Algorithms Middle Examination
Nov. 9, 2012
(10:10 am ~ 12:30 am)

1. (10%) Please complete the following table

	Insertion_sort	Selection_sort	Merge_sort	Heap_sort	Quick_sort
Average case		$O(N^2)$			
Worst case	$O(N^2)$				
Stable or not					

2. (10%) Let $p(n) = \sum_{i=0}^{d} a_i n^i$, where $a_d > 0$, be a degree-d polynomial in n, and let k be a constant. Use the definitions of the asymptotic notation to prove the following property. If $k \le d$, then $p(n) = \Omega(n^k)$.

3. (7%) Solve the recurrence $T(n) = 3T(\sqrt{n}) + \log n$ by making a change of variables.

4. (8%) Using Substitution Method to find the solution of $T(n)$.

$$T(n) = T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + 1, \quad T(1) = 1$$

5. (7%) Give an $O(n \lg k)$-time algorithm to merge k sorted lists into one sorted list where n is the total number of elements in all the input lists.

6. (10%) Please prove that the average running time of Quicksort is $O(n \lg n)$.

7. (8%) Please give an optimal decision tree with four elements a, b, c, and d.

8. (5%) It is known that $\Omega(n \log n)$ is a lower bound for sorting. However, we have seen algorithms like counting sort or radix sort which can sort n items in $O(n)$ time. Is there a contradiction? If not, why? Explain?

9. (10%) Describe a method to find the kth smallest elements of a set of n distinct integers in $O(n)$ time.

10. (5%) Please give two key factors that an optimizations problem must have in order for dynamic programming to apply.

11. (10%) Please use the bottom-up approach of Dynamic Programming to find the optimal order, and its cost, for computing the product $A_1A_2A_3A_4A_5$, where A_1= (10 x 15), A_2 = (15 x 8), A_3 = (8 x 4), A_4 = (4 x 10), and A_5 = (10 x 20). (You need to show your answer in two-dimensional tables.)

12. (10%) Determine an LCS of two sequences X = abcbdaa and Y = dcbadbca. (You need to show your answer in two-dimensional tables.)