國立清華大學試卷

記			分		
1	-0	2	-0		
3	-0	4	-0		
5	-0	6	-0		
7	-0	8	-0		
9		10			
11		12			
13		14			
15		16			
17		18			
19		20			
總　分	100				

所　系　資工

科　目　工數

學　號　104062229

姓　名　毛羿宜

日　期　11/4

1. $yy' = 2x\sec(4y)$

$\frac{y}{\sec(4y)} \cdot y' = 2x$

$\int y\cos(4y)\,dy = \int 2x\,dx$

$\int y\cos(4y)\,dy = x^2 + c$

$\frac{1}{4}y\sin 4y + \frac{1}{16}\cos 4y = x^2 + c$ -0

$\frac{1}{16}(-1) = \frac{4}{9} + c$

$c = -\frac{1}{16} - \frac{4}{9} = -\frac{73}{144}$

$\frac{1}{4}y\sin 4y + \frac{1}{16}\cos 4y = x^2 - \frac{73}{144}$ #.

2. $y' = \frac{2x+2y}{-3x-3y+7}$ $\Rightarrow$ $(2x+2y)dx + (3x+3y-7)dy = 0$

$2x+2y = z$ $\quad dz = 2dx + 2dy \Rightarrow dy = \frac{z-2dx}{2}$

$3x+3y = \frac{3}{2}z$

$z\,dx + (\frac{3}{2}z - 7)\frac{dz - 2dx}{2} = 0$

$4z\,dx + (3z-14)(dz - 2dx) = 0$

$4z\,dx + (3z-14)dz - 2(3z-14)dx = 0$

$(-2z+28)dx + (3z-14)dz = 0$

$\frac{3z-14}{z-14}\,dz = 2dx$

$\int \frac{3z-14}{z-14}\,dz = 2x + C$

$u = z-14 \quad du = dz$

$\int \frac{3u+28}{u}\,du = 2x + C$ -0

$3u + 28\ln u = 2x + C$

$3(z-14) + 28\ln(z-14) = 2x + C$

$3(2x+2y-14) + 28\ln(2x+2y-14) = 2x + C$

$3x + 3y - 21 + 14\ln|2x+2y-14| = x + C$ #.

3. $y' + \frac{2}{x} y = \frac{5}{x^2} y^2$

$y^{-2} y' + \frac{2}{x} y^{-1} = \frac{5}{x^2}$

$u = y^{-1} \qquad \frac{du}{dx} = -y^{-2} y'$

$-u' + \frac{2}{x} u = \frac{5}{x^2}$

$u' - \frac{2}{x} u = -\frac{5}{x^2}$

$I = x^{-2}$

$u \cdot x^{-2} = \int -5x^{-4}\, dx$

$u \cdot x^{-2} = \frac{1}{3} \cdot 5x^{-3} + C$

$u = \frac{1}{3} \cdot 5x^{-3} \cdot x^2 + Cx^2$

$\frac{1}{y} = \frac{5}{3} x^{-1} + Cx^2$ #

4. $M(x,y)dx + N(x,y)dy = 0$

$IM(x,y)dx + IN(x,y)dy = 0$

$\frac{\partial IM}{\partial y} = \frac{\partial IN}{\partial x}$

$\frac{\partial I}{\partial y} \cdot M + I \frac{\partial M}{\partial y} = \frac{\partial I}{\partial x} N + I \frac{\partial N}{\partial x}$

$I\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{\partial I}{\partial x} N - \frac{\partial I}{\partial y} M$

when $I(x,y) = I(x-y)$

$u = x - y, \quad \frac{\partial I}{\partial x} = \frac{dI}{du} \cdot \frac{\partial u}{\partial x} = \frac{dI}{du} \cdot 1 = \frac{dI}{du}$

$\frac{\partial I}{\partial y} = \frac{dI}{du} \cdot \frac{\partial u}{\partial y} = \frac{dI}{du} \cdot (-1) = -\frac{dI}{du}$

$I\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{dI}{du} N + \frac{dI}{du} M$

$\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N + M}\right) du = \frac{dI}{I}$

$f(x-y)$ $\int \frac{dI}{I} = \int \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N+M}\right) d(x-y)$

$I = e^{\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N+M} d(x-y)} = e^{\int f(x-y)\, d(x-y)}$.

