

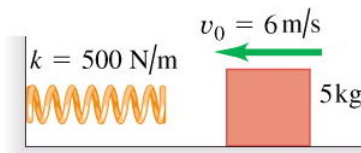
0. 【5%】依下面說明在答案卷上作答者，可得 5 分

- (i) 答案卷第一張為封面，第一張正反兩面不要作答。  
 (ii) 由第二張紙開始算起，第一頁依空格號碼順序寫下所有填充題答案，寫在其他頁不記分。  
 (iii) 計算題之演算過程與答案依題號順序寫在第二頁以後，每題從新的一頁寫起。

注意：單位沒寫一格扣一分，constant (MKS):  $g=9.8\text{m/s}^2$ ,  $m_p=\text{proton mass}=1.67\times 10^{-27}\text{ kg}$ ,  $1\text{eV}=1.6\times 10^{-19}\text{ J}$ ,  $c=3\times 10^8\text{ m/s}$

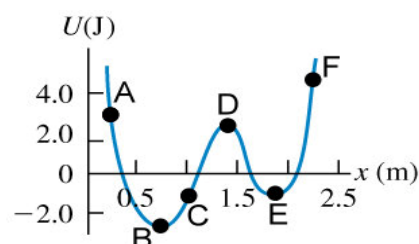
### Part I. 填充題 (每格 3 分, 共 75 分)

■A 5-kg block is moving at 6 m/s along a frictionless, horizontal surface toward a massless spring with force constant  $k = 500\text{ N/m}$  that is attached to a wall. (a) Find the maximum distance the spring will be compressed (1). (b) If the spring is to compress by no more than



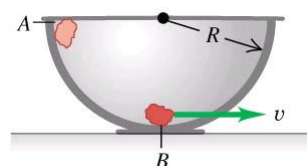
0.2 m, what should be the maximum value of the initial velocity  $v_0$ ? (2)

■A particle moves along  $x$ -axis while acted on by a single conservative force parallel to the  $x$ -axis. The force corresponds to the potential-energy function graphed in the figure. The particle is released from rest at point A. (a) What is the direction of the force on the particle at point C? (positive or negative) (3) (b) At which point, is the kinetic energy of the particle a maximum? (4) (c) Which point(s) correspond to unstable equilibrium? (5)

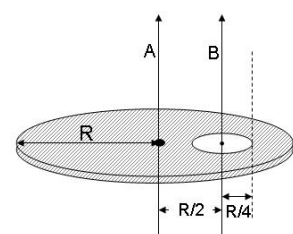


■If the speed of an object is doubled, the magnitude of its relativistic momentum (A) increases by a factor greater than 2 (B) increases by a factor of 2 (C) increases by a factor between 1 and 2 (D) the answer depends on the initial speed. (6)

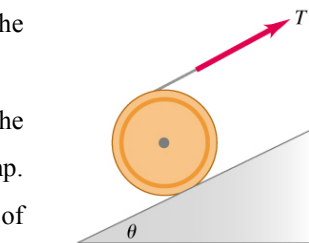
■A small rock with mass 0.12 kg is released from rest at point A, which is at the top edge of a hemispherical bowl with radius  $R = 0.52\text{ m}$  and fixed in space. Assume that the rock slides rather than rolls. The work done by friction on the rock when it moves from point A to point B at the bottom of the bowl is 0.22 J. Between points A and B, how much work is done on the rock by the normal force (the force that is normal to the wall of the bowl)? (7) What is the speed of the rock as it reaches point B? (8) Just as the rock reaches point B, what is the normal force on it due to the bottom of the bowl? (9)



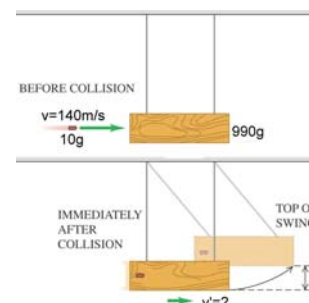
■A thin, flat, uniform disk with radius  $R$  has a circular hole of radius  $R/4$ , centered at a point  $R/2$  from the disk's center. The mass of this disk with a hole is  $M$ . Find the moment of inertia if it rotates about an axis (a) through the original center of the disk (axis A) (10) (b) through the center of the hole (axis B) (11)



■A uniform solid cylinder of mass  $M$  is supported on a ramp that rises at an angle  $\theta$  above the horizontal by a wire that is wrapped around its rim and pulls on it tangentially parallel to the ramp. The cylinder is in equilibrium under gravity, friction and wire tension  $T$ . (a) What is the direction of the friction, uphill or downhill? (12) (b) Find  $T$  in terms of  $M$ ,  $g$ ,  $\theta$ . (13) (c) what is the minimum coefficient of static friction  $\mu_s$  between the cylinder and the ramp? (14)



