Communication Test 1

1. Complete the following:

   (1) $a\cos(x) - b\sin(x) =$

   (2) $\sin(x)\sin(y) =$

   (3) $\sin^2(x) = \frac{1}{2}(\qquad\qquad)$

   (4) $e^{j\theta} + e^{-j\theta} =$

   (5) $1 - 2\sin^2(x) =$

2. Let $T = \frac{1}{f_c}$. Prove the following:

   (1) $\int_0^T \sin(2\pi k f_c t)\sin(2\pi n f_c t)dt = 0$ if $k \neq n$

   (2) $\int_0^T \sin(2\pi k f_c t)\cos(2\pi n f_c t)dt = 0$ for all k and n.

sin

cos

3. Let

$$v_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$v_2 = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

   (a) Show that these two vectors are orthogonal.

   (b) The above two vectors are obtained by sampling two sinusoidal functions. State the functions for v_1 and v_2 respectively.

4. Let $v_1 = (1,3,-2,-3)$ and $v_2 = (2,-1,1,-1)$.

   (a) Show that these two vectors are orthogonal.

   (b) $v_3 = av_1 + bv_2 = (-1,11,-8,-7)$. Determine a and b.

$a+2b=-1$

$3a-b=11$

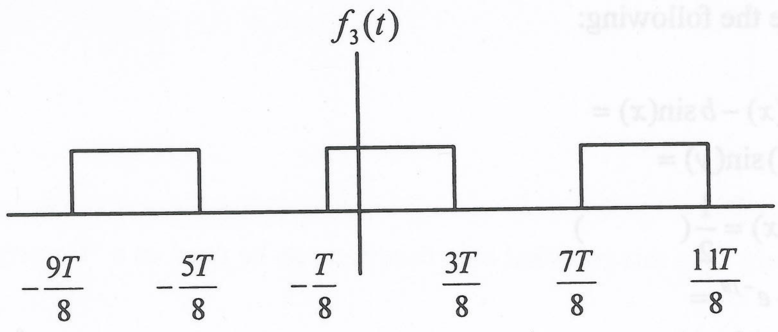
$6a-2b=22$

$7a=21$

$a=3$

$a=-2$

5. Consider the following function:



(a) Show that in this case, the Fourier series expansion coefficients are as followings:

$$a_0 = \frac{1}{2}$$

$$a_k = \frac{1}{\pi k}\left(\sin\left(\frac{k3\pi}{4}\right) + \sin\left(\frac{k\pi}{4}\right)\right)$$

$$b_k = \frac{-1}{\pi k}\left(\cos\left(\frac{k3\pi}{4}\right) - \cos\left(\frac{k\pi}{4}\right)\right)$$

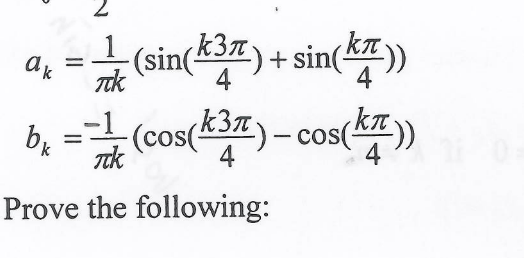
(b) Prove the following:

$$\sin\left(\frac{k3\pi}{4}\right) + \sin\left(\frac{k\pi}{4}\right) = 2\sin\left(\frac{k\pi}{2}\right)\cos\left(\frac{k\pi}{4}\right)$$

sin 1

6. Consider the following rectified sine function. Find its Fourier series coefficients:

$\left(\frac{k\pi}{4} + \frac{2k\pi}{4}\right)$



sin $= 1 - \cos 2x = \sin^2 x$

Show

(a) $a_0 = \frac{1}{\pi}$.

(b) $a_1 = 0$.

$1 + \cos 2x = \cos$

(c) For $k > 1$:

$$a_k = \frac{1}{\pi(1 - k^2)}(\cos(k\pi) + 1).$$

(d) $b_1 = \frac{1}{2}$.

(e) For $k > 1$:

$$b_k = 0.$$

(f) Explain why this rectified sine function can be used as a "frequency doubler".

7. Show that for Fourier series expansion, $a_k \cos(2\pi k f_0 t) + b_k \sin(2\pi k f_0 t)$ can be expressed as a cosine function.

8. Using the knowledge that in the Fourier series expansion in complex exponential, $X_k = \frac{1}{2}(a_k - jb_k)$ and $X_{-k} = \frac{1}{2}(a_k + jb_k)$, show that

$$X_{-k}e^{-j2\pi k f_0 t} + X_k e^{j2\pi k f_0 t}$$

can be expressed as a cosine function.

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$e^{j\theta} = \cos\theta + j\sin\theta$