



0. 【5 分】依下面說明在答案卷上作答者，可得 5 分

- (i) 答案卷第一張為封面。第一張正反兩面不要作答。
- (ii) 由第二張紙開始算起，第一頁依空格號碼順序寫下所有填充題答案，寫在他頁不記分。
- (iii) 計算題之演算過程與答案依題號順序寫在第二頁以後，每題從新的一頁寫起。

Constants: $g = 10 \text{ m/s}^2$; $\log(10/3) = 0.52$; The density of air: 1.3 kg/m^3 ; the sound speed: 340 m/s .

Part I. 填充題 (每格 3%, 共 51%) 如有單位，必須寫出。

■ Which of the functions represent traveling waves? 【1】 (a) $A \cos[(kx + \omega t)^2/2]$, (b) $A \cos[(kx)^2 - (\omega t)^2]$, (c) $A \exp[-\alpha(x - vt)^{3/2}]$, (d) $A \sin^2[2\pi(-t + x/v)]$, (e) $A \exp(-\beta t) \sin(kx - \omega t)$, (f) $A(x + vt)^3$

■ When a 3-kg crown is immersed in water, it has an apparent weight of 27 N. The density of the crown is 【2】.

■ A 50-g block is attached to a horizontal spring ($k = 20 \text{ N/m}$) and its motion is described by $x = A \cos(\omega t)$, where $A = 30 \text{ cm}$. (a) At $t = T/6$, where T is the period, the kinetic energy is 【3】; (b) at $x = 2A/3$, the potential energy is 【4】; (c) the minimum time at which the kinetic and potential energies are equal is 【5】.

■ A police siren emits sound at a primary frequency of 1000 Hz. Two police cars with sirens wailing move at 40 m/sec, one toward you and the one away from you. (a) The frequency you hear from the car coming to you is 【6】; (b) The beat frequency is 【7】. (四捨五入到整數位)

■ The wave function of a wave is $y(x, t) = 4 \cos(0.2x + 40t + 0.8)$, where x and y are in centimeters, and t is in seconds. Then, (a) the wavelength is 【8】; (b) the period is 【9】; (c) the wave velocity is 【10】; (d) at $x = 1 \text{ cm}$ and $t = 0.5 \text{ s}$, the particle velocity is 【11】; (f) when the wave travels at a medium with linear mass density of 2 g/m , the average power is 【12】.

■ A speaker emits 0.1 W of acoustic power at frequency of 200 Hz. Estimate the following values by using $\pi = 3$. At a distance of 5 m, (a) the intensity is 【13】; (b) the intensity level is 【14】; (c) the pressure amplitude is 【15】; (d) the displacement amplitude is 【16】.

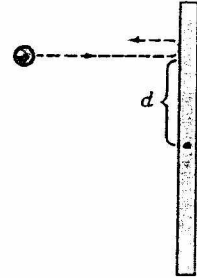
■ A girl of mass m stands still at the rim of a stationary circular platform (disk) of mass M and of radius R freely pivoted at its center ($I = MR^2/2$). She walks at the speed of v relative to earth along the rim. Then the angular velocity of the platform is 【17】.

1.

Get an expression for the energy that can send a satellite (m) initially at rest on the earth's surface (M , radius R_E) (a) vertically to a height of H ; (b) into orbit at the same height. (c) For what value of H , in terms of R_E , would the value for part (b) be triple of that in (a). Don't worry about the earth's rotation. [4%, each]

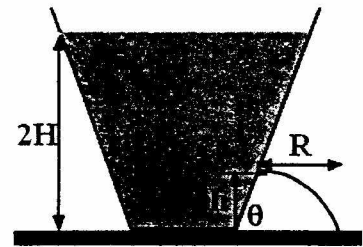
2.

There is a spot ("sweet spot") of distance d from the center at which you strike the ball to minimize the impact on your hands (assumed to be at the very end). To simplify, consider a baseball bat to be a uniform rod of length L and mass M ($I_{\text{end}} = ML^2/3$). A ball of mass m is moving perpendicular to the bat at a speed of v as in the figure. Assume the collision to be elastic. Find out (a) the total angular around the end of the bat; (b) the distance d of the sweet spot, and (c) the velocity of the center of mass of the bat and the velocity of the ball right after the collision with the ball. [3%, 3%, 6%]



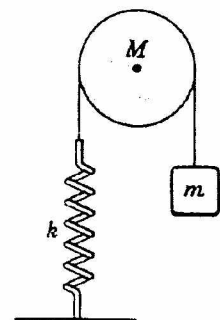
3.

The height of water in a huge tank is $2H$, which has a tiny hole above the ground by a height of h . The water coming from this hole can reach a distance of R measured from the hole. The angle between the wall of the tank and the group is 45° . Neglect the movement at the water surface in the tank. (a) What is R ? Replace half of water with oil of density of 0.5 such that the each depth of water and oil is H . (b) If the hole is located at the lower-half part (in height) of the tank, what is R ? (3) If at the upper-half part of the tank, what is R ? [4%, each]



4.

