Probability & Statistics

1. (20%) The manufacturer of microwave ovens measured the radiation emitted through the open doors of the 20 ovens and recorded the values in the following table.

    0.09   0.29   0.13   0.44   0.62   0.08   0.73   0.37   0.04   0.01

    0.18   0.48   0.36   0.06   0.21   0.88   0.58   0.16   0.14   0.05

    (a) Draw the relative frequency histogram using 5 intervals with class marks at 0.10, 0.30, 0.50, 0.70, 0.90 and with class width 0.1.

    (b) Compute the sample mean and the sample variance of these grouped data.

2. (25%) The annual incomes (in thousands of dollars) of 15 families in a metropolitan region are listed in the following. The data were collected to determine the profitability of locating a shopping mall in the region.

    11   51   43   42   53   112   23   38   67   65   81   58   31   71   47

    (a) Construct an ordered stem-and-leaf display.

    (b) Find the first, second, and third quartiles of the sample.

    (c) Find the midrange, range, interquartile range, sample mean, and sample variance.

    (d) Draw a box-and-whisker diagram

    (e) What are the inner fences, outer fences, suspected outliners, and outliers for the above example?

3. (15%) Consider the $n=10$ observations:

    0.38   0.99   0.71   0.65   0.34   0.93   0.86   0.33   0.79   0.75

    Use the q-q plot to check whether these could be observations from the theoretical model given by the p.d.f. $f(x)=2x$, $0 \le x < 1$.

4. (20%) A quality control engineer observes that a printed circuit board with i defects ($i=0, 1, 2, 3$) seems to occur with a frequency inversely proportional to $i+1$; that is, the sample space corresponding to this experiment consists of the finite set of points $S= \{ s_0, s_1, s_2, s_3 \}$, where s_i is the event that the circuit board has i defects and $P(s_i)=c/(i+1)$, $i=0, 1, 2, 3$

    (a) Compute the constant c.

    (b) What is the probability that a circuit board has no defects?

5. (20%) An urn contains 3 balls marked LOSE and three balls marked WIN. You and an opponent take turns selecting at random a single ball from the urn without replacement. The person who selects the third WIN ball wins the game. It does not matter who selected the first two WIN balls.

    (a) If you draw first, find the probability that you win the game on your second draw.

    (b) If you draw first, find the probability that your opponent wins the game on his second draw.

    (c) If you draw first, what is the probability that you win?

    (d) If your opponent draws first, what is the probability that you win?