

CS 3130 Introduction to Computer-Aided Design of Integrated Circuits

Mid-term Exam (Time allowed: 80 mins.)

1. The on-set of a Boolean function f is shown on the right.

a. Express f as a sum of minterms. (5 pts.)

b. List all prime implicants of f . (10 pts.)

c. Find a minimum cover for f . (5 pts.)

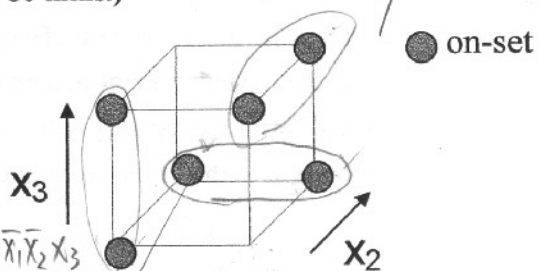
a) $f = \bar{x}_1 \bar{x}_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3$

b) x_1, x_2

$x_1 \backslash x_2$	00	01	11	10
0	1	1	0	1
1	0	1	1	1

Ans: $\bar{x}_1 \bar{x}_2, \bar{x}_2 x_3, x_1 x_3$
 $x_1 x_2, x_2 \bar{x}_3, \bar{x}_1 \bar{x}_3$

c) $\{ \bar{x}_1 \bar{x}_2, x_1 x_3, x_2 \bar{x}_3 \}$
 $f = \bar{x}_1 \bar{x}_2 + x_1 x_3 + x_2 \bar{x}_3$



2. a. What is the difference between implicants and prime implicants? (5 pts.)

b. People typically divide high-level synthesis into three subtasks, what are they? (6 pts.)

c. Explain what is a critical path in a combinational circuit. (5 pts.)

a) implicant: 只要其包含的黑點都在 on-set 或 dc-set 即可
 prime implicant: 此種 implicant 不可被其他 implicant 包含

b) Allocation, Scheduling, Assignment

c) critical path 上的 delay 會 bound 住整體 circuit 的 performance, 即為擁有最大從 input 到 output delay time 的 path.

3. Suppose the delay of a 3-input OR gate is 3ms and the delay of a 2-input OR gate is 1.5ms.

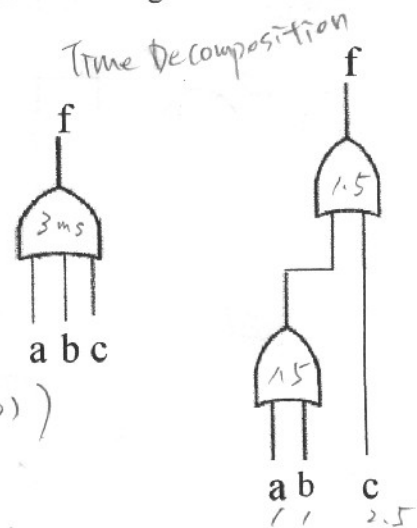
Give a situation for which the arrival time of output f is better in the right hand circuit than in the left hand circuit below. (10 pts.)

assume input arrival time

$$\begin{cases} a = 1 \text{ ms} \\ b = 1 \text{ ms} \\ c = 2.5 \text{ ms} \end{cases}$$

\Rightarrow 左圖 $\text{time}(f) = 3 + \max(a, b, c)$
 $= 3 + 2.5 = 5.5$

右圖 $\text{time}(f) = 1.5 + \max(c, 1.5 + \max(a, b))$
 $= 1.5 + \max(2.5, 1.5 + 1)$
 $= 1.5 + 2.5 = 4 \Rightarrow \text{better!!}$



4. a. Which of the following expressions are cube-free? (8 pts.)

- (i) $abc+de$ (ii) $ab+bc$ (iii) $ab(a+d)(e+d')$ (iv) $(a+bc)(d+e)+f$

b. Let $x=abcdg+abcd+abce+abcf+ag$.

(i) Construct the cube-literal matrix of x . (6 pts.)

(ii) Find all prime rectangles in the cube-literal matrix and give the kernel co-kernel pairs corresponding to the prime rectangles. (10 pts.)

Ans: (i), (iv)
(since iii) 中 b 可整除, (iii) ab 可整除, 非 cube-free)

b₁
ii)

	a	b	c	d	e	f	g
abcdg	1	1	1	1	0	0	1
abcd	1	1	1	1	0	0	0
abce	1	1	1	0	1	0	0
abcf	1	1	1	0	0	1	0
ag	1	0	0	0	0	0	1

(i) \downarrow
 $(\{1,2\}, \{1,2,3,4\}) \Rightarrow$ kernel: $g+1$ ✓
co-kernel: $abcd$ ✓

