

3:20 ~ 5:10 p.m., November 12, 2004

- (20%) Suppose there are only nine white-eyed fruit flies in a container of 40 flies. An inspector selects five flies randomly without replacement to examine.
  - (5%) Find the probability of at least four white-eyed fruit flies in this inspection.
  - (5%) How many items should be examined so that the probability of finding at least one white-eyed fruit fly exceeds 0.8?
  - (10%) If the selection is done at random but with replacement, repeat the questions (a) & (b).
- (15%) Four distinct integers are chosen at random and without replacement from the set of the ten positive integers,  $\{1, 12, 2, 24, 4, 34, 6, 68, 8, 9\}$ . Let the random variable  $X$  be the second largest of these four numbers. Find its p.m.f.
- (15%) Suppose the moments of  $X$  are  $E(X^m) = (m+1)!2^m, m = 1, 2, 3, \dots$ . Find  $P(X > 7.8)$ .
- (25%)  $X$  has a Poisson distribution.
  - (10%) If  $P(X = 1) = 2P(X = 2)$ , compute  $P(X = 4)$  and  $P(|X - \mu| < 2\sigma)$ .
  - (5%) If  $P(X = 0) = 0.135$ , find  $P(X = 3)$ .
  - (10%) If  $f(x) = \frac{4}{x} f(x-1), x = 1, 2, \dots$ , find  $Var(X)$ .
- (10%)  $X$  has a gamma distribution with  $\alpha = 3$  and  $\theta = 2$ , find  $P(10.64 < X \leq 16.81)$ .
- (15%) If  $X$  is  $b(n, p)$ , show that  $E\left(\frac{X}{n}\right) = p$  and  $E\left(\left(\frac{X}{n} - p\right)^2\right) = \frac{p(p-1)}{n}$ .