

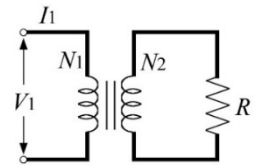


- (i) 依空格號碼順序在第二張正面寫下所有填充題答案，不要寫計算過程。  
(ii) 依計算題之順序在第二張反面以後寫下演算過程與答案，每題從新的一頁寫起。

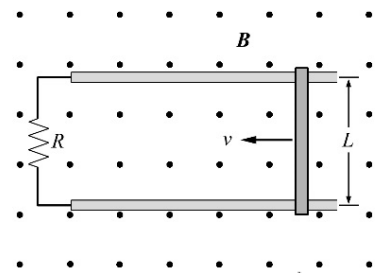
Constants:  $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ ,  $g = 9.8 \text{ m/s}^2$

### Part I. Filling the blank (5 points per blank)

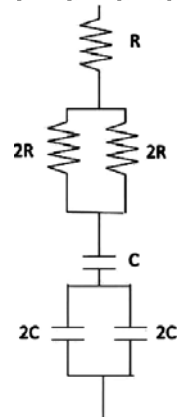
- A transformer consists of  $N_1$ -turn primary coil and  $N_2$ -turn secondary coil. The secondary circuit is completed by a resistor  $R$ , as shown in right figure. If the current and terminal voltage in the primary coil are  $I_1$  and  $V_1$ , the ratio  $V_1 / I_1 =$  **【1】**  $R$ .



- A pair of parallel conducting rails, a resistor  $R$  across the rails, and a conducting bar of negligible resistance are arranged in a uniform field  $\mathbf{B}$  as shown in right figure. The bar is pulled along the rails with velocity  $\mathbf{v}$  to the left. Take  $L = 10 \text{ cm}$ ,  $B = 0.50 \text{ T}$ ,  $R = 2.0 \Omega$ , and  $v = 2.0 \text{ m/s}$ , the magnetic force on the bar is **【2】** N. (direction must be included).



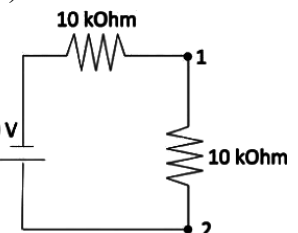
- A series  $RLC$  circuit, has  $R = 18 \text{ k}\Omega$ ,  $C = 14 \mu\text{F}$ , and  $L = 0.20 \text{ H}$ , is connected across an AC power at  $120 \text{ V rms}$  and  $60 \text{ Hz}$ . The resonance frequency is **【3】** Hz. The average power delivered to the entire circuit is **【4】** W.



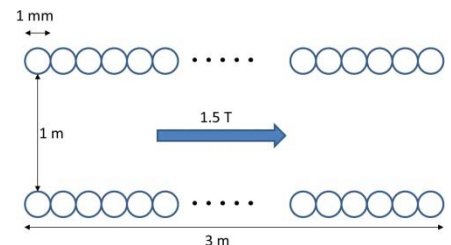
- The right diagram shows a network of resistances and capacitances. The total resistance is **【5a】**  $R$  and the total capacitance **【5b】**  $C$ .

- Which of the following descriptions are NOT related to Faraday law? (more than one answer)  
(a) Back emf, (b) Diamagnetism, (c) Eddy current, (d) Ferromagnetism, (e) Levitation(浮升) of a superconductor in a  $\mathbf{B}$  field, (f) Operation of a transformer, (g) Paramagnetism **【6】**

- The potential difference between the points 1 and 2 (see right image) is **【7a】** V. With a non-ideal voltmeter (internal resistance of  $100 \text{ k}\Omega$ ), you measure **【7b】** V.

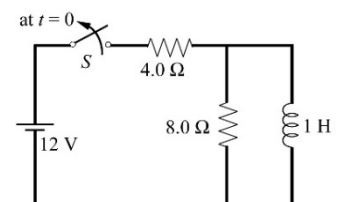


- A rectangular coil with sides  $a=5 \text{ cm}$  and  $b=10 \text{ cm}$  and carrying a current of  $1 \text{ A}$  is placed inside a magnetic field of  $1 \text{ T}$ . The maximum torques acting on the coil is: **【8】** Nm

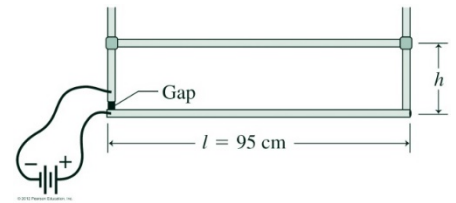


- In a MRI Scanner, a coil generates a magnetic field of  $1.5 \text{ T}$ . The coil has a length of  $3 \text{ m}$  and a diameter of  $1 \text{ m}$ . The coil wire has a diameter of  $1 \text{ mm}$  and is densely packed. The required current to create the magnetic field is  $I =$  **【9】** A.

