(1) (10%) Let
$$b_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$
 $b_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $b_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and let L be the linear transformation from

$$R^2$$
 into R^3 defined by $L(x) = x_1b_1 + x_2b_2 + (x_1 + x_2)b_3$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Find the matrix A

representing L with respect to the bases $\ [e_1,e_2\]$ into $\ [b_1,b_2,b_3\]$.

(2) (10%) Let L be the linear transformation from
$$R^3$$
 into R^3 defined by
$$L(x) = (2x_1 - x_2 - x_3, 2x_2 - x_1 - x_3, 2x_3 - x_1 - x_2)^T$$

(a) Determine the standard representation A of L and (b) use it to find $L([1,1,1]^T)$.

3.(20%) Let L be the linear operator mapping from R^3 into R^3 defined by L(x) = Ax, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix} \text{ and let } v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

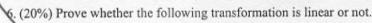
- (a) Find the transition matrix V corresponding to the change of basis from $[v_1, v_2, v_3]$ to $[e_1, e_2, e_3]$ and
- (b) Use it to determine matrix B representing L with respect to $[\nu_1, \nu_2, \nu_3]$.
- 4. (20%) Let L be the operator on P_3 defined by L(p(x)) = xp'(x) + p''(x)
- (a) Find the matrix A representing L with respect to $[1, x, x^2]$.
- (b) Find the matrix B representing L with respect to $[1, x, 1 + x^2]$.
- (c) Find the matrix S such that $B = S^{-1}AS$.

(d) If
$$p(x) = a_0 + a_1(x) + a_2(1+x^2)$$
, calculate $L^2(p(x))$.

5. (20%)Let L be the linear operator mapping from R^3 into R^2

L:
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} x_2 + x_1 \\ x_3 + x_2 \end{bmatrix}$$

- (a) Find Ker(L).
- (b) Find L(R3).



(a)L:
$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow [x_1, x_2, x_1^2 + x_2^2]^T$$
 (b) L: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow [x_1, x_2, x_1 + 2x_2]^T$

