國立清華大學試卷

記		分			
1	10	2	10		
3	10	4	6		
5	10	6	10		
7	10	8	10		
9	8	10	10		
11		12			
13		14			
15		16			
17		18			
19		20			
總分	94				

所系 資工系

科目 通訊概論

學號 [redacted]

姓名 [redacted]

日期 12/3

1.

(a) $F(\cos(2\pi f_0 t))$

$= F(\frac{1}{2}(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}))$

$= \frac{1}{2}F(e^{j2\pi f_0 t}\cdot 1) + \frac{1}{2}F(e^{-j2\pi f_0 t}\cdot 1)$

$= \frac{1}{2}\delta(f+f_0) + \frac{1}{2}\delta(f-f_0)$ // $F(1) = \delta(f)$

$\Rightarrow F^{-1}(\frac{1}{2}(\delta(f+f_0) + \delta(f-f_0))) = \cos(2\pi f_0 t)$

(b)

$F(x(t)) = X(f)$

$\Rightarrow F(x(t-t_0)) = \int_{-\infty}^{\infty} x(t-t_0)\cdot e^{-j2\pi f t}\, dt$

$= \int_{-\infty}^{\infty} x(\tau) e^{-j\pi f(\tau+t_0)}\, d\tau$ // 令 $\tau = t-t_0$

$= e^{-j2\pi f t_0}\cdot \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f \tau}\, d\tau$

$= e^{-j2\pi f t_0} X(f)$

(c)

$\int_{-\infty}^{\infty} \cos(2\pi\alpha t)\, x(t)\, dt$

$= \int_{-\infty}^{\infty} \frac{1}{2}(e^{-j2\pi\alpha t} + e^{j2\pi\alpha t})\, x(t)\, dt$

$= \frac{1}{2}[\, e^{-j2\pi\alpha t}\cdot \underbrace{\int_{-\infty}^{\infty} x(t)\, dt}_{X(f)} + e^{j2\pi\alpha t} \underbrace{\int_{-\infty}^{\infty} x(t)\, dt}_{X(f)}\,]$

$= \frac{1}{2}(X(f-\alpha) + X(f+\alpha))$

$\Rightarrow F^{-1}(\frac{1}{2}X(f-\alpha) + \frac{1}{2}X(f+\alpha)) = \cos(2\pi\alpha t)\, x(t)$

1. d.

$$\int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi f t}\, dt = e^{-j2\pi f \cdot 0} = 1 \qquad /* \int_{-\infty}^{\infty} x(t)\,\delta(t)\, dt = x(0) */$$

$$\Rightarrow F(\delta(t)) = 1$$

2. (a)

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t}\, dt$$

$$= \int_{-T_s}^{T_s} e^{-j2\pi f t}\, dt$$

$$= \frac{1}{-j2\pi f}\left(e^{-j2\pi f T_s} - e^{j2\pi f T_s}\right)$$

$$= \frac{1}{\pi f} \sin(2\pi f T_s)$$

(b).

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t}\, dt$$

$$= \int_{-T_s}^{0} -e^{-j2\pi f t}\, dt + \int_{0}^{T_s} e^{-j2\pi f t}\, dt$$

$$= \frac{1}{j2\pi f}\left(1 - e^{j2\pi f T_s}\right) + \frac{1}{j2\pi f}\left(1 - e^{-j2\pi f T_s}\right)$$

$$= \frac{1}{j2\pi f}\left(2 - \left(e^{j2\pi f T_s} + e^{-j2\pi f T_s}\right)\right)$$

