Algorithms first examination

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1. (a) Write an algorithm to solve the sorting problem.
   (b) What are the best and worst case instances of your algorithm respectively?
   (c) Analyze the best case complexity of your algorithm.
   (d) Analyze the worst case complexity of your algorithm
   (e) Analyze the average case complexity of your algorithm.
   (f) Is your algorithm optimal in its best, worst and average cases respectively?

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2. Prove that the worst case lower bound of the convex hull problem is nlogn.

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3. Given At-Most-4-satisfiability problem is NP-complete.
   Prove that 3-satisfiability problem is also NP-complete.

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4. For each of the following statements, determine wheather it is correct or not. Please explain if it is not correct.

   (a) Cook's theorem tells us that every NP-complete problem takes exponential number of steps to solve this problem in the worst case.

   (b) Cook's theorem tells us that some NP-complete problems take exponential number of steps to solve these problems in the worst case.

   (c) Every NP-complete problem can be solved by a polynomial time deterministic algorithm in average if and only if P=NP is proved.

   (d) Every NP-complete problem can be solved by a polynomial time deterministic algorithm in average once P=NP is proved.

(e) If a problem is proved to be an NP-complete problem, then at present it always takes exponential number of steps to solve this problem for all kinds of inputs.

(f) The lower bound of NP-complete problem is exponential if and only if P .NEQ. NP is proved.

(g) The lower bound of NP-complete problem is exponential once if P .NEQ. NP is proved.

(h) The lower bound of all NP-hard problems is exponential if and only if P .NEQ. NP is proved.

(i) The lower bound of all NP-hard problems is exponential once if P .NEQ. NP is proved.

(j) Suppose that it is proved that the lower bound of the satisfiability problem is polynomial, we can conclude that P=NP.

(k) It is proved that the problem of determining whether a given number is prime or not can be solved in polynomial time by a deterministic algorithm.

(l) It is proved that the problem of determining whether a given number is prime or not is NP-complete.

(m) It is possible that NP-complete .EQ. NP-hard.

(n) If the NP-complete problems can be solved in polynomial time by deterministic algorithms, then so are the NP-hard problems.

(o) If the NP-hard problems can be solved in polynomial time by deterministic algorithms, then so are the NP-complete problems.