Exam IV for CS 3332 機率統計

3:20 ~ 5:10 p.m., January 14, 2005

1. (20%) Let X_1 and X_2 be independent Poisson random variables with respective means $\lambda_1 = 2$ and $\lambda_2 = 1$. Suppose $Y_1 = \min\{X_1, X_2\}$ and $Y_2 = X_1 + X_2$. Find (a) $P(Y_1 \geq 2)$, (b) $P(Y_2 \geq 1)$, (c) $Var(Y_2)$, (d) $Var(X_1 X_2)$.

2. (20%) Given a fair four-sided die, let Y be the no. of rolls needed to observe each face at least once.
   (a) Argue that $Y = X_1 + X_2 + X_3 + X_4$, where X_i has a geometric distribution with $p_i = \frac{5-i}{4}$, $i = 1, 2, 3, 4$, and X_1, X_2, X_3, X_4 are independent.
   (b) Find the mean and variance of Y.
   (c) Find the m.g.f. of Y.
   (d) Find $P(Y = y), y = 4, 5, 6, 7$.

   $\frac{4}{4} \; \frac{3}{4} \; \frac{2}{4} \; \frac{1}{4}$

3. (20%) Suppose $X_1, X_2, \ldots, X_n$ are a random sample of size n from the normal distribution $N(\mu, \sigma^2)$. If $W = YZ$, where $Y = \sum_{i=1}^n a_i X_i$, and $Z = \sum_{i=1}^n b_i X_i$, (a_i and b_i are real constants). Find (a) $E(W)$, (b) $Var(W)$. Note: Y and Z may be stochastically dependent.

4. (20%) Suppose X_1, X_2, X_3, X_4 are a random sample of size 4 from $\chi^2(2)$. Estimate $P(1 \leq \bar{X} < 3)$ using (a) the Central Limit Theorem, (b) Chebyshev's inequality, respectively.

5. ~~(10%) Suppose $X_1, X_2, \ldots, X_n$ are a random sample of size n from the normal distribution $N(\mu, \sigma^2)$.~~ Show that the sum $W_n = \sum_{i=1}^n X_i$ does not have a limiting distribution.

6. (10%) Let Y be $b(n, 0.55)$. Find the smallest value of n so that (approximately) $P\left(\frac{Y}{n} > \frac{1}{2}\right) \geq 0.95$.

$P(\frac{Y}{n} - 0.55$

$p(\frac{Y}{n} < \frac{1}{2}) < 0.05$

$\left|\frac{Y}{n} - 0.55\right| \leq \frac{1}{2} - 0.55$

Y_2 X_1

$X_1 \geq 1$

$Y_2 \geq 1$

$5(e^{-2})\left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!}\right)$

$\frac{2^0}{0!} \quad \frac{2^1}{1!} \quad \frac{2^2}{2!}$

$1 \quad 2 \quad 2$

$1 + 2 + \frac{4}{2}$

$e^{-1}\left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!}\right)$