$$= \frac{1}{j2\pi f}\left(2 - 2\cos(2\pi f T_s)\right)$$

$$= \frac{1 - \cos(2\pi f T_s)}{j\pi f}$$

$$= \frac{2}{j\pi f} \sin^2(\pi f T_s)$$

3. case 1 : $u < -3$ $\quad x(u) * y(u) = 0$

case 2 : $-3 \le u < -1$

$$x(u) * y(u) = \int_{-2}^{t+1} 1 \cdot 1 \, dt$$

$$= t \Big|_{-2}^{t+1}$$

$$= (t+1) - (-2)$$

$$= t + 3$$

case 3 : $-1 \le u < 1$

$$x(u) * y(u) = 2 \cdot 1 = 2$$

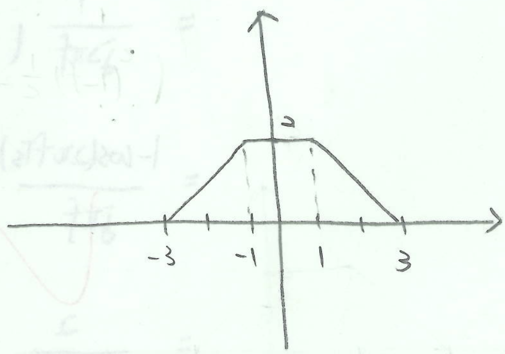
case 4 : $1 \le u < 3$

$$x(u) * y(u) = \int_{t-1}^{2} 1 \cdot 1 \, dt$$

$$= t \Big|_{t-1}^{2}$$

$$= 2 - (t-1)$$

$$= 3 - t$$



case 5 : $u \ge 3$ $\quad x(u) * y(u) = 0$

← 隔壁有喔

4. (a)

$a_0 = \cos(\frac{\pi}{3}) = \frac{1}{2}$

$a_1 = \cos(\frac{2\pi}{4} + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$ * $e^{-j\frac{2\pi}{n}} = e^{-j\frac{\pi}{2}} = -j$

$a_2 = \cos(\frac{4\pi}{4} + \frac{\pi}{3}) = -\frac{1}{2}$ * $A_i = \sum_{k=0}^{3} a_k \cdot e^{-j\frac{2\pi i k}{n}}$

$a_3 = \cos(\frac{6\pi}{4} + \frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$A_0 = a_0 + a_1 + a_2 + a_3 = 0$

$A_1 = a_0 - a_1 j - a_2 + a_3 j = 1 + \sqrt{3} j$

$A_2 = a_0 - a_1 + a_2 - a_3 = 0$

$A_3 = \overline{A_1} = 1 - \sqrt{3} j$

(b)

$a_0 = \frac{1}{4}(A_0 + A_1 + A_2 + A_3)$ use eq. in (b)

$= \frac{1}{4} \cdot 2 = \frac{1}{2}$ * $a_i = \frac{1}{4} \sum_{k=0}^{3} A_k \cdot e^{j\frac{2\pi i k}{n}}$

$a_1 = \frac{1}{4}(A_0 + A_1 j - A_2 - A_3 j)$ * $e^{j\frac{2\pi}{n}} = e^{j\frac{\pi}{2}} = j$

$= \frac{1}{4}(j - \sqrt{3} - j - \sqrt{3})$

$= \frac{1}{4}(-2\sqrt{3}) = -\frac{\sqrt{3}}{2}$

$a_2 = \frac{1}{4}(A_0 - A_1 + A_2 - A_3)$

$= \frac{1}{4} \cdot (-2) = -\frac{1}{2}$

$a_3 = \frac{1}{4}(A_0 - A_1 j - A_2 + A_3 j)$

$= \frac{1}{4}(-j + \sqrt{3} + j + \sqrt{3})$

$= \frac{1}{4}(2\sqrt{3}) = \frac{\sqrt{3}}{2}$

5. $x(t) = m(t)$

$y(t) = \cos(2\pi f_c t)$

$$F(x(t)y(t)) = X(f) * Y(f)$$

$$= M(f) * [\tfrac{1}{2}\delta(f-f_c) + \tfrac{1}{2}\delta(f+f_c)]$$

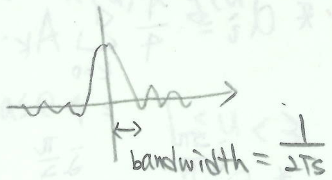
$$= \int_{-\infty}^{\infty} M(v)\cdot[\tfrac{1}{2}\delta(v-f+f_c) + \tfrac{1}{2}\delta(v-f-f_c)]\,dv$$

$$= \tfrac{1}{2}[M(f-f_c) + M(f+f_c)]$$

" $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\,dt = x(t_0)$ "

