

$$\sqrt{10^2 + 25} = \sqrt{125}$$

$$\sqrt{10+4}$$

The 5th Exam on Linear Algebra Jan 10th, 2005

(1) (10%) Find the eigenvalues and the corresponding eigenvectors of

$$A = \begin{bmatrix} 4 & -5 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} -2x+z \\ -y \\ x-2z \end{bmatrix}$$

(2) (10%) Show that $A = \begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & b \end{bmatrix}$ is defective.

$$-2x^2 + xz - y^2 + xz - 2z^2$$

(3) (10%) Compute e^A for the following matrix $A = \begin{bmatrix} -2 & -1 \\ 6 & 3 \end{bmatrix}$

$$-2(x^2 + xz + z^2) - y^2$$

4. (15%) Solve the initial problem $Y' = AY$, $Y(0) = Y_0$, $Y_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$, $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

5. (15%) Find an orthogonal or unitary matrix that diagonalize A, where

$$A = \begin{bmatrix} 1 & 3+i \\ 3-i & 4 \end{bmatrix}$$

$$ce^{\lambda t}$$

6. (15%) Find the singular value decomposition of $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$.

7. (15%) Factor the following matrix into LDL^T , where L is lower triangular with 1's on the diagonal and D is a diagonal matrix

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 10 \end{bmatrix}$$

8. (10%) Find a suitable change of coordinates (i.e. rotation and/or translation) so that the resulting conic section is in standard form $x^2 + 2xy + y^2 + 3x + y - 1 = 0$.

9. (10%) Determine $A = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -2 \end{bmatrix}$ is positive definite, negative definite or indefinite.

$$-2x + z$$

$$A = \frac{A-C}{R} \quad x^2 + xy + yx + y^2$$