloor$

    MERGE-SORT(A, p, q)

    MERGE-SORT$(A, q + 1, r)$

    MERGE(A, p, q, r) → $O(r-p+1)$

else

1&2-5a

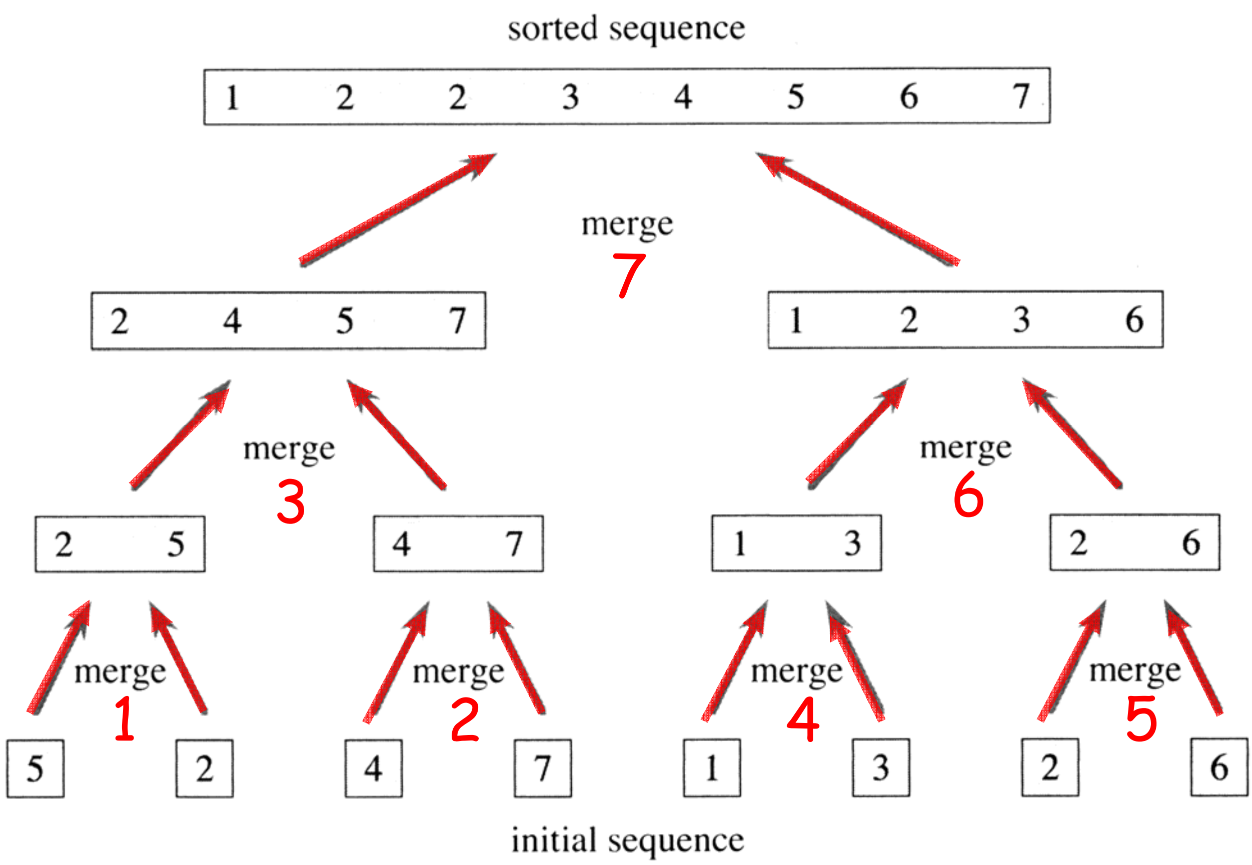


Figure 2.4 The operation of merge sort on the array $A = \langle 5, 2, 4, 7, 1, 3, 2, 6 \rangle$. The lengths of the sorted sequences being merged increase as the algorithm progresses from bottom to top.

Analysis: (*recurrence*)

constant time

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 2T(n/2) + \underline{\Theta(n)} & \text{if } n > 1 \end{cases}$$

How to merge in Θ(n) time?

$$= \Theta(n \log n)$$

Homework: Pro. 2-1 and 2-4.