# Algorithm Mid-term 2004/4/20 (二) 10:10~12:00

1. (10%)(a) What is P , NP , NP-Complete and NP-Hard problem ?
   Draw a plot to explain their relation.
   (3%)(b) What is Cook's Theorem.

2. (10%) Assume we have known the 3-SAT problem is a NP-Complete problem, prove k-SAT a NP-Complete problem for general $k \geq 4$.

3. (10%) If $T(n)= 3n^2+n\log n+(\log n)^2$ which statement are true?
   - (a) $T(n)= \Theta(n^2)$
   - (b) $T(n)=O(n\log n)$
   - (c) $T(n)= \Theta(n^2\log n)$
   - (d) $T(n)=\Omega(n^2)$
   - (e) $T(n)=O(n^2)$

4. (10%)There is an algorithm X :
   Input : A graph G=(V , E) , while V is the vertex set and E is the edge set.
   Output : A minimal spanning tree of this graph.

   Sorting the edges in ***E*** in decrease.
   Repeat
   Step 1.Set ***i***= 0
   Step 2.delete ***i***'th edge ***e*** of E from ***G*** , ***E = E – e***
   Step 3. ***i = i + 1***
   Step 4.if ***G'*** is connective then ***G = G'*** , else restore edge ***e*** in ***G (E=E+e)***
   Until size of ***E*** = size of ***V - 1***

   Prove the above algorithm can solve MST correctly (hint : like kruskal's algorithm)

5. (15%) Questions : True or False (Explain your answer if it is false.)
   - (1) If a problem is NP-complete, then it must be in NP and in NP-hard.
   - (2) If we can find a polynomial algorithm to solve one NP-complete problem in average case, then NP-complete problem = P problem.
   - (3) As so far, it seems that $P \neq NP$.
   - (4) TSP is a NP-hard problem.
   - (5) If a problem A can be reduced to Merge-Sort in $\Theta(n\log n)$ time, then we can say that the lower bound of the time complexity of A is $\Omega(n\log n)$.

6. (10%) Figure 1. is a connected network for a project with 11 tasks or activities A1….A11 with length. There are 9 events V1….V9. The events V1 and V9 may be interpreted as "start project" and "finish project" respectively. The length of a path is the sum of the times of activities on this path.

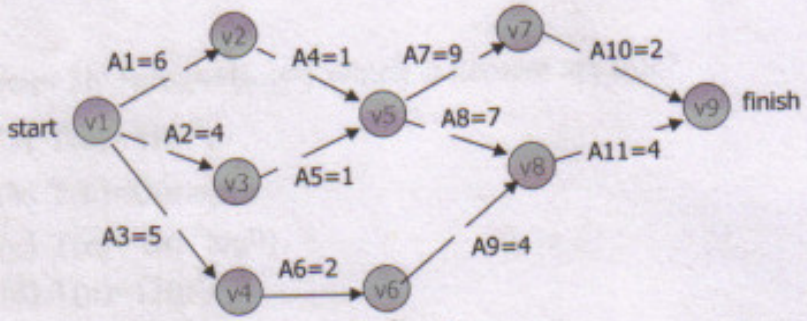


figure 1

(a).(5%) Compute the earliest time of each vertex **E(vi)** and latest time of each vertex **L(vi)**

(b). (5%) Indicate the critical points (vertices) in the graph and any one critical path of the graph

7. (10%) Write a non-deterministic algorithm to solve TSP problem.

8. Please prove that the lower bound of convex hull problem is $\Omega(n\log^n)$

9. (10%) Give a symbol set and corresponding frequency, construct a Huffman code Tree.

| Symbols: | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| frequency: | 3 | 15 | 8 | 6 | 30 | 22 | 4 |

10. (2%) Write comments about this class.