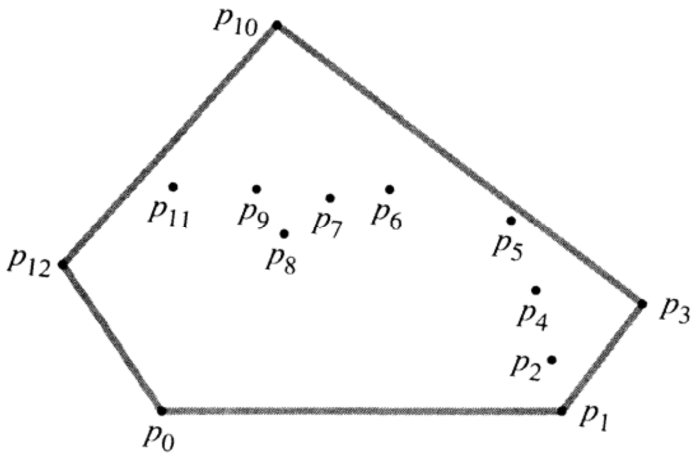
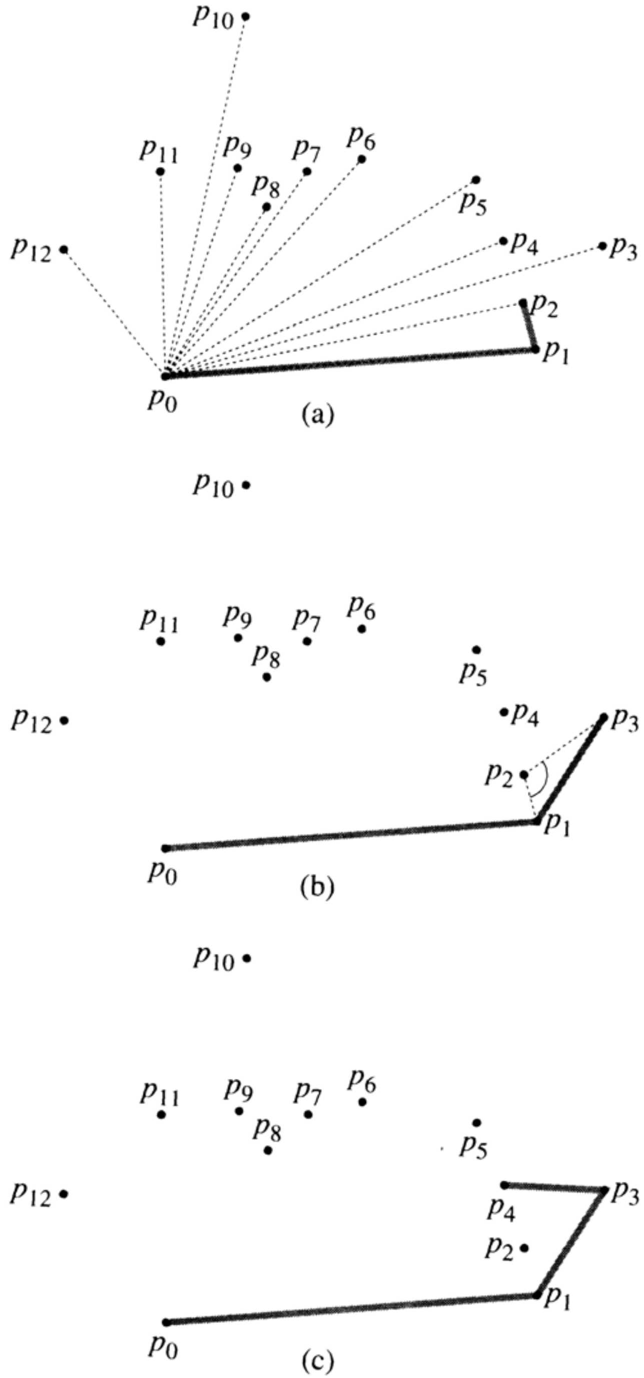
----------------------------- 33 -----------------------------
Computational Geometry

