國立清華大學試卷

記			分		
1		2			
3		4			
5		6			
7		8			
9		10			
11		12			
13		14			
15		16			
17		18			
19		20			
總 分	100				

所 系 資工

科 目 機率

學 號 104062229

姓 名 毛翊宣

日 期 3/22

1.

(a) $\frac{4! \times 4! \times 2}{8!}$ #

(b) $\frac{4! \times 5!}{8!}$ #

(c) $\frac{4! \times 2! 2! 2! 2!}{8!}$ #

2. (1) 10^{20} #

(2) $C^{10+20-1}_{20} = C^{29}_{20}$ #.

(3) $C^{10+10-1}_{10} = C^{19}_{10}$ #.

3. (a) $P(E_i) = \frac{(7-1)!}{7!} = \frac{6!}{7!} = \frac{1}{7}$ #

(b) $P(E_i E_j) = \frac{(7-2)!}{7!} = \frac{5!}{7!} = \frac{1}{42}$ #

(c) $P(E_1 \cup E_2 \cup \dots \cup E_7) = \frac{6!}{7!} \times 7 - C^7_2 \times \frac{5!}{7!} + C^7_3 \times \frac{4!}{7!} - C^7_4 \times \frac{3!}{7!} + C^7_5 \times \frac{2!}{7!} - C^7_6 \times \frac{1!}{7!} + C^7_7 \times \frac{0!}{7!}$

$= 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!} + \frac{1}{7!}$ #.

4.

(a) $0.2 \times 0.06 + 0.35 \times 0.04 + 0.43 \times 0.02$

$= 0.012 + 0.014 + 0.1009$

$= 0.035$ #

(b) $\frac{0.35 \times 0.04}{0.035} = 0.4$ #

5、

(a) a sum of 5 occurs in the first roll: $\frac{4}{6\times6} = \frac{4}{36} = \frac{1}{9}$ ※

a sum of 7 occurs in the first roll: $\frac{6}{6\times6} = \frac{1}{6}$ ※

(b) Let E be the event a sum of 5 occurs before a sum of 7,
F be the event a sum of 5 occurs in the first roll,
G be the event a sum of 7 occurs in the first roll,
O be the event that the sum is not 5 or 7 in the first roll.

$$P(E) = P(E|F)P(F) + P(E|G)P(G) + P(E|O)P(O)$$

$$P(E) = 1 \cdot \frac{4}{36} + 0 \cdot \frac{6}{36} + P(E|O)\frac{26}{36}$$

$$P(E) = \frac{1}{9} + P(E) \cdot \frac{26}{36}$$

$$\frac{10}{36}P(E) = \frac{1}{9}$$

$$P(E) = \frac{1}{9} \times \frac{36}{10} = \frac{2}{5}$$ ※

6、(a)

Let $x \in (\bigcup_{i=1}^{n} E_i)^c$

$x \notin \bigcup_{i=1}^{n} E_i$

$\Rightarrow x \notin E_1$ and $x \notin E_2$ and $x \notin E_3$ ... and $x \notin E_n$

$\Rightarrow x \in E_1^c$ and $x \in E_2^c$ and $x \in E_3^c$ ... and $x \in E_n^c$

$\Rightarrow x \in \bigcap_{i=1}^{n} E_i^c$

Let $y \in \bigcap_{i=1}^{n} E_i^c$

$\Rightarrow y \in E_1^c$ and $y \in E_2^c$ and $y \in E_3^c$ ... and $y \in E_n^c$

$\Rightarrow y \notin E_1$ and $y \notin E_2$ and $y \notin E_3$ ... and $y \notin E_n$

$\Rightarrow y \notin (E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n)$

$\Rightarrow y \notin \bigcup_{i=1}^{n} E_i$

$\Rightarrow y \in (\bigcup_{i=1}^{n} E_i)^c$

$\therefore$ for any element $x \in (\bigcup_{i=1}^{n} E_i)^c$, $x \in \bigcap_{i=1}^{n} E_i^c$, and for any element $y \in \bigcap_{i=1}^{n} E_i^c$, $y \in (\bigcup_{i=1}^{n} E_i)^c$, then, $(\bigcup_{i=1}^{n} E_i)^c = \bigcap_{i=1}^{n} E_i^c$ ※

(b)

$P(E_1 \cup E_2 \cup \cdots \cup E_n)$

$= 1 - P((E_1 \cup E_2 \cup E_n)^c)$

$= 1 - P(E_1^c \cap E_2^c \cap E_3^c \cap \cdots \cap E_n^c)$ (by De Morgan's first Law)

because $\{E_1, E_2, \cdots, E_n\}$ is independent,

$\{E_1^c, E_2^c, E_3^c, \cdots, E_n^c\}$ is independent,

then

$P(E_1^c \cap E_2^c \cap \cdots \cap E_n^c) = P(E_1^c)\, P(E_2^c) \cdots P(E_n^c)$

$= (1-P(E_1))(1-P(E_2)) \cdots (1-P(E_n))$

$= \prod_{i=1}^{n} (1-P(E_i))$

$\Rightarrow P(E_1 \cup E_2 \cup \cdots \cup E_n)$

$= 1 - \prod_{i=1}^{n} (1-P(E_i))$ #

7.

$(a_1 a_2 a_4 a_6$ is closed$) + (a_1 a_3 a_5 a_6$ is closed$) - (a_1 a_2 a_3 a_4 a_5 a_6$ is closed$)$

$= p^4 + p^4 - p^6$

$= 2p^4 - p^6$ #