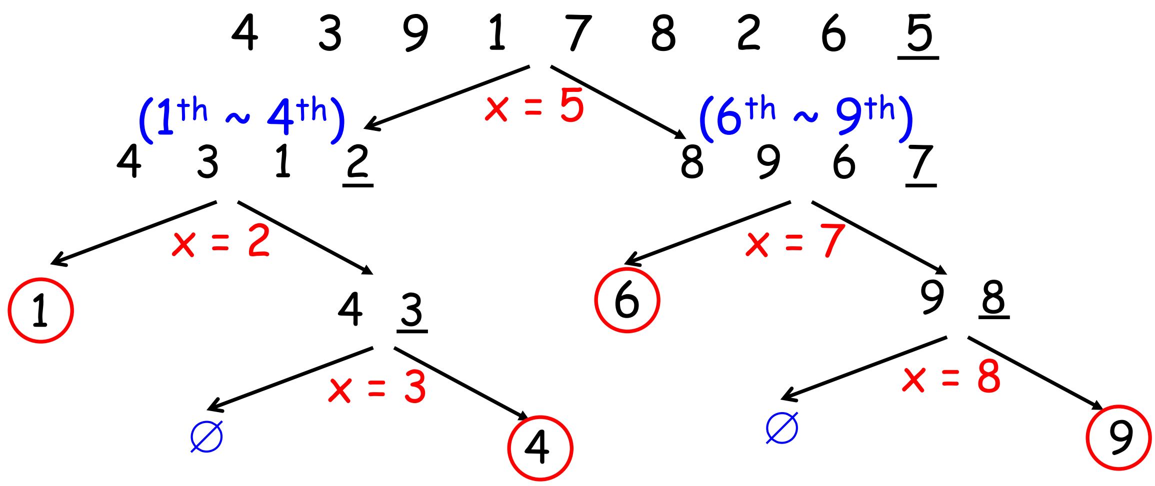
$A = 4^{th}\ 3^{rd}\ 9^{th}\ 1^{st}\ 7^{th}\ 8^{th}\ 2^{nd}\ 6^{th}\ 5^{th}$ 7-5a

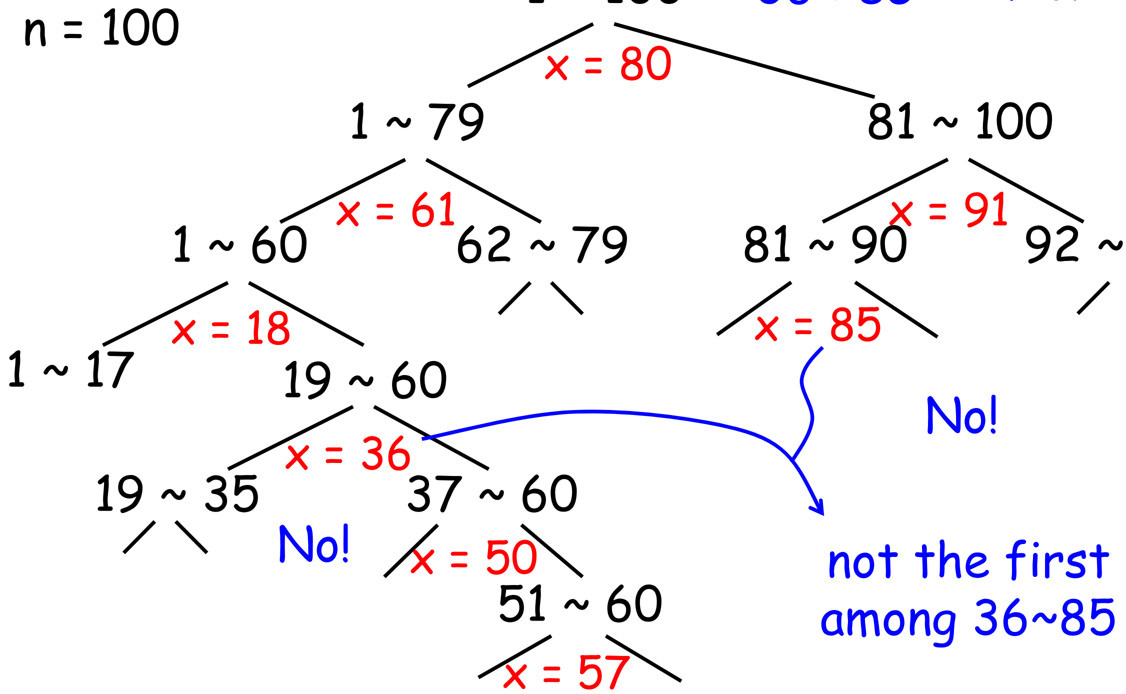


*Every number serves as a pivot at most once!

