Prob. 6-3

a.

2	3	4	5
8	9	12	14
16	∞	∞	∞
∞	∞	∞	∞

b. 1. If $Y[1, 1] = \infty$, then $Y[i, 1] = \infty$ for $1 < i \le m$

$Y[1, j] = \infty$ for $1 < j \le n$

By similar approach, we can prove that all entries in Y are ∞.

2. If $Y[m, n] < \infty$, then $Y[i, n] < \infty$ for $1 \le i < m$

$Y[m, j] < \infty$ for $1 \le j < n$.

By similar approach, we can prove that all entries are less than ∞.

c.

EXTRACT-MIN(Y)

begin

$min \leftarrow Y[1, 1]$;

$Y[1, 1] \leftarrow \infty$;

YOUNG(Y, 1, 1);

return *min*;

end.

YOUNG(Y, i, j)

begin

$min\_i \leftarrow i$;

$min\_j \leftarrow j$;

if ($i + 1 \le m$) and ($Y[i + 1, j] < Y[i, j]$) then

$min\_i \leftarrow i + 1$;

$min\_j \leftarrow j$;

endif

if ($j + 1 \le n$) and ($Y[i, j + 1] < Y[min\_i, min\_j]$) then

$min\_i \leftarrow i$;

$min\_j \leftarrow j + 1$;

endif

if ($min\_i \neq i$) or ($min\_j \neq j$) then

SWAP $Y[i, j]$ and $Y[min\_i, min\_j]$;

YOUNG(Y, min_i, min_j);

```
        endif
    end.
```

每次只花 $O(1)$的時間就會向右或向下走一格

worst case 會走到右下角的(m, n)

$\therefore\ T(p) = T(p-1) + O(1)$ where $p = m + n$

By iteration method, we can get $T(p) = O(m + n)$.

d. 方法與 c. 類似,從右下方塞數字進去接著往左上方走,略

e. 做 n^2 次的 EXTRACT-MIN: $n^2 * O(n + n)$

total time complexity is $O(n^3)$

f.

```
CHECK(Y, x)
    begin
        i ← 1;
        j ← n;
        while ((i ≤ m) and (j > 0))
            if (Y[i, j] ≥ x)
                if (Y[i, j] = x) then
                    return true;
                else
                    j ← j – 1;
                endif
            else
                i ← i + 1;
            endif;
        return false
    end.
```

從右上方開始走，數字一樣就找到了，比 x 大就往左，比 x 小就往下，由於每次轉向另一個方向的全部數字就可以宣告全部不可能,所以不可能發生左右走上下走的回頭情況。

最多只會向左走 n 格, 向下走 m 格, time complexity is $O(m + n)$

Prob. 7-1

a.

13	19	9	5	12	8	7	4	11	2	6	21
i										j	

6	19	9	5	12	8	7	4	11	2	13	21
	i								j		

6	2	9	5	12	8	7	4	11	19	13	21
								j	i		

b.

Let i_k be the value of i after kth iteration, j_k be the value of j after kth iteration.
$i_k \geq i_1 \geq p$ and $j_k \leq j_1 \leq r$ for all possible $k \geq 1$

If $i_k < j_k$, there exists an integer i_k', $i_k < i_k' \leq r$, such that $A[i_k'] \geq x$
and an integer j_k', $p \leq j_k' < j_k$, such that $A[j_k'] \leq x$ ....(1)

proof of (1):

basis:

$A[p] = x$, $i_0 < p < r$ and $A[p] \geq x$, $p \leq p < j_0$ and $A[p] \leq x$

$\Rightarrow$ basis hold

induction:

Assume $i_k < i_k' \leq r$, $A[i_k'] \geq x$

Assume $p \leq j_k' < j_k$, $A[j_k'] \leq x$

$\Rightarrow$ by observation, $i_{k+1} \leq i_k'$ and $A[i_{k+1}] \geq x$

$\Rightarrow$ by observation, $j_{k+1} \geq j_k'$ and $A[j_{k+1}] \leq x$

If $i_{k+1} < j_{k+1}$, $A[i_{k+1}]$ and $A[j_{k+1}]$ swap by the algorithm.

$\Rightarrow$ There exists $i_{k+1}' = j_{k+1}$ such that $i_{k+1} < i_{k+1}' \leq r$ and $A[i_{k+1}'] \geq x$
and $j_{k+1}' = i_{k+1}$ such that $p \leq j_{k+1}' < j_{k+1}$ and $A[j_{k+1}'] \leq x$

By mathematical induction, (1) hold.

By (1), if $i_k < j_k$, i_k' and j_k' exists.

By observation, $i_{k+1} \leq i_k' \leq r$, $p \leq j_k' \leq j_{k+1}$, $i_{k+1} > i_k$, and $j_{k+1} < j_k$ for $k \geq 0$

$i_k \geq i_1 \geq p$ and $j_k \leq j_1 \leq r$ for all possible $k \geq 1$

$\Rightarrow$ $p \leq i_k \leq r$ and $p \leq j_k \leq r$ for all possible $k \geq 1$

$\Rightarrow$ Q.E.D.

c.

By (b), $p \le j$ hold.

Since $A[p] \ge x$, $i_1 = p$.

By observation, $j_{k+1} < j_k$ and $j_1 \le r$

If $j_1 < r$, $p \le j < r$ hold

If $j_1 = r$, since $i_1 = p$ and $p < r$, it terminates when $k > 1$

$j_k \le j_2 < j_1 = r$ for $k \ge 2$

$\Rightarrow$ $p \le j < r$ hold.

$\Rightarrow$ Q.E.D.

d.

Every element of $A[p..i_k - 1] \le$ x, and every element of A$[j_k + 1..r] \ge x$ after kth iteration................(2)

proof of (2):

basis:

$A[p..i_0 - 1]$ = empty set, and $A[j_0 + 1..r]$ = empty set

$\Rightarrow$ hold

induction:

Assume every element of $A[p..i_k - 1] \le x$ and every element of $A[j_k + 1..r] \ge x$

If $i_k < j_k$, $A[i_k]$ and $A[j_k]$ swap, $A[i_k] \le$ x and $A[j_k] \ge x$

By observation, every element of $A[i_k + 1..i_{k+1} - 1] \le$ x and every element of $A[j_{k+1} + 1..j_k - 1] \ge$ x

$\Rightarrow$ Every element of $A[p..i_{k+1} - 1] \le x$ and every element of $A[j_{k+1} + 1..r] \ge x$

By mathematical induction, (2) hold.

When it terminates, $i_k \ge j_k$.

If $i_k = j_k$, $A[i_k] = x$

$\Rightarrow$ By (2), every element of $A[p..i_k] \le x$ and every element of $A[j_k..r] \ge x$

$\Rightarrow$ every element of $A[p..j_k] \le$ every element of $A[j_k + 1..r]$

$\Rightarrow$ hold

If $i_k > j_k$

$\Rightarrow$ By (2), every element of $A[p..i_k - 1] \le x$ and every element of $A[j_k + 1..r] \ge x$

$i_k > j_k$, $i_k - 1 \ge j_k$

$\Rightarrow$ every element of $A[p..j_k] \leq$ every element of $A[j_k + 1..r]$

$\Rightarrow$ Q.E.D.

e.

Quicksort (A, p, r)

  if $p < r$ then

    $q \leftarrow$ *Hoare-Partition* (A, p, r)

    Quicksort (A, p, q)

    Quicksort (A, q+1, r)