Calculus 2016 Midterm Exam

2016/11/22

1.(10%) State and prove the product rule of limit.

2.(10%) Suppose $f:[0,1]\to[0,1]$ is continuous, then prove $\exists c\in[0,1]$ such that $f(c)=c$.

3.(10%) If $f:\mathbb{R}\to\mathbb{R}$ is continuous and the range $f(\mathbb{R})=\{f(x)\mid x\in\mathbb{R}\}$ is contained in $\mathbb{Z}$, then prove f is a constant function.

4.(10%) Prove $x^2\geq\sin x-\frac{1}{4}$, $\forall x\in[\frac{1}{2},1]$.

5.(10%) If $f(x)=\begin{cases}\tan x, & x\in\mathbb{Q},\\ \sin^2 x, & x\notin\mathbb{Q},x\in\mathbb{R},\end{cases}$ then prove or disprove $\lim_{x\to 0}f(x)$ exists.

6.(10%) If $f(x)=\frac{1}{2}x^2+\cos x-1$, $\forall x\in(-\frac{\pi}{2},\frac{\pi}{2})$, then find the local extreme points.

7.(10%) $f(x)=\begin{cases}x^3-x+2, & x\leq 0,\\ x^2-2x+2, & x>0.\end{cases}$ Find the increasing intervals and the decreasing intervals.

8.(10%) $f(x)=\dfrac{\tan(x^2+1)+x^5}{\sec^2(x^3+1)}$, find $f'(x)$.

9.(10%) If $f'(0)=2$, then find $\lim_{x\to 0}\dfrac{f(x)-f(2\sin x)}{x}$.

10.(10%) $\sqrt{\sin y}-2x^2y^3+\dfrac{x}{y+1}=\csc(y^2+1)$, then find $\dfrac{dy}{dx}$.

11.(5%) 有一天，老師一大早去小雜貨店買報紙，店員跟老師說了一句什麼話?