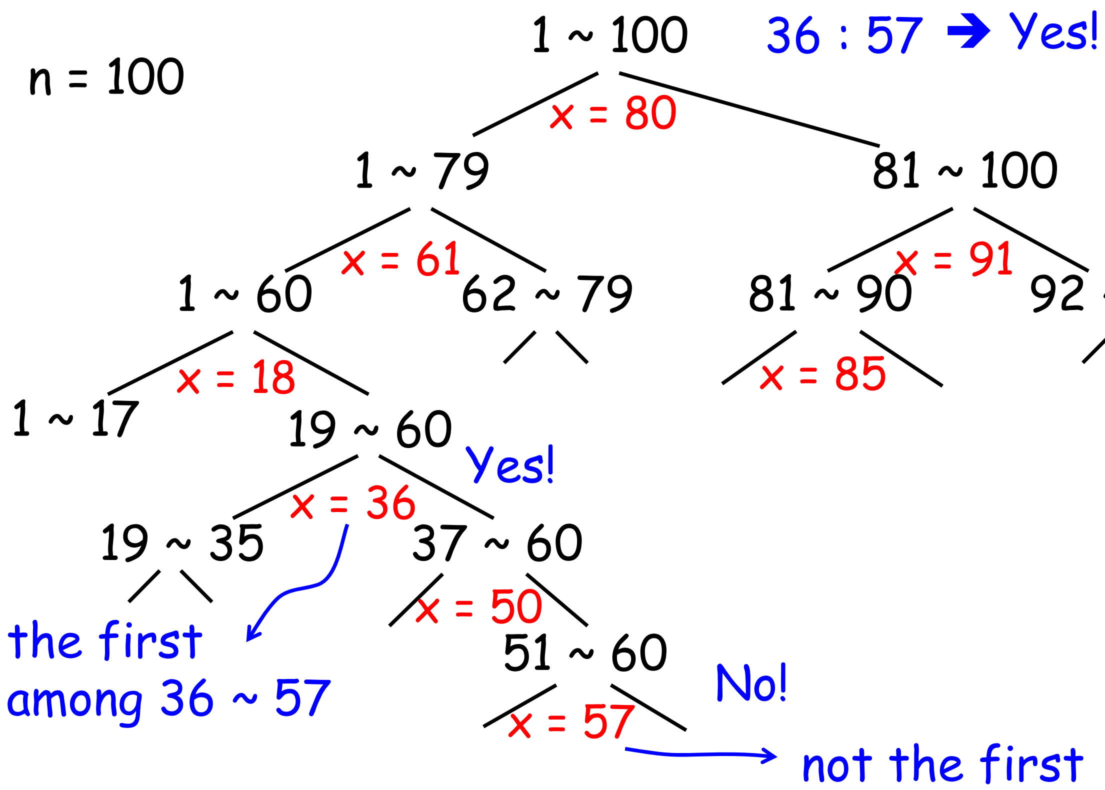
*Each call is on "consecutive" numbers!

at most n internal nodes
at most n+1 leaves ⇨ at most 2n+1 calls to Quicksort

n = 100 36 : 85 ➔ No! 7-5b



7-5c



7-5d

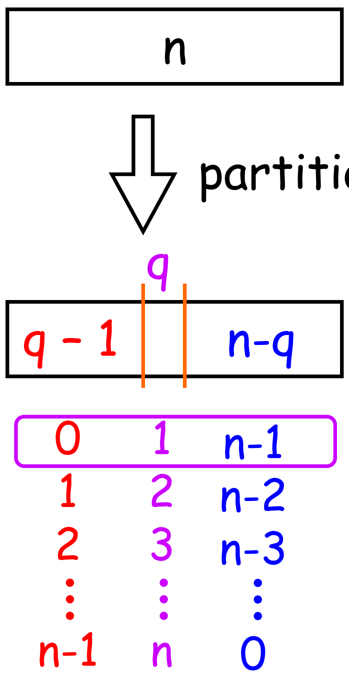
$i^{th} : j^{th}$

i : j if and only if

no one in [i+1, j-1] becomes
a pivot before i and j

$$\Rightarrow \text{probability} = \frac{|\{i, j\}|}{|\{i, i+1, \ldots, j\}|} = \frac{2}{j - i + 1}$$

Quick Sort: Average Case



$$E(n) = (n-1) + \begin{cases} 1/n * (\cancel{E(0)} + E(n-1)) \\ 1/n * (E(1) + E(n-2)) \\ 1/n * (E(2) + E(n-3)) \\ \quad\vdots \qquad\qquad \vdots \\ 1/n * (E(n-1) + \cancel{E(0)}) \end{cases}$$

$$= (n-1) + \frac{1}{n}\sum_{q=1}^{n}(E(q-1) + E(n-q))$$

$$= \underbrace{(n-1)}_{n,\ bn,\ \Theta(n)} + \frac{2}{n}\sum_{k=1}^{n-1} E(k)$$

Classic approach:

$$E(n) = \begin{Bmatrix} \Theta(n) \\ bn \\ n \\ n-1 \end{Bmatrix} + \frac{2}{n}\sum_{k=1}^{n-1} E(k)$$

guess $E(n) = O(n \lg n)$
prove by substitution method
(very hard to understand!!!)

Knuth's approach:

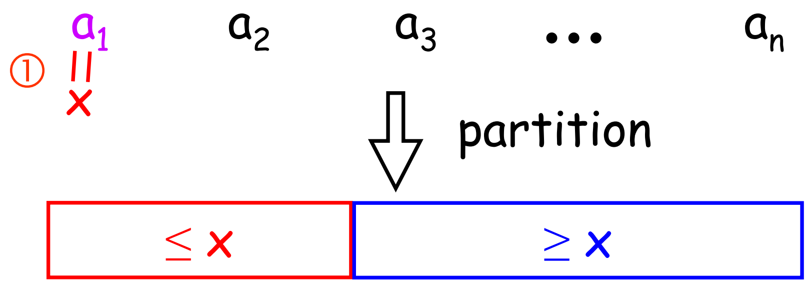
$$E(n) = n+1 + \frac{2}{n}\sum_{k=1}^{n-1} E(k) \longrightarrow E(n) = \frac{n+1}{n}E(n-1) + 2$$

compute $E(n) \approx 2n \lg n$
by iteration method!

Randomized-variable approach:

$$E(n) = O(n + X) = O(n + \sum_{i,j} Pro(i : j))$$

assume independent



② *a_1 may be either □ or □

*guarantee $|\square| \geq 1$

$|\square| \geq 1$