CS2102 Digital Logic Design
Exam 1 : 10:10 – 12:00, October 21, 2014

1. (a) Express the following unsigned numbers in decimal: $(110011010)_2$ and $(735.64)_8$ (6%)
   (b) Represent the unsigned decimal numbers 637 and 599 in BCD, and then show the steps necessary to form their sum. (8%)

2. The following is a string of ASCII characters whose bit patterns have been converted into hexadecimal for compactness: 44 CC E4 C0 A6 D4 48 F5. Of the eight bits in each pair of digits, the leftmost is a parity bit. The remaining bits are the ASCII code.
   (a) Convert the string to bit form and decode the ASCII. (8%)
   (b) Determine the parity used: odd or even? (2%)

American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	–	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	_	o	DEL

3. (a) Convert decimal -73 and $+29$ to binary, using the 8-bit signed-2's-complement representation. (4%)
   (b) Perform the binary equivalent of $(-73) + (+29)$ and $(+29) - (-73)$ and convert the answers back to decimal. (6%)

4. $F = x'y'z' + w'x'yz' + wx'yz'$
   (a) Draw a logic diagram that implements F using AND, OR, and NOT gates. Assume that only w, x, y and z are available as input signals. (5%)
   (b) Simplify F to a minimum number of literals. (5%)

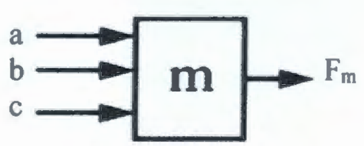
5. Use the brief notation Σ or Π to answer the following questions.
   (a) Express $F_1(a, b, c, d) = (a'+b')c'd$ in the sum-of-minterms form. (5%)
   (b) Express $F_2(a, b, c, d) = ab'd+c'(a+d)$ in the product-of-maxterms form. (5%)
   (c) Express $F = F_1F_2$ in the sum-of-minterms form. (5%)

6. (a) Determine whether $y'z' + yz' + x'z = x' + xz'$ is true or false, and justify your answer. (5%)
   (b) Show that the dual of the exclusive-NOR is equal to its complement. (5%)

$x'y+xy'$

7. (a) Give a simplified sum-of-products form for $F_1(x, y, z) = \Pi(1, 5)$ using the K-map method. What are the essential prime implicants of F_1? (5%)
   (b) Give a simplified sum-of-products form for $F_2(A, B, C, D) = A' + AB + B'C + ACD$ using the K-map method. Then implement the simplified function with a two-level NAND gate circuit by assuming that both the normal and complement inputs are available. (10%)
   (c) Give a simplified product-of-sums form for the following Boolean function F_3 that contains five don't-care conditions using the K-map method. Then implement the simplified function with a two-level NOR gate circuit by assuming that both the normal and complement inputs are available. (10%)
   $F_3(A, B, C, D) = \Sigma(2, 4, 7, 10, 12, 14)$
   $d(A, B, C, D) = \Sigma(0, 3, 6, 8, 13)$

8. The block diagram and the truth table of the three-input minority gate are shown below. Prove the three-input minority gate is a universal gate by implementing two-input AND, two-input OR and inverter gates using three-input minority gates only. You can assume that 0 and 1 are available as inputs of minority gates. (6%)



three-input minority gate

a	b	c	F_m
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$((A'C)'+D)'$
$((A'C)'+D)'$
$((A'+C')'+D)'$

$A'C+D'$
$(A+C')'+D'$
$F_3' = (D'+A'C)'$
$= D\cdot(A'C)'$
$= D\cdot(A+C')$
$(AD+C'D)'$

$a'b'+b'c'+a'c'$

(a) $1\times2^1+1\times2^3+1\times2^4+1\times2^7+1\times2^8$

$= 2+8+16+128+256$

$= 410$ #

$7\times8^2+3\times8^1+5\times8^0+6\times8^{-1}+4\times8^{-2}$

$= 448+24+5+0.75+0.0625$

$= 477+0.8125$

$= 477.8125$ #

(b) 6 3 7   5 9 9

= 0110 0011 0111   0101 1001 1001

```
      1      1
   0110   0011   0111
 + 0101   1001   1001
 --------------------
   1100   1101   0000
 + 0110   0110   0110   (add 6)
 --------------------
 1 0010   0011   0110
```

= 1 2 3 6 #

2.

