CALCULUS (JAN. 12, 2006)

1. (20) Evaluate the following integrals:

(a) $\int \dfrac{1}{1+e^x}\,dx$,   (b) $\int \sec^5 x\,dx$,   (c) $\int x^2(\ln x)^2\,dx$,   (d) $\int_0^\infty \dfrac{1}{x^{1/2}+x^{3/2}}\,dx$.

2. (10) Find the following limits :

(a) $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2} - \dfrac{1}{1-\cos x}\right)dx$,   (b) $\lim\limits_{x\to\infty}(e^x - 2x)^{\frac{1}{x}}$.

3. (10) Find the area of the surface of revolution the curve $y = \cosh x$ for $x \in [0,a]$ around the x-axis.

4. (10) Find the centroid of the region bounded by $x = 2y^4$ and $x = y^2 + 1$.

5. (10) A spherical tank of radius R is initially full of oil of density ρ. Find the the total work done in pumping all the oil from the sphere to a height of $2R$ above the top of the tank.

6. (10) Find the area bounded by the curve $y = \dfrac{1}{(x+1)^2(x^2+1)}$ and the x-axis for $x \in [0,1]$.

7. (10) Evaluate the integral $\int \dfrac{1}{2+\cos x+\sin x}\,dx$.

8. (10) Solve the initial value problem $y' - (\tan x)y = \cos x$, $y(0) = 1$.

9. (10) Solve the initial value problem $4y'' + 4y' + 101y = 0$, $y(0) = 10$, $y'(0) = 20$.