CALCULUS (JUNE 22, 2006)

1. (10) Evaluate the following integrals,
(a) $\iint_R x\,dA$ where R is given by $0 \le y \le \sin x,\ 0 \le x \le \pi$. (b) $\int_0^1 \int_y^1 e^{-x^2}\,dx\,dy$.

2. (10) Find the volume of the region given by $0 \le z \le r^2$ and in one loop of $r^2 = 2\sin\theta$.

3. (10) Find the centroid of the region given by $x^2 \le y \le 4$ with $\delta(x, y) = y$.

4. (10) Find the potential function of the vector field
$\mathbf{F} = (y\cos z - yze^x)\mathbf{i} + (x\cos z - ze^x)\mathbf{j} - (xy\sin z + ye^{\overset{x}{\cancel{z}}})\mathbf{k}$.

5. (10) Find the centroid of the ice-cream cone bounded by $\phi = \frac{\pi}{6}$ and $\rho = 4\cos\phi$ with density $\delta(x, y, z) = z$.

6. (20) Find the volume of the region in the first octant given by
$1 \le xy \le 4,\ 1 \le yz \le 9,\ 4 \le xz \le 9$.

7. (10) A sphere of radius 1 is interior and tangent to a sphere of radius 2.
Find the average distance from the tangent point to all points between two spheres.

8. (10 Find the area of the region bounded by $x = \cos^3 t,\ y = \sin^3 t,\ 0 \le t \le 2\pi >$

9. (10) Evaluate $\iint_S x^2 + y^2 dS$ where S is the part of $x^2 + y^2 + z^2 = 25$ above $z = 3$.

10. (10) Evaluate $\iint_S \mathbf{F} * \mathbf{n}dS$ where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + 4\mathbf{k}$ and S is the boundary of the region $x^2 + y^2 \le z \le 5$.

11. (10) Let T be a region in space with volume V, boundary surface S and centroid (a, b, c). Use the divergence theorem to show that $c = \frac{1}{2V} \iint_S z^2\,dx\,dy$.

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