B

九十四學年第二學期 普通物理 B 第一次段末考試題

[Benson Ch. 22 – 28] 2006/3/31, ***8:30AM – 10:00AM***

0.【5%】依下面說明在答案卷上作答者，可得 5 分

(i) 答案卷第一張爲封面。第一張正反兩面**不要作答**。

(ii) 由第二張紙開始算起，第一頁依空格號碼順序寫下所有填充題答案，**寫在其他頁不記分**。

(iii) 計算題之演算過程與答案依題號順序寫在第二頁以後，**每題從新的一頁寫起**。

Part I. 填充題 (每格 3%, 共 45%)

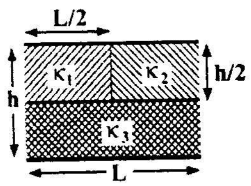

Fig.1

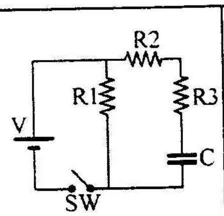

Fig.2

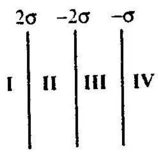

Fig.3

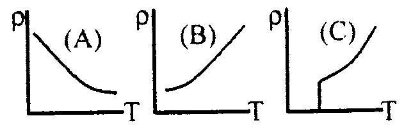

Fig.4

■ A parallel-plate capacitor is filled with dielectric materials of constant κ_1、κ_2、κ_3, as Fig 1. The capacitance is 【1】. Express your answer in terms of C_0, the capacitance with no dielectric.

■ For the circuit powered by a battery of V, as Fig.2, the switch SW is closed at t = 0 (a) the initial current through R3 is 【2】, (b) the final steady state current through R1 is 【3】, (c) when the switch SW is reopened, the time constant is 【4】.

■ Two charge metal spheres with radii R_1, R_2 are connected by a long conducting wire. The electrical field on the surface of R_1 is E_1. The surface electrical field of R_2 is 【5】.

■ Resistivity vs. temperature for various materials is shown as Fig. 4. Write down a correct sequence for superconductor-semiconductor-metal. 【6】.

■ A hypothetical potential function is $V(x, y, z) = xy + (yz)^{-1} - \exp[-(x^2 + z^2)]$. The electrical field at z-direction is 【7】.

■ A galvanometer has a full-scale deflection for a current of 5 mA, the coil has a resistance of 10 Ω. Modify the instrument as following : (a) an ammeter that can measure up to 1A, we need a shunt resistor of 【8】 Ω in parallel, (b) an voltmeter that can measure up to 50V, we need a resistor of 【9】 Ω in series, (c) If it is modified as an ohmmeter that powered by a 3 V battery, the minimum scale of resistance is 【10】 Ω.

■ Three infinite nonconductive sheets are charged with 2σ, − 2σ, − σ, as Fig. 3. The electrical field (a) in the region II is 【11】, (b) in the region III is 【12】.

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■ A point charge − 4q is at $x = 0$ while a second charge + 9q is at $x = 1$ m. The field strength is zero at x = __【13】__ .

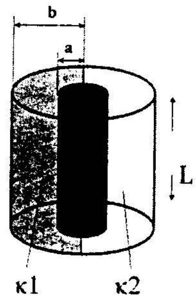
■ A dipole consists of + Q at $(a, a, 0)$ and − Q at $(-a, -a, 0)$. At $(100a, 100a, 0)$ and $(200a, -200a, 0)$, the electrical fields are E_1 and E_2, respectively. E_1 / E_2 = __【14】__ . Apply an electrical field $\mathbf{E} = E_0\,\boldsymbol{k}$ on the dipole. The torque the dipole experienced is __【15】__ .

Part II. 計算題 (共 55%)

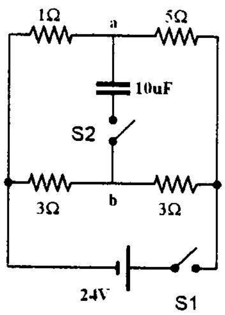
1. A nonconductive cylinder of radius R and infinite length is charged with density $\rho(r) = Ar$, where A is constant, and r is the distance to the central axis. Find the electrical field at (a) $r < R$ and (b) $r > R$.. (7%, 6%)

2. A nonconductive sphere of radius R has total charge Q uniformly distributed throughout its volume. Find (a) the electrical field $E(r)$ at $r < R$, (b) the electrical potential $V(r)$ at $r < R$, (c) the potential energy of the sphere (note: express your answer in terms of Q and R). (5%, 5%, 5%)

3. A coaxial cable of length L is used as a capacitor, which consists of a central conductor of radius a surrounded by a cylindrical shell of radius b. Neglect the edge effect on the two ends of cable. (a) Find the total capacitance when the gap between the central conductor and shell is empty. (b) When the gap is filled with two dielectric materials κ1 = 1.5 and κ2 = 2, equally as right figure. Find the capacitance. (7%, %5)



4. In the circuit as right figure, switch S1 is initially closed and S2 is open. (a) Find $V_a - V_b$. (b) After S2 is also closed and reaches steady state, find $V_a - V_b$. (c) S1 is opened and S2 is left closed, find the time constant for the capacitor discharge. (5%, 5%, 5%)



(A)

【1】 $E_1 \dfrac{R_1}{R_2}$	【2】 $\dfrac{2\kappa_3(\kappa_1+\kappa_2)}{2\kappa_3+\kappa_1+\kappa_2}C_0$	【3】 50/995 = 0.0503	【4】 9990
【5】 590	【6】 $\dfrac{V}{R_2+R_3}$	【7】 $\dfrac{V}{R_1}$	【8】 $C(R_1+R_2+R_3)$
【9】 $-2z\exp[-(x^2+z^2)]+\dfrac{1}{yz^2}$	【10】 (C)(A)(B)	【11】 $5\sigma/(2\varepsilon_0)$, $\rightarrow$	【12】 $\sigma/(2\varepsilon_0)$, $\rightarrow$
【13】 16	【14】 $2aQE_0(\mathbf{i}-\mathbf{j})$	【15】 –2m	

