$S = \{5+3n \mid 0 \le n \le 18\}$. Let $a, b \in S$. $a = 5+3n$. $b = 5+3m$

$a+b = 10+3(n+m) = 70 \Rightarrow n+m = 20$

Let $S_2 = \{\{0\}, \{1\}, \{2,18\}, \{3,17\} \ldots\ldots \{9,11\}, \{10\}\}$

$n, m \in S_2$. From pegionhole principle, we have to chose at least **12** elements to ensure there will be at least two whose sum is 70.

(a) $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow (p \vee q) \wedge (p \vee \neg q)$

$\Leftrightarrow p \vee (q \wedge \neg q) \Leftrightarrow p \vee F_0 \Leftrightarrow p$ ※

(b) $\neg[\neg[(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow [(p \vee q) \wedge r] \wedge q$

$\Leftrightarrow [(p \vee q) \wedge q] \wedge r \Leftrightarrow q \wedge r$ ※

+4

(a) (i) If f is onto and one-to-one, there exist a $g(b)$ that

$g \circ f = I_A$. $f \circ g =$

(ii) Define $g: B \to A$. $g(b) = a$, $b = f(a)$. $a \in A$. $b \in B$.

For every b there is only an a that $f(a) = b$

and $(g \circ f)(a) = g(f(a)) = g(b) = a \Rightarrow g \circ f = I_A$

(b) (i) If f is invertible, $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$ and Forevery $b \in B$

there is an $a \in A$ that $f(a) \neq b$.

(ii) Invertible $\Rightarrow$ there is a g that $g(f(a)) = a$

one-to-one: $f(a_1) = f(a_2) \Rightarrow g(f(a_1)) = g(f(a_2)) \Rightarrow a_1 = a_2$ ※

onto: for every $b \in B$. $g(b) = a = g(f(a)) \Rightarrow b = f(a)$

$\Rightarrow f$ is onto.

4. (a) (i) False. $\frac{8}{3}$ is not an integer.

(ii) True.

(iii) True.

(iv) False. if $x=0$.

(v). False. $\frac{0}{0}$ doesn't exist.

(vi) True. $x=1$

(vii) False. if $x=0$

(viii) False. if $y=-x$.

-3

(b) ? -2.

5. (a) $A\times B=\{(1,2),(2,2),(3,2),(4,2),(1,7),(2,7),(3,7),(4,7)\}$

(b) $A\cup(B\times C)=\{1,2,3,4,(2,3),(2,4),(2,7),(7,3),(7,4),(7,7)\}$

(c) $(A\times C)\cap(B\times C)=(A\cap B)\times C=\{(2,3),(2,4),(2,7)\}$

6. (a)

	x	a	b	c
x	x	a	b	c
a	a	_	_	_
b	b	_	_	_
c	c	_	_	_

4^9 #

(b) $3+\frac{9-3}{2}=6$

4^6 #

$3^2 \times 3$

$81 \times 3 = 243$

216 96 150

7. (a) $q\to p$ # (b) $(s\wedge r)\to q$ #

8. $85085=5\times7\times11\times13\times17$

$243-96+3=150$

(a) $S(5,3)=\left[\sum_{k=0}^{3}(-1)^k\binom{3}{3-k}(3-k)^5\right]\times\frac{1}{3!}=(3^5-3\times2^5+3)\times\frac{1}{6}$

$=25$ #

(b) $S(5,2)2!+S(5,3)3!+S(5,4)4!+S(5,5)5!$

$=30+150+240+120=540$ #

9. (a) $p\wedge(\neg q\rightarrow\neg p)\wedge(\neg q\vee r)]\rightarrow r$

$\Leftrightarrow[p\wedge(q\vee\neg p)]\wedge(\neg q\vee r]\rightarrow r$

$\Leftrightarrow[(p\wedge q)\vee(p\wedge\neg p)]\wedge(\neg q\vee r)\rightarrow r$

$\Leftrightarrow(p\wedge q)\wedge(\neg q\vee r)\rightarrow r$

$\Leftrightarrow\neg[(p\wedge q)\wedge(\neg q\vee r)]\vee r$

$\Leftrightarrow[\neg(p\wedge q)\vee(q\wedge\neg r)]\vee r$

$\Leftrightarrow(\neg p\vee\neg q)\vee[(q\vee r)\wedge(\overset{T_0}{\underline{\neg r\vee r}})]$

$\Leftrightarrow\neg p\vee(\neg q\vee q)\vee r\Leftrightarrow\neg p\vee T_0\vee r\Leftrightarrow T_0.$ #

(b) 1. $p\rightarrow(q\rightarrow r)$

2. $p\vee s$ $\quad<\begin{matrix}\neg s\\ p\end{matrix}$

~~3. p~~

4. $q\rightarrow r$

5. $\neg r\rightarrow\neg q$

6. $t\rightarrow q$

7. $\neg q\rightarrow\neg t$

~~8.~~

$\neg r\rightarrow\neg t$