Exam on Differential Equation Dec. 1 , 2009

1.(10%) Consider the following linear system v

$$m\frac{dy^2}{dt^2}+b\frac{dy}{dt}+ky=0. \quad \frac{dy^2}{dt^2}=-\frac{b}{m}\frac{dy}{dt}-\frac{k}{m}y$$

Convert the above equation to a linear system such that dY/dt = AY, Y= [y, v],^{T}

where $\frac{dy}{dt}=v$. What is A?

2. (20%) Consider the following linear system

$$\frac{dY}{dt}=\begin{bmatrix}-4 & 1\\ 2 & -3\end{bmatrix}Y,\quad Y(0)=\begin{bmatrix}1\\0\end{bmatrix}, Y(t)=\begin{bmatrix}x(t)\\y(t)\end{bmatrix}$$

(a) Find the eigenvalues of the systems. $-2, -5.$

(b) Find the general solution of the systems. $k_1 e^{-2t}\begin{bmatrix}1\\2\end{bmatrix}+k_2 e^{-5t}\begin{bmatrix}1\\-1\end{bmatrix}$

(c) Solve the initial-value problem $k_1=\frac{1}{3}, k_2=\frac{2}{3}$

3. (20%) Consider the following linear system

$$\frac{dY}{dt}=\begin{bmatrix}0-\lambda & 1\\ 1 & -0\lambda\end{bmatrix}Y,\quad Y(0)=\begin{bmatrix}1\\0\end{bmatrix}, Y(t)=\begin{bmatrix}x(t)\\y(t)\end{bmatrix}$$

(a) Find the eigenvalues of the system ± 1

(b) Find the eigenvectors $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}$

(c) Solve the initial-value problem $\frac{1}{2}e^{t}\begin{bmatrix}1\\1\end{bmatrix}+\frac{1}{2}e^{-t}\begin{bmatrix}1\\-1\end{bmatrix}$

4. (20%)Consider the following linear system

$$\frac{dY}{dt}=\begin{bmatrix}-1 & 2\\ 0 & -1\end{bmatrix}Y,\quad Y(0)=\begin{bmatrix}0\\1\end{bmatrix}, Y(t)=\begin{bmatrix}x(t)\\y(t)\end{bmatrix}$$

(a) Find the eigenvalues of the system -1

(b) Find the eigenvectors $\begin{bmatrix}1\\0\end{bmatrix}$

(c) Solve the initial-value problem $e^{-t}\begin{bmatrix}0\\1\end{bmatrix}+te^{-t}\begin{bmatrix}2\\0\end{bmatrix}$

5. Consider the following linear system

$$\frac{dY}{dt}=\begin{bmatrix}-2-\lambda & 2\\ 1 & -1-\lambda\end{bmatrix}Y,\quad Y(0)=\begin{bmatrix}0\\1\end{bmatrix}, Y(t)=\begin{bmatrix}x(t)\\y(t)\end{bmatrix}$$

(a) Find the eigenvalues of the system $0, -3$

(b) Find the eigenvectors $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\-1\end{bmatrix}$

(c) Solve the initial-value problem $\frac{2}{3}\begin{bmatrix}1\\1\end{bmatrix}-\frac{1}{3}e^{-3t}\begin{bmatrix}2\\-1\end{bmatrix}$

6 .(10%) Consider the following linear system

$$\frac{dy^2}{dt^2}+25y=0.$$

(a)Determine the general solution of the system. $y(t)=k_1\cos 5t+k_2\sin 5t$

(b) Determine the solution for y(0)=0 and y'(0)=1. $y(t)=\frac{1}{5}\sin 5t$