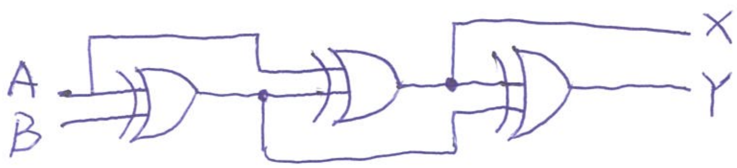
CS2101 Midterm I

1:20-3:00pm, Thursday, April 16, 2015

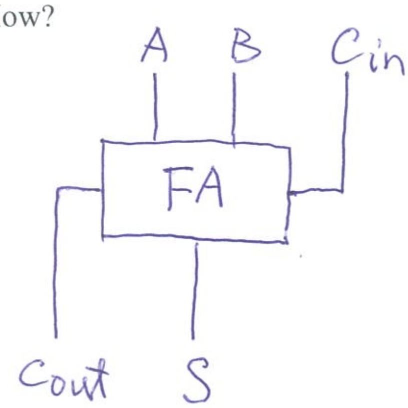
1. 10% Binary representations
   i. Convert binary number 010101_2 to decimal number
   ii. Represent decimal number 1357_{10} in 12-bit binary number
   iii. Represent decimal number -1357_{10} in 12-bit 2's complement binary number
2. 10% Prove that ~(A & B) = ~A | ~B /* De Morgan's Theorem */
3. 20% In the K-Map below
   i. Identify all prime implicants
   ii. Identify all essential prime implicants
   iii. Give a minimized Boolean expression of the function
   iv. Implement the function using AND, OR and NOT gates

ab \ cd	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	0	1	1	0
10	1	X	1	1

4. 10% Analyze the circuit schematic below. Express its function in Truth Table.



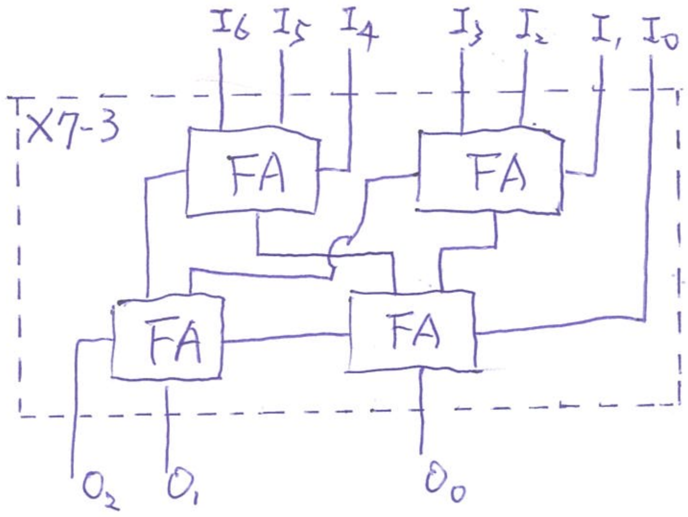
5. 10% We can implement a 1-bit full adder(FA) by composition of a majority function circuit and an odd function circuit.
   i. Why?
   ii. And How?



6. 15% Below is a 7-input 3-output function called X7-3. Its input are vector of 7 bits and output 3-bit binary number.

   i. What is the function of X7-3?

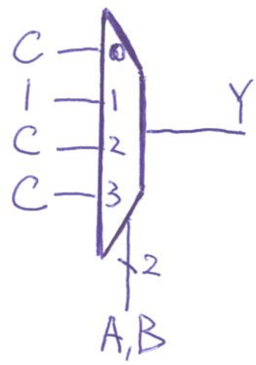
   ii. Construct an X15-4 using X7-3s (and some additional FAs) as building blocks.



7. 15% In addition to data selection, multiplexers can implement arbitrary function. Given a 4-to-1 multiplexer (binary select) below

   i. What is function Y = f(A, B, C)?

   ii. Rewire the multiplexer to implement a 3-input NOR function.



8. 10% In an n-bit adder/subtractor circuit operating on 2's complement binary numbers, how is overflow detected?

