# Concrete Mathematics

## Midterm Exam

(Nov. 8, 2004)

1. (20 points) Please prove $\varepsilon_p(n!) = \sum_{0 \le i \le k} n_i \cdot \frac{p^i - 1}{p-1}$, where $n = (n_k, n_{k-1}, \cdots, n_0)_p$.

2. (20 points) Please solve the recurrence

$$\begin{cases} f(0) = 0 \\ f(n) = f(\left\lceil \frac{n-1}{2} \right\rceil) + f(\left\lfloor \frac{n-1}{2} \right\rfloor) + 2n \quad for\ \text{n>0} \end{cases}$$

3. (20 points) Please evaluate $\sum_{1 \le k \le n} k^2 H_k$.

4. (20 points) Please find the recurrence relation for $J_4(n)$.

5. (20 points) Please find the closed form for $\sum_{0 < k \le 2m} \left\lfloor \frac{nk + x}{2m} \right\rfloor$.