35

Test 2 for CS2334(01)

November 28, 2016

58

StudentName : 潘信宏 *ID* : 104062205 *GroupN* : 7 *Index* : 5, 35

10

I.(20%) Mark ○ if the statement is *true*, and mark × otherwise.

(X) **(a)** Let $A \in R^{m \times n}$, then the nullspace of A must be a subspace of R^n and the column space of A must be a subspace of R^m.

(O) **(b)** Let $S = \{[x, y]^t | \, x = 2y\} \subset R^2$, then S is a vector subspace of R^2 and $dim(S) = 1$.

(X) **(c)** Let V and W be subspaces of vector space R^2 with $V \cap W = \{0\}$ and $dim(V) = dim(W) = 1$, then $V \cup W$ is a vector subspace of R^2.

(X) **(d)** If $A \in R^{n \times n}$ is nonsingular, then $Null(A) = \{0\}$.

(O) **(e)** Let S and T be vector subspaces of R^n, then $S + T$ is a vector subspace of R^n and $dim(S + T) = dim(S) + dim(T)$.

(O) **(f)** If $span(\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n) = R^m$, then $n \geq m$.

(O) **(g)** If nonzero vectors $\mathbf{x}$ and $\mathbf{y}$ have $\|\mathbf{x}\|_2 = \|\mathbf{y}\|_2$, then $(\mathbf{x} + \mathbf{y}) \perp (\mathbf{x} - \mathbf{y})$.

(O) **(h)** Let $U = span([1, 0, 1]^t)$ and $V = span([0, 1, 0]^t)$ be vector subspaces of R^3, then $U = V^{\perp}$.

(X) **(i)** Every square matrix A can be factored as the product of Q and R, where Q is orthogonal and R is $upper - \Delta$.

(X) **(j)** Let $H_1, H_2, \cdots, H_m \in R^{n \times n}$ be Householder matrices, define $H = \prod_{j=1}^{m} H_j$, then H is symmetric and orthogonal.

(O) **(k)** ~~A~~ A set of nonzero mutually orthogonal vectors must be linearly independent.

$(H_1 + H_2)(H_1^T + H_2^T)^T$

$\cancel{H_1H_1^T} + H_1H_2^T + H_2H_1^T + \cancel{H_2H_2^T}$

II.(28%) Answer each of the following questions.

28

(A) Express $\mathbf{x} = [2, 1, 3]$ as a linear combination of $\mathbf{u} = [1, 1, 1]$, $\mathbf{v} = [1, 1, 0]$, $\mathbf{w} = [1, 0, 0]$.

$x = 3u - 2v + w$

(B) Let $\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z} \in R^4$ be orthonormal vectors, then $\|\mathbf{w} - 3\mathbf{x} + 5\mathbf{y} - \mathbf{z}\|_2 = \underline{6}$

$\sqrt{1+9+25+1} = 6$

(C) Let

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 3 \\ 1 & 2 & 1 & 5 \end{bmatrix}$$

Then $dim(Null(A)) = \underline{1}$, and and the rank of A is $\underline{3}$.

(D) Let $\mathbf{x} = [1, -3, 5, 1]^t$, then $\|\mathbf{x}\|_1 = \underline{10}$, $\|\mathbf{x}\|_2 = \underline{6}$, $\|\mathbf{x}\|_\infty^2 = \underline{25}$.

(E) Let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in R^2$ and let $L : R^2 \to R^2$ be a linear transformation such that $L([1, 2]^t) = [-2, 3]^t$, $L([1, -1]^t) = [5, 2]^t$. Then $L([7, 2]^t) = \underline{[14, 17]^t}$

(F) Let $L : R^3 \to R^2$ be a linear transformation such that $L([1, 0, 0]^t) = [1, 0]^t$, $L([0, 1, 0]^t) = [1, -1]^t$, $L([0, 0, 1]^t) = [1, 1]^t$. Then $Ker(L) = \{\alpha[-2, 1, 1]^t\}$

(G) Let $\mathbf{a} = [1, 1, 1]^t$, $\mathbf{x} = [1, 3, 8]^t$, then the projection vector of $\mathbf{x}$ along the line $\mathbf{a}$ is

$(4, 4, 4)$

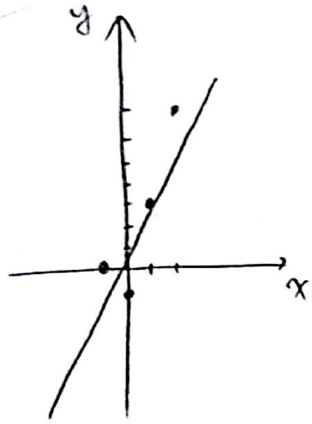
0

III.(12%) Find the best least squares fitting line for the data set $\{[-1, 0]^t, [0, 1]^t, [1, 3]^t, [2, 8]^t\}$ and plot your solution associated with the above data points.