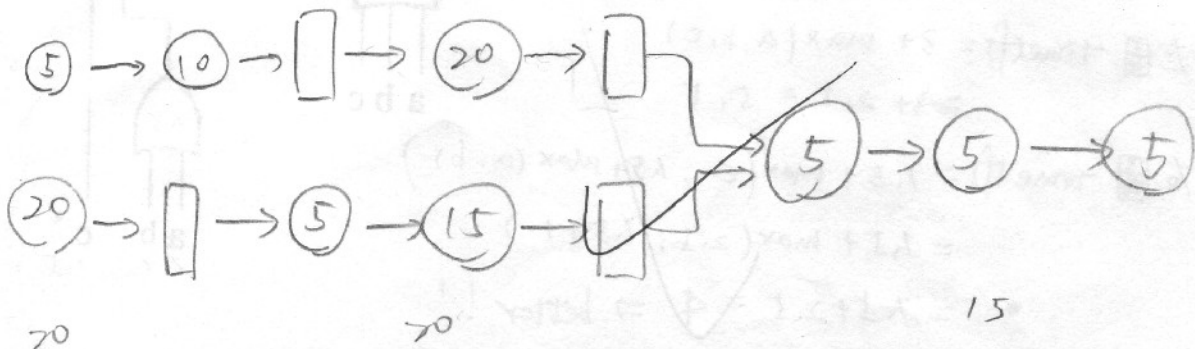
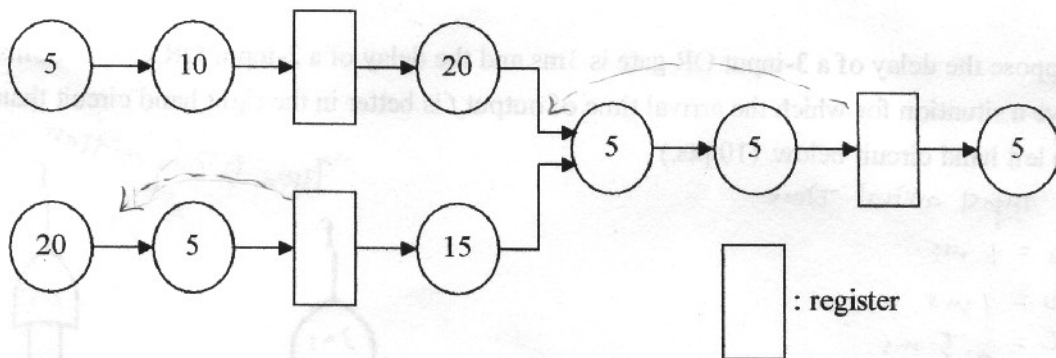
$(\{1,2,3,4\}, \{1,2,3\}) \Rightarrow$ kernel: $dg+de+ef$ ✓
co-kernel: abc ✓

$(\{1,5\}, \{1,7\}) \Rightarrow$ kernel: $1+bcd$ ✓
co-kernel: ag ✓

$(\{1,2,3,4,5\}, \{1\}) \Rightarrow$ kernel: $bcdg+bcd+bc+bcf+g$ ✓
co-kernel: a ✓

kernel	co-kernel
1	abce
1	abcf
1	abcdg

5. Consider the following circuit where the numbers inside a node denotes its delay. Is it possible to reduce the cycle time to 20 by retiming without increasing the number of pipeline stages? Justify your answer (i.e., either give a possible solution or show that it cannot be done). (15 pts.)

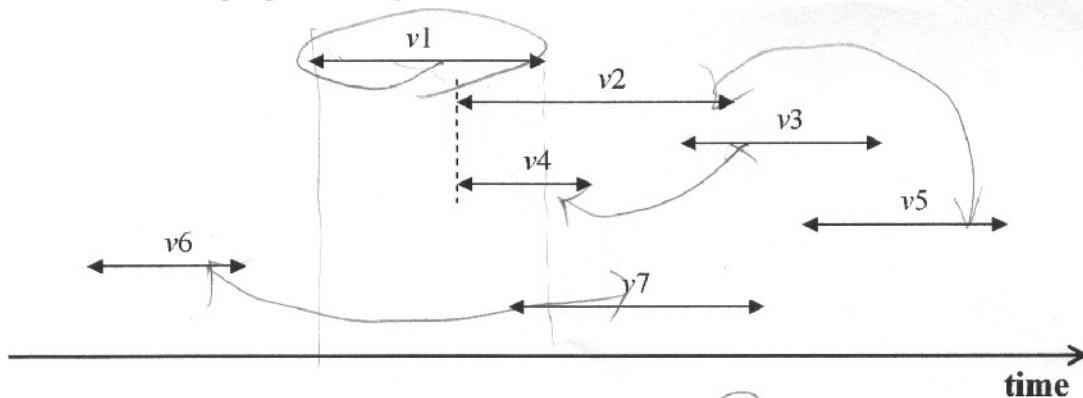


此解仍為 3 個 stage, cycle 可降至 20. (只是需要多 1 個 register)

6. Suppose the durations of tasks v1 to v7 are as shown below.

a. Draw a compatibility graph for v1 to v7. (7 pts.)

b. Hence find a clique partitioning for v1 to v7 with the minimum number of cliques. (8 pts.)



a) lifetime 未 cover 的狀況

$v1 = v3, v5, v6,$

$v2 = v5, v6,$

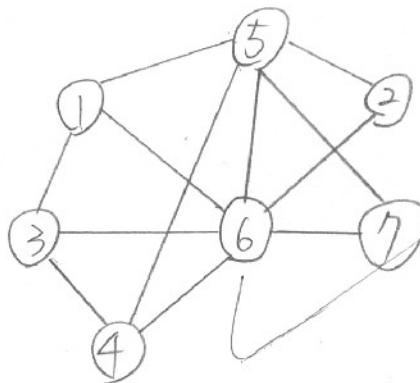
$v3 = v1, v4, v6,$

$v4 = v3, v5, v6,$

$v5 = v1, v2, v4, v6, v7$

$v6 = v1, v2, v3, v4, v5, v7$

$v7 = v5, v6$



edge 的兩端能
許 share 同一 register

b) 找 complete sub-graph (找 3 個以上的, 2 個的有 edge 即可)

1-3-6

1-5-6

2-5-6

3-4-6

4-5-6

5-6-7

一定有 6, 此拆法不見得好 (用了其中一個, 會使得某 2 個 task 獨立一組)

1. 較多 edge 的 5, 6 配較少的 2, 7; 剩下的 1, 3, 4, 不得已拆 2 組

2. $\{5, 2\}, \{6, 7\}, \{3, 4\}, \{1\}$

~~$\{2, 5, 6\}, \{7\}$~~