NP-Completeness (Branch-and-bound)

34-1x

Polynomial time: $O(n^k)$ for some constant k, n is the problem size. polynomial → easy, can solve it

Exponential time: $O(2^n)$, $O(3^{n^3})$, ...
non-polynomial → hard, cannot solve it

non-deterministic algorithm: an algorithm which

1. guesses an answer, and then
2. verifies the answer.

34-1a

non-deterministic sort ?

polynomial deterministic

計算

P: the set of problems that can be solved in $O(n^k)$ time (using a deterministic algorithm).

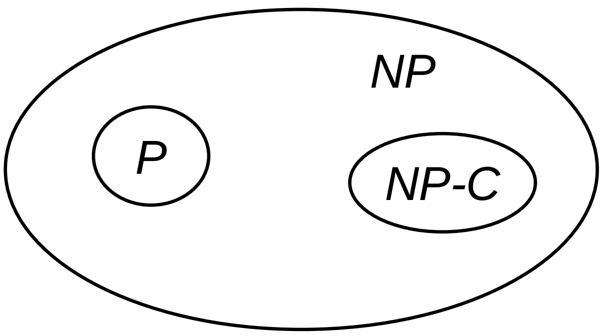
NP: the set of problems that can be solved in $O(n^k)$ time using a non-deterministic algorithm. (Or, problems whose answers can be verified in $O(n^k)$ time.)

驗算

34-1b

polynomial

non-deterministic



34-1y

Decision problem: return Yes/No!

Reduction: transform a problem into another. 34-2a

* Selection ⇨ Sorting
* Decision version ⇨ Optimization version
* if A ⇨ B, then usually B is harder (B $\geq_{hard}$ A) 34-2de
* if A $⇨^p$ B and B $⇨^p$ C, then A $⇨^p$ C 34-2e'

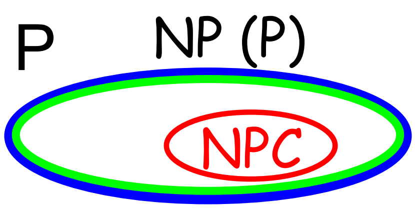
  reduction (hardness) is transitive 34-2fg

NP-Complete: a problem *A* is in *NP-C* iff (i) *A* is in *NP*, and (ii) all problems in *NP* can be reduced to it in $O(n^k)$ time.

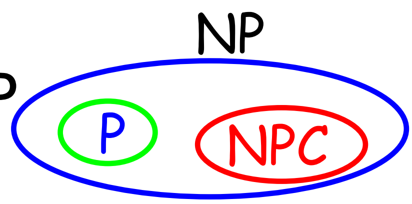
* all NP-C problems are of the same difficulty 34-2h
* all NP $⇨^p$ A ≅ an NP-C $⇨^p$ A ≅ all NP-C $⇨^p$ A

* If any problem in *NP-C* can be solved in $O(n^k)$ time, then *P=NP*. ☆ It is believed (not proved) that *P≠NP*.

* if NP = P



if NP ≠ P

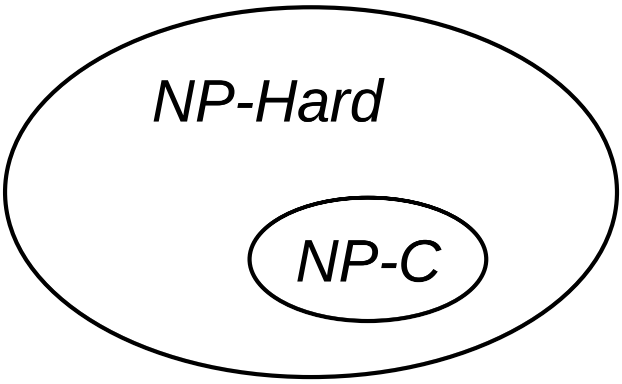


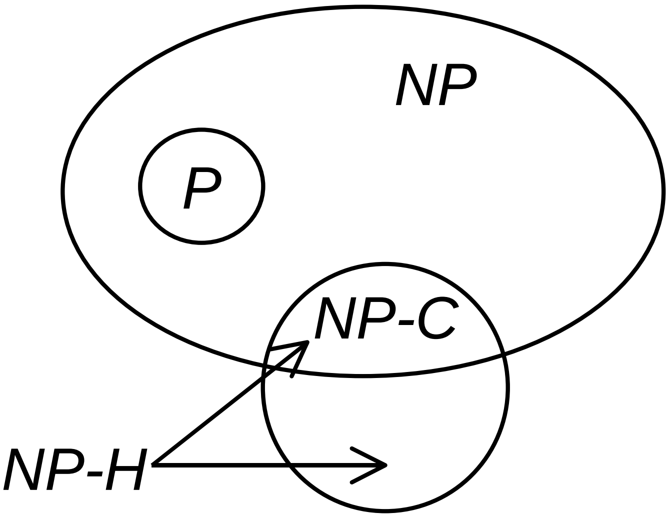
NP-Hard: a problem *A* is in *NP-H* iff

(1) *A* is at least as hard as problems in *NP-C*.

