Homework

Name ______________________________

1. Let $\{x_n\}$ be a Cauchy sequence. Suppose that for every $\varepsilon > 0$ there is some $n > 1/\varepsilon$ such that $|x_n| < \varepsilon$. Prove that $x_n \to 0$.

2. Let $x_{n+1} = f(x_n) = rx_n(1-x_n)$ and $0 < x_0 < 1$. Show that if $1 < r < 3$ then $x_n \to x^* = \frac{r-1}{r}$ and if $0 < r < 1$ then $x_n \to 0$.

3. Let $a, b > 0$ and define $a_1 = a$, $b_1 = b$, $a_{n+1} = \frac{a_n+b_n}{2}$ and $b_n = \sqrt{a_n b_n}$. Show that
(i) $a_n \geq a_{n+1} \geq b_{n+1} \geq b_n$.
(ii) The sequences $\{a_n\}$ and $\{b_n\}$ converge to the same limit.(It is called Gauss arithmetic-geometric mean of a and b.)

4. Let $\{b_n\}$ be a bounded sequence of nonnegative numbers and $0 < r < 1$ and define $s_n = b_1 r + b_2 r^2 + \cdots + b_n r^n$. Use Monotone Convergence Thm to prove that $\{s_n\}$ converges.

5. Suppose the sequence $\{a_n\}$ is monotone. Prove that $\{a_n\}$ converges if and only if $\left\{(a_n)^2\right\}$ converges. Show that the result does not hold without the monotonicity assumption.

6. Every sequence has a monotone subsequence.

7. Show that a monotone sequence is bounded if it has a bounded subsequence.

8. Suppose that the sequnce $\{a_n\}$ is monotone and that it has a convergent subsequence. Show that $\{a_n\}$ converges.

9. Is $\{x_n\}$, where $x_n = 1 + \frac{1}{2!} + \cdots + \frac{1}{n!}$, a Cauchy sequence?

10. If $\lim\limits_{n\to\infty} x_n = x$, then $\lim\limits_{n\to\infty} |x_n| = |x|$.