44	CC	E4	C0	A6
01000100	11001100	11100100	11000000	10100110
D	L	d	@	&

D4	48	F5
11010100	01001000	11110101
T	H	u

(a) DLd@&THu

(b) even #

3. (a)

$-73 = 10110111$ ✓ $29 = 00011101$ ✓

$73 = 01001001$

$\frac{73}{2} = 36 \ldots 1$

$\frac{36}{2} = 18 \ldots 0$

$\frac{18}{2} = 9 \ldots 0$

$\frac{9}{2} = 4 \ldots 1$

$\frac{4}{2} = 2 \ldots 0$

$\frac{2}{2} = 1 \ldots 0$

$\frac{1}{2} = 0 \ldots 1$

$\frac{29}{2} = 14 \ldots 1$

$\frac{14}{2} = 7 \ldots 0$

$\frac{7}{2} = 3 \ldots 1$

$\frac{3}{2} = 1 \ldots 1$

$\frac{1}{2} = 0 \ldots 1$

$29 = 00011101$

$-73 = 10110111$ #

(b) $(-73)+(+29)$

$$\begin{array}{r} 10110111 \\ +\,00011101 \\ \hline 11010100 \end{array}$$

$\Rightarrow 00101100$

$= -44$ ✓ #

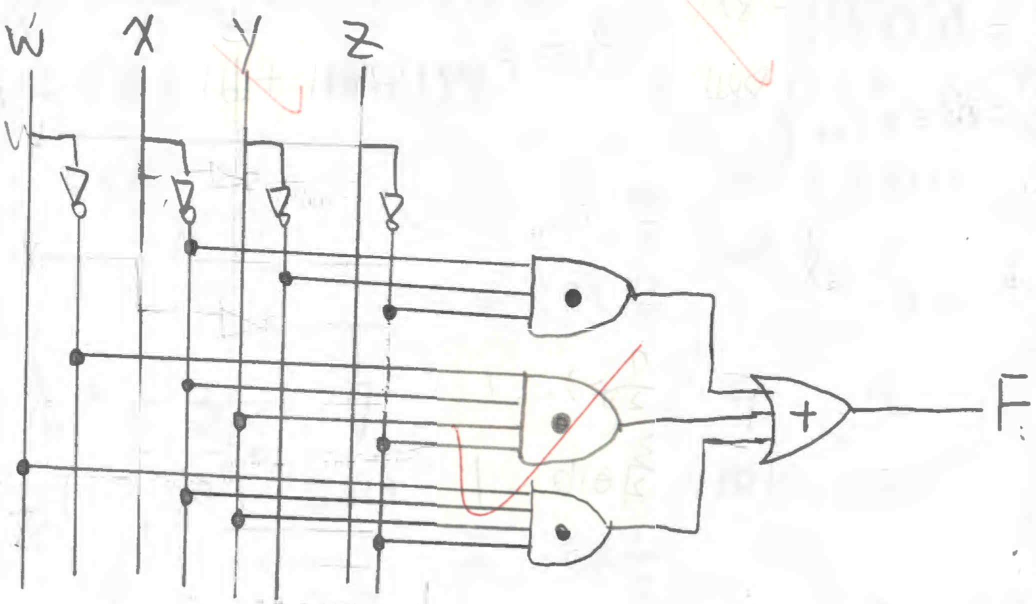
$(+29)-(-73) = (+29)+(+73)$

$$\begin{array}{r} 00011101 \\ +\,01001001 \\ \hline 01100110 \end{array}$$

$= 102$ # ✓

4.

