Subject: Close-book final of CS3334-01 (Engineering Mathematics), Jan. 14, 2005

Six hints and rules for a fair examination:

一、Close books, close everything, and turn off all personal electronics including the cell phone's sound and vibration; calculators can be left on.

二、Please raise your hand with patience when you want to talk to the teacher or a TA.

三、Please try to understand and answer the questions properly.

四、Please express your "thought process" succinctly to avoid jump-to-conclusion answer; please make each of your final answers stands out clearly and explicitly to avoid grading errors.

五、Between 10:10AM and 12:00noon, you are welcome and allowed to talk to the teaching assistants. No chatting or any other form of communications between classmates will be allowed.

六、A "model" is not the same as an "initial value problem".

七、Ten credits for each question.

1. Solve the initial-value problem:
$dy/dt = -y + u_3(t),\ y(0) = 2.$

2. Solve the initial-value problem:
$dy/dt = -y + t^2,\ y(0) = 1.$

3. For the following differential equation:
$d^2y/dt^2 + 4y = \cos 2t,\ y(0) = -2,\ y'(0) = 0$
(a) compute the Laplace transform of both sides of the equation;
(b) find the solutions by using the inverse Laplace transform.

4. For the function:

$$d^2y/dt^2 + 4dy/dt + 13y = e^{-t}.$$

Derive the general solution.

5. For the function:

$$d^2y/dt^2 + 3dy/dt + 2y = 2 \sin t.$$

Derive the general solution.

6. A linear system is modeled as:

$$\begin{pmatrix} \dfrac{dx}{dt} \\ \\ \dfrac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(a) Compute the two eigenvalues for the above;
(b) For the two eigenvalues, compute the associated eigenvectors, respectively;
(c) Compute the general solution;
(d) If the initial condition is **Y(t=0)** = (5 0), solve the IVP by generating a solution.

7. A linear system is modeled as:

$$\begin{pmatrix} \dfrac{dx}{dt} \\ \\ \dfrac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

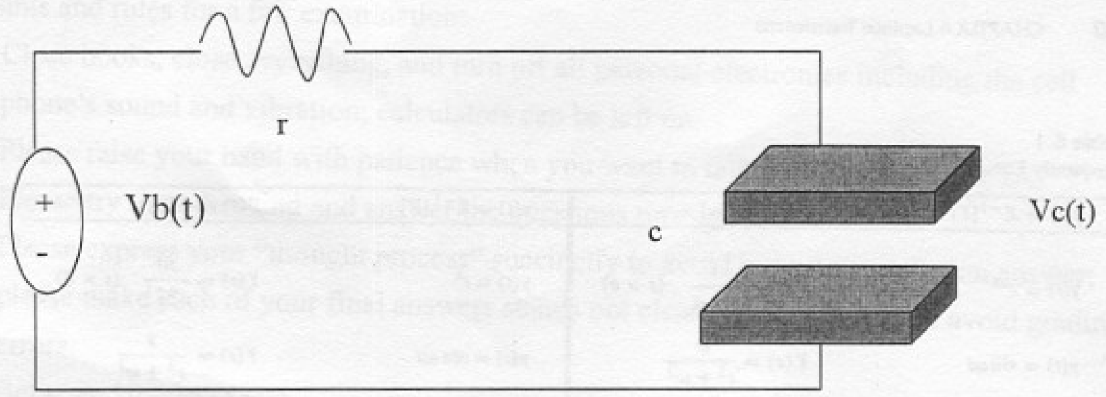
(a) Compute the two eigenvalues that are conjugated complex numbers;
(b) Selecting one of the two eigenvalues, compute its eigenvector;
(c) Compute the general solution that has real-numbered parameters;
(d) If the initial condition is **Y(t=0)** = (0.5 0.5), solve the initial-value problem (IVP) by generating a solution that has only real numbers.

8. We consider the system:

$$\begin{cases} dx(t)/dt = 2x(t) + y(t) \\ \\ dy(t)/dt = -y(t). \end{cases}$$

(a) Derive the general solution.
(b) If the initial condition is (x(0) , y(0)) = (-1 , 3), determine the solution for the IVP.

