# Problems for Exam 1

**1.** Given $sinh(x) = (e^x - e^{-x})/2$ for $-\infty < x < \infty$.

**(a)** Find the Taylor expansion of $sinh(x)$ about 0.

**(b)** Find the inverse function $sinh^{-1}(x)$.

**2.** Prove that if $n\epsilon < 0.01$, then $(1+\epsilon)^n - 1 < 0.01006$.

**3.** Let $L, M \in R^{n\times n}$ be unit lower-$\Delta$, show that

**(a)** $LM$ is unit lower-$\Delta$.

**(b)** $L^{-1}$ is unit lower-$\Delta$.

**4.** Prove that if $A \in R^{n\times n}$ is a nonsingular matrix and $A$ has an LU-decomposition in which L is unit lower-$\Delta$ and U is upper-$\Delta$, then L and U are unique.

**5.** Prove that if $\|A\| < 1$, then $\|(I+A)^{-1}\| \le (1 - \|A\|)^{-1}$

**6.** Let $T \in R^{n\times n}$ be a tridiagonal matrix which is also diagonally dominant.

**(a)** Show that a diagonally dominant matrix is nonsingular.

**(b)** Give an efficient algorithm to do $T = LU$.

**(c)** How many operations are needed for your algorithm?

**7.** Let $\|A\|_1 = max_{\|\mathbf{x}\|_1=1}\{\|A\mathbf{x}\|_1\}$ If $A \in R^{m\times n}$, then $\|A\|_1 = max_{1\le j\le n}\left[\sum_{i=1}^m |a_{ij}|\right]$

**8.** Let $\mathbf{x}, \mathbf{y} \in R^{n\times n}$ be unit vectors. Find a Householder matrix $H$ such that $H\mathbf{x} = \mathbf{y}$.

**9.** For solving $A\mathbf{x} = \mathbf{b}$, find the iteration matrix $B \in R^{n\times n}$ in the Jacobi method and that in the Gauss-Seidel method when

$$A = \begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}$$