Test 1 for CS2334 (01)

October 13, 2008

1.(10%) Mark ○ if the statement is *true*, otherwise mark × if the statement is *false*.

X (a) If $A, B \in R^{n\times n}$ are nonsingular, then $(A+B)^{-1} = B^{-1} + A^{-1}$.

O (b) The product of unit lower-Δ matrices is also unit lower-Δ.

O (c) Let $A \in R^{n\times n}$, then $det(\alpha A) = \alpha^n det(A)$, where α is a constant.

O (d) $J \in R^{n\times n}$, where $J = [a_{ij}]$ with $a_{i,j} = 1$ if $j - i = 1$ else $a_{ij} = 0$, then J^n is a zero matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ & 0 & 1 \\ & & 0 \end{bmatrix}$$

X (e) The product of two elementary matrices is also an elementary matrix.

2.(30%) A linear system of equations is given below.

$$\begin{aligned} 3x + y - z &= 0 \\ -6x \quad + 2z &= -4 \\ 3x - 3y \quad &= 9 \end{aligned}$$

(a) Express this system as $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = [x, y, z]^t$. Show the augmented matrix for this system.

(b) Use Gaussian elimination and back substitution to solve this system of equations.

$x=1$
$y=-2$
$z=1$

(c) Find $A = LU$, where L is unit lower-Δ and U is upper-Δ.

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d) Find $det(A)$.

6

3.(30%) Given

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ 2 & 3 & 2 \end{bmatrix}$$

(a) Find $det(M_{21})$, $det(M_{22})$, and $det(M_{23})$.

$-8 \quad -2 \quad 5$

(b) Find the cofactors A_{21}, A_{22}, and A_{23}.

$8 \quad -2 \quad -5$

(c) Compute $det(A)$ from the results of (b) and find A^{-1}.

-3

$$\frac{1}{3}\begin{bmatrix} 13 & -8 & -14 \\ -4 & 2 & 5 \\ -7 & 5 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -1 \\ 0 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix} \quad \det(A) = -5$$

4.(20%) Given

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 7 \\ 1 & 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 7 \\ -5 & 3 & 5 \end{bmatrix}.$$

(a) Find an elementary matrix E such that $EA = B$. $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) Find an elementary matrix F such that $AF = C$. $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5.(10%) Let $P, Q, R \in R^{3\times 3}$ be defined as

$$P = I - 2\mathbf{e}_2\mathbf{e}_1^t, \quad Q = I + 3\mathbf{e}_3\mathbf{e}_1^t, \quad R = I - 4\mathbf{e}_3\mathbf{e}_2^t$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Write the result of $(RQP)^{-1}$ in a matrix form.

$11-8-3=0$

$13-14+6$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 4 & 1 \end{bmatrix}$$