Test 3 for CS2334

December 6, 2004

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(10 pts) 1. *Mark* ○ *if the statement is* true, *and mark* × *otherwise.*

10

O (a) A set of nonzero orthogonal vectors are linearly independent.

X (b) A set of nonzero orthonormal vectors in R^n must be a basis.

O (c) Every square matrix can be factored as QR, where Q is orthogonal and R is upper-Δ.

X (d) If $\mathbf{x}, \mathbf{y} \in R^n$ and $\langle \mathbf{x}, \mathbf{y} \rangle = 1$, then $\mathbf{x}$ and $\mathbf{y}$ are linearly independent.

X (e) If U, V, W are vector subspaces of R^n such that $U \perp V$ and $V \perp W$, $U \perp W$.

O (f) If $A \in R^{m \times n}$, then AA^t and A^tA have the same rank.

O (g) Let $Q_1, Q_2, \cdots, Q_m \in R^{n \times n}$ be orthogonal, then $\prod_{i=1}^{m} Q_i$ is also orthogonal.

X (h) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be linearly independent vectors in R^3, then any Gram-Schmidt orthogonalization process constructs the unique orthonormal basis.

X (i) A Householder matrix is symmetric, orthogonal, and has determinant 1.

X (j) Let $\mathbf{x}, \mathbf{y} \in R^n$ such that $\langle \mathbf{x}, \mathbf{y} \rangle = 0$. Then $\mathbf{x}$ and $\mathbf{y}$ are orthonormal.

O (k) In R^n, if $\mathbf{p}$ is the projection of $\mathbf{b}$ along the line $\mathbf{a}$, then $\mathbf{a}^t(\mathbf{b} - \mathbf{p}) = 0$.

(10 pts) **2.** *Choose the best solution in the following questions.*

7 3 (a) Let $\mathbf{u}, \mathbf{v}, \mathbf{w} \in R^n$ be orthonormal vectors, then $\|2\mathbf{u} - 4\mathbf{v} + 4\mathbf{w}\|_2 =$

(1) 4, (2) 5, (3) 6, (4) 7, (5) none.

4 (b) Let $\mathbf{x} = [1, 2, 1, 2]^t$, $\mathbf{y} = [1, -1, -1, 1]^t$, then the angle between $\mathbf{x}$ and $\mathbf{y}$ is

(1) $\frac{\pi}{6}$, (2) $\frac{\pi}{4}$, (3) $\frac{\pi}{3}$, (4) $\frac{\pi}{2}$, (5) none.

1 (c) Let $V = \{[b, 0, a]^t \mid a, b \in R\} \subset R^3$, then $dim(V^{\perp}) =$?

(1) 1, (2) 2, (3) 3, (4) 4, (5) none.

1 (d) Let $\mathbf{u} = [1, 2, 3, 4]^t$, then the rank of $\mathbf{u}\mathbf{u}^t$ is

(1) 1, (2) 2, (3) 3, (4) 4, (5) none.

4 (e) Let $A \in R^{m\times n}$ have rank r, then $dim(Null(A)) + dim(R(A)) =$?

(1) $m - r$, (2) $n - r$, (3) m, (4) n, (5) none.

4 (f) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, the least squares solution of $A\mathbf{x} = \mathbf{b}$ is

(1) $[-1, -1]^t$, (2) $[0, 1]^t$, (3) $[1, 0]^t$, (4) $[1, 1]^t$, (5) none.

3 (g) Let $H \in R^{n\times n}$ be a Householder matrix and define (i) orthogonal, (ii) symmetric, (iii) $\|H\mathbf{x}\|_2 = 1$ for $\mathbf{x} \in R^n$. What statements of (i), (ii), (iii) are true?

(1) (i),(ii) only, (2) (i),(iii) only, (3) (i),(ii),(iii), (4) (ii),(iii) only, (5) (iii) only.

5 (h) Let $Q \in R^{n\times n}$ be orthogonal, then $det(Q) =$?