A block of mass m and a string with elastic constant k are connected via a string that hangs over a pulley ($I = MR^2/2$) of mass M and radius R , as shown here. The string does not slip. (a) What's the equilibrium extension x_0 of this block-spring system? When the block is pulled down from the equilibrium and then is released. At an arbitrary point, (b) what are the expressions of the kinetic and potential energies of this system? (c) What is the angular frequency of the oscillations? [3%, 6%, 4%]



3.

$$(a) P_0 + 2\rho_1 gH + 0 = P_0 + \rho_1 gh + \frac{\rho_1 v^2}{2}, \text{ where } \rho_1 = 1. \therefore v = \sqrt{2g(2H - h)}$$

The ejected water takes time $t = \sqrt{2h/g}$ to reach the ground. $\therefore R = vt = 2\sqrt{h(2H - h)}$

$$(b) \rho_2 gH + \rho_1 gH = \rho_1 gh + \frac{\rho_1 v^2}{2}, \therefore v^2 = 2g[(\frac{\rho_2}{\rho_1} + 1)H - h], \text{ where } \rho_2 = 0.5. R = vt = 2\sqrt{h(\frac{3}{2}H - h)}$$

$$(c) \rho_2 gH + \rho_1 gH = \rho_1 gH + \rho_2 g(h - H) + \frac{\rho_2 v^2}{2}, \therefore v^2 = 2g[2H - h], R = vt = 2\sqrt{h(2H - h)}$$

4.

(a) x_0 is the equilibrium extension where gravitational and spring forces balance,

$$mg = kx_0, \therefore x_0 = mg/k$$

(b) If set $U_g = 0$ at the natural length of spring ($x = 0$), the energy of the system is

$$E = -mgx + \frac{1}{2} kx^2 + \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2. \text{ Since the string does not slip, } v = \omega R.$$

For a block on a vertical spring, $F = -k(x - x_0) = -kx'$.

Thus the potential of this block-spring system can be written as

$$U = \frac{1}{2} k(x - x_0)^2 = \frac{1}{2} kx'^2, \text{ while the kinetic energy is } K = \frac{1}{2} (m + M/2)v^2$$

$$(c) E = K + U, dE/dt = 0 \Rightarrow (m + \frac{M}{2})v \frac{dv}{dt} + kx'v = 0,$$

$$\text{where } v = dx'/dt. \Rightarrow (m + \frac{1}{2} M) d^2x'/dt^2 + kx' = 0$$

$$\therefore \omega^2 = 2k/(M + 2m)$$

A

【1】 acdf	【2】 10^4 kg/m^3	【3】 0.675 J	【4】 0.4 J
【5】 $\pi/80 \text{ sec}$	【6】 1133 Hz	【7】 238 Hz	【8】 $10\pi \text{ cm}$
【9】 $\pi/20 \text{ sec}$	【10】 -200 cm/s	【11】 $-160\sin(21) \text{ cm/s}$	【12】 5.12 mW
【13】 $3.33 \times 10^{-4} \text{ W/m}^2$	【14】 85.2 dB	【15】 $\sqrt{2.94 \times 10^{-1}} \text{ Pa}$	【16】 $\sqrt{1.05 \times 10^{-12}} \text{ m}$
【17】 $2mv/MR$			

B

【1】 $2mv/MR$	【2】 1133 Hz	【3】 238 Hz	【4】 10^4 kg/m^3
【5】 0.675 J	【6】 0.4 J	【7】 $\pi/80 \text{ s}$	【8】 acdf
【9】 $10\pi \text{ cm}$	【10】 $\pi/20 \text{ sec}$	【11】 -200 cm/s	【12】 $-160\sin(21) \text{ cm/s}$
【13】 5.12 mW	【14】 $3.33 \times 10^{-4} \text{ W/m}^2$	【15】 85.2 dB	【16】 $\sqrt{2.94 \times 10^{-1}} \text{ Pa}$
【17】 $\sqrt{1.05 \times 10^{-12}} \text{ m}$			

1.

$$E_i = -GmM/R_E$$

$$(a) E_a = -GmM/(R_E + H), \text{ thus } \Delta E = GmM[1/R_E - 1/(R_E + H)]$$

$$(b) E_b = -GmM/2(R_E + H), \text{ thus } \Delta E = GmM[1/R_E - 1/2(R_E + H)]$$

$$(c) E_b = 3 E_a, H = 3/2 R_E$$

2.

(a) The total angular momentum of the system is solely determined by the ball before the collision, $l = mv(d + L/2)$.

(b) After the collision, the speeds of the ball and the bat are u and v_c , respectively.

$$(1) \text{ Linear momentum: } mv + 0 = -mu + Mv_c$$

$$(2) \text{ Angular momentum: } mv(d + L/2) + 0 = -mu(d + L/2) + (ML^2/3)\omega$$

$$(3) \text{ Kinetic energy: } mv^2/2 = mu^2/2 + \frac{1}{2}(ML^2/3)\omega^2$$

To minimize the impact, it requires $v_c = (L/2)\omega$.

$$\text{From (1)/(2), } \frac{m(u + v)}{m(u + v)(d + L/2)} = \frac{Mv_c}{\frac{1}{3}ML^2\omega^2} \Rightarrow d = L/6$$

$$(c) \text{ Simplify (3) } \Rightarrow v^2 = u^2 + \frac{M}{3m}v_c^2.$$

$$\text{Combined with (1), solve } v_c \text{ and } u; v_c = \frac{6mv}{3M + 4m}, u = \frac{3M - 4m}{3M + 4m}$$