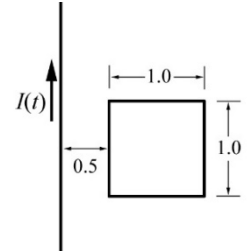
- The switch  $S$  in the circuit shown in right figure is closed for a long time. It is opened at time  $t = 0$ , the current through  $8.0 \Omega$ -resistor  $I(t) =$  **【10】** A.



- A current of 70 A is driven through the shown structure. The upper bar ( $m = 30\text{g}$ ) can vertically move. The equilibrium position  $h$  is **【11】** cm. Note: consider gravitational force and the magnetic interaction between the lower and upper bar.



- A square conducting loop of side 1.0 m lies in a plane containing a long, straight wire as shown in right figure. The current in the long wire  $I(t)$  comes from a charging  $RC$  circuit with a time constant 2.0 s, and at time  $t = 2.0$  s,  $I = 4.0$  A. The induced emf in the loop at time  $t = 4.0$  s is **【12】** V.



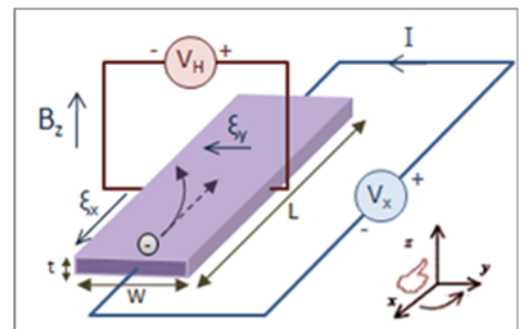
- A capacitor is connected across a 60-Hz, 110-V rms power line, and an rms current of 0.20 A flows. If the capacitor is replaced by an inductor, and the same current flows. The inductance is **【13】** H.
- The highvoltage powerline (resistance of 1 Ohm) between a power plant and the next town is operated at 110 kV and transports a power of 44 MW. (a) The power consumption of the powerline is **【14a】** MW. If the line would be operated at 11 kV, the power consumption would be **【14b】** MW. (Note: Ignore the reduced voltage due to the powerline).

- The right figure shows a double pipe system (Radius in units of  $r$ ). Each tube carries a current  $I$ . If the two currents are flowing into the same direction, the magnetic field at point (0  $r$ ) is **【15a】**, at point (3  $r$ ) **【15b】** and at point (6  $r$ ) **【15c】**. If the two currents flow into opposite directions the respective values are **【15d】**, **【15e】**, and **【15f】**



## Part II Problems (10 points per problem)

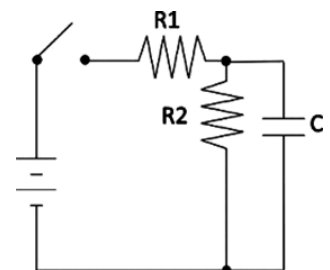
1. The right image shows the geometry of a Hall probe: a current  $I$  flows through a semiconductor (length  $L$ , width  $w$ , thickness  $t$ ) and a magnetic field  $B$  is applied perpendicular to  $I$ . If the charge carrier in the semiconductor has density  $n$  (number/volume), and each carrier has charge  $q$ , what is the Hall Voltage  $V_H$  in terms of  $I$ ,  $B$ ,  $n$  and other given quantities?



2. A conducting disk with radius  $a$ , volume  $V$ , and resistivity  $\rho$  is inside a solenoid of circular cross section. The axis of disk coincides with that of the solenoid. The magnetic field in the solenoid is given by  $B(t) = bt$ , where  $b$  is a constant. Find (a) the magnitude of induced electric field  $E(r)$  in the disk as a function of the distance  $r$  from the disk center. (b) the power dissipation in the entire disk.

(Hint: You may use the microscopic version of Ohm law  $\mathbf{E} = \rho \mathbf{J}$ , where  $\mathbf{J}$  is the current density.)

3. (a) Evaluate the current  $I_c$  (as a function of time), which flows into the capacitor with time  $t$  after closing the switch. The battery has emf  $\mathcal{E}$ . (Hint: use Kirchhoff's Law to write down equations for loops and nodes, and solve for the charge  $Q$  in the capacitor using  $I_c = dQ/dt$ .)



- (b) What is the time constant  $\tau$  for charging the capacitor?

