Probability & Statistics

2005.1.7

1. (10%) Let $f(x, y)=\frac{3}{2}$, $x^2 \le y \le 1$, $0 \le x \le 1$, be the joint p.d.f. of X and Y. Find
   (a) $f_1(x)$, the marginal p.d.f. of X.
   (b) $f_2(y)$, the marginal p.d.f. of Y.
   (c) $P(X>Y)$
2. (10%) Let X_1, X_2 be independent random variables, each with the binomial p.m.f.
$$f(x)=C_x^2(\frac{1}{3})^x(\frac{2}{3})^{2-x},\ x=0, 1, 2.$$
   (a) Find the joint p.m.f. of $Y=X_1$ and $W=X_1+X_2$.
   (b) Determine the marginal p.m.f. of W.
   (c) Compute the correlation coefficient of Y and W.
3. (10%) Let X_1, X_2, X_3 denote a random sample of size n=3 from a distribution with the geometric p.m.f. $f(x)=(\frac{3}{4})(\frac{1}{4})^{x-1}$, x=1, 2, 3, ... That is, X_1, X_2, and X_3 are mutually independent and each has this geometric distribution.
   (a) Determine $P(X_1+X_2+X_3=6)$.
   (b) If Y equals the minimum of X_1, X_2, X_3, find $P(Y>2)$.
4. (10%) Let X_1 and X_2 be two independent random variables with respective means μ_1 and μ_2 and variances σ_1^2 and σ_2^2. What is the mean and the variance of $Y=2X_1X_2$.
5. (10%) Suppose that the distribution of the weight of a prepackaged 1-pound bag of carrots is $N(1.18, 0.07^2)$ and the distribution of the weight of a prepackaged 3-pound bag of carrots is $N(3.22, 0.09^2)$. Selecting bags at random, find the probability that the weight of one 3-pound bag exceeds the sum of three 1-pound bags.
6. (10%) Let X equal the weight of a fat-free Fig Newton cookie. Assume that the distribution of X is $N(14.22, 0.0854)$. These cookies are sold in packages that have a label weight of 340 grams. Assuming that a package is filled with a random sample of cookies, how many cookies should be put into a package to be quite certain (say with a probability of at least 0.98) that the total weight of the cookies exceeds 340 grams? Also keep in mind that extra cookies in a package decreases profit.
7. (10%) If X and Y are independent uniform (0, 1) random variables, find E[$|X-Y|^{\alpha}$] for α>0.
8. (15%) A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints is $N(21.37, 0.16)$.
   (a) Let X denotes the weight of a single mint selected at random from the production line. Find $P(X< 20.857)$.
   (b) During a particular shift 100 mints are selected at random and weighted. Let Y equal the number of these mints that weigh less than 20.857 grams. Find approximately $P(Y\le 5)$.
   (c) Let $\overline{X}$ equal the sample mean of the 100 mints selected and weighted on a particular shift. Find $P(21.31\le \overline{X} \le 21.39)$.
9. (15%) Suppose a random sample of size n=19 is taken from a normal distribution with σ^2=9. Compute the probability that the sample standard deviation s lies between 2 and 4.