Calculus II Final C

2015/06/16

Name:______________ Student ID:______________

1. (7%) 數學家Galois 怎麼死的?

16, 17 2.(14%) Test the series for absolute convergence, conditional convergence, or divergence.

(a) $\sum_{n=0}^{\infty} \frac{n^3}{n!}$ Y16 (b) $\sum_{n=3}^{\infty} (-1)^n \frac{1}{n \ln n[\ln(\ln n)]}$ Y17

35 3.(7%) Find the Taylor series of the function $f(x) = x^2 \cos x$ at $x = 0$ $\Sigma \frac{(-1)^n}{(2n)!} x^{2n} \times x^2$

Y12 4. (7%) Find the absolute maximum of the function $f(x, y) = \frac{x^2 y}{y^2 + 1}$ on $[-1,1]\times[-1,1]$.

Y51 5. (7%) Let $u = x^2 - xy$; $x = s\cos t$, $y = t\sin s$. Find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$.

6. (28%) Calculate: $\frac{\pi}{4} \sim 0$ $1 \sim 0$ $\int x \tan^{-1} x^2 dx$ Y41 tan

(a) $\int_0^1 x \tan^{-1} x^2 dx$ (b) $\int_0^{\frac{1}{2}} \int_0^{\sqrt{1-x^2}} xy\sqrt{x^2+y^2}\, dydx$ Y46 (c) $\lim_{x\to 0}(x \ln|\sin x|)$

39 (d) $\iiint_T z\sqrt{x^2+y^2+z^2}\, dxdydz$ $T: 0 \le x \le \sqrt{9-y^2}, 0 \le y \le 3, 0 \le z \le \sqrt{9-(x^2+y^2)}$

7. (14%) Determine whether the following statements are true or false:

24 (a) If $\sum_{n=0}^{\infty} a_n$ absolutely converges, then $\sum_{n=0}^{\infty} a_n$ converges.

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24. (b) If $\sum_{n=0}^{\infty} a_n$ converges to zero, then $\{(a_n)^2\}_{n=0}^{\infty}$ also converges to zero.

8. (7%) Find the directional derivative of the function $f(x, y) = x^2 + x\sin\frac{\pi y}{4} + z^3$ at $(1,-1,1)$ in the direction of $(1,1,0)$.

$\tan^{-1} x$

9. ~~8~~. (14%) Find the interval of convergence.

Y22 (a) $\sum_{n=1}^{\infty} \frac{\ln n}{n} x^n$ (b) $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!} x^{2n}$ Y27

Answer Options

Y1. $\sqrt{2}-\frac{1}{2}+\frac{\pi}{4}$

Y2. $\sqrt{2}-\frac{1}{2}-\frac{\pi}{4}$

Y3. $\sqrt{2}-\frac{1}{2}+\frac{\pi}{8}$

Y4. $2\sqrt{2}-1+\frac{\pi}{4}$

Y5. $2\sqrt{3}+\frac{\pi}{4}$

Y6. $\frac{\pi}{8}-\frac{\ln 2}{4}$

Y7. $\frac{\pi}{8}+\frac{\ln 2}{4}$

Y8. $\frac{\pi}{8}$

Y9. $\frac{\pi \ln 2}{8}$

Y10. $\frac{\pi}{4}+\frac{\ln 2}{2}$

Y11. $\frac{1}{3}$

Y12. $\frac{1}{2}$

Y13. $\frac{5}{2}$

Y14. 2

Y15. $\frac{3}{2}$

Y16. absolute convergence

Y17. conditional convergence

Y18. divergence

Y19. $[-1,1]$

Y20. $(-1,1)$

Y21. $[-2,2)$

Y22. $[-1,1)$

Y23. $(-1,0)$

Y24. True

Y25. False

Y26. $(-\infty,0)$

Y27. $(-\infty,\infty)$

Y28. $x=0$

Y29. $(0,\infty)$

Y30. $[-2,2]$

Y31. $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$

Y32. $\sum_{n=0}^{\infty}\frac{x^{n+2}}{(n)!}$

Y33. $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$

Y34. $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+3}$

Y35. $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n+2}$

Y36. $\frac{3125\pi}{20}$

Y37. $\frac{81\pi}{20}$

Y38. $\frac{49\pi}{20}$

Y39. $\frac{243\pi}{20}$

Y40. $\frac{625\pi}{20}$

Y41. $\frac{19}{480}$

Y42. $\frac{19}{240}$

Y43. $\frac{17}{240}$

Y44. $\frac{17}{480}$

Y45. $\frac{13}{480}$

Y46. 0

Y47. ∞

Y48. 1

Y49. -1

Y50. $-\infty$

Y51. $\frac{\partial u}{\partial s}=2s\cos^2 t-t\sin s\cos t-st\cos t\cos s$

$\frac{\partial u}{\partial t}=st\sin s\sin t-2s^2\sin t\cos t-s\cos t\sin s$

Y52. $\frac{\partial u}{\partial s}=2s\cos^2 t-t\sin s\cos t-st\cos t\cos s$

$\frac{\partial u}{\partial t}=\sin s\sin t-s\sin t\cos t-s\cos t\sin s$

Y53. $\frac{\partial u}{\partial s}=3s\cos^2 t-t\sin s\cos t-st\cos t\cos s$

$\frac{\partial u}{\partial t}=st\sin s\sin t-3s^2\sin t\cos t-s\cos t\sin s$

Y54. $\frac{\partial u}{\partial s}=s\cos t-t\sin s\cos t-s\cos t\cos s$

$\frac{\partial u}{\partial t}=st\sin s\sin t-2s^2\sin t\cos t-s\cos t\sin s$

Y55. $\frac{\partial u}{\partial s}=st\sin s\sin t-2s^2\sin t\cos t-s\cos t\sin s$

$\frac{\partial u}{\partial t}=2s\cos^2 t-t\sin s\cos t-st\cos t\cos s$