or (2) all problems in *NP* can be reduced to *A* in $O(n^k)$ time. all NP $⇨^p$ *A*

or (3) a problem in *NP-C* can be reduced to *A* in $O(n^k)$ time. an (all) NP-C $⇨^p$ *A*





34-3ab

NP-Complete problems:

$\begin{cases} 0\text{: false} \\ 1\text{: true} \end{cases}$

Cook Thm. (Turing award)

* *(Circuit) satisfiability problem* (*SAT*):

$$((a \rightarrow b) \vee \neg((\neg a \leftrightarrow c) \vee d)) \wedge \neg b$$

2^n assignments (Clearly, SAT $\in$NP)

* *3-CNF satisfiability problem* (*3SAT*):

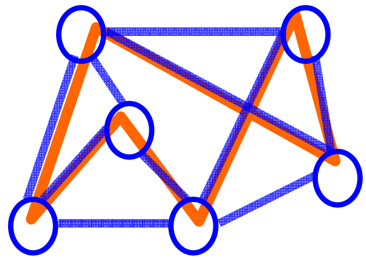
$$(a \vee b \vee c) \wedge (a \vee \neg d \vee e) \wedge (b \vee f \vee a)$$

(conjunctive normal form)

* *The subset-sum (partition) problem*: partition a set of (real) numbers into two subsets of the same sum.

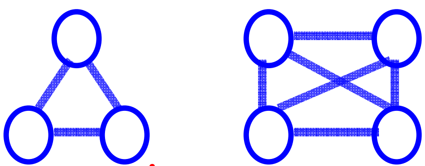
visit each vertex exactly once !

* *The hamiltonian-cycle problem*

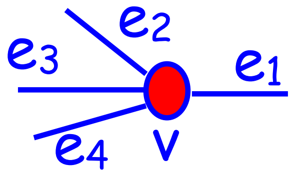


* *The traveling-salesperson problem (TSP)*

* *The clique problem*

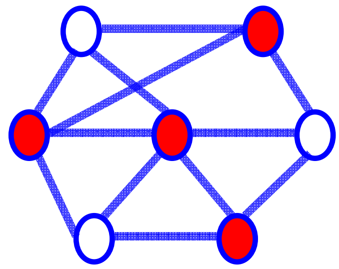


completely connected subgraph

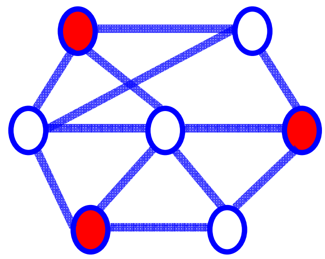


* *The longest "simple" path problem*

* *The vertex-cover problem*



* *The independent set problem*



* *The graph coloring problem*

* 2SAT $\in$ *P*. (2CNF $\in$ *P*)

34-4a 34-4x 34-4b

Question 1: Determine whether the following statements are correct of not.

(1) If a problem is *NP-Complete*, then it can not be solved by any polynomial time algorithm in worst cases.

N

(2) If a problem is *NP-Complete*, then we have not found any polynomial time algorithm to solve it in worse cases.

y

(3) If a problem is *NP-Complete*, then it is unlikely that a polynomial time algorithm can be found in the future to solve it in worst cases.

y

(4) If a problem is *NP-Complete*, then it is unlikely that we can find a polynomial time algorithm to solve it in average cases.

N?

y

(5) If we can prove that the lower bound of an *NP-Complete* problem is exponential, then we have proved that *NP*≠*P*.

Question 2: Determine whether the following statements are correct of not, and justify your answer.

N

(a) Any *NP-hard* problem can be solved in polynomial time if there is an algorithm that can solve the satisfiability problem in polynomial time.

