Probability (CS 3332)

Mid-term Exam 1 (March 23, 2016)

Preliminary problems

1. (15%) Let A and B be two events. Prove that

$$P(AB) \geq P(A) + P(B) - 1.$$

2. (15%) A fair die is tossed six times. What is the probability of getting exactly two 6's?

3. (15%) An urn contains five white and three red chips. Each time we draw a chip, we look at its color. If it is red, we replace it along with two new red chips, and if it is white, we replace it along with three white chips. What is the probability that, in successive drawing of chips, the colors of the first four chips alternate?

4. (15%) Suppose that three numbers are selected one by one, at random and without replacement from the set of numbers $\{1, 2, 3, \ldots, n\}$. What is the probability that the third number falls between the first two if the first number is smaller than the second?

Challenging problems

5. (10%) Mary has two books. She is taking books along on her holiday vacation. With probability 0.5, she will take only the first book; with probability 0.4, she will take only the second book; and with probability 0.3, she will take both books. What is the probability that she will take neither of the two books?

6. (20%) Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. Let E_i be the event that the ith hand has exactly one ace, where $i = 1, 2, 3, 4$. Find $P(E_1E_2E_3E_4)$.

7. (10%) A total of n balls are sequentially and randomly chosen, without replacement, from an urn containing r red and b blue balls ($n \leq r + b$). Given that k out of n balls are blue, what is the conditional probability that the first ball chosen is blue?