CS 333400 - Engineering Mathematics

Midterm II @ 11/12/2010

※ 答案紙上需寫下計算過程，否則不予計分。

Table 3.1

$f(t)$	$F(s) = \mathfrak{L}[f(t)](s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$t^n (n = 1, 2, 3, \cdots)$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
te^{at}	$\frac{1}{(s-a)^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$

1. (10%) Solve the initial value problem using Euler Equation.

$$x^2y'' + 7xy' + 13y = 0 \,;\, y(-1) = 1 \,,\, y'(-1) = 3$$

2. (10%) Find the general solution using the method of variation of parameters.

$$y'' + 9y = 12\sec(3x)$$

3. (10%) Solve the initial value problem using the method of undetermined coefficients.

$$y'' - 4y = -7e^{2x} + x \,;\, y(0) = 1 \,,\, y'(0) = 3$$

4. (10%) Solve the initial value problem. $x^2y'' - 2xy' + 2y = 10\sin(\ln(x))\,;\; y(1) = 3 \,,\, y'(1) = 0$

5. (10%) Find the general solution $(3x+2)^2y'' + 3(3x+2)y' - 36y = 3\ x^2$ +4x +1

6. (10%) Use the linearity of the Laplace transform, and Table 3.1, to find the Laplace transform of the function.

$$t - \cos(5t)$$

7. (10%) Use the linearity of the inverse Laplace transform and Table 3.1 to find the (continuous) inverse Laplace transform of the function.

$$\frac{2s-5}{s^2+16}$$

8. (10%) Suppose that *f*(*t*) is defined for all $t \geq 0$. Then *f* is *periodic* with period *T* if $f(t+T) = f(t)$ for all $t \geq 0$, then *f* has Laplace transform : $\mathcal{L}[f](s) = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$. Find $\mathcal{L}[f]$.

f has the graph of Figure 3.3.

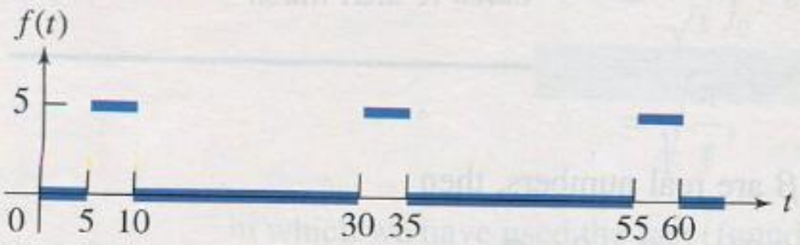

FIGURE 3.3

9. (10%) Use the Laplace transform to solve the initial value problem. $y' - 2y = 1 - t\,;\ y(0) = 4$

10. (10%) Find the Laplace transform of the function. $f(x) = \begin{cases} 1 & \text{for } 0 \leq t < 7 \\ \cos(t) & for\ t \geq 7 \end{cases}$