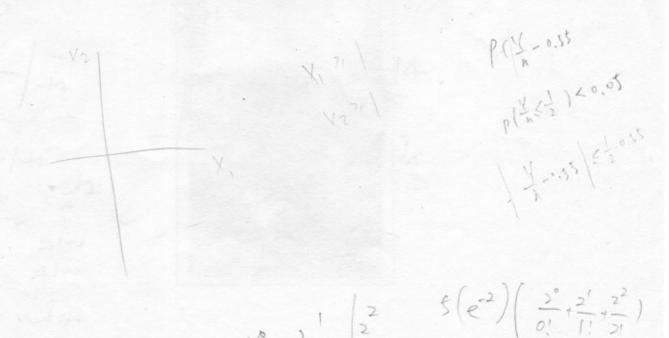
## Exam IV for CS 3332 機率統計

## 3:20 ~ 5:10 p.m., January 14, 2005

- 1. (20%) Let  $X_1$  and  $X_2$  be independent Poisson random variables with respective means  $\lambda_1 = 2$  and  $\lambda_2 = 1 \text{ . Suppose } Y_1 = \min\{X_1, X_2\} \text{ and } Y_2 = X_1 + X_2 \text{ . Find (a) } P(Y_1 \geq 2) \text{ , (b) } P(Y_2 \geq 1) \text{ , (c) } Var(Y_2) \text{ , (d) } Yar(Y_2) \text{ .}$  $Var(X_1X_2)$ .
- 2. (20%) Given a fair four-sided die, let Y be the no. of rolls needed to observe each face at least once.
  - (a) Argue that  $Y = X_1 + X_2 + X_3 + X_4$ , where  $X_i$  has a geometric distribution with  $p_i = \frac{5-i}{4}$ , i = 1, 2, 3, 4, and  $X_1, X_2, X_3, X_4$  are independent. 1 3 7 7
  - (b) Find the mean and variance of Y.
  - (c) Find the m.g.f. of Y.
  - (d) Find P(Y = y), y = 4,5,6,7.
- 3. (20%) Suppose  $X_1, X_2, \dots, X_n$  are a random sample of size n from the normal distribution  $N(\mu, \sigma^2)$ . If W = YZ, where  $Y = \sum_{i=1}^{n} a_i X_i$ , and  $Z = \sum_{i=1}^{n} b_i X_i$ , ( $a_i$  and  $b_i$  are real constants). Find (a) E(W), (b) Var(W). Note: Y and Z may be stochastically dependent.
- (20%) Suppose  $X_1, X_2, X_3, X_4$  are a random sample of size 4 from  $\chi^2(2)$ . Estimate  $P(1 \le \overline{X} < 3)$  using (a) the Central Limit Theorem, (b) Chebyshev's inequality, respectively.
- 5. (10%) Suppose  $X_1, X_2, \ldots, X_n$  are a random sample of size n from the normal distribution  $N(\mu, \sigma^2)$ . Show that the sum  $W_n = \sum_{i=1}^n X_i$  does not have a limiting distribution.
- 6. (10%) Let Y be b(n, 0.55). Find the smallest value of n so that (approximately)  $P\left(\frac{Y}{n} > \frac{1}{2}\right) \ge 0.95$ .



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