(1)

We know that $\dim(E_{\lambda_1}) = \mathrm{n} - 1$,

then, $\dim(E_{\lambda_2}) \geq 1$

Because A is n*n matrix, the dimension of $[L_A]_\beta$ is at most n.

=> $\mathrm{n} \geq \dim(E_{\lambda_1}) + \dim(E_{\lambda_2}) \geq n$

=> $\dim(E_{\lambda_1}) + \dim(E_{\lambda_2}) = n$

By theorem 5.1, we can know that if L_A has an ordered basis $\beta = \{v_1, v_2, v_3 \dots, v_n\}$, then L_A is diagonalizable, and hence of A is diagonalizable.

(2)

$$\mathrm{U}_1 = \mathrm{T} + \mathrm{T}^* = (\mathrm{T} + \mathrm{T}^*)^{**} = (\mathrm{T}^* + \mathrm{T}^{**})^* = (T^* + T)^* = \mathrm{U}_1^*,$$

and

$$\mathrm{U}_2 = \mathrm{TT}^* = (\mathrm{TT}^*)^{**} = (\mathrm{T}^{**}\mathrm{T}^*)^* = (\mathrm{TT}^*)^* = \mathrm{U}_2^*.$$

(3)(a)

Yes, four rules of inner product definition are satisfied.

(3)(b)

No, in this case $\forall \vec{\mathrm{v}} \neq 0$, $< \vec{\mathrm{v}}, \vec{\mathrm{v}} > \, > 0$ is not always correct.

(4)

$||x + y||^2 = (x + y)(x + y) = xx + xy + yx + yy = \|x\|^2 + \|y\|^2$,

where x and y are orthogonal vectors in V, so their inner product is zero.

then the answer $||x + y||^2 = ||x||^2 + ||y||^2$ be proved.