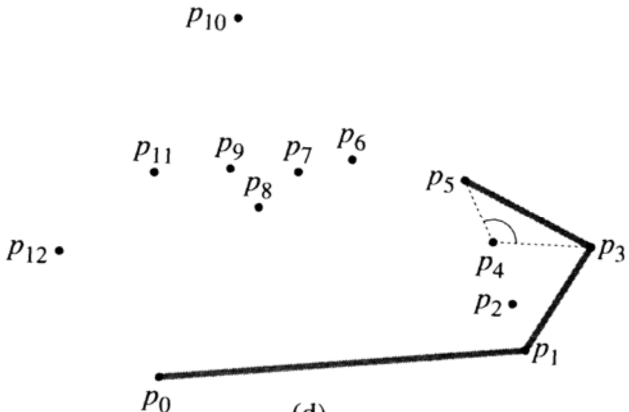
33.3 Finding the convex hull

The *convex hull* of a set *Q* of points is the smallest convex polygon *P* for which each point in *Q* is either on the boundary of *P* or in its interior.



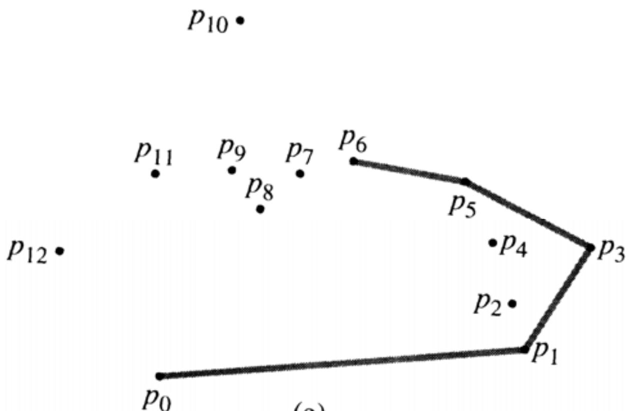
Graham's scan (Output the vertices in counterclockwise order.)





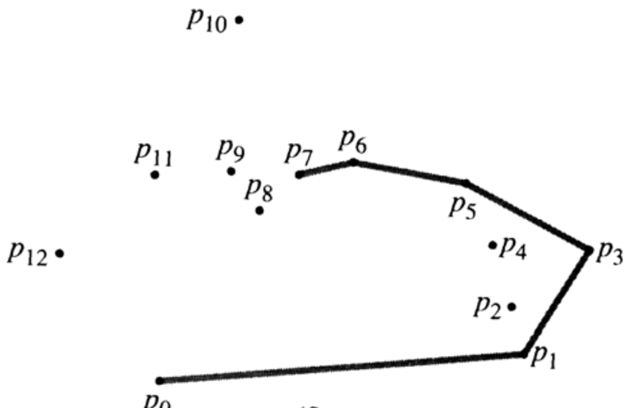
p_{10}
p_{11} p_9 p_7 p_6
p_8
p_5
p_{12}
p_4 p_3
p_2
p_1
p_0

(d)



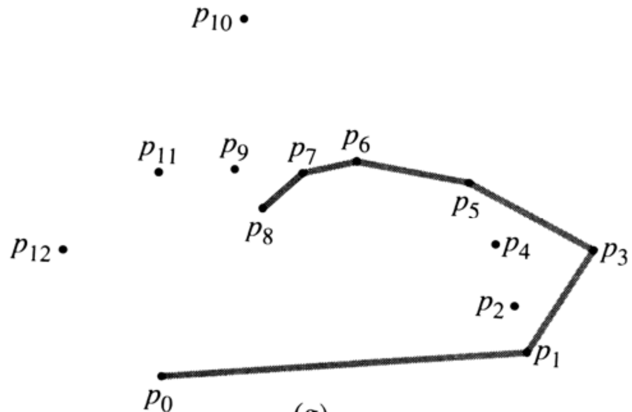
p_{10}
p_{11} p_9 p_7 p_6
p_8
p_5
p_{12}
p_4 p_3
p_2
p_1
p_0

(e)