(B)

【1】 $\dfrac{2\kappa_3(\kappa_1+\kappa_2)}{2\kappa_3+\kappa_1+\kappa_2}C_0$	【2】 $\dfrac{V}{R_2+R_3}$	【3】 $\dfrac{V}{R_1}$	【4】 $C(R_1+R_2+R_3)$
【5】 $E_1 \dfrac{R_1}{R_2}$	【6】 (C)(A)(B)	【7】 $-2z\exp[-(x^2+z^2)]+\dfrac{1}{yz^2}$	【8】 50/995 = 0.0503
【9】 9990	【10】 590	【11】 $5\sigma/(2\varepsilon_0)$, $\rightarrow$	【12】 $\sigma/(2\varepsilon_0)$, $\rightarrow$
【13】 –2m	【14】 16	【15】 $2aQE_0(\mathbf{i}-\mathbf{j})$	

Part II

(1)

Using Gauss' s Law

While r < R

$$2\pi r L E = \frac{1}{\epsilon_0}\int_0^r A r L\, 2\pi r\, dr = \frac{2 A L \pi}{\epsilon_0}\int_0^r r^2\, dr = \frac{2 A L \pi}{3\,\epsilon_0}\, r^3$$

$$E = \frac{A}{3\,\epsilon_0}\, r^2 \;\ldots\ldots\ldots\,(a)$$

While r > R

$$2\pi r L E = \frac{1}{\epsilon_0}\int_0^R A r L\, 2\pi r\, dr =$$

$$\frac{2 A L \pi}{\epsilon_0}\int_0^r r^2\, dr = \frac{2 A L \pi}{3\,\epsilon_0}\, R^3$$

$$E = \frac{A}{3\,\epsilon_0}\,\frac{R^3}{r} \;\ldots\ldots\ldots\,(b)$$

(2)

$$\rho = \frac{Q}{(\frac{4}{3}\pi R^3)}$$

For r < R

$$4\pi r^2 E = \frac{1}{\epsilon_0}\int_0^r \rho\, 4\pi r^2\, dr = \frac{1}{\epsilon_0}\,\frac{r^3}{R^3}\,Q$$

$$E = \frac{1}{4\pi\epsilon_0}\,\frac{rQ}{R^3}\left(\text{or } k\,\frac{rQ}{R^3}\right)\ldots\ldots(a)$$

For r > R

$$E = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{r^2}$$

$$V(r) = -\int_\infty^r E(r)\,dr = -\left(\int_\infty^R E(r)\,dr + \int_R^r E(r)\,dr\right)$$

$$= -\left(\int_\infty^R \frac{1}{4\pi\epsilon_0}\,\frac{Q}{r^2}\,dr + \int_R^r \frac{1}{4\pi\epsilon_0}\,\frac{rQ}{R^3}\,dr\right)$$

$$= -\frac{Q}{4\pi\epsilon_0}\left(\frac{-1}{R} + \frac{r^2 - R^2}{2R^3}\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\,\frac{3R^2 - r^2}{2R^3}\;\left(\text{or } \frac{kQ(3R^2 - r^2)}{2R^3}\right)\ldots\ldots(b)$$

Consider the total energy in the entire space by $u = \epsilon_0 E^2/2$

$$U = \int_0^\infty \frac{1}{2}\,\epsilon_0\,E(r)^2\,4\pi r^2\,dr$$

$$= 2\pi\left(\int_r^R \left(k\,\frac{rQ}{R^3}\right)^2 r^2\,dr + \int_R^\infty \left(k\,\frac{Q}{r^2}\right)^2 r^2\,dr\right)$$

$$= 2\pi k^2\,\epsilon_0\,Q^2\left(\int_0^R \frac{r^4}{R^6}\,dr + \int_R^\infty \frac{1}{r^2}\,dr\right)$$

$$= \frac{kQ^2}{2}\left(\frac{R^5}{5R^6} + \frac{1}{R}\right)$$

$$= \frac{3kQ^2}{5R}\ldots\ldots(c)$$

(3)

By Gauss' s Law

$$E\,L\,2\pi r = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0}\,\frac{Q}{Lr}$$

$$V_b - V_a = -\int_a^b \frac{1}{2\pi\epsilon_0}\,\frac{Q}{Lr}\,dr$$

$$= -\frac{1}{2\pi\epsilon_0}\,\frac{Q}{L}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}\left(\text{or } \frac{L}{2k\ln(b/a)}\right)\ldots\ldots(a)$$

$$C' = \frac{C}{2}\,1.5 + \frac{C}{2}\,2$$

$$= 1.75\,C = \frac{3.5\pi\epsilon_0 L}{\ln(b/a)} = \frac{0.875\,L}{k\ln(b/a)}\ \ldots\ldots..(b)$$

(4)

(a) S2 open, no current through capacitor, both currents through 3Ω and 5Ω are 4 A. $V_a = 24 - 4\times5 = 4$ (V), $V_b = 24 - 4\times3 = 12$ (V). Therefore, $V_a - V_b = -8$ (V).

(b) S2 close and steady state, no current through capacitor, Therefore, $V_a - V_b = -8$ (V).

(c) $R_a = 4\ \Omega$, $R_b = 8\ \Omega$, thus $R^{-1} = (1/4) + (1/8)$, $R = 8/3\ \Omega$. Thus $RC = 10 \times 8/3\ \mu s = 26.7\ \mu s$.