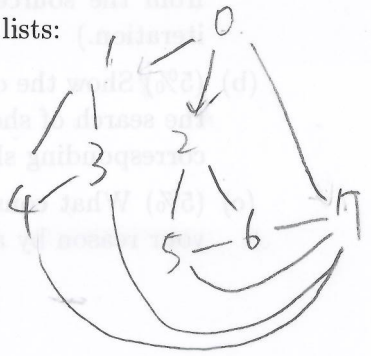
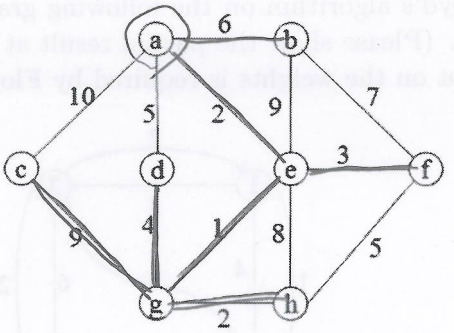
Data Structures
Spring 2009, Final Exam, June 18
7:00-9:00PM

1. (10%) A graph G is represented by the following adjacency lists:
   vertex 0 → 1 → 2 → 7
   vertex 1 → 0 → 3 → 4
   vertex 2 → 0 → 5 → 6
   vertex 3 → 1 → 4 → 7
   vertex 4 → 1 → 3 → 7
   vertex 5 → 2 → 6 → 7
   vertex 6 → 2 → 5 → 7
   vertex 7 → 0 → 3 → 4 → 5 → 6



   (5%) What is the sequence of BFS starting from vertex 0?
   (5%) What is the sequence of DFS starting from vertex 7?

2. (15%) Consider the following undirected graph.
   Find the minimum-cost spanning tree by using the following algorithms. (Please give partial results at the end of each iteration.)
   (a)(5%)Kruskal's algorithm.
   (b)(5%)Prim's algorithm.
   (c)(5%)Sollin's algorithm.



3. (15%) Please answer the following questions about sorting.
   (a) (2%) What is stable sort ? Give one sorting algorithm that is not stable.
   (b) (2%) Which sorting algorithm is most suitable for a sequence that is partially sorted ?
   (c) (2%) Which sorting algorithm is suitable for external sort ?
   (d) (2%) What is the permutation of a sequence {1, 2, 3, 4, 5, 6} that exhibits the worst case behavior of quick sort?
   (e) (2%) What is the lower bound on sorting problem ?
   (f) (5%) Here are 12 integers: 42, 85, 2, 45, 34, 26, 253, 128, 87, 54, 8, 102. Sort them using *quick sort*. Give the status of the list at the end of each phase.

4. (5%) We are to build the optimum Huffman code for the following set of letters. Their appearance frequencies are given as follows. Let left and right branches are encoded as 0, and 1, respectively. What is the code for C ?

letter	A	B	C	D	E	F	G	H
frequency	2	2	3	3	4	6	10	20

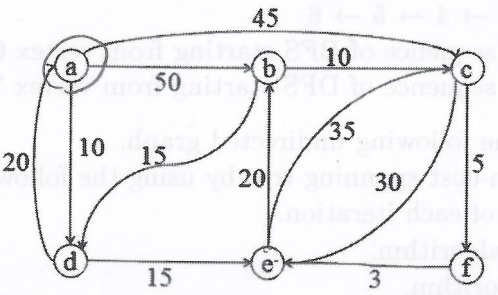
4 6 8 12 18

5. (15%) Please use Dijkstra's algorithm to answer the following questions.

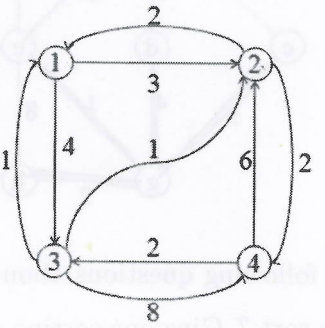
   (a) (5%) Please use Dijkstra's algorithm on the following graph to find the shortest paths from the source node a to all other nodes. (Please show the partial result at each iteration.)

   (b) (5%) Show the contents of array path[] which is used to record the previous node during the search of shortest paths. That is, for any given node, we can trace back to find the corresponding shortest path easily by path[].

   (c) (5%) What constraint on the weights is required by Dijkstra's algorithm? Please give your reason by an example ?



6. (7%) Please use Floyd's algorithm on the following graph to find the distance matrix of all-pairs shortest paths. (Please show the partial result at each iteration.)
   (3%)What constraint on the weights is required by Floyd's algorithm?



7. (10%) A gate-level circuit can be modelled as a graph $G = (V, E)$, where a gate g_i corresponds to a node n_i in V, an output of g_i to input of g_j corresponds to an edge from n_i to n_j in E, and the delay d_i of g_i corresponds to the weight w_i of n_i. Write an algorithm to find the longest delay of the circuit.

8. (10%) Write an $O(|V| + |E|)$ C-like algorithm $HasCycle(V, E)$ which returns the value *true* if the directed graph with vertex set V and edge set E has a cycle and $false$ otherwise.

9. (10%) Let $S_1, S_2, ..., S_m$ be sets of integers in the range 1 to n and $\sum_{i=1}^{m} |S_i| = n$ where $|S_i|$ is the size of set S_i. Given $S = S_1 \bigcup S_2 \bigcup ... \bigcup S_m$, describe an $O(n)$ time algorithm to sort all S_i's (i.e., the elements in each S_i are sorted.) and show that your algorithm indeed achieves this time complexity.