

Part I – A

1. $x=(2n+1)\pi$	2. 2A	3. B	4. $\sqrt{\frac{GM}{R_E + h}}$ $-\sqrt{\frac{GM}{R_E + 2h}}$	5. $\frac{GMm}{R_E + h}$ $-\frac{GMm}{R_E + 2h}$	6. B	7. $\frac{(\rho_{Hg} - \rho)h_2}{\rho}$	8. C
9. f+3	10. 45°	11. $5\omega^3 A^2 / k$	12. $10\pi\omega^2 A^2 / k$	13. $2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$	14. $2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$	15. $20\text{Log} \frac{r_2}{r_1}$	16. $-3Gm^2/l$

Part I – B

1. $-3Gm^2/l$	2. $\frac{(\rho_{Hg} - \rho)h_2}{\rho}$	3. $5\omega^3 A^2 / k$	4. $10\pi\omega^2 A^2 / k$	5. B	6. C	7. f+3	8. $\sqrt{\frac{GM}{R_E + h}}$ $-\sqrt{\frac{GM}{R_E + 2h}}$
9. $\frac{GMm}{R_E + h}$ $-\frac{GMm}{R_E + 2h}$	10. B	11. $2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$	12. $2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$	13. 45°	14. $20\text{Log} \frac{r_2}{r_1}$	15. $x=(2n+1)\pi$	16. 2A

Part II

1. (a) For the satellite $\sum F = ma$ $\frac{GmM_E}{r^2} = \frac{mv_0^2}{r}$

$$v_0 = \left(\frac{GM_E}{r} \right)^{1/2}$$

(b) Conservation of momentum in the forward direction for the exploding satellite:

$$(\sum mv)_i = (\sum mv)_f$$

$$5mv_0 = 4mv_i + m \cdot 0$$

$$v_i = \frac{5}{4} v_0 = \left[\frac{5}{4} \left(\frac{GM_E}{r} \right)^{1/2} \right]$$

- (c) With velocity perpendicular to radius, the orbiting fragment is at perigee. Its apogee distance and speed are related to r and v_i by $4mr v_i = 4mr_f v_f$ and $\frac{1}{2} 4mv_i^2 - \frac{GM_E 4m}{r} = \frac{1}{2} 4mv_f^2 - \frac{GM_E 4m}{r_f}$.

Substituting $v_f = \frac{v_i r}{r_f}$ we have $\frac{1}{2} v_i^2 - \frac{GM_E}{r} = \frac{1}{2} \frac{v_i^2 r^2}{r_f^2} - \frac{GM_E}{r_f}$. Further, substituting $v_i^2 = \frac{25}{16} \frac{GM_E}{r}$ gives

$$\frac{25}{32} \frac{GM_E}{r} - \frac{GM_E}{r} = \frac{25}{32} \frac{GM_E r}{r_f^2} - \frac{GM_E}{r_f}$$

$$\frac{-7}{32r} = \frac{25r}{32r_f^2} - \frac{1}{r_f}$$

Clearing of fractions, $-7r_f^2 = 25r^2 - 32rr_f$ or $7\left(\frac{r_f}{r}\right)^2 - 32\left(\frac{r_f}{r}\right) + 25 = 0$ giving

$$\frac{r_f}{r} = \frac{+32 \pm \sqrt{32^2 - 4(7)(25)}}{14} = \frac{50}{14} \text{ or } \frac{14}{14}. \text{ The latter root describes the starting point. The outer end of the orbit has } \frac{r_f}{r} = \frac{25}{7}; \quad \boxed{r_f = \frac{25r}{7}}.$$

2. (a) Suppose the flow is very slow: $\left(P + \frac{1}{2} \rho v^2 + \rho g y\right)_{\text{river}} = \left(P + \frac{1}{2} \rho v^2 + \rho g y\right)_{\text{rim}}$

$$P + 0 + \rho g 500 = P_0 + 0 + \rho g 1500$$

$$\text{ans : } P = P_0 + 10000\rho$$

- (b) The volume flow rate is

$$10 \frac{m^3}{s} = \pi \frac{d^2 v}{4}, \rightarrow \text{ans : } v = \frac{40}{\pi d^2}$$

- (c) Imagine the pressure as applied to stationary water at the bottom of the pipe:

$$P + 0 + \rho g 500 = P_0 + \frac{1}{2} \rho v^2 + \rho g 1500$$

$$P = P_0 + 10000\rho + \frac{1}{2} \rho \left(\frac{40}{\pi d^2}\right)^2 \rightarrow \text{ans} = \frac{1}{2} \rho \left(\frac{40}{\pi d^2}\right)^2$$

$$3. (a) F = -2T \frac{y}{\sqrt{L^2 + y^2}} \approx -2T \frac{y}{L}$$

$$(b) \quad F = ma \rightarrow -2T \frac{y}{L} = m \frac{d^2 y}{dt^2} \rightarrow -\frac{2T}{mL} y = m \frac{d^2 y}{dt^2} \rightarrow \omega = \sqrt{\frac{2T}{mL}}$$

$$4. \quad I = \frac{1}{2} \rho \omega^2 s_{\max}^2 v = 2\pi^2 \rho v f^2 s_{\max}^2$$

$$(a) \quad \frac{I_2}{I_1} = \frac{2\pi^2 \rho v f'^2 s_{\max}^2}{2\pi^2 \rho v f^2 s_{\max}^2} \rightarrow I_2 = \frac{f'^2}{f^2} I_1$$

$$(b) \quad \frac{I_2}{I_1} = \frac{2\pi^2 \rho v (f/2)^2 (2s_{\max})^2}{2\pi^2 \rho v f^2 s_{\max}^2} \rightarrow I_2 = I_1$$