(a)



#

(b)

$F = x'y'z' + w'x'yz' + wx'yz'$

$= x'y'z' + (w'+w)(x'yz')$

$= x'y'z' + x'yz'$

$= (x'z')(y'+y)$

$= x'z'$

#

5. (a) $(a'+b')c'd$

$= a'c'd + b'c'd$

$= a'bc'd + a'b'c'd + ab'c'd + a'b'c'd$

 0101 $\quad$ 0001 $\quad$ 1001 $\quad$ 0001

 5 $\quad$ 1 $\quad$ 9 $\quad$ 1

$F_1 = \Sigma(1, 5, 9), = m_1 + m_5 + m_9$

#

(b) $ab'd + c'(a+d)$

$= ab'd + ac' + c'd$ $\qquad 5 \; 0101$

$= ab'cd + ab'c'd + abc'd + ab'cd + abc'd' + ab'c'd' + abc'd + ab'c'd + a'bc'd$

$\;\; 1011 \quad 1001 \quad 1101 \quad 1011 \quad \quad \quad 1101 \quad 1001$

$\;\; 11 \quad 9 \quad 13 \quad 11 \quad 14 \quad 10 \quad 13 \quad 9 \quad + a'b'c'd$

0001

1

$F_2 = \Sigma(1, 5, 9, 8, 11, 13, 14) = m_1 + m_5 + m_9 + m_8 + m_{11} + m_{13} + m_{14}$

$= \Pi(0, 2, 3, 4, 6, 7, 10, 14, 15) = M_0 + M_2 + M_3 + M_4 + M_6 + M_7 + M_8 + M_{14} + M_{15}$

#

(c)

$F = F_1 \cdot F_2$

$= \Sigma(1, 5, 9) = m_1 + m_5 + m_9$ #

6. (a) $y'z' + yz' + x'z = z'(y+y') + x'z$

$= z' + x'z$

$= (x+x')z' + x'z$

$= xz' + x'z' + x'z$

$= xz' + x'(z'+z)$

$= x' + xz'$

$y'z' + yz' + x'z = x' + xz'$ is true #

b) $(x\oplus y) = x'y + xy' \overset{\text{dual}}{=} (x'+y)(x+y')$

$(x\oplus y)' = xy + x'y'$

$x'y + xy' \overset{\text{dual}}{=} (x'+y)(x+y') = x'y' + xy$ #

欲求 $(\text{XNOR})' = \text{dual XNOR}$

$(x\oplus y)' = x'y' + xy$

$\text{dual} = (x+y)(x'+y')$

$(x'y'+xy)'$

$= (x+y)(x'+y')$

−5

7. (a) $= \Sigma(0,2,3,4,6,7)$ $\pi(1,5)$

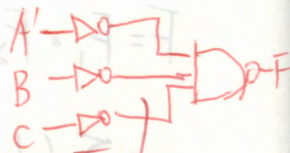
			y	y
	1		1	1
x	1		1	1
		z	z	

$EPI = y, z'$

(b)

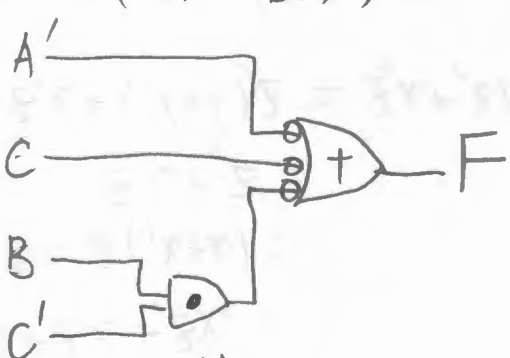
AB \ CD			C	C	
	1	1	1	1	
	1	1	1	1	B
A	1	1	1	1	B
A			1	1	
		D	D		

$F_2 = A' + B + C$



$F_2 = A' + C + BC'$

$= (A \cdot C' \cdot (BC')')'$



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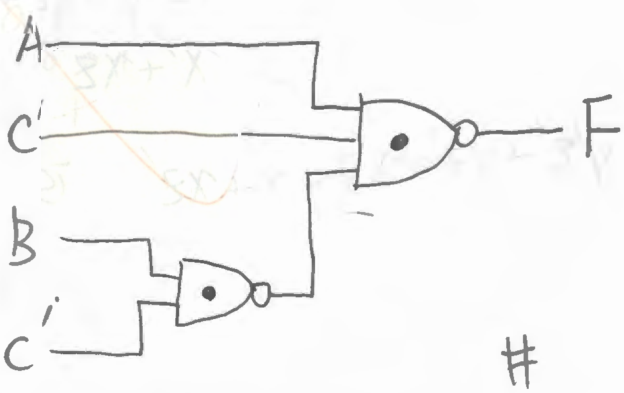


