Exam on Differential Equation Jan. 9, 2015

- 1. (25%)
- (a) (10%) Compute the Laplace transform of the solution of the initial-value problem.

$$\frac{d^2y}{dt^2} + 441y = u_0(t), y(0) = 1, y'(0) = 1.$$

(b) (5%) Explain whether the system is stable or not based on the solution in (a).

(c) (10%)
$$\frac{d^2y}{dt^2} + 441y = u_0(t) - u_5(t), y(0) = 1, y'(0) = 1.$$

- 2. (15%)
- (a) (5%) Write the integral function of the convolution f*g for the given functions f

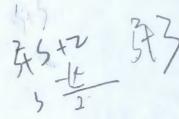
and g:
$$f(t) = \cos t$$
 and $g(t) = e^{-t}$.

- (b) (10%) Compute the convolution of f*g.
- 3. (20%)
- (a) (10%) Compute the solution of the initial-value problem

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = \delta_2(t), y(0) = 1, y'(0) = 0.$$

(b) (10%) Compute the solution of the initial-value problem

$$\frac{d^2y}{dt^2} + 9y = e^{-3t}, y(0) = 1, y'(0) = 0.$$



4. (20%) Solve the initial-value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = u_2(t)e^{-(t-2)}\cos 3(t-2), y(0) = 1, y'(0) = 1$$

5. (20%) Compute the Laplace transform of the solution of the initial-value problem.

$$\frac{d^2y}{dt^2} + 9y = \cos 3t + \sin 2t, y(0) = 1, y'(0) = 0.$$

f(t)	F(s)	f(t)	F(s)
e ^{at}	$\frac{1}{s-a}(s>0)$	t ⁿ	$\frac{n!}{s^{n+1}}(s>0)$
sin <i>wt</i>	$\frac{\omega}{s^2 + \omega^2}$	cosat	$\frac{s}{s^2 + \omega^2}$
$e^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$	e ^{at} cos wt	$\frac{s-a}{(s-a)^2+\omega^2}$
$t \sin \omega t$	$\frac{2\omega s}{\left(s^2+\omega^2\right)^2}$	t cos \omega t	$\frac{s^2 - \omega^2}{\left(s^2 + \omega^2\right)^2}$
$u_a(t)$	$\frac{e^{-as}}{s}(s>0)$	$\delta_a(t)$	e^{-as}
$u_a(t)f(t-a)$	$e^{-as} F(s)$	$e^{at}f(t)$	F(s-a)