CALCULUS (NOV. 21,2005)

1. (15) Find the following limits and give your reason

$$\text{(a) } \lim_{x\to 0} x^2 \csc 3x \cot 2x, \qquad \text{(b) } \lim_{x\to 0} x^{\frac{1}{3}} \sin\frac{5}{x}, \qquad \text{(c) } \lim_{x\to\infty} \sqrt{x^2+2} - x.$$

2. (10) Compute y' if (a) $y = \sec(\sin x)$, (b) $y = \sqrt{1+\cot 5x}$.

3. (10) Show that the part of the graph of $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 1$ in the first quadrant is decreasing and concave upward.

4. (10) Show that the equation $x^5 + x^3 + x + 100 = 0$ has exactly one real root.

5. (10) Show that among all open cylindrical can with given fixed total surface area, the one with maximal volume has height equal the the radius of its base.

6. (10) A rocket that is launched vertically is tracked by a radar station located on the ground 4 mi from the launch site. What is the vertical speed of the rocket at the instant its distance from the radar station is 5 mi and this distance is increasing at the rate 3600 mi/hr.

7. (10) A rectangle has area 64 square cm. a straight line is drawn from one corner of the rectangle the the midpoint of the far side. What is the possible minmum length of such a line.

8. (10) Find the possible maximal volume of a cylindrical can inside a ball of radius R.

9. (10) Discuss and sketch the graph of $y = \frac{x^2}{x^2-x-2}$.

10. (10) Use the $\epsilon-\delta$ method to show that if $f(x) \le 0$ for all $x \neq a$ and $\lim_{x\to a} f(x) = L$, then $L \le 0$.(Hint : choose a good ϵ to derive a contradition.)