1. <i> $010101_2 = 2^4 + 2^2 + 2^0 = 16 + 4 + 1 = 21_{10}$

<ii>

$$
\begin{array}{r|l}
2 & 1357 \\
2 & 678 \cdots 1 \\
2 & 339 \cdots 0 \\
2 & 169 \cdots 1 \\
2 & 84 \cdots 1 \\
2 & 42 \cdots 0 \\
2 & 21 \cdots 0 \\
2 & 10 \cdots 1 \\
2 & 5 \cdots 0 \\
2 & 2 \cdots 1 \\
 & 1 \cdots 0
\end{array}
$$

$1357_{10} = 0101\ 0100\ 1101_2$

Verification

$2^{10} + 2^8 + 2^6 + 2^3 + 2^2 + 2^0$

$= 1024 + 256 + 64 + 8 + 4 + 1$

$= 1357$

<iii> -1357_{10}

1's Complement of $0101\ 0100\ 1101$

$= 1010\ 1011\ 0010$

2's Complement $= 1010\ 1011\ 0011$

2. Prove by Truth Table

A	B	A&B	$\sim(A\&B)$	$\sim A$	$\sim B$	$\sim A \mid \sim B$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$\sim(A\&B) = \sim A \mid \sim B$

3.

ab \ cd	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	0	1	1	0
10	1	X	1	1

$\bar b \bar d$, $\bar b c$, cd, $a\bar b$, ad

⟨i⟩ Prime Implicants

$a\bar b$, ad, cd, $\bar b \bar d$, $\bar b c$

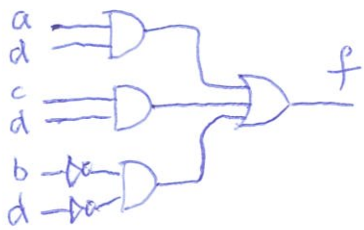
⟨ii⟩ Essential P I

ad, cd, $\bar b \bar d$

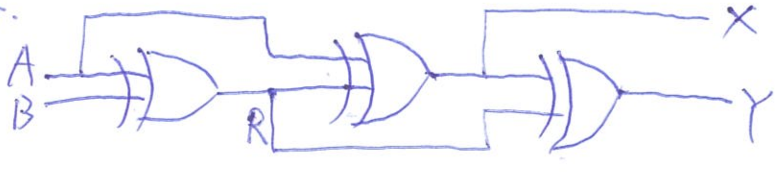
⟨iii⟩

$f = ad + cd + \bar b \bar d$

⟨iv⟩



4.



A	B	R = A^B	X = A^R	Y = X^R
0	0	0	0	0
0	1	1	1	0
1	0	1	0	1
1	1	0	1	1

$X = B$

$Y = A$

This circuit SWAP two bits.

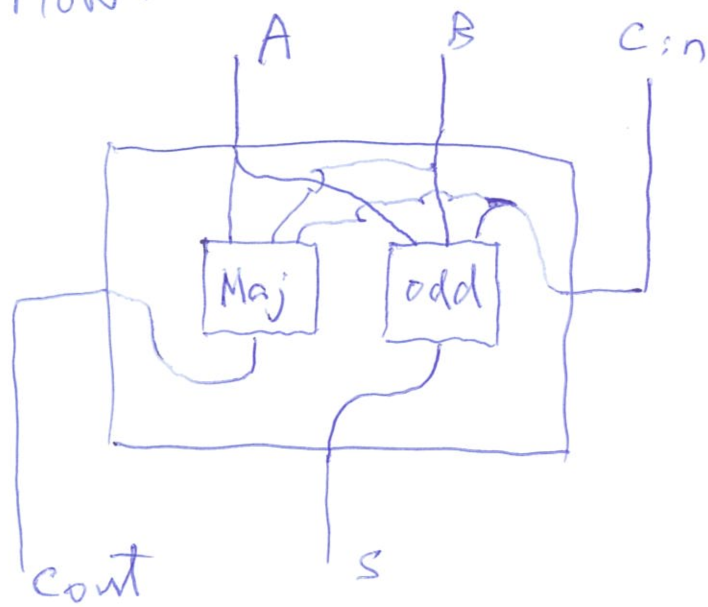
5. Full Adder

A	B	Cin	Cout	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Why:

