Calculus Midterm
2014/11/11

1. (10 points) Prove that if $f : [0,1] \to [0,1]$ is continuous, then there exists $c \in [0,1]$ such that $f(c) = c$.

2. (10 points) State and prove the ~~product~~ reciprocal rule of limit.

3. (10 points) If $f(x) = \begin{cases} x^2 & , x \in \mathbb{Q} \\ \sin^3 x & , x \notin \mathbb{Q} \text{ and } x \in \mathbb{R} \end{cases}$, then prove or disprove that $\lim_{x \to 0} f(x) = 0$.

4. (10 points) Show that $\tan x \geq x^2$, $\forall x \in \left[0, \frac{\pi}{8}\right]$.

5. (10 points) Prove or disprove that $\lim_{x \to 0} \sin^3 x \sin\left(\frac{1}{x^5} + 4\right)$ exists.

6. (15 points)

   (a) If $f(x) = \sec^6\left(\sin^3\left(x^3 + 2\right)\right)$, then compute $f'(x)$.

   (b) If $\tan\left(x - y^2\right) = \left(x^2 + 4\right)^{20}$, then compute $\frac{dy}{dx}$.

   (c) Let $f(x) = \frac{\sin^2\left(x^2 - 1\right)}{1 + x^4}$, then compute $f''(x)$.

7. (10 points) If $f\left(\frac{x-1}{\cos^2 x + 1}\right) = x^2$, then compute $f'(0)$.

8. (10 points) Prove that if $\lim_{x \to c} f(x) = L \neq 0$ and $\lim_{x \to c} g(x) = 0$, then $\lim_{x \to c} \frac{f(x)}{g(x)}$ does not exists.

9. (10 points) Compute $\lim_{x \to 0} \frac{\cos x - 1}{x^2}$.

10. (5 points) How did the police officer recognize that professor's classmate lived in Bellingham of the U.S. or in Bellingham of Canada ?