

## Perform Union on the list containing $x$

Case 1: update  $x \Rightarrow L_{\text{NEW}} \geq 2 L_{\text{OLD}}$

Case 2: not update  $x \Rightarrow L_{\text{NEW}} \geq L_{\text{OLD}} + 1$

$\Rightarrow$  IF  $x$  has been update  $k$  times,  $|L_x| \geq 2^k$

$\Rightarrow t(x) = k \leq \lg |L_x| \leq \lg n$

$\Rightarrow$  Time all Union =  $\sum_x t(x) \leq \sum_x \lg n \leq n \lg n$

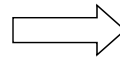
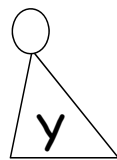
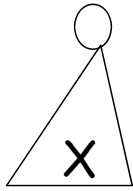
# Union by Rank and Union by Size

21-8a

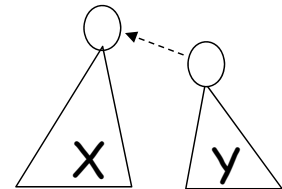
UR:

rank = 5

rank = 3

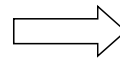
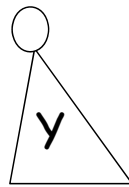
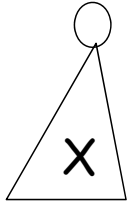


rank = 5

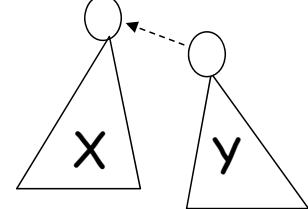


rank = 5

rank = 5



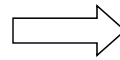
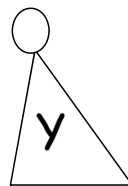
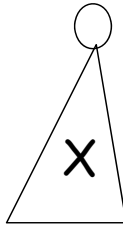
rank = 5 + 1 = 6



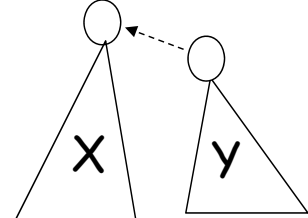
US:

size = 9

size = 7



size = 9 + 7 = 16



21-8b

may be approximate

$3 \times 2 = 6$  possibilities

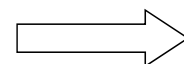
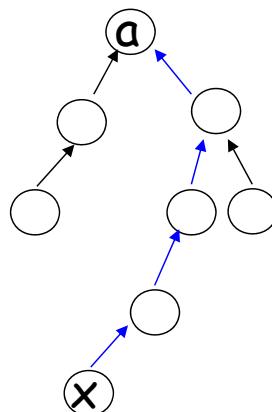
{ UR (union by rank)  
US (union by size)

× PC (path compression)

exact

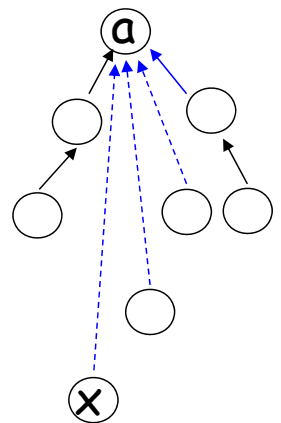
\* US is as good as UR

rank = 4  
size = 8



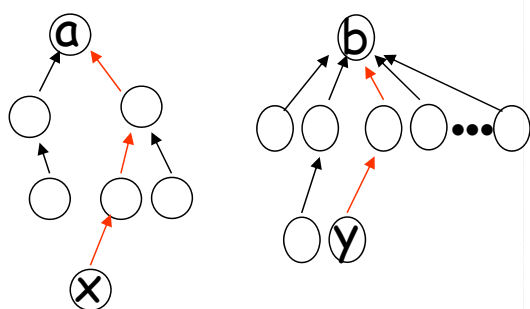
Find(x)  
(with PC)

rank = 4  
size = 8

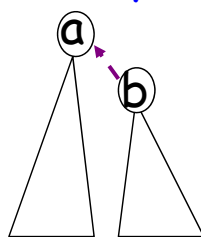


# Union(x, y)

21-8c

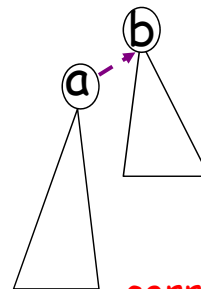


UR only



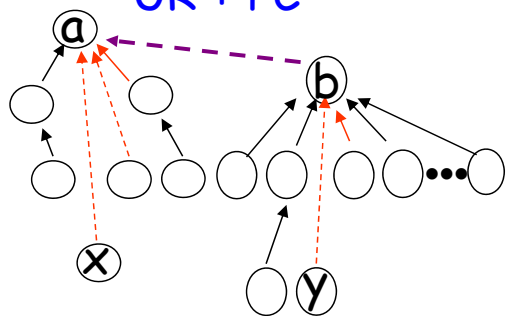
correct?

US only



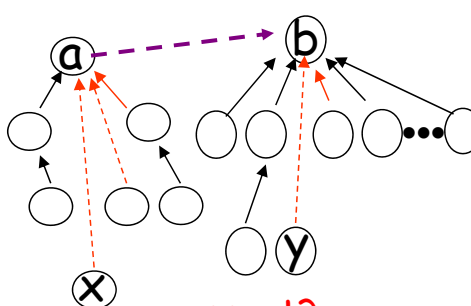
correct?

UR + PC



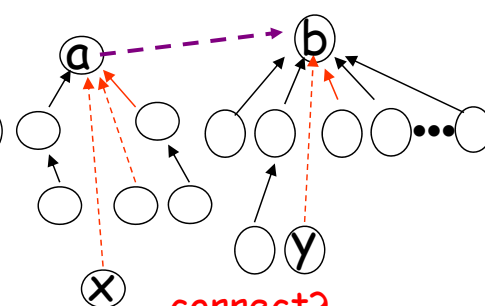
correct?

US + PC



correct?

PC only



correct?

## Data Structures for Disjoint Sets

21-10a

Traditional: try to  
reduce single OP  
worst-case

Amortized: try to reduce overall time

Worst-case

Procedure	✓ 2-3 trees	✗ Linked Lists	✓ Forests
MAKE-SET	$O(1)$	$O(1)$	$O(1)$
UNION	$O(\lg n)$	$O(n)$	$O(\lg n)$
FIND-SET	$O(\lg n)$	$O(1)$	$O(\lg n)$

Amortized

Procedure	✗ 2-3 trees	✓ Linked Lists	★ Forests
MAKE-SET	$O(1)$	$O(1)$	$O(1)$
UNION	$O(\lg n)$	$O(\lg n)$	$O(\alpha(n))$
FIND-SET	$O(\lg n)$	$O(1)$	$O(\alpha(n))$

much simpler

better & simpler