CS 2336 Discrete Mathematics: 1st Exam Solution

1. 12

   {5} {8} {11,59}.....{35}

   至少要選 12 組

2 a)p

   b)q^r

過程在課本 p.63 p.64

3.

A function f: $A \to B$ is invertible if it is one-to-one and onto.

pf. 1. It suffices to show there is a function g: $B \to A$ such that $g \circ f =$ and $f \circ g =$.

2. Since f is onto, for each $b \in B$ there is an $a \in A$ with $f(a)=b$.

3. Define the function g: $B \to A$ by $g(b)=a$.

4. Clearly, $g(f(a))=g(b)=a$, implying $g \circ f = I_A$.

5. And, $f(g(b))=f(a)=b$, implying $f \circ g = I_B$.

A function f: $A \to B$ is invertible only if it is one-to-one and onto.

pf. 1. Let g be the invertible function of f. To show f is one-to-one, it suffices to show $a_1, a_2 \in A$ with $f(a_1)=f(a_2) \Rightarrow a_1=a_2$.

2. $f(a_1)=f(a_2)$

$\Rightarrow g(f(a_1))=g(f(a_2))$

$\Rightarrow a_1=a_2$

3. To show f is onto, it suffices to show for any $b \in B$, $b=f(a)$ for some $a \in A$.

4. $b=f(a)$

$\Rightarrow g(b)=g(f(a))=a$.

4.

(a) (multiple 不像 divide 有不能除 0 的問題)

   i. X

   ii. O

iii. O
iv. O
v. O
vi. O
vii. O
viii. X (x=-y)

(b)

i. X
ii. O
iii. O
iv. O
v. O
vi. O
vii. X (y!=0)
viii. O (沒有-y 的情況)

5. (參考習題 5-1-1)

(a) {(1,2) (1,7) (2,2) (2,7) (3,2) (3,7) (4,2) (4,7)}

(b) {1, 2, 3, 4, (2,3) (2,4) (2,7) (7,3) (7,4) (7,7)}

(c) {(2,3) (2,4) (2,7)}

6. (參考投影片 ch5, p37, p38)

(a) 4^9

(b) 4^6

7.

(a) $q \rightarrow p$

(b) $(r \wedge s) \rightarrow q$

8.

Since 85085 = 5 x 7 x 11 x 13 x 17

(a) S(5, 3) = 25

(b) 2! x S(5, 2) + 3! x S(5, 3) + 4! x S(5, 4) + 5! x S(5, 5) = 540

9.

(a)

(1) $\neg q \rightarrow \neg p$ Premise

(2) $p \rightarrow q$ Step (1) and $(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$

	(3)	p	Premise
	(4)	q	Step (2) and Step (3) and Rule of Detachment
	(5)	¬ q V r	Premise
	(6)	q → r	Step (5) and $(\neg q \vee r) \Leftrightarrow (q \rightarrow r)$
	(7)	r	Step (4) and Step (6) and Rule of Detachment
(b)			
	(1)	¬ s	Premise
	(2)	p V s	Premise
	(3)	p	Step (1) and Step (2) and Rule of Disjunctive Syllogism
	(4)	p → (q → r)	Premise
	(5)	q → r	Step (3) and Step (4) and Rule of Detachment
	(6)	t → q	Premise
	(7)	t → r	Step (5) and Step (6) and Law of the Syllogism
	(8)	¬ r → ¬ t	Step (7) and $(t \rightarrow r) \Leftrightarrow (\neg r \rightarrow \neg t)$