6. 要用 high bit rate 傳 代表 T_s 要小，但 bandwidth $= \frac{1}{2T_s}$ 就会

变很大 ⇒ 要 handle 的 frequency 就会变很多 (wide-band)



7. ① 有 carrier signal 要做 demodulation 時，可以用簡單的

R-C circuit 做 (Envelope detector)

② 用 $1+k_a m(t)$ 会比 $k_a m(t)$ 更容易做到

(在 modulate 的時候)

8. (a) $a_0 = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$a_1 = \cos(\frac{\pi}{2}+\frac{\pi}{6}) = -\frac{1}{2}$

$a_2 = \cos(\pi+\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$

$a_3 = \cos(\frac{3\pi}{2}+\frac{\pi}{6}) = \frac{1}{2}$

$e^{-j\frac{2\pi}{n}} = e^{-j\frac{\pi}{2}} = -j$

$A_0 = a_0+a_1+a_2+a_3 = 0$

$A_1 = a_0 - a_1 j - a_2 + a_3 j$

$= \sqrt{3} + j$

$A_2 = a_0 - a_1 + a_2 - a_3 = 0$

$A_3 = \overline{A_1} = \sqrt{3} - j$

(b) 對 A_k 乘上 H 得 $\begin{cases} -j & i>0 \\ j & i<0 \end{cases}$

$A_0' = 0$

$A_1' = 1 - \sqrt{3} j$

$A_2' = 0$

$A_3' = 1 + \sqrt{3} j$

⇒ 求 $a_0' = \frac{1}{4}(A_0'+A_1'+A_2'+A_3') = \frac{1}{4}\cdot 2 = \frac{1}{2}$

$a_1' = \frac{1}{4}(A_0'+A_1' j - A_2' - A_3' j) = (j+\sqrt{3} - j + \sqrt{3})\frac{1}{4}$

$= \frac{1}{4}(2\sqrt{3}) = \frac{\sqrt{3}}{2}$

$a_2' = \frac{1}{4}(A_0' - A_1' + A_2' - A_3') = \frac{1}{4}\cdot(-2) = -\frac{1}{2}$

$a_3' = \frac{1}{4}(A_0' - A_1' j - A_2' + A_3' j) = (-j - \sqrt{3} + j - \sqrt{3}) = \frac{(-2\sqrt{3})}{4} = \frac{-\sqrt{3}}{2}$

$a_k' = \frac{1}{4}\sum_{k=0}^{3} A_k' e^{j2\pi\frac{ik}{n}} = \sin(\frac{2\pi k}{4}+\frac{\pi}{6})$ for $k = 0,1,2,3$

9.

(a) 乘上 $\cos(2\pi f_c t)$

$$s'(t) = y(t)\cos(2\pi f_c t)$$

$$= \frac{1}{2}A_c[(1+k_a m(t))\cos(2\pi f_c t)](\cos(2\pi(2f_c+f_{IF})t)+\cos(2\pi f_{IF}t))$$

用 IF filter 就可以取出 $\frac{1}{2}A_c(1+k_a m(t))\cos(2\pi f_c t)\cos(2\pi f_{IF} t)$

Envelope Detector

+3

(b) 因為沒有 IF 的話就必須要做很多好的 band pass filter，很貴。故加上 IF，band pass filter 不用做得很窄，就算 pass 了隔壁的 carrier，因為它無法通過 IF filter 所以沒有關係。故有了 IF，只需一個很好的 IF filter 加上很多個普通的 band pass filter 就可以(便宜)。

10. (a) $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$

$f_i(t) = f_c + k_f m(t)$

$\Rightarrow \theta_i(t) = 2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(t)\,dt$

$\Rightarrow S(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(t)\,dt]$

(b) $\xrightarrow{S(t)}$ $\boxed{\text{微分器}}$ $S'(t) \rightarrow$ $\boxed{\text{Envelope detector}}$ $\rightarrow m(t)$

‧對 $s(t)$ 做微分得 $s'(t)$

$S'(t) = -A_c(2\pi f_c + 2\pi k_f m(t))\sin[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(t)\,dt]$

‧amplitude 是 $a + b\,m(t)$

$\Rightarrow$ 用 envelope detector 取出 message