EE2336 02 Discrete Mathematics Midterm I   March 30 2005

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(1) Note that there is only one page in total.

(2) Please remember to write down your name and ID.

True or False (30pts)

1. Let A and B be sets. The Cartesian products $A \times B = B \times A$ if and only if $A = B$.

2. $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

3. A function f is one-to-one, or invertible, if and only if $f(x) = f(y)$ implies that $x = y$ for all x and y in the domain of f.

4. "Pig can fly only if 7 is a prime number." The truth value of this implication is true.

5. $\forall x \, \exists y \, P(x, y)$ is false if for every x there is a y for which $P(x, y)$ is false.

6. $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

7. The complement of the set A is equal to the difference of the universal set U and A, that is, $\bar{A} = U - A$.

8. If a function f is onto, its range is equal to the codomain.

9. The statement "There is exactly one student who passes the exam in the class" can be expressed as $\exists x \, \forall y \, P(x) \wedge ((y \neq x) \rightarrow \neg P(y))$ where $P(x)$ is "x passes the exam" and the universe of discourse for x and y consists of all students in the class.

10. The following statement is true: $\{\{\varnothing\}\} \subset \{\{\varnothing\}, \{\varnothing\}\}$.

Answer the Question

1. (10pts) Let A, B and C be sets. Draw the Venn diagrams for $((A - C) - (B - C)) \cup ((B \cap C) - (A \cap C))$.

2. (10pts) Draw graphs of the function $f(x) = \lceil x/2 \rceil + \lfloor x/2 \rfloor$ when $-6 \leq x \leq 6$.

3. (10pts) Construct the truth table of the compound proposition $((p \rightarrow q) \rightarrow r) \leftrightarrow s$.

4. (10pts) Let x be a real number. Show that $\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + 1/3 \rfloor + \lfloor x + 2/3 \rfloor$

5. (15pts) Prove or disprove that if n is a positive integer, then n^2 is even if and only if $9n + 8$ is even.

6. (12pts) Prove or disprove that $\{[p \vee (s \wedge q)] \wedge [(q \wedge r) \vee \neg p]\} \rightarrow [q \wedge (s \vee r)]$ is a tautology by using a series of logical equivalences, instead of using the truth table.

7. (8pts) Express the statement using predicates, quantifiers and logical connectives, if necessary: "There are exactly two students who pass the exam in the class."

Good luck and happy examining!