[CS 2336]Discrete Mathematics: Autumn 2009

2^{rd} exam (close book)
Examination Date: Nov. 30, 2009
Time: 13:10-15:00

1. a) it's suffice to show that $\mathcal{R}$ is (i) reflexive, (ii) symmetric, (iii) transitive. Let $(x,y) \in A$, $x+y = x+y \Rightarrow (x,y)\mathcal{R}(x,y) \Rightarrow$ reflexive. Let $(x_1,y_1),(x_2,y_2) \in A$, $x_1+y_1 = x_2+y_2$, $x_2+y_2 = x_1+y_1 \Rightarrow (x_1,y_1)\mathcal{R}(x_2,y_2),(x_2,y_2)\mathcal{R}(x_1,y_1) \Rightarrow$ symmetric. Let $(x_1,y_1),(x_2,y_2),(x_3,y_3) \in A,(x_1,y_1)\mathcal{R}(x_2,y_2),(x_2,y_2)\mathcal{R}(x_3,y_3) \Rightarrow x_1+y_1 = x_2+y_2 = x_3+y_3, so x_1+y_1 = x_3+y_3 and (x_1,y_1)\mathcal{R}(x_3,y_3) \Rightarrow$ transitive. Since $\mathcal{R}$ is reflexive, symmetric, and transitive, it is an equivalence relation.

   b) $[(1,3)] = \{(1,3),(3,1),(2,2)\}, [(2,4)] = \{(2,4),(4,2),(3,3)\}, [(1,1)] = \{(1,1)\}$.

   c) $\{(1,1)\} \cup \{(1,2)(2,1)\} \cup \{(1,3)(3,1)(2,2)\} \cup \{(1,4)(4,1)(2,3)(3,2)\} \cup \{(2,4)(4,2)(3,3)\} \cup \{(3,4)(4,3)\} \cup \{(4,4)\}$.

2. a) yes, the glb ϕ and lub $1,2,3$ are in A.

   b) no, the glb 1 is not in A.

   c) yes, a ordered relation is lattice. (for example: N)

3. a) Spanning graph has the same vertices set, and each edge of G can be exist or not. So, there are 7 edges in G, the number of spanning sub-graphs is 2^7.

   b) Connected means all vertices are in a same sub-graph. So, only edge $\overline{gf}, \overline{gh}, \overline{fh}$ can be removed. Furthermore, cannot removing more then one edge or the graph will be disconnected. So we can choose removing $\overline{gf}, \overline{gh}, \overline{fh}$, or not removing any edges, total 4 connected subgraphs.

   c) f must be isolated means that $\overline{fg}, \overline{fh}, \overline{df}$ cannot exist. So, there remains 4 edges to choose. The nubmer of sub-graph is 2^4.

4.

Name	Repeated Edges	Repeated Virtices	Open/Close
Trail	No	Yes	Open
Circuit	No	Yes	Close
Path	No	No	Open
Cycle	No	No	Close

5. a) 2^{10}

   b) 2^{14}

   c) $2^5 \times 3^9$

   d) 2^5

   e) 1

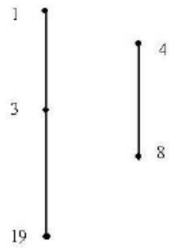
   f) $\sum_{i=1}^{5} S(5,i) = 52$

   g) $\sum_{i=1}^{3} S(3,i) = 5$

6. $\delta|V| \leq \sum_{v \in V} \deg(v) \leq \Delta|V|$. Since $2|E| = \sum_{v \in V} \deg(v)$, it follows that $\delta|V| \leq 2|E| \leq \Delta|V|$, so $\delta \leq 2(e/n) \leq \Delta$.

7. R^{11}={ (a,a), (a,c), (a,e), (a,d), (b,e), (b,d), (c,d), (c,e), (d,d), (d,e)}

8.



9.

Theorem 7.6

If $\mathfrak{R}$ is an equivalence relation on a set A, and $x, y \in A$, then (a) $x \in [x]$; (b) $x \mathfrak{R} y$ if and only if $[x] = [y]$; and (c) $[x] = [y]$ or $[x] \cap [y] = \emptyset$.

a) 1. $x \mathfrak{R} x$ because $\mathfrak{R}$ is an equivalence relation
2. This implies $x \in [x]$
b) (⇒) 1. If $x \mathfrak{R} y$, let $w \in [x]$.
2. Then, $w \mathfrak{R} x$
3. $w \mathfrak{R} y$ because $\mathfrak{R}$ is transitive
4. Hence $w \in [y]$ and $[x] \subseteq [y]$.
5. $y \mathfrak{R} x$ because $\mathfrak{R}$ is symmetric
6. if $t \in [y]$, then $t \mathfrak{R} y$ and by the transitive property, $t \mathfrak{R} x$.
7. Hence $t \in [x]$ and $[y] \subseteq [x]$. Consequently, $[x] = [y]$.

Theorem 7.6 (2)

If $\mathfrak{R}$ is an equivalence relation on a set A, and $x, y \in A$, then (a) $x \in [x]$; (b) $x \mathfrak{R} y$ if and only if $[x] = [y]$; and (c) $[x] = [y]$ or $[x] \cap [y] = \emptyset$.

b) (⇐) 1. let $[x] = [y]$.
2. Since $x \in [x]$ by part (a), then $x \in [y]$ or $x \mathfrak{R} y$.
c) 1. We assume that $[x] \neq [y]$ and show how it then follows that $[x] \cap [y] = \emptyset$.
2. If $[x] \cap [y] \neq \emptyset$, then let $v \in A$ with $v \in [x]$ and $v \in [y]$.
3. Then $v \mathfrak{R} x$, $v \mathfrak{R} y$
4. $x \mathfrak{R} v$ because $\mathfrak{R}$ is symmetric
5. $(x \mathfrak{R} v$ and $v \mathfrak{R} y) \Rightarrow x \mathfrak{R} y$ because $\mathfrak{R}$ is transitive
6. $x \mathfrak{R} y \Rightarrow [x] = [y]$ by part (b).
7. This contradicts the assumption that $[x] \neq [y]$