#

5. $y = S + \frac{1}{z}$

$y' = s' + \frac{-z'}{z^2}$

$$s' + \frac{-z'}{z^2} = P(x)\left(S+\frac{1}{z}\right)^2 + Q(x)\left(S+\frac{1}{z}\right) + R(x)$$

$$s' + \frac{-z'}{z^2} = P\left(S^2 + \frac{2S}{z} + \frac{1}{z^2}\right) + Q\left(S+\frac{1}{z}\right) + R$$

$$S' + \frac{-z'}{z^2} = (PS^2 + QS + R) + \frac{2PS+Q}{z} + \frac{P}{z^2}$$

$\because$ S is a non-homogeneous solution of $y' = P(x)y^2 + Q(x)y + R(x)$

$\Rightarrow S' = PS^2 + QS + R$

$$\therefore \frac{-z'}{z^2} = \frac{2PS+Q}{z} + \frac{P}{z^2}$$

$$-z' = (2PS+Q)z + P$$

$z' + (2PS+Q)z = -P \Rightarrow$ first linear ODE #

6. $(xy^2 + 2x^2y^3 + 3xy)\,dx + (x^2y + 2x^3y^2)\,dy = 0$

$\frac{\partial M}{\partial y} = 2xy + 6x^2y^2 + 3x$ $\quad\quad$ $\frac{\partial N}{\partial x} = 2xy + 6x^2y^2$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{Ny - Mx} = \frac{3x}{-3x^2y} = -\frac{1}{xy}$$

$$I = e^{\int -\frac{1}{xy}\,dxy} = (xy)^{-1}$$

$\Rightarrow (y + 2xy^2 + 3)\,dx + (x + 2x^2y)\,dy = 0$

$\int (y + 2xy^2 + 3)\,dx = xy + x^2y^2 + 3x + f(y) = \phi(x,y)$

$$\frac{\partial \phi(x,y)}{\partial y} = x + 2x^2y + f'(y) = x + 2x^2y$$

$f(y) = c$

$\phi(x,y) = xy + x^2y^2 + 3x = C$ #

7. $E = iR + \frac{q}{C}$

$i = q'$

$E = q'R + \frac{q}{C}$

$q' + \frac{1}{RC} q = \frac{E}{R}$

$q \cdot e^{\frac{t}{RC}} = \int \frac{E}{R} \cdot e^{\frac{t}{RC}}$

$q \cdot e^{\frac{t}{RC}} = \frac{E}{R} \cdot RC \cdot e^{\frac{t}{RC}} + k$

$q = CE + k e^{-\frac{t}{RC}}$

$q(0) = 0 = CE + k \quad \Rightarrow k = -CE$

$q = CE(1 - e^{-\frac{t}{RC}})$

$i = q' = -CE(-\frac{1}{RC}) e^{-\frac{t}{RC}}$

$i = \frac{E}{R} e^{-\frac{t}{RC}}$

$80 - 72 = i \cdot R$

$i = \frac{8}{R} = \frac{80}{R} e^{-\frac{t}{RC}}$

$\frac{1}{10} = e^{-\frac{t}{0.2}}$

$\ln \frac{1}{10} = -\frac{t}{0.2}$

$t = 0.20 \ln 10$

$t \doteqdot 0.46$ #

8.

$$Q' = Q_1 \cdot R - \frac{Q}{L} \times R$$

$$Q' = Q_1 R - \frac{R}{L} Q$$

$$Q' + \frac{R}{L} Q = Q_1 R$$

$$Q \cdot e^{\frac{R}{L}t} = \int Q_1 R \cdot e^{\frac{R}{L}t}$$

$$Q \cdot e^{\frac{R}{L}t} = Q_1 R \, d\left(\frac{L}{R}\right) e^{\frac{R}{L}t} + C$$

$$Q = Q_1 L + C \cdot e^{-\frac{R}{L}t}$$

$$Q(0) = Q_1 L + C = Q_0$$

$$C = Q_0 - Q_1 L$$

$$Q(t) = Q_1 L + (Q_0 - Q_1 L) e^{-\frac{R}{L}t}$$ #.