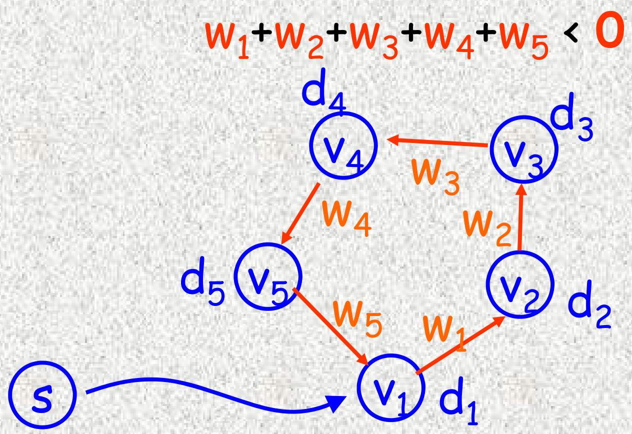
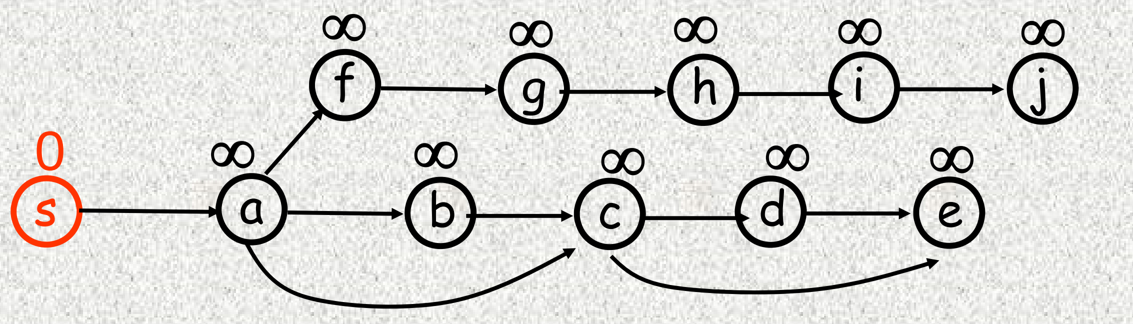
Bellman-Ford with negative cycles

Assume that G contains a negative cycle C reachable from s

After n-1 iterations, every $d_i \neq \infty$





$d(a) \neq \infty$ after ? iteration

$d(b) \neq \infty$ after ? iteration

$d(d) \neq \infty$ after ? iteration

$d(j) \neq \infty$ after ? iteration

⇨ after n-1 iterations,
any vertex v reachable from s has $d(v) \neq \infty$

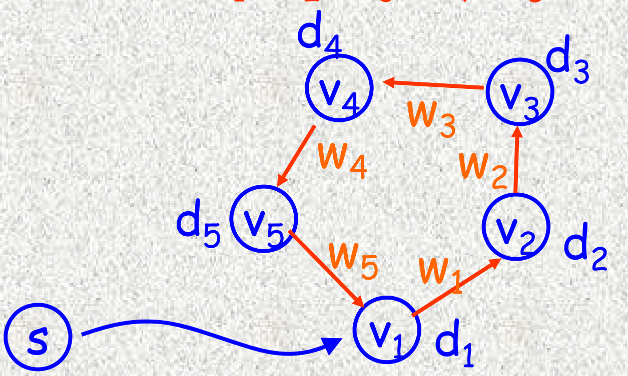
Bellman-Ford with negative cycles

Assume that G contains a negative cycle C reachable from s

After n-1 iterations, every $d_i \neq \infty$

At iteration n, **at least one v_i** accepts "Relax"

$w_1+w_2+w_3+w_4+w_5 < 0$



By contradiction:

$$
\begin{array}{rl}
 & \cancel{d_1} + w_1 \geq \cancel{d_2} \\
 & \cancel{d_2} + w_2 \geq \cancel{d_3} \\
 & \cancel{d_3} + w_3 \geq \cancel{d_4} \\
 & \cancel{d_4} + w_4 \geq \cancel{d_5} \\
+ & \cancel{d_5} + w_5 \geq \cancel{d_1} \\
\hline
 & w_1+w_2+w_3+w_4+w_5 \geq 0
\end{array}
$$

$\Rightarrow$ C is not negative !!!