[CS 2336]Discrete Mathematics: Autumn 2009

2^{rd} exam (close book)
Examination Date: Nov. 30, 2009
Time: 13:10-15:00

1. Let $A = \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$, and define $\mathcal{R}$ on A by $(x_1, y_1)\mathcal{R}(x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.

   a) Verify that $\mathcal{R}$ is an equivalence relation on A. **(5%)** reflexive. symmetric transi
   b) Determine the equivalence classes $[(1, 3)], [(2, 4)]$, and $[(1, 1)]$. **(3%)**
   c) Determine the partition of A induced by $\mathcal{R}$. **(2%)**

2. The poset $(A, \mathcal{R})$ is called a *lattice* if for all $x, y \in A$ the elements lub$\{x, y\}$ and glb$\{x, y\}$ both exist in A. Which of the following posets is a lattice? Please explain your reasons.

   a) Let $u = \{1, 2, 3\}$ and A be a power set of u. $\mathcal{R}$ is the subset relation on A. **(3%)**
   b) $\mathcal{R}$ is the "(exactly) divides" relation applied to $\{2, 3, 5, 6, 7, 11, 12, 35, 385\}$. **(3%)**
   c) $\mathcal{R}$ is the "less than or equal to" relation on Z, the set of integer. **(3%)**

3. Please answer the following questions:

   a) How many spanning subgraphs are there for the graph G in Fig. 1? **(3%)**
   b) How many connected spanning subgraphs are there in part (a)? **(3%)**
   c) How many of the spanning subgraphs in part (a) have vertex f as an isolated vertex? **(3%)**

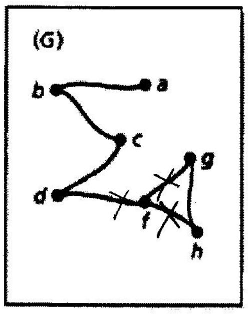


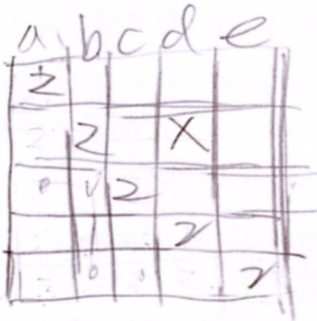
Fig. 1

4. Fill in the blanks of the following table. **(12%)**

Name	Repeated Edges	Repeated Virtices	Open/Close
Trail			
Circuit			
Path			
Cycle			

5. Let $R = \{a, b, c, d, e\}$, determine the number of relations on R that are

   a) reflexive and symmetric. **(2%)**
   b) symmetric and contain (b, d). **(2%)**
   c) antisymmetric and contain $(b, \ d)$. **(2%)**
   d) symmetric and antisymmetric. **(2%)**
   e) reflexive, symmetric, and antisymmetric. **(2%)**
   f) equivalence relations. **(2%)** reflexive. symmerityc
   g) equivalence relations satisfy $a, b \in [d]$. **(2%)** transitive



$25 \cdot 3^{10}$

6. If G is an undirected graph with n vertices and e edges, let $\delta = \min_{v \in V}\{\deg(v)\}$ and let $\Delta = \max_{v \in V}\{\deg(v)\}$. Prove that $\delta \leq 2(e/n) \leq \Delta$. **(10%)**

7. For $A = \{a, b, c, d, e\}$, let $\mathcal{R} = \{(a, a), (a, c), (b, c), (c, d), (c, e), (d, d), (d, e)\}$ be a relation on A. Draw the directed graph G on A that is associated with $\mathcal{R}^{11}$. **(10%)**

8. Let $A = \{1, 3, 4, 8, 19\}$, and define on A by $x\mathcal{R}y$ if $x - y$ is a nonnegative even integer. Draw the Hasse diagram for the poset$(A, \mathcal{R})$. **(10%)**

9. Let $\mathcal{R}$ be an equivalence relation on a set A, and $x, y \in A$.

   reflexive. symmetric. transitive

   a) Prove that $x \in [x]$. **(4%)**
   b) Prove that $x\mathcal{R}y$ if $[x] = [y]$. **(4%)**
   c) Prove that $x\mathcal{R}y$ only if $[x] = [y]$. **(4%)**
   d) Prove that $[x] = [y]$ or $[x] \bigcap [y] = \emptyset$. **(4%)**

$a \in [x]$ a & y

$x \in [x]$
$y \in [y]$

if $x R y$

$a R x$ $a R y$

$\Rightarrow x R a$

a