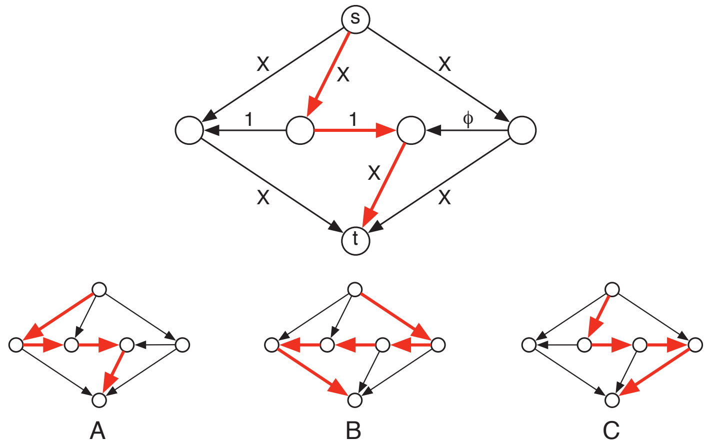
maximum flow. Each iteration requires $O(E)$ time, to create the residual graph G_f and perform a whatever-first-search to find an augmenting path. Thus, in the words case, the Ford-Fulkerson algorithm runs in $\boxed{O(E|f^*|)}$ time.

If we multiply all the capacities by the same (positive) constant, the maximum flow increases everywhere by the same constant factor. It follows that if all the edge capacities are *rational*, then the Ford-Fulkerson algorithm eventually halts. However, if we allow irrational capacities, the algorithm can loop forever, always finding smaller and smaller augmenting paths. Worse yet, this infinite sequence of augmentations may not even converge to the maximum flow! One of the simplest example of this effect was discovered by Uri Zwick.

Consider the graph shown below, with six vertices and nine edges. Six of the edges have some large integer capacity X, two have capacity 1, and one has capacity $\phi = (\sqrt{5}-1)/2 \approx 0.618034$, chosen so that $1 - \phi = \phi^2$. To prove that the Ford-Fulkerson algorithm can get stuck, we watch the residual capacities of the three horizontal edges as the algorithm progresses. (The residual capacities of the other six edges will always be at least $X - 3$.)



Uri Zwick's non-terminating flow example, and three augmenting paths.

The Ford-Fulkerson algorithm starts by choosing the central augmenting path, shown in the large figure above. The three horizontal edges,, in order from left to right, now have residual capacities 1, 0, ϕ. Suppose the horizontal residual capacities are ϕ^{k-1}, 0, and ϕ^k for some non-negative integer k.

1. Augment along B, adding ϕ^k to the flow; the residual capacities are now $\phi^{k+1}, \phi^k, 0$.

2. Augment along C, adding ϕ^k to the flow; the residual capacities are now $\phi^{k+1}, 0, \phi^k$.

3. Augment along B, adding ϕ^{k+1} to the flow; the residual capacities are now $0, \phi^{k+1}, \phi^{k+2}$.

4. Augment along A, adding ϕ^{k+1} to the flow; the residual capacities are now $\phi^{k+1}, 0, \phi^{k+2}$.

Thus, after $4n+1$ augmentation steps, the residual capacities are $\phi^{2n-2}, 0, \phi^{2n-1}$. As the number of augmentation steps grows to infinity, the value of the flow converges to

$$1 + 2\sum_{i=1}^{\infty} \phi^i = 1 + \frac{2}{1-\phi} = 4 + \sqrt{5} < 7,$$

even though the maximum flow value is clearly $2X + 1$.