■A bullet, with mass 10 g, and velocity 140 m/s, is fired into a block of wood with mass 990 g, suspended by a string, and makes a completely inelastic collision with it. After impact of the bullet, the block swings up to a maximum height  $y$ . (a) What is the velocity of the block and the bullet just after the impact? (15) (b) What is the height  $y$ ? (16)

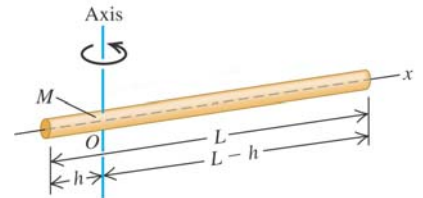


■A solid ball and a hollow ball, originally at rest, both roll without slipping down a ramp, which is inclined at an angle  $\beta$  to the horizontal. Which ball rolls down faster? (17)

■Which of the following statements about conservative and nonconservative forces is (are) correct? (no partial point) (A) A conservative force always acts to push the system toward lower potential energy. (B) A fast moving baseball encounters a dragging force:  $\vec{F} = -k\vec{v}$  where  $\vec{v}$  is its velocity and  $k$  is a constant. This dragging force is a nonconservative force. (C) For a nonconservative force  $\vec{F}(x, y, z)$ , you can still always find a potential function  $U(x, y, z)$  such that  $\vec{F} = -\vec{\nabla} \cdot U$  (18)

■A muon has a half life of  $1.56 \times 10^{-6}$  s as measured in a reference frame in which it is at rest. (It means 50% of the muons decay to something else after  $1.56 \mu\text{s}$  on average.) If muons are created at very high altitude above the earth surface and moving at  $0.99c$  relative to earth (a) what is its half life measured by an observer on earth? (19) (b) On average, how far (as measured by an observer on earth) can the muons travel when 50% of them have decayed? (20)

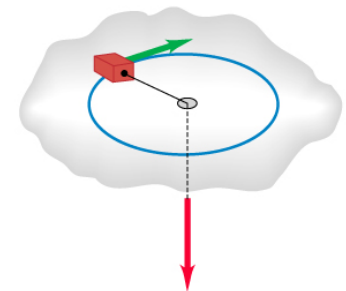
■A uniform thin rod has mass  $M$  and length  $L$ . (a) Compute the moment of inertia about an axis through  $O$ , at a distance  $h$  from one end. (In terms of  $M$ ,  $L$ ,  $h$ ) (21) (b) If  $h = L/2$ , what is the moment of inertia  $I$ ? (In terms of  $M$ ,  $L$ ) (22)



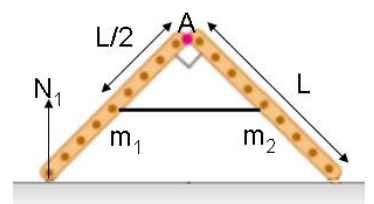
■Two protons are moving apart from each other, each moving at half the speed of light relative to the laboratory. What speed does one proton measure from the other proton? (23) What is the relativistic kinetic energy of each proton as measured by an observer at rest in the laboratory? (24) What is the relativistic kinetic energy of the other proton as measured by an observer riding along with one of the protons? (25)

## Part II 計算題: 須寫下詳細的計算過程 (共 30 分)

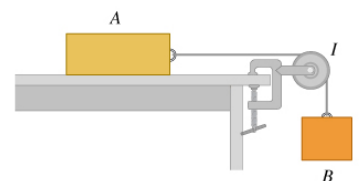
1. A small block on a frictionless, horizontal surface has a mass of 1 kg. It is attached to a massless cord passing through a hole in the surface. The block is originally revolving at a distance of 1 m from the hole with an angular speed of 10 rad/s. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.5 m. Model the block as a particle. (a) Is angular momentum of the block conserved? Why? (b) What is the new angular speed? (c) How much work was done in pulling the cord? (Hint: find the change in kinetic energy)



2. Two uniform ladders, both with length  $L$ , are hinged at point  $A$  and tied together by a horizontal rope at its center. The angle between the ladders is 90 degree. The ladders have mass  $m_1$  and  $m_2$  respectively. Assume the floor is frictionless. (a) Find the upward force  $N_1$  at the bottom of the left ladder. (b) Find the tension in the rope. (c) Find the magnitude of the force one ladder exerts on the other at point  $A$ . Express your answer in terms of  $m_1$ ,  $m_2$ ,  $g$ , and  $L$ .



3. Two blocks with mass  $m_A$  and  $m_B$  are connected by a light rope passing over a pulley. The pulley has radius  $R$  and a moment of inertia  $I$ . The rope does not slip over the pulley, and the pulley spins on a frictionless axle. The coefficients of static and kinetic friction between block  $A$  and the tabletop are  $\mu_s$  and  $\mu_k$  respectively. The system is released from rest. Because  $m_B > m_A\mu_s > m_A\mu_k$ , block  $B$  starts to descend.