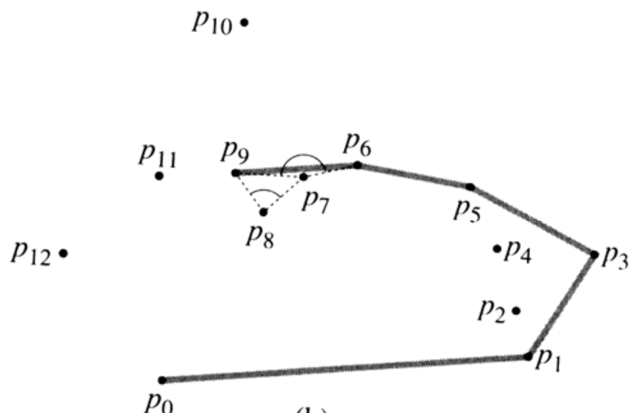
p_{10}
p_{11} p_9 p_7 p_6
p_8
p_5
p_{12}
p_4 p_3
p_2
p_1
p_0

(f)



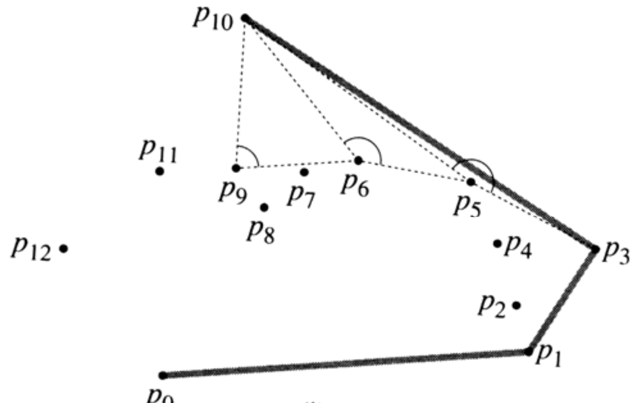
p_{10}
p_{11} p_9 p_7 p_6
p_8
p_5
p_{12}
p_4 p_3
p_2
p_1
p_0

(g)



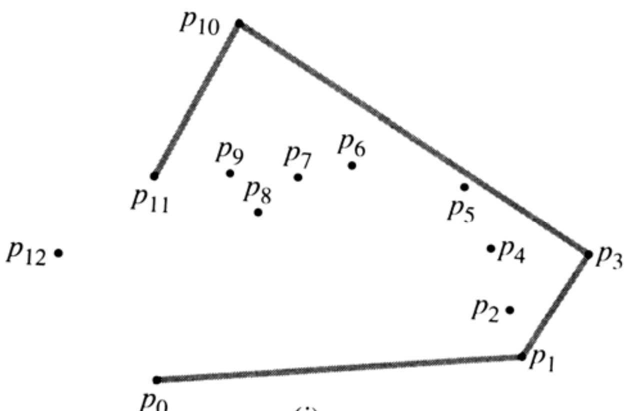
p_{10}
p_{11} p_9 p_6
p_7
p_8
p_5
p_{12}
p_4 p_3
p_2
p_1
p_0

(h)

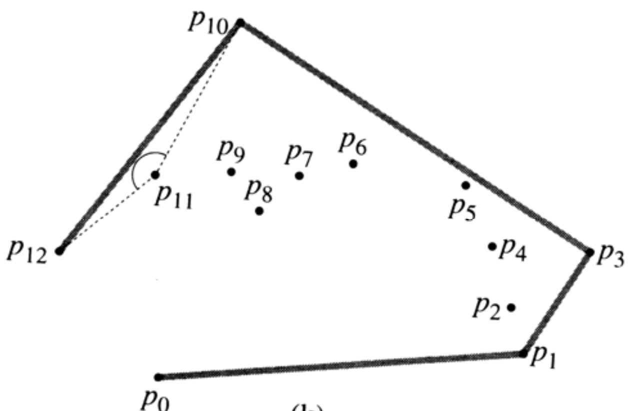


p_{10}
p_{11}
p_9 p_7 p_6
p_8
p_5
p_{12}
p_4 p_3
p_2
p_1
p_0

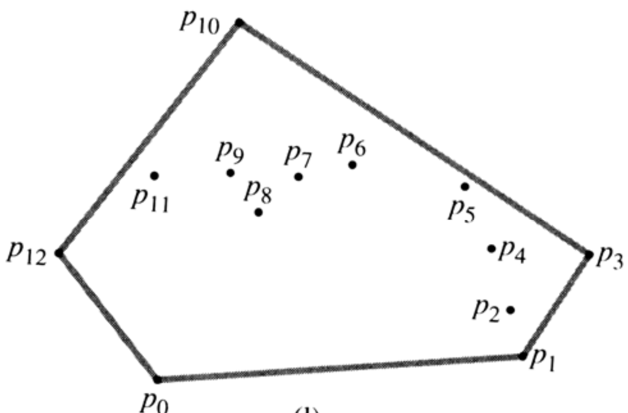
(i)