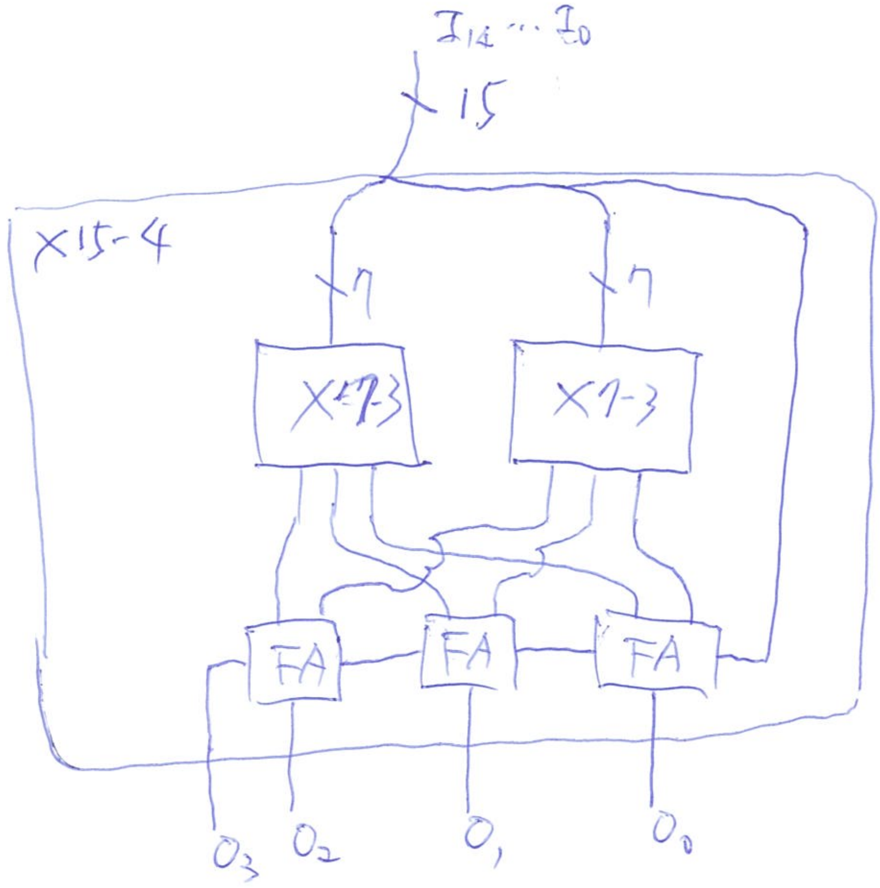
- Cout is 1 when two or more 1's in the input
  ⇒ Majority Function
- S is 1 when one or three 1's in the input.
  ⇒ ODD Function

How:



6. <1> it counts number of 1's in the Input

<2>

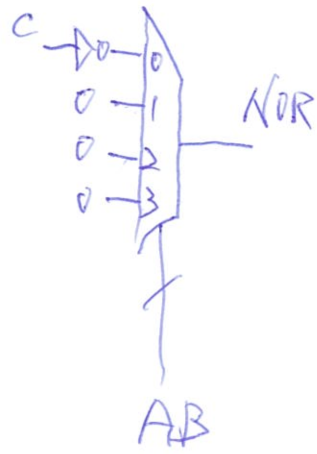


7. <1>

A	B	C	Y	Y
0	0	0	C	0
0	0	1	C	1
0	1	0	1	1
0	1	1	1	1
1	0	0	C	0
1	0	1	C	1
1	1	0	C	0
1	1	1	C	1

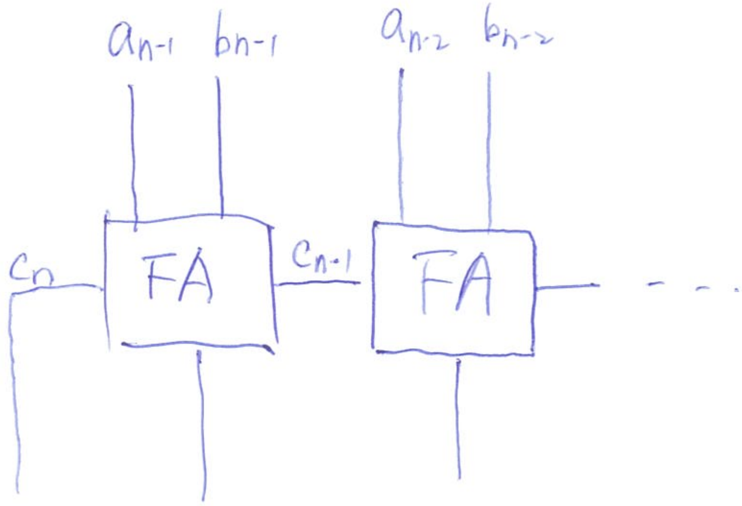
<ii>

A	B	C	NOR(ABC)	
0	0	0	1	$\overline{C}$
0	0	1	0	
0	1	0	0	0
0	1	1	0	
1	0	0	0	0
1	0	1	0	
1	1	0	0	0
1	1	1	0	



Y is a prime number detector

8.



A	B	
+	−	No OVF
+	+	$a_{n-1}=0$, $b_{n-1}=0$, OVF when $c_{n-1}=1$, $c_n=0$
−	−	$a_{n-1}=1$, $b_{n-1}=1$, OVF when $c_{n-1}=0$, $c_n=1$
−	+	No OVF

$\Rightarrow$

OVF detection ckt