N

(b) Any *NP-Complete* problem can be solved by a polynomial time deterministic algorithm in average if and only if *NP*=*P* is proved.

y

(c) The clause-monotone satisfiability problem is *NP-Complete*, where a formula is called monotone if each clause of the formula contains either only positive variables or only negative variables.

Question 3: Determine whether the following statements are correct of not.

(1) The NP problems consist of only decision problems.
N

(2) If a problem is NP-complete, then it can not be solved by any polynomial time algorithm in worst cases.
N

(3) If we prove that problem *A* can polynomial-time reduce to satisfiability problem, then problem *A* is NP-complete.
N

(4) If a problem *A* is polynomial time reducible to problem *B* and *B* has a polynomial time algorithm, then problem *A* has a polynomial time algorithm.
Y

(5) If an NP-complete problem can be solved in polynomial time, then NP $\neq$ P.
N

(6) The problem of determining whether an integer number is a prime number is an NP-complete problem.
N

*Big CS News in 2001

(7) Any NP-hard problem can be solved in polynomial time if there is an algorithm that can solve the satisfiability problem in polynomial time.
N

(8) The hamiltonian-path problem can be solved in polynomial time on directed acyclic graphs
Y

(9) 3-CNF (the satisfiability problem, in which
N? each clause has exactly three literals) is reducible to 2-CNF problem.

By yourself, using 34-def.

Question 4: Suppose problem P_1 can be reduced to another problem P_2 in $O(n^2)$ time, where n is the input size. Answer the following questions and justify your answer briefly.

Y (a) If P_1 is *NP-hard*, is P_2 *NP-hard*?

N (b) If P_2 is *NP-hard*, is P_1 *NP-hard*?

N (c) Suppose P_1 can be solved in $O(f(n))$ time. Is it possible to derive a time lower-bound or a time upper-bound for P_2? If it is possible, what is the time bound?

Approximation Algorithm: Let A^* be the optimal solution of an input. An approximation algorithm will produce a solution A such that

$|A^*-A|/A^* \leq \varepsilon$, e.g. ε(n) = 0.5 -> error within 50%

where ε is called the *relative error bound*.

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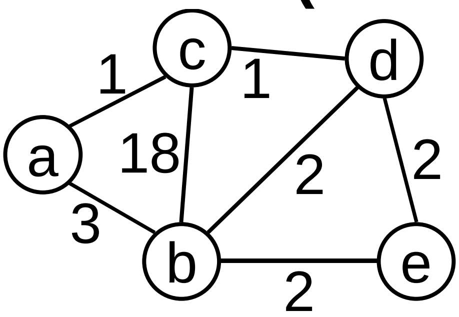
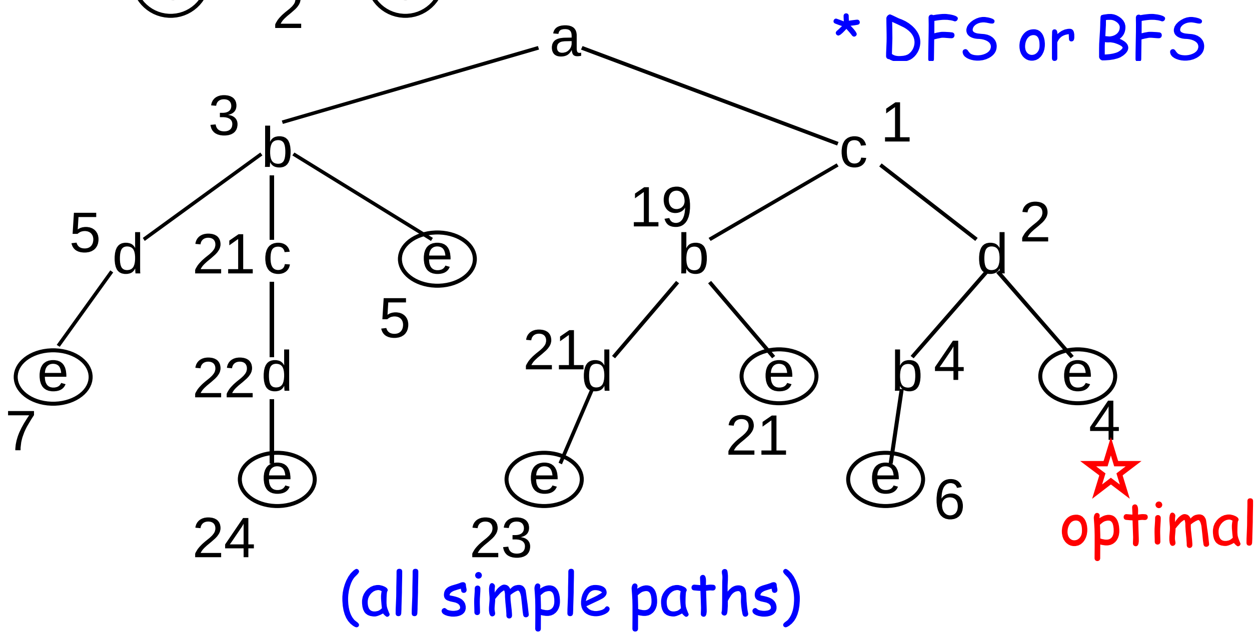
Heuristic Algorithm: may produce a good solution but no guarantee on the error bound.