(j)



(k)



(l)

GRAHAM-SCAN(Q)

1 let p_0 be the point in Q with the minimum y-coordinate,
  or the leftmost such point in case of a tie
2 let $\langle p_1, p_2, \ldots, p_m \rangle$ be the remaining points in Q,
  sorted by polar angle in counterclockwise order around p_0
  (if more than one point has the same angle, remove all but
  the one that is farthest from p_0)
3 let S be an empty stack
4 PUSH(p_0, S)
5 PUSH(p_1, S)
6 PUSH(p_2, S)
7 **for** $i = 3$ **to** m
8   **while** the angle formed by points NEXT-TO-TOP(S),
        TOP(S), and p_i makes a nonleft turn
9   POP(S)
10   PUSH(p_i, S)
11 **return** S

Time complexity:

Line 1: $O(n)$
Line 2: $O(n\lg n)$
Line 3~6: $O(1)$
Line 8~9: $O(n)$
(at most n *PUSH* $\rightarrow$ at most n *POP*)
Line 10: $O(n)$

* **Correctness**: See textbook.

Jarvis's March (using a technique known as ***package wrapping*** or ***gift wrapping***)

Step 1: Find the lowest point x and the highest point y.
Step 2: Compute the right chain
$(p_0=x, p_1, \ldots, p_k=y)$.
Step 3: Compute the left chain
$(p_k=y, p_{k+1}, \ldots, p_h=x)$.

* In $O(1)$ time, we can compare the polar angles of two points. (How ??? See Section 33.1)

* In $O(n)$ time, we can determine the point with smallest (or largest) polar angle with respect to a given point.

* Since each computation of p_i take $O(n)$ time, $T(n)=O(nh)$.

* In the worst case, $h=n$ and thus $T(n)=O(n^2)$.

* Jarvis's march is better than Graham's scan if $h=o(\lg n)$.

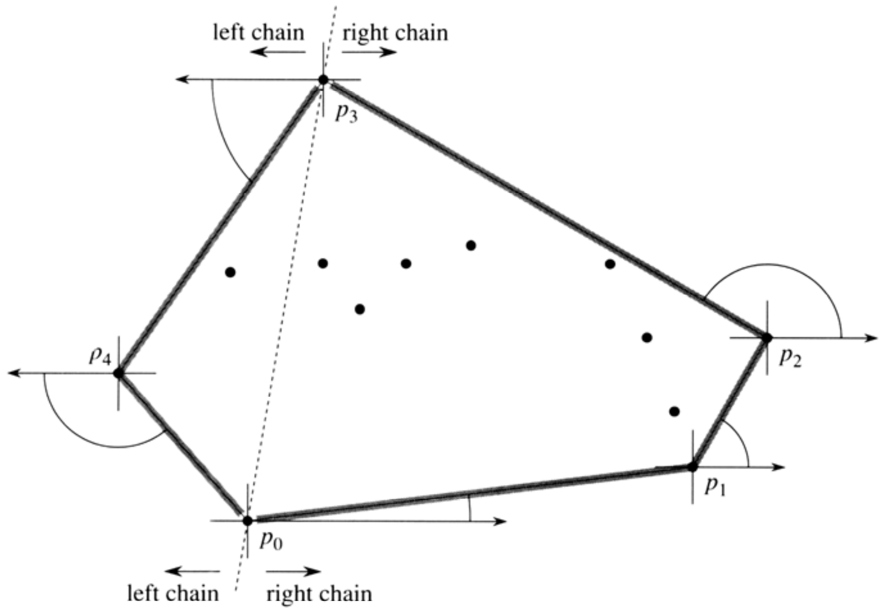


Figure 33.9 The operation of Jarvis's march. The first vertex chosen is the lowest point p_0. The next vertex, p_1, has the smallest polar angle of any point with respect to p_0. Then, p_2 has the smallest polar angle with respect to p_1. The right chain goes as high as the highest point p_3. Then, the left chain is constructed by finding smallest polar angles with respect to the negative x-axis.

Homework: None.