EE2336 02 Discrete Mathematics Final Exam

June 22 2005

---

(1) Note that there are three pages in total.
(2) Please remember to write down your name and ID.

True or False (30pts)

F 1. A connected multigraph has an Euler circuit iff it has exactly 2 vertices of odd degree.

T 2. For all $n \geq 3$, C_n is bipartite if n is even, and C_n is not bipartite if n is odd. Cu

F 3. A full m-ary tree with i internal vertices contains $mi + 1$ leaves. vertice

T 4. For a binary tree of height h, the number of leaves $l \leq 2^h$. If the binary tree is full and balanced, then $h = \lceil \log_2 l \rceil$.

T 5. Let G be a directed multigraph with vertex set V and edge set E. Then $\sum_{v \in V} \deg^+(v) + \sum_{v \in V} \deg^-(v) = 2|E|$.

F 6. The transitive closure of a relation R equals the composite relations R^*.

F 7. The recurrence relation $b_n = b_{n-1} + 5\, b_{n-1} + 2\, b_{n-1}^{\;2} + 2$ is non-homogeneous, non-linear, and not with constant coefficients.

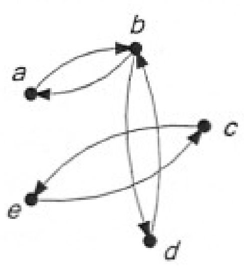
F 8. A pseudograph may have multiple edges between two vertices, but self-loop is not allowed.

T 9. The equivalence classes form a partition of a set S. That is, for a partition $\{A_i \mid i \in I, I \text{ is the index set}\}$, $(\forall i \in I\ (A_i \neq \phi)) \wedge (\forall i \neq j, ij \in I\ (A_i \cap A_j = \phi)) \wedge \bigcup_{i \in I} A_i = S$.

T 10. Both adjacency matrices and incidence matrices can represent multigraphs and loops.

Answer the Question

1. (8pts) Find the solution of the recurrence relation $a_n = 8a_{n-1} - 20a_{n-2} + 16a_{n-3} + 2^n$ with $a_0 = 6, a_1 = 20, a_2 = 28, a_3 = 32$.
2. (12pts) Given a relation R with the directed graph shown,



(a) (2pts) Is it reflexive? State your reason. If not reflexive, find the reflexive

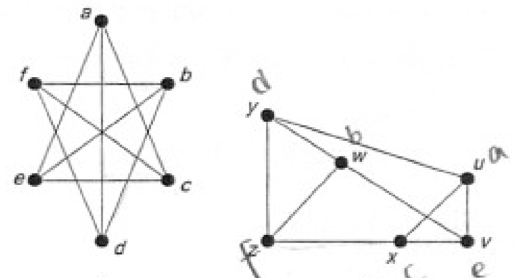
---

closure of R with the directed graph. Note that the reason is a must for this and the following questions.

(b) (8pts) Find the transitive closure of R with the directed graph.

(c) (2pts) Is the transitive closure of R an equivalence relation? State your reason. reflexive. symmetric. transitive

3. (8pts) Determine whether the graphs G and H are isomorphic. If no, show the invariant(s) they do not meet. If yes, find the isomorphism f. You must state the reason by examining the invariants.



G H

4. (9pts) Suppose that the function f satisfies the recurrence relations

$$f(n) = 2f(\sqrt{n}) + 1$$ whenever n is a perfect square greater than 1 and $f(2) = 1$.

(a) (3pts) Find $f(16)$.

(b) (6pts) Find a big-O estimate for $f(n)$.

Hint: Make the substitution $m = \log n$.

Master Theorem: Let f be an increasing function $f(n)$ that, for all $n=b^k$ for all $k \in \mathbf{Z}^+$, satisfies the recurrence relation: $f(n) = af(n/b) + cn^d$ where $a \geq 1$, integer $b>1$, real $c>0$, $d \geq 0$. Then

$$f(n) \in \begin{cases} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

$f(\sqrt{10})$ $f(10^m) = 2f(10^{\frac{m}{2}}) + 1$ $m = \log \sqrt{n}$ $(\log n)^{\frac{1}{2}}$ $\sqrt{2^m}$

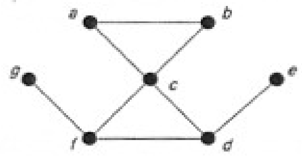
5. (10pts) There are four coins. One of them may be counterfeit or not. If it is counterfeit, it may be lighter or heavier than others. We now want to use a balance scale to determine whether there is a counterfeit coin, and if there is, whether it is lighter or heavier.

(a) (4pts) Using a decision tree, how many leaves are there for this problem? What is the minimum number of weighings needed?

(b) (6pts) Draw the decision tree to describe how to find the counterfeit coin and determine whether it is lighter or heavier using the minimum number of weighings.

6. (6pts) Prove that a tree with n vertices has $n - 1$ edges.

7. (8pts) Use adjacency matrix to prove that the length of the shortest paths between every two vertices of the following graph is less than or equal to 3.



8. (10pts) How many vertices and how many edges do these graphs have? Also identify the names of these graphs. Note that you have to describe how to figure out the numbers briefly, rather than just write them down!

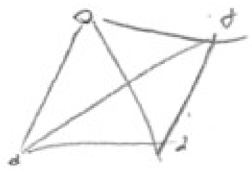
   (a) (2pts) K_n

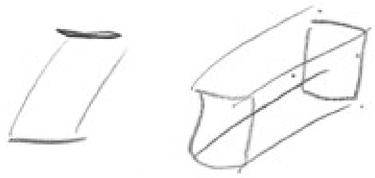
   (b) (2pts) C_n Vertice && edge

   (c) (2pts) W_n

   (d) (2pts) $K_{m,n}$ complete bipartite

   (e) (2pts) Q_n cube





2 2

1 4 12

2^1 2^2 2^3

2^{n-1}

$2(2^{n-1}) + 2^{n-1}$

$2 \cdot 4 + 4$

$2^n + 2^{n-1}$

$2^{n-1}(2+1)$

Good luck and happy examining!