(1) 1, (2) -1, (3) n, (4) $\sqrt{n}$, (5) none.

3 (i) Let $\mathbf{a} = [1, 1, 1]^t$, $\mathbf{b} = [2, 4, 6]^t$, then the projection of $\mathbf{b}$ onto the line $\mathbf{a}$ is

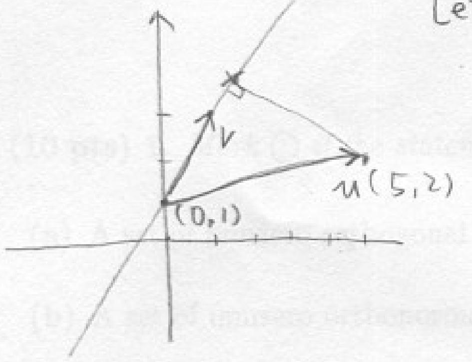
(1) $\mathbf{a}$, (2) $2\mathbf{a}$, (3) $4\mathbf{a}$, (4) $6\mathbf{a}$, (5) none.

2 (j) Let $f, g \in C[-1, 1]$, and define the inner product $\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx$, then $\langle sin2\pi x, sin2\pi x\rangle$

(1) 0, (2) 1, (3) 2π, (4) 4π, (5) none.

3

(3 pts) 3. Find the point on the line $y = 2x + 1$ that is closest to $[5, 2]^t$.



Let $u = [5-0,\ 2-1]^t = [5, 1]^t$

$v = [1, 2]^t$ 代表 $y = 2x+1$ 的走向

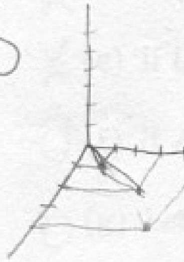
則 $w = \frac{v^T u}{v^T v} v = \frac{7}{5} v = [\frac{7}{5}, \frac{14}{5}]^t$

∴ the point $\Rightarrow [0,1]^t + [\frac{7}{5}, \frac{14}{5}]^t = [\frac{7}{5}, \frac{19}{5}]^t$

#

3

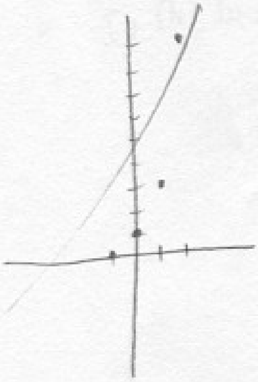
(3 pts) 4. Let $\mathbf{a}_1 = [1, 1, 0]^t$, $\mathbf{a}_2 = [2, 3, 0]^t$, and $\mathbf{b} = [4, 5, 6]^t$. Find the projection vector of $\mathbf{b}$ onto the plane that is spanned by the vectors $\mathbf{a}_1 = [1, 1, 0]^t$ and $\mathbf{a}_2 = [2, 3, 0]^t$.



$\Rightarrow (4, 5, 0)^t$ #

1

(4 pts) 5. (a) Find the best least squares fitting line to the data $[-1, 0]^t$, $[0, 1]^t$, $[1, 3]^t$, $[2, 9]^t$, (b) plot your linear function from (a) along with the data on a coordinate system.



令 line $\Rightarrow y = ax + b$

$ax - y + b = 0$

$$\left(\frac{-a+b}{\sqrt{a^2+1}}\right)^2 + \left(\frac{b-1}{\sqrt{a^2+1}}\right)^2 + \left(\frac{a+b-3}{\sqrt{a^2+1}}\right)^2 + \left(\frac{2a+b-9}{\sqrt{a^2+1}}\right)^2$$

$$= \frac{a^2-2ab+b^2}{a^2+1} + \frac{b^2-2b+1}{a^2+1} + \frac{a^2+2ab+b^2-6a-6b+9}{a^2+1} + \frac{4a^2+4ab+b^2-36a-18b+81}{a^2+1}$$

X