- How much work is done on block  $B$  by gravity if block  $B$  moves a distance  $d$  downward?
- How much work is done on block  $A$  by friction if block  $B$  moves a distance  $d$  downward?
- Calculate the speed of block  $B$  as a function of the distance  $d$  that it has descended.

(Express your answer in terms of  $m_A$ ,  $m_B$ ,  $R$ ,  $I$ ,  $\mu_k$ ,  $d$ ,  $g$ . In part (a) and part (b), be careful about the sign of work.)

(1)	0.6 m
(2)	2 m/s
(3)	negative
(4)	B
(5)	D
(6)	A
(7)	0 Joule
(8)	2.6 m/s
(9)	2.7 N
(10)	$\frac{247}{480}MR^2$
(11)	$\frac{383}{480}MR^2$
(12)	uphill
(13)	$\frac{Mg}{2}\sin\theta$
(14)	$\frac{1}{2}\tan\theta$
(15)	1.4 m/s
(16)	0.1 m
(17)	The solid ball
(18)	AB (全對才給分)
(19)	11.06 $\mu\text{s}$
(20)	3284 m
(21)	$\frac{1}{3}M(L^2 - 3Lh + 3h^2)$
(22)	$\frac{1}{12}ML^2$
(23)	$\frac{4}{5}c$
(24)	145 MeV or $2.32 \times 10^{-11}$ Joule
(25)	625 MeV or $1 \times 10^{-10}$ Joule

1.(a) Yes, angular momentum is conserved. The tension passes through the center of rotation so this force exerts no torque.

(b)  $L_1=L_2$ ,  $I\omega_1=I\omega_2$  Block treated as a point mass, so  $I=mr^2$

$$mr_1^2\omega_1 = mr_2^2\omega_2 \quad \text{so} \quad \omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1 = \left(\frac{1}{0.5}\right)^2 \times 10 = 40 \text{ rad/s}$$

$$(c) \quad K_1 = \frac{1}{2}I\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = 0.5 \times 1 \times 1^2 \times 10^2 = 50 \text{ J}$$

$$K_2 = \frac{1}{2}I\omega_2^2 = \frac{1}{2}mr_2^2\omega_2^2 = 0.5 \times 1 \times 0.5^2 \times 40^2 = 200 \text{ J}$$

so the work done by tension =  $K_2-K_1=200-50 = 150 \text{ J}$

$$2. (a) \text{ take torques about the right end. } N_1 \times \sqrt{2}L = m_1g\left(\frac{L}{\sqrt{2}} + \frac{L}{2\sqrt{2}}\right) + m_2g\left(\frac{L}{2\sqrt{2}}\right)$$

$$\text{The upward force at the bottom of the left ladder is } N_1 = \frac{3}{4}m_1g + \frac{1}{4}m_2g$$

(b) find the torques on the left ladder using point A as origin.

$$N_1 \times \frac{L}{\sqrt{2}} = m_1g \times \frac{L}{2\sqrt{2}} + T \times \frac{L}{2\sqrt{2}}$$

$$\text{Take the result of } N_1 \text{ from part (a), one obtains the tension } T = \frac{1}{2}m_1g + \frac{1}{2}m_2g$$

$$(c) \text{ the horizontal component of the force } F_H \text{ at point A must be equal to the tension } T = \frac{1}{2}m_1g + \frac{1}{2}m_2g$$

The vertical component  $F_V$  can be calculated using force balance on the left ladder.

$$N_1 = m_1g + F_V$$

$$\text{So } F_V = N_1 - m_1g = \frac{1}{4}m_2g - \frac{1}{4}m_1g$$

$$\text{The magnitude of force at point A} = \sqrt{F_H^2 + F_V^2} = \frac{g}{4}\sqrt{5m_1^2 + 5m_2^2 + 6m_1m_2}$$

3. (a) Work done on block B by gravity =  $m_Bgd$

(b) Work done on block A by friction =  $-\mu_k m_Agd$

(c) Use work-energy theorem

$$K_1 + m_Bgd - \mu_k m_Agd = K_2 \quad \text{where } K_1=0 \text{ and } K_2 = \frac{1}{2}m_Av_2^2 + \frac{1}{2}m_Bv_2^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2}I\left(\frac{v_2}{R}\right)^2$$

$$0 + m_Bgd - \mu_k m_Agd = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2}I\left(\frac{v_2}{R}\right)^2$$

$$v_2 = \sqrt{\frac{2(m_Bgd - \mu_k m_Agd)}{m_A + m_B + \frac{I}{R^2}}} = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + \frac{I}{R^2}}}$$