Quicksort: Average Case

$$E(n) = n\text{-}1 + \frac{1}{n}\sum_{q=1}^{n}\{\, E(q-1) + E(n-q)\,\} \qquad = n\text{-}1 + \frac{2}{n}\sum_{k=1}^{n-1} E(k)$$

兩邊都要做

substitution method or Knuth's approach $\Rightarrow$ $O(n \lg n)$

Selection: Average Case

永遠都做大的那一邊 (多算沒關係)

$$E(n) = n\text{-}1 + \frac{1}{n}\sum_{q=1}^{n}\{\, E(\max\{q-1, n-q\})\,\}$$

substitution method $\Rightarrow$ $O(n)$

只做一邊，機率按比例

$$E(n) = n\text{-}1 + \frac{1}{n}\sum_{q=1}^{n}\left\{\frac{q-1}{n-1}E(q-1) + \frac{n-q}{n-1}E(n-q)\right\}$$

Knuth's approach $\Rightarrow$ $O(n)$

Selection: Average Case

只做一邊，機率按比例

$$E(n) = n\text{-}1 + \frac{1}{n}\sum_{q=1}^{n}\left\{\frac{q-1}{n-1}E(q-1) + \frac{n-q}{n-1}E(n-q)\right\}$$

$$E(n) = n\text{-}1 + \frac{1}{n(n-1)}\{\, 0E(0) + 1E(1) + 2E(2) + \ldots + (n\text{-}1)E(n\text{-}1) +$$
$$(n\text{-}1)E(n\text{-}1) + (n\text{-}2)E(n\text{-}2) + \ldots + 0E(0)\,\}$$

$$E(n) = n\text{-}1 + \frac{2}{n(n-1)}\sum_{k=1}^{n-1} kE(k)$$

Knuth's approach

$$E(n) = n+1 + \frac{2}{n(n-1)}\sum_{k=1}^{n-1} kE(k) \qquad (1)$$

不換也可以，但推導比較比較不簡潔漂亮

9-3c

$$E(n) = n+1 + \frac{2}{n(n-1)} \sum_{k=1}^{n-1} kE(k) \quad \text{------} \quad (1)$$

$$n(n-1)E(n) = (n+1)n(n-1) + 2 \sum_{k=1}^{n-1} kE(k) \quad \text{------} \quad (2)\text{: } (1) \times n(n-1)$$

$$(n-1)(n-2)E(n-1) = n(n-1)(n-2) + 2 \sum_{k=1}^{n-2} kE(k) \quad \text{------} \quad (3)\text{: } (2) \text{ with } n = n-1$$

$$n(n-1)E(n) = n(n-1)(3) + 2(n-1)E(n-1) + (n-1)(n-2)E(n-1)$$

$$n(n-1)E(n) = 3n(n-1) + n(n-1)E(n-1)$$

$$E(n) = 3 + E(n-1) = 3n = O(n)$$ (by iteration method)

9-3d

1. $\underline{4\ 1\ 6\ 3\ 1}\ \underline{2\ 5\ 7\ 8\ 7}\ \underline{6\ 5\ 9\ 1\ 7}\ \underline{1\ 3\ 5\ 7\ 6}\ \underline{9\ 2\ 4\ 3\ 5}$ $(r = 5)$

2. $$\begin{array}{cccccccccc} & 1 & & 2 & & 1 & & 1 & & 2 \\ & 1 & & 5 & & 5 & & 3 & & 3 \\ m_1 & 3 & m_2 & 7 & m_3 & 6 & m_4 & 5 & m_5 & 4 \\ & 4 & & 7 & & 7 & & 6 & & 5 \\ & 6 & & 8 & & 9 & & 7 & & 9 \end{array}$$

$M = \{3, 7, 6, 5, 4\}$

3. $m = \text{Select}(M, \lceil |M|/2 \rceil) = 5$ (median of medians)

