x^4+18x^2+81

$\tan+1$, $\tan^6\theta$, $3\sec\theta$

Calculus 2 Midterm

2015 年 4 月 21 日

1. (10 points) State and prove the reciprocal rule of the sequence.

2. (30 points) Calculate the following indefinite integrals.

(1) $\int \cos(\ln x)\,dx$ (2) $\int x3^{x^2}\,dx$ (3) $\int \arcsin x\,dx$

(4) $\int \frac{x^6}{\left(x^2+9\right)^2}\,dx$ (5) $\int \frac{1}{\sqrt{4x^2-9}}\,dx$

$\sqrt{\frac{4}{9}x^2-1}$ $x>\frac{3}{2}$

$\sec^2\theta=\tan^2\theta+1$

$x=\frac{3}{2}\sec\theta$ $\frac{1}{3\sqrt{\sec^2\theta-1}}d\theta\cdots$

$\frac{1}{\cos\theta}=\frac{2}{3}x$ $\frac{dx}{d\theta}=\frac{3}{2}\sec$ $\cos\theta=\frac{3}{2x}$ $\sin^2$

$u^{\frac{1}{2}}$ $2u^{\frac{1}{2}}$

3. (15 points) $\begin{cases} a_1=1 \\ a_n=\sqrt{2+a_{n-1}} \end{cases}$, n=2, 3, 4,$\cdots$. Prove that the sequence $\{a_n\}$ is convergent an find its limit.

4. (10 points) Prove that $\lim_{n\to\infty}\frac{1}{\sqrt{n}}=0$. ① ε n_0

$\int x^{-2}dx$ $\frac{1}{-3}x^{-3}$

5. (10 points) Does the improper integral $\int_{-1}^{5}\frac{x}{x^2-9}dx$ converges or diverges? If it converges, find its limit.

$(x+3)(x-3)$

x^{-1}

6. (10 points) Compute $\lim_{x\to\infty}\frac{1}{e^x}\int_0^x e^{t^2}\,dt$ if the limit exists. L'Hopital

$\frac{1}{n}\ln\sin\frac{1}{n}$ 0

7. (5 points) Compute $\lim_{n\to\infty}\left(\sin\frac{1}{n}\right)^{\frac{1}{n}}$ if the limit exists. 0

8. (10 points) Do the following improper integrals converge or diverge? Comparison

(1) $\int_1^\infty \frac{\sin^2 2x}{x^2}\,dx$ (2) $\int_e^\infty \frac{1}{\sqrt{x^3+1}\ln x}\,dx$

$\frac{1}{\sqrt{x^3+1}\ln x}<\frac{1}{\sqrt{x^3+1}}<\frac{1}{\sqrt{x^3}}$

9. (5 points) What is the name of your teaching assistance?

吴宗軒

$10180\,81$

$x^{-\frac{3}{2}}$ $x^{-\frac{1}{2}}$ $\frac{1}{\sqrt{x}}$ $-2x^{-\frac{1}{2}}$

$(1-\sin^2\theta)^2$

$\sin^4\theta-2\sin^2\theta-1$