

CALCULUS (APRIL 13, 2006)

1. (30) Determine the absolute convergence, conditional convergence or divergence of the following series and give your reason.

(a) $\sum_2^\infty \frac{(-1)^n}{n \ln n}$, (b) $\sum_1^\infty \frac{\cos n}{n^2+1}$, (c) $\sum_1^\infty \sin(\frac{1}{n^2})$, (d) $\sum_2^\infty (\ln \frac{1}{n})^n$, (e) $\sum_1^\infty \frac{((n+2)!)^2}{3^n (n!)^2}$.

2. (10) Determine the set of convergence of the following power series.

(a) $\sum_1^\infty \frac{(3-x)^n}{n^2}$, (b) $\sum_1^\infty (\frac{x^2+1}{3})^n$.

3. (10) Determine the following limits and give your reason

(a) $\lim_{x \rightarrow 0} \frac{1-\cos x}{x(e^x-1)}$, (b) $\lim_{x \rightarrow 1} \frac{\ln(x^2)}{x-1}$.

4. (10) Find the Maclaurin series of the following functions, give the general terms.

(a) $\sinh(x^2)$, (b) $\frac{1}{(1+x^2)^2}$.

5. (10) Find the limits of the following sequence if they exist, and give your reason

(a) $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^{2n}$, (b) $\lim_{n \rightarrow \infty} \frac{\sinh x + x^2}{\cosh x - e^x}$. $x = n$ or $x \rightarrow \infty$

6. (10) Using power series method to solve the initial value problem

$y'' + y' + y = 0$, $y(0) = 1$, $y'(0) = 0$.

7. (10) Find the area inside the large loop but outside the small loop of the curve

$r = 1 + 2 \cos \theta$.

8. (10) Find the area of the surface of revolution of the parametric curve

$x = \sinh t$, $y = \cosh t$, $0 \leq t \leq 1$ about the x-axis