$y = ?x + ?$

incomplete !

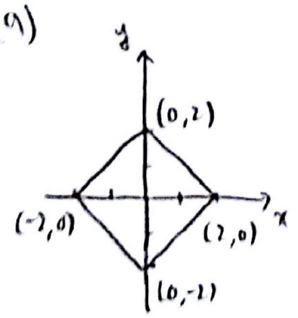
?



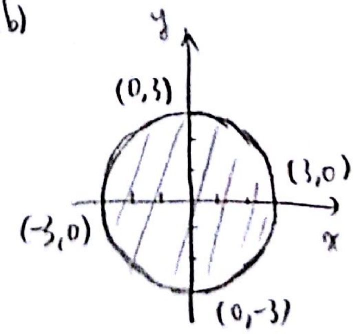
10

IV.(10%) Let $\mathbf{v} = [x, y]^t \in R^2$, draw the figure and mark its (x,y) coordinates of intersections with x and y axis respectively for (a) $\|\mathbf{v}\|_1 = 2$, (b) $\|\mathbf{v}\|_2 \le 3$, and (c) $\|\mathbf{v}\|_\infty \le 4$.

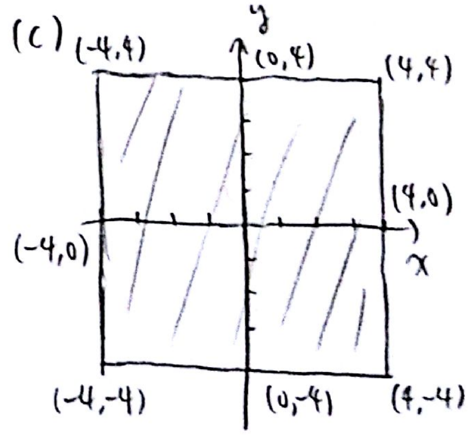
(a)



(b)



(c)



V.(10%) Let $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k$ be linearly independent vectors in R^n and let $A \in R^{n\times n}$ be a nonsingular matrix. Define $\mathbf{y}_j = A\mathbf{x}_j$ for $1 \le j \le k$. Prove that $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k$ are linearly independent vectors.

A is a nonsingular matrix $\Rightarrow \det(A) \neq 0 \Rightarrow y = AX$ 有唯一解

$\Rightarrow$ if X is L.I. then y is L.I.

VI.(20%) Let $\mathbf{w} \in R^n$ be a unit vector, that is, $\|\mathbf{w}\|_2 = 1$, and define a Householder matrix $G = I - 2\mathbf{w}\mathbf{w}^t$. Denote $\sigma = \|\mathbf{x}\|_2$, where $\mathbf{x} = [x_1, x_2, \cdots, x_n]^t \in R^n$.

10

(a) Show that G is *symmetric* and *orthogonal*.

(b) Let $\mathbf{v} = \mathbf{x} - \sigma\mathbf{e}_1$ and $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|_2}$, define $H = I - 2\mathbf{u}\mathbf{u}^t$. Show that $H\mathbf{x} = \sigma\mathbf{e}_1$.

(a) symmetric: $G^t = (I - 2ww^t)^t = I^t - 2(w^t)^t w^t = I - 2ww^t = G$ #

orthogonal: $GG^t = (I - 2ww^t)(I - 2ww^t)$

$= I - 4ww^t + 4ww^tww^t = I - 4ww^t + 4ww^t = I$ #

(b) $u = \dfrac{x - 6e_1}{\|x - 6e_1\|_2}$

$\cancel{Hx = 6e_1,\ x = H^{-1} \cdot 6e_1} = 6He_1 = 6\left(I - \frac{2}{\|x-6e_1\|}(x - 6e_1)(x - 6e_1)^t\right)e_1$

X