Brute-force (search): Try all possible answers.



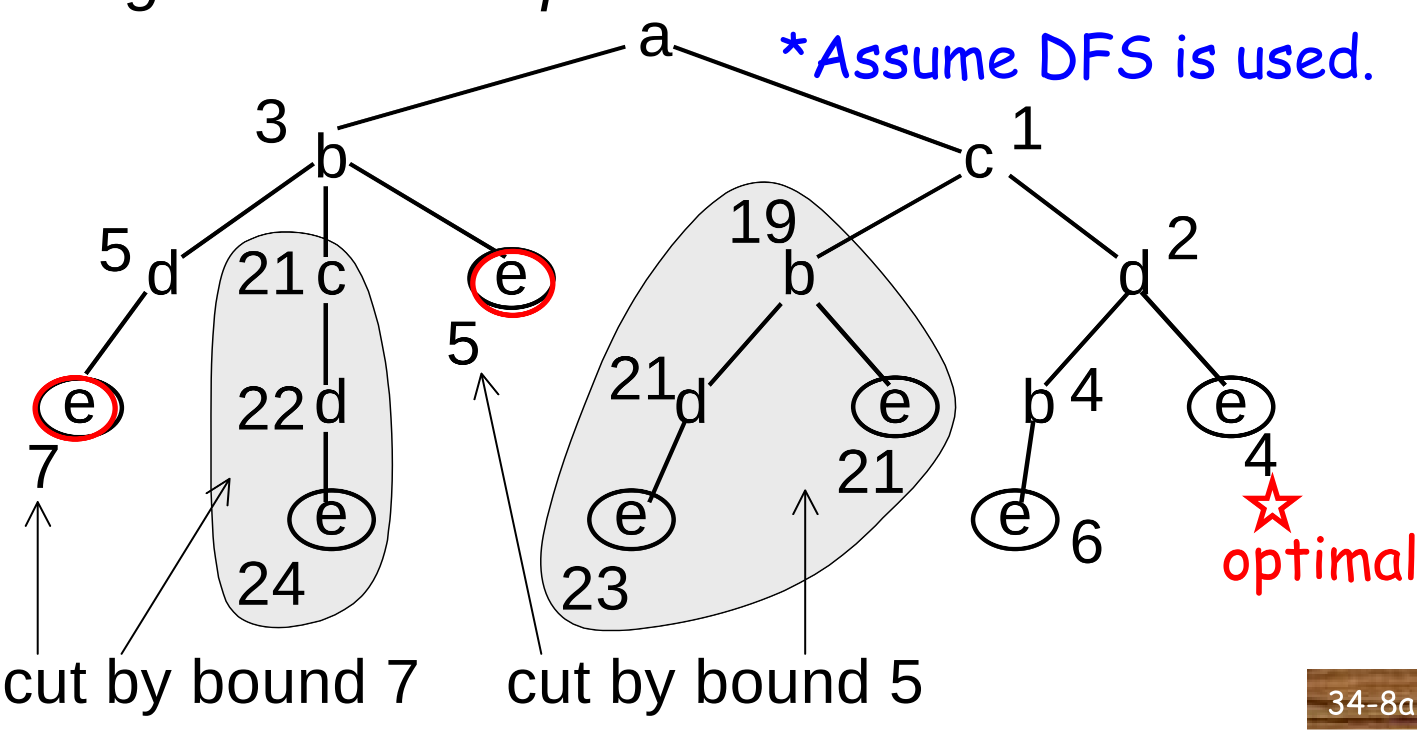
: Find a shortest path from a to e.

34-8x



Branch-and-Bound search: *Brute-force + Intelligent cuts to impossible answers.*

34-8z



34-8a

Homework: None.