Part I Answer (有效位數不扣分)

A 【1】	(d) (g) (多選一個或少一個給三分)	= B 【6】	B 【1】	$(N_1/N_2)^2$
A 【2】	$\mu_0 \cdot e^{-1} \cdot \ln 3 / \pi = 1.6 \times 10^{-7}$	= B 【12】	B 【2】	$2.5 \times 10^{-3}$ N, toward right (方向沒寫扣一分)
A 【3】	$3.0e^{-8.0t}$ , (3 給分, 題目似乎是問 at $t = 0$ )	= B 【10】	B 【3】	95 Hz (寫 598 扣一分, 因當成 $\omega$ )
A 【4】	$110/(24\pi) = 1.46$	= B 【13】	B 【4】	0.8
A 【5】	$2.5 \times 10^{-3}$ N, toward right (方向沒寫扣一分)	= B 【2】	B 【5】	(a): 2R (b): 4/5C (各 2.5 分)
A 【6】	$(N_1/N_2)^2$	= B 【1】	B 【6】	(d) (g) (多選一個或少一個給三分)
A 【7】	95 Hz (寫 598 扣一分, 因當成 $\omega$ )	= B 【3】	B 【7】	(a): 75 V (b): 71.4 V (各 2.5 分)
A 【8】	0.8	= B 【4】	B 【8】	0.005
A 【9】	(a): 2R (b): 4/5C (各 2.5 分)	= B 【5】	B 【9】	1200 (or 1194)
A 【10】	(a): 75 V (b): 71.4 V (各 2.5 分)	= B 【7】	B 【10】	$3.0e^{-8.0t}$ , (3 給分, 題目似乎是問 at $t = 0$ )
A 【11】	(a) 0.16 MW (b) 16 MW (各 2.5 分)	= B 【14】	B 【11】	0.32
A 【12】	$a = d = f = 0$ $b = c = e = \mu_0 I / (6 \pi r)$ (錯一個扣一分, 扣到五分完為止)	= B 【15】	B 【12】	$\mu_0 \cdot e^{-1} \cdot \ln 3 / \pi = 1.6 \times 10^{-7}$
A 【13】	0.005	= B 【8】	B 【13】	$110/(24\pi) = 1.46$
A 【14】	1200 (or 1194)	= B 【9】	B 【14】	(a) 0.16 MW (b) 16 MW (各 2.5 分)
A 【15】	0.32	= B 【11】	B 【15】	$a = d = f = 0$ $b = c = e = \mu_0 I / (6 \pi r)$ (錯一個扣一分, 扣到五分完為止)

Part II Answer

【A1 = B2】

(a) 五分, (b) 五分。(b)部分視情況部份給分。

(a) In the loop of radius  $r$ ,

$$|\mathcal{E}(r)| = \left| -\frac{d\Phi_B}{dt} \right| = \left| -\frac{d}{dt}(\pi r^2 \cdot bt) \right| = \pi b r^2 = \left| \oint \vec{E}(r) \cdot d\vec{r} \right| = E(r) \cdot 2\pi r$$

Therefore,  $E(r) = br/2$ .

(b) The current density in the loop of radius  $r$  is  $J(r) = E(r)/\rho = br/2\rho$ .

The power dissipated in this loop is

$$dP = J(r) \cdot dA(r) \cdot |\mathcal{E}(r)| = \frac{b}{2\rho} r \cdot h dr \cdot \pi b r^2 = \frac{\pi h b^2}{2\rho} r^3 dr, \text{ where } h \text{ is the height of disk.}$$

The total power dissipation in the disk  $P = \int_0^a dP = \frac{\pi h a^4 b^2}{8\rho} = \frac{a^2 b^2}{8\rho} V$ .

【A2 = B3】

式子正確列出得五分，解出  $I_c(t)$  得三分，正確寫出 time constant 得二分。

$$\varepsilon - I_1 R_1 - I_2 R_2 = 0 \quad \text{---(1)}$$

$$Q/C + I_2 R_2 = 0 \quad \text{----(2)}$$

$$I_1 = I_2 + I_C \quad \text{-----(3)}$$

Eliminate  $I_2$  by  $I_2 = I_1 - I_C$ , into (1) and (2), you get  $\varepsilon - I_1 - (I_1 - I_C)R_2 = 0$  ---(4) and  $Q/C + (I_1 - I_C)R_2 = 0$  -----(5)

From (4) (5), eliminate  $I_1$ , get  $\varepsilon - I_C R_1 = (R_1 R_2 / R_2 C) Q$  -----(6)

Differentiate both side of (6) and use  $I_C = dQ/dt$ , find  $\frac{dI_C}{dt} = -\frac{R_1 + R_2}{R_1 R_2 C} I_C$ , this is a differential equation for  $I_C$ , and

the solution is  $I_C = I_0 e^{-\frac{R_1 + R_2}{R_1 R_2 C} t}$ , at  $t = 0$ ,  $I_C = \varepsilon / R_1 = I_0$ , and the time constant  $\tau = R_1 R_2 C / (R_1 + R_2)$

【A3 = B1】

$$qE = qvB; V_H = E/w \Rightarrow V_H = wvB \quad \text{-----(1)}$$

$$I = qnvA = qnv w \cdot t \quad \text{-----(2)}$$

$$(1) + (2) \Rightarrow V_H = IB / (qnt)$$