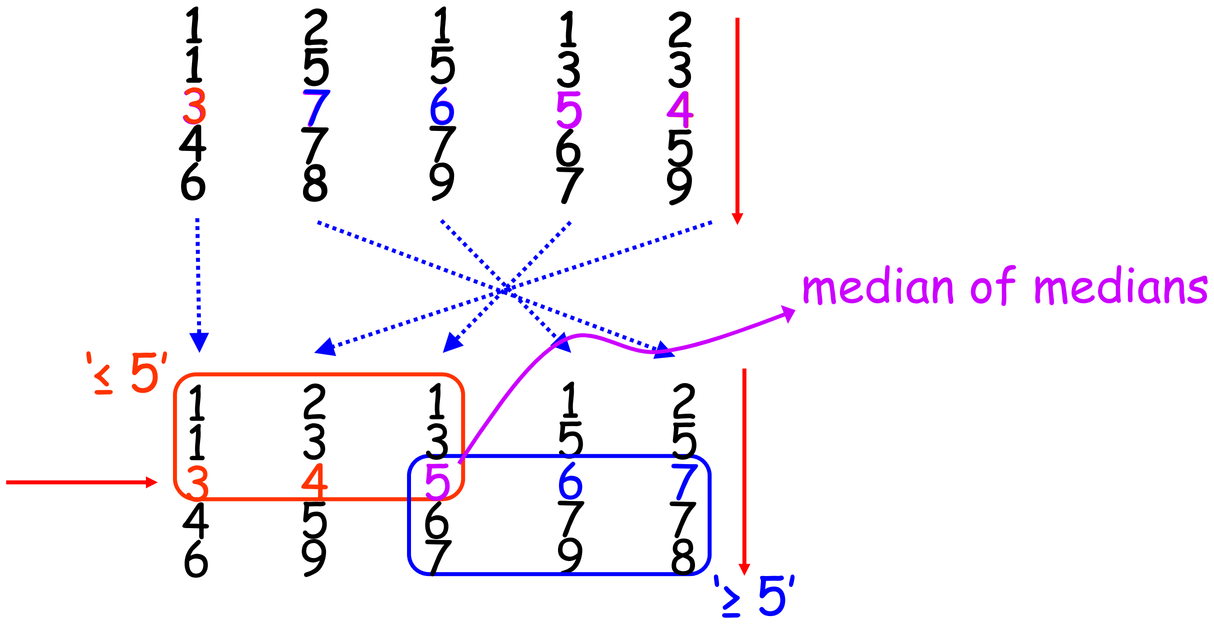
4. $S_1 = \{4\ \ 1\ \ 3\ \ 1\ \ 2\ \ 1\ \ 1\ \ 3\ \ 2\ \ 4\ \ 3\}$
$S_2 = \{5\ \ 5\ \ 5\ \ 5\}$
$S_3 = \{6\ \ 7\ \ 8\ \ 7\ \ 6\ \ 9\ \ 7\ \ 7\ \ 6\ \ 9\}$

$\boxed{|S_1| = 11,\ |S_2| = 4,\ |S_3| = 10}$

5. $\boxed{\text{case 1. } i = 7}$ Select(S_1, 7)

$\boxed{\text{case 2. } i = 13}$ return $m = 5$

$\boxed{\text{case 3. } i = 22}$ Select(S_3, 22 – 15)



⇨ '≤ 5' 超過 $\frac{1}{4}$ ⇨ '> 5' 最多 $\frac{3}{4}$ ⇨ $|S_3| \le \frac{3}{4}n$

⇨ '≥ 5' 超過 $\frac{1}{4}$ ⇨ '< 5' 最多 $\frac{3}{4}$ ⇨ $|S_1| \le \frac{3}{4}n$

Divide & Conquer

D & C

1. partition the input into same subproblems
2. recursively solve the subproblems
3. combine subsolutions

(Ex. merge-Sort)

$$T(n) = \sum T(n_i) + C(n)$$

Partition

1. break the problem into independent subproblems
2. solve the subproblems

(Ex. quick-sort)

$$T(n) = P(n) + \sum T_i(n_i)$$

Prune & Search

repeatedly remove invalid candidates

(Ex. selection)
(Ex. binary search)

$$T(n) = r(n) + T(n')$$