

CS 3332 Probability and Statistics

Final Exam

Time: 3:20pm-5:20pm, Jan. 12, 2007

(Totally 5 problems, 100 pts)

1. (20 pts) Give a Yes/No answer to each of the following statement with explanations. (Give your reasons in 1 or 2 sentences to support your answers.)
 - A. If each observation in the sample is multiplied by a positive constant c , the new sample variance is c times its original value.
 - B. If X is a random variable of a normal distribution with mean -2 and variance 100 , then the random variable $0.1 * X + 0.2$ has the standard normal distribution.
 - C. Let X_1, X_2, \dots, X_n denote a random sample of size n and \bar{X} be the sample mean. The statistic $S'^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ is an unbiased estimator to the population variance σ^2 .
 - D. If X is a normal distributed random variable with mean μ and variance σ^2 , then the random variable X^2 has a chi-squared distribution.
 - E. Increasing the sample size in the statistical hypothesis testing will reduce the type I error α and type II error β simultaneously.
2. (20 pts) Assume each observation $X_i, i=1, \dots, 100$, is drawn from a population with a continuous uniform distribution in $[0, 1]$.
 - (a) (4 pts) Give the formula for sample mean \bar{X} and sample variance S^2 for this problem.
 - (b) (8 pts) Compute the mean and variance for the sample mean \bar{X} .
 - (c) (4 pts) Try to give an approximate distribution for \bar{X} as best as you can. Explain your reason.
 - (d) (4 pts) Compute the probability that the sample mean value is larger than 0.55 .
3. (20 pts) : Assume the lifetime of a specific brand of CPUs follows a normal distribution. If five of this brand of CPUs are randomly chosen and they have lifetimes of $2.5, 3.2, 3.8, 4.2$, and 4.0 years.
 - (a) (4 pts) Compute the sample mean and sample variance.
 - (b) (8 pts) Find a 98% confidence interval for the mean of this population.
 - (c) (8 pts) Find a 98% confidence interval for the variance of this population.

4. (20 pts) A random sample of 100 students is selected for a survey and 56 of them are found to support a new regulation in the school.
- (a) (6 pts) Find a 96% confidence interval for fraction of all the students in this school favoring this new regulation.
 - (b) (4 pts) Write the result of the 96% confidence interval estimation in (a) with a mathematical equation. Note that the 96% confidence should be inside the equation.
 - (c) (4 pts) What can we assert with 96% confidence about the size of the error if we estimate the fraction of students favoring this new regulation to be 0.56?
 - (d) (6 pts) How large a sample is needed if we wish to be 96% confident that the estimation error is within 0.02 of the true fraction of all the students?
5. (20 pts) A manufacturer of a specific type of electronic components claim his products have a mean lifetime 30000 hours. Consider the problem of testing the hypothesis of $\mu = 30000$ hours against $\mu \neq 30000$ hours with a 0.01 level of significance. A random sample of 100 components is selected to test the hypothesis. Assume the population standard deviation is 1000 hours.
- (a) (6 pts) Determine the test statistic and the associated critical region.
 - (b) (6 pts) Test the hypothesis if a random mean of the 50 components is computed to be 29700 hours.
 - (c) (8 pts) What is the P-value associated with the sample mean observation given in (b)?