CS 3332 Probability and Statistics
Midterm Exam II
Time: 3:20pm-5:10pm, Dec. 1, 2006
(Totally 6 problems, 120 pts in 2 pages)

1. (20 pts) Give a Yes/No answer to each of the following statement with explanations. (Give your reasons in 1 or 2 sentences to support your answers.)

A. The variance of the continuous random variable with a uniform distribution on the interval [0, 4] is 4/3.

B. Chi-square distribution is a special case of Gamma distribution.

C. If X_1 and X_2 are both normally distributed with means μ_1 and μ_2, respectively, and variances σ_1^2 and σ_2^2, respectively, then $X_1 + X_2$ is a normal random variable

with mean = $\mu_1 + \mu_2$ and variance = $\sigma_1^2 + \sigma_2^2$.

D. For a random variable X, the moment generating function for the random variable $(X + a)$ is the same as the moment generating function for X, where a is a constant number.

E. If X is a normal distributed random variable with mean μ and variance σ^2, then

the random variable $\dfrac{(X - \mu)^2}{\sigma^2}$ is also a normal distribution.

2. (20 pts) Suppose the probability of an electronic device manufactured by a certain company is 0.01. A quality control strategy is used to reject a shipment of electronic devices if two or more defective units are found after checking 100 units sampled from this shipment.

(a) (8 pts) What is the probability that a shipment is rejected? Write down the exact formula without calculating out the exact number.

(b) (6 pts) Is it reasonable to use Poisson distribution to approximate the distribution of the random variable associated with the number of defective units found from 100 units? Why? (Hint: $p(x, \lambda t) = \dfrac{e^{-\lambda t}(\lambda t)^x}{x!}, x = 0, 1, 2, \cdots$)

(c) (6 pts) Apply the approximation in (b) to compute the probability that a shipment is rejected. Try to work out the number as best as you can. You can write your answer containing the constant e. (Hint: e = 2.71828…)

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3. (20 pts) Let the cumulative standard normal distribution function be given by

$$\Phi(r) = \int_{-\infty}^{r} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$ Given a normally distributed random variable X with mean 70 and standard deviation 10, find the following value. You can write your answer in terms of the function Φ :

(a) (6 pts) P(X < 60)

(b) (6 pts) P(X > 90)

(c) (8 pts) the value of k such that P (X < k) = 0.1.

4. (20 pts) Consider a random variable X with a binomial distribution given by b(x; n, p).

(a) (6 pts) Prove that the moment generating function for X is $M_X(t) = (pe^t + q)^n$.

(b) (8 pts) Compute the 1st and 2nd moments about the origin from its moment generating function.

(c) (6 pts) Compute its mean and variance from the computed moments in (b).

5 (20 pts) Suppose that every three months, on average, an earthquake occurs in California. Assume the earthquake occurrence is a Poisson process. Let X be a random variable for the time (in months) until the next earthquake.

(a) (10 pts) What type of distribution function best describes the random variable X? Write down the probability distribution function for X.

(b) (10 pts) What is the probability that the next earthquake occurs between three and seven months?

6 (20 pts) Let X_1 and X_2 be independent random variables each having the probability distribution

$$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & x > 0 \\ 0 & elsewhere \end{cases}$$

(a) (15 pts) Compute the probability distribution function of $Y = X_1 + X_2$.

(b) (5 pts) What type of probability distribution is your answer in (a)?