CS2102 01 Digital Logic Design Midterm 1 (10:10am-12:00pm, Oct 28th)

Truth or False (24%):
(Write down T or F. You don't need to give the reason.)

1. ______ According to the duality principle, $x + 1 = 1$ and $x \cdot 1 = x$ are dual functions.
2. ______ The addition of a positive number and a negative number, both in 2's complement form, will not produce an overflow.
3. ______ The n-bit signed-2's complement system can represent 2^n different integers, ranging from -2^{n-1} to $2^{n-1} - 1$.
4. ______ Any Boolean function can be represented by either of the 2 unique canonical forms, namely, sum-of-products or product-of-sums forms.
5. ______ The AOI implementation represents the AND-OR-INVERT implementation, which can be obtained by the complement of sum-of-products form.
6. ______ Each K-map defines a unique Boolean function.
7. ______ It is possible that nonstandard forms can have fewer literals than standard forms.
8. ______ AND gate is a universal gate because any Boolean function can be implemented by AND gates only.

Answer the following questions: (76%)
(Write down your intermediate results. Do not give the final answer only.)

1. (11%) **[Conversion]** Number conversions:
   (a) (4%) Convert the $(452.4)_8$ to hexadecimal.
   (b) (4%) Convert $(32.0625)_{10}$ to octal number.
   (c) (3%) Represent –73 in the 8-bit signed-magnitude representation.
2. (6%) **[2's Complement]** Show the detailed procedure of the following additions with 7-bit binary numbers, using 2's complement for negative numbers. You should also translate the result to decimal, and indicate if there is an overflow.
   (a) (3%) (–15) + (–50);
   (b) (3%) (+28) – (+49).
3. (5%) **[Canonical]** Use the brief notation $\sum$ or Π to express the Boolean function $F(a,b,c,d) = (ad + b'c + bd')(b + d)$ in **product-of-maxterms**.
4. (5%) **[Multi-level]** Draw the multi-level NAND circuit for $F = w(x + z') + xz'$ by using NAND gates only. You don't need to simplify the Boolean function. Assume that both the normal and complement inputs are available.
5. (13%) **[K-Map]** For the Boolean function $F(w,x,y,z) = xy'z + x'y'z + w'xy + wx'y + wxy$,
   (a) (5%) Use K-map to simplify F to **sum-of-products form**;
   (b) (5%) Draw the two-level AND-OR logic diagram of F. Assume that both the normal and complement inputs are available.
   (c) (3%) Draw the OAI implementation of F. Assume that both the normal and complement inputs are available.
6. (10%) **[EPI]** Consider the Boolean function $F(A,B,C,D) = \prod(3,4,6,11,12,14)$.

(a) (5%) List all the prime implicants (PIs) and essential prime implicants (EPIs).

(b) (5%) List all the possible simplified functions of F in **sum-of-products** form.

7. (10%) **[Don't-Care]** Simplify the Boolean function $F(A,B,C,D) = \sum(1,5,9,11,15)$ with the don't-core condition $d(A,B,C,D) = \sum(0,3,6,8,13)$ in **product-of-sums** form. And then implement the simplified function with a two-level NOR circuit. Assume that both the normal and complement inputs are available.

8. (6%) **[Boolean Algebra]** Prove the validity of Theorem 6(b): $x(x+y) = x$ by using Boolean algebra.

Table 2.1
Postulates and Theorems of Boolean Algebra

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

Show your proof by using postulates and theorems other than Theorem 6(a)(b) in the table. You should identify specific postulate or theorem to justify each step (**do not** use Theorem 6(a)(b)). Do not use the truth table.

9. (10%) **[Design]** Design a voting machine F for four people, say, A, B, C and D. Each person sets his/her input as 1 when he/she agrees with some certain topic to be voted. Otherwise, he/she votes for 0 if he/she disagrees. The output of the machine will be 1 if three (75%) or four (100%) persons agree with the topic.
   (a) (4%) List the truth table of the voting machine F.
   (b) (3%) Use K-map to derive the simplified **sum-of-products** function.
   (c) (3%) Draw the corresponding NAND-NAND logic diagram. Assume that both the normal and complement inputs are available.

Good luck and happy examination!!

If you have too much time left, there is a joke for you:

A professor was giving a big test one day to his students. Once the test was over the students all handed the tests back in. The professor noticed that one of the students had attached a $1000 bill to his test with a note saying "Ten dollars per point."

The next class the professor handed the tests back out. This student got back his test, an envelope with $620 change, and a note saying "Thanks..."