二上(3張)

-45 +15 +30
9 +75 -6

Exam on Differential Equation Oct. 31, 2014

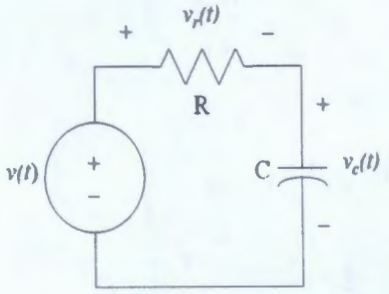
$y(t) = 2t^3 + \frac{9}{2}t^2$

1. (10%) $\frac{dy}{dt} - \frac{2y}{t} = 2t^2, y(-2) = 2$ $6t^2 + 9t - 4t^2 = 9t = 2t^2$

2. (20%)(a) (15%)Solve the initial-value problem $\frac{dy}{dt} = 5(1-y)y, y(0) = 1$ $\frac{1-y}{y} = \frac{1}{ke^{5t}}$

   (b) (5%) what are the equilibrium points ? 0, 1

3. (25%) Given the following RC circuit with v(t)= 5, R=2 and C=0.5 and $v_c(0) = 0$. Note that $\frac{dv_c(t)}{dt} = \frac{v(t) - v_c(t)}{RC}$.



$$i(t) = \frac{dq}{dt} = C\frac{dv_c}{dt}$$

$$q(t) = \int_{-\infty}^{t} i(x)dx$$

   (a) (5%)Compute the voltage $v_c(t)$ across the capacitor $5 - 5e^{-t}$

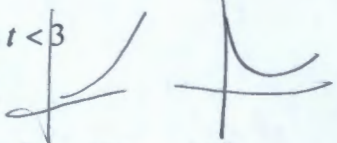
   (b) ((5%)Compute the current $i(t)$ in the circuit. $\frac{5}{2}e^{-t}$

   (c)(5%)Compute the voltage $v_c(t)$ across the capacitor in the circuit. If $(5-5e^{-t})e^{-t}$

$$v(t) = \begin{cases} 5, & for \quad 0 \le t < 3 \\ 0, & for \quad t \ge 3. \end{cases};$$

(d) (5%)Draw the solution curve for $v_c(t)$ vs. t for $0 \le t < 3$

(e)(5%)Draw the solution curve for $v_c(t)$ vs. t. for $3 \le t$



4. (10%) Find the general solution of the equation $\frac{dy}{dt} + 5y = \cos t + \cos 3t$. $ke^{-5t} + \frac{5}{26}\cos t + \frac{1}{26}\sin t + \frac{3}{34}\cos 3t + \frac{3}{34}\sin 3t$

5. (20%) (a)(10%)Solve the initial-value problem $y(t) = -e^{-t}$

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = 0, y(0) = -1, y'(0) = 1$$

   (b)10% Assume that $Y = \begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} y \\ \frac{dy}{dt} \end{bmatrix}$. Write the matrix form for the differential equation Y'=AY and Solve the initial problem of Y $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

6. (15%) (a) (10%)Find the particular solution $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 2\sin 3t$

   (b)(5%)What is the solution y(t) as t is positive and very large

$\frac{5}{39}\cos 3t - \frac{1}{39}\sin 3t + k_1e^{2t} + k_2e^{3t}$ $-\frac{5}{4}\cos \to \frac{1}{4}\sin + \frac{25}{12}$ sin $- \frac{5}{12}$ cos