

CS 3332 Probability and Statistics
Midterm Exam I
Time: 2:10pm-3:10pm, Oct. 24, 2006
(Totally 5 problems, 100 pts in 2 pages)

1. (20 pts) Give a Yes/No answer to each of the following statement with explanations. (Giving reasons to support your answers.)
 - A. If A and B are two events and $p(A|B) = p(B|A) = 1$, then $A = B$.
 - B. If the events A and B are independent, then A' and B' are independent, too.
 - C. If events A and B are mutually exclusive and event C is nonempty, then we have

$$P(A \cup B|C) = P(A|C) + P(B|C)$$
 - D. For a random variable X, the correlation coefficient between X and $cX+2$ is 1, where c is any nonzero constant.
 - E. For a random variable X with a *symmetric* probability distribution function around its mean, we have $P(X > \mu_X + 3\sigma_X) > \frac{1}{18}$.
2. (20 pts) The lung cancer is known to be found in men over 65 with probability 10%. A blood test has been used to detect this disease with a 8% false negative (i.e., the test incorrectly gives a negative result) rate and a 4% false positive (i.e. the test incorrectly gives a positive result) rate. Note that the positive result means the disease is found in the test, while the negative result means the disease is not found in the test.
 - (a) (10 pts) What is the probability that a man over 65 receives a negative test result?
 - (b) (10 pts) If a 70-year-old man took the test and received a negative result, what is the probability that he has the lung cancer disease?
3. (20 pts) The joint probability mass function of the discrete random variables X and Y is given as follows:

$$f(x, y) = \begin{cases} \frac{x+y}{36} & x = 1, 2, 3 \text{ and } y = 1, 2, 3 \\ 0 & \text{elsewhere} \end{cases}$$
 - (a) (6 pts) Determine the marginal probability distribution function of Y.
 - (b) (6 pts) Compute the mean and variance of Y.
 - (c) (8 pts) Compute the expectation $E(X | Y=3)$

4. (20 pts) Consider two random variables X and Y satisfying the following equations:

$$E[(X - 2)^2] = 4, \quad E[(X - 4)^2] = 8$$

$$E[(Y - 3)^2] = 10, \quad E[(Y - 6)^2] = 13$$

Their correlation coefficient $\rho_{XY} = 0.5$

- (a) (5 pts) Determine the mean and variance of X.
 - (b) (5 pts) Determine the mean and variance of Y.
 - (c) (5 pts) Determine the covariance between X and Y
 - (d) (5 pts) Compute the covariance between $X+2Y$ and $X-3Y$
5. (20 pts) The joint probability density function for two continuous random variables X and Y is given as follows:

$$f(x, y) = \begin{cases} cxy & 0 < x < 3 \text{ and } 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) (5 pts) Determine the constant c to make it a joint probability density function.
- (b) (5 pts) Determine the marginal probability distribution function of X.
- (c) (5 pts) Determine the marginal probability distribution function of Y.
- (d) (5 pts) Determine the covariance between X and Y.