9. Using $Qc(t) = c * Vc(t)$, $dQ(t)/dt = I(t)$, $V_r(t) = r * I_r(t)$, and the following simple circuit:



(a) Due to Kirchhoff's Voltage Law (KVL, $Vb = V_r + Vc$) and KCL, explain and prove that the above circuit can be modeled by the following ordinary differential equation (ODE):

$$dVc(t)/dt = (Vb(t) - Vc(t)) / (r*c).$$

(b) If Vb(t) is a constant direct current battery at 1.0 volt and $Vc(t=0) = 0$, please write down the initial-value problem (IVP) that consists of an ODE and the initial condition.

(c) Please analytically solve the IVP in (b)

10. A cup of hot chocolate is initially 180°F and is left in a room with an ambient temperature of 80°F. Suppose that at time t=0 it is cooling at a rate of 20°F per minute.

(a) Assume that Newton's law of cooling applies: The rate of cooling is proportional to the difference between the current temperature and the ambient temperature .Write an initial-value problem that models the temperature of the hot chocolate.

(b) How long does it take the hot chocolate to cool to a temperature of 110°F?

T ln

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$\frac{3}{10} = e^{-x/5}$

$x = 5 \ln \frac{10}{3}$

602 CHAPTER 6 Laplace Transforms

Table 6.1
Frequently Encountered Laplace Transforms.

$y(t) = \mathcal{L}^{-1}[Y]$	$Y(s) = \mathcal{L}[y]$	$y(t) = \mathcal{L}^{-1}[Y]$	$Y(s) = \mathcal{L}[y]$
$y(t) = e^{at}$	$Y(s) = \dfrac{1}{s-a} \quad (s > a)$	$y(t) = t^n$	$Y(s) = \dfrac{n!}{s^{n+1}} \quad (s > 0)$
$y(t) = \sin \omega t$	$Y(s) = \dfrac{\omega}{s^2+\omega^2}$	$y(t) = \cos \omega t$	$Y(s) = \dfrac{s}{s^2+\omega^2}$
$y(t) = e^{at} \sin \omega t$	$Y(s) = \dfrac{\omega}{(s-a)^2+\omega^2}$	$y(t) = e^{at} \cos \omega t$	$Y(s) = \dfrac{s-a}{(s-a)^2+\omega^2}$
$y(t) = t \sin \omega t$	$Y(s) = \dfrac{2\omega s}{(s^2+\omega^2)^2}$	$y(t) = t \cos \omega t$	$Y(s) = \dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$y(t) = u_a(t)$	$Y(s) = \dfrac{e^{-as}}{s} \quad (s > 0)$	$y(t) = \delta_a(t)$	$Y(s) = e^{-as}$

Table 6.2
Rules for Laplace Transforms:
Given functions $y(t)$ and $w(t)$ with $\mathcal{L}[y] = Y(s)$ and $\mathcal{L}[w] = W(s)$ and constants α and a.

Rule for Laplace Transform	Rule for Inverse Laplace Transform
$\mathcal{L}\left[\dfrac{dy}{dt}\right] = s\mathcal{L}[y] - y(0) = sY(s) - y(0)$	
$\mathcal{L}[y+w] = \mathcal{L}[y] + \mathcal{L}[w] = Y(s) + W(s)$	$\mathcal{L}^{-1}[Y+W] = \mathcal{L}^{-1}[Y] + \mathcal{L}^{-1}[W] = y(t) + w(t)$
$\mathcal{L}[\alpha y] = \alpha \mathcal{L}[y] = \alpha Y(s)$	$\mathcal{L}^{-1}[\alpha Y] = \alpha \mathcal{L}^{-1}[Y] = \alpha y(t)$
$\mathcal{L}[u_a(t) y(t-a)] = e^{-as}\mathcal{L}[y] = e^{-as} Y(s)$	$\mathcal{L}^{-1}[e^{-as} Y] = u_a(t) y(t-a)$
$\mathcal{L}[e^{at} y(t)] = Y(s-a)$	$\mathcal{L}^{-1}[Y(s-a)] = e^{at}\mathcal{L}^{-1}[Y] = e^{at} y(t)$