The midterm Examination of Probability (10/31/06)

誓言：我在測驗間絕不提供與接受別人的協助。

學號：　　　　　簽名：

請將題目與答案本一併繳回。

題目若需計算過程，請陳列於答案本中，若沒有計算過程該題將不予計分！

計分方式：1-20 題，每題三分;21-25 題，每題八分。

測驗時間：10：10—12:30

PartI 每題三分

1. Experience shows that 20% of the people reserving tables at a certain restaurant never show up. If the restaurant has 30 tables and takes 33 reservations, what is the probability that it will be able to accommodate everyone?

2. If a point selected at random in the unit square(whose vertices are $(0,0),(0,1),(1,0),(1,1)$) is known to be in the region bounded by $x=1, y=0, y=x$, find the probability that it is also in the triangle bounded by $x=0, y=0, x+y=1$.

3. A college is composed of 70% men and 30% women. It is known that 55% of the men and 15% of the women smoke cigarettes. What is the probability that a student observed smoking a cigarette is a man?

4. Suppose n balls are distributed in $n+1$ boxes. What is the probability that exactly one box is empty?

5. Two boxes each have 7 balls labeled $1,2,...,7$. A random sample of size 4 is drawn without replacement from each box. Find the probability that the samples contain exactly 2 balls having the same numbers in common.

6. A box has b black balls and r red balls. Balls are drawn from the box one at a time without replacement. Find the probability that the first black ball selected is drawn at the nth trial.

7. Let N be a positive integer and let

$$f(x)=\begin{cases} cx3^x, & x=1,2,...,N \\ 0, & \text{otherwise.} \end{cases}$$

find the value of c such that f is a probability density.

8. Let M be a positive integer. Let X be a geometrically distributed random variable having parameter p. Let $Y=X$ if $X<M$ and let $Y=M$ if $X \geq M$; that is, $Y=\min(X,M)$. Compute the density f_Y of Y.

(背面尚有試題)

$\boxed{9.}$ Let X and Y be independent random variables each having a geometric density with parameter p. Set $M = \min(X,Y)$ and $Z = Y - X$. Please find the probability $P(M = 5, Z = -10)$.

$\boxed{10.}$ Consider an experiment having three possible outcomes that occur with probabilities $1/3$, $1/6$, and $1/2$, respectively. Suppose 10 independent repetitions of the experiment are made and let X_1, X_2 denote the number of times the 1st, 2nd outcome occurred respectively. What is the probability $P(X_1 + X_2 = 6)$?

$\boxed{11.}$ A die is rolled until an even number appears. How many rolls are required so that the probability of getting an even number is at least 0.9?

$\boxed{12.}$ Let X be uniformly distributed on $\{2,4,...,2N\}$. Find $\Phi_X(t)$.

$\boxed{13.}$ Suppose X and Y are two independent random variables such that $EX^4 = 3,\ EY^2 = 2, EX^2 = 1, EY = 1$. Compute $\mathrm{Var}(X^2Y)$.

$\boxed{14.}$ Let X , Y , and Z be independent random variables having finite positive variances σ_1^2, σ_2^2 and σ_3^2 respectively. Find the correlation between X-$2Y$ and Y-Z.

$\boxed{15.}$ Let X be a geometrically distributed random variable with parameter p and let $M > 0$ be an integer. Set $Y = \max(M, X)$. Compute the mean of Y.

$\boxed{16.}$ Let X and Y be independent random variables each having a geometric density with parameter p. Find $E[Y \mid X + Y = 24]$.

(Hint: $E[\ Y \mid X = x\ ] = \sum_y y\, P(Y = y \mid X = x)$)

$\boxed{17.}$ Let X be a nonnegative integer-valued random variable whose probability generating function is given by $\Phi_X(t) = e^{\lambda(t^2-1)}$,where $\lambda > 0$. Find f_X .

$\boxed{18.}$ Let X be such that $P\{X = 1\} = p = 1 - P\{X = -1\}$. Find $c \neq 1$ such that $E\,[c^X] = 1$.

$\boxed{19.}$ Suppose $0 < P(A) < 1$ and $P(A \mid B) = 1$. Find $P(B^c \mid A^c)$.

$\boxed{20.}$ Give an example of random variable such that in general the bound given by Chebyshev's inequality cannot be improved.

（下頁尚有試題）

Part II 每題八分

$\boxed{21.}$ Give a detailed definition of nonnegative integers random variables.

$\boxed{22.}$ Hypergeometric distribution. Suppose we have a population of n objects, r of which are of type one and $n-r$ of type two. A sample of size n is drawn without replacement from this population. Let S_n denote the number of objects of type one that are obtained. Compute the Variance of S_n.

$\boxed{23.}$ Let S_n be Poisson Random variable with parameter n. Prove that

$\lim P(|\frac{S_n}{n}-1|\geq\delta)=0$ as $n\to\infty$ for every $\delta>0$. (Hint: Weak law of large numbers)

$\boxed{24.}$ Let $\{X_n;n\geq 1\}$ be independent nonnegative integer valued random variables have a common density. Set $S_0=0$ and $S_n=X_1+\ldots+X_n, n\geq 1$. Let N be a nonnegative integer valued random variable and suppose that N, $X_1, X_2, \ldots$ are independent. Then $S_N=X_1+\ldots+X_N$ is the sum of random number of random variables. Show that the probability generating function of S_N is given by $\Phi_{S_N}(t)=\Phi_N(\Phi_{X_1}(t))$, $-1\leq t\leq 1$.

$\boxed{25.}$ Make a census on some kind of disease in a community with large population. Now check blood for N citizens in two ways. (1) Each person each time, so need check N times. (2) Check the mixture of blood of a group of k people. If the outcome reports no virus, that means all these k people are not of this disease; while if the outcome reports virus, then each person from this group is checked again, so k people need checked $k+1$ times in this way. Discuss which way may decrease the number of checks? (Hint: consider the expectation.)