#

(c)

AB \ CD				
	X		X	1
	1		1	X
	1	X		1
	X			1

$F_3 = D' + A'C$

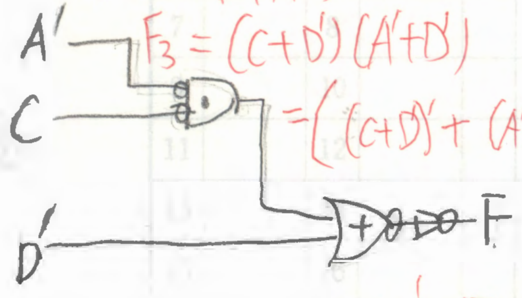
$= (((A+C')' + D')')'$

product-of-sums

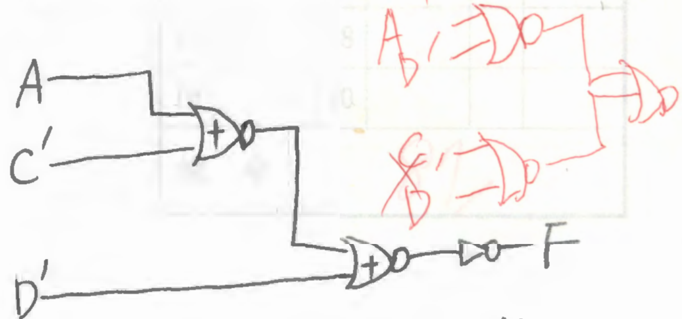
$F_3' = C'D + AD$

$F_3 = (C+D')(A'+D')$

$= [(C+D')' + (A'+D')']'$



-6



#

8.

a \ bc	00	01	11	10
0	1	1		1
1	1			

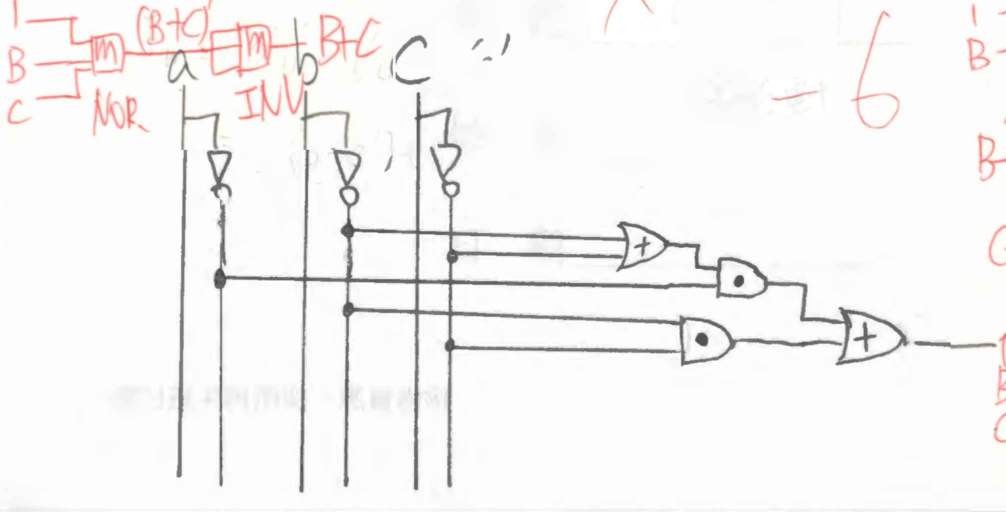
$F_m = a'b' + b'c' + a'c'$

$= a'(b'+c') + b'c'$

OR $(B+C) = [(B+C)']'$

X

-6



INVERTER

AND Let A=1 then $F_m = B'C'$