Exam on Differential Equation Jan. 9, 2015

1. (25%)

(a) (10%) Compute the Laplace transform of the solution of the initial-value problem.

$$\frac{d^2y}{dt^2}+441y=u_0(t), y(0)=1, y'(0)=1.$$

(b) (5%) Explain whether the system is stable or not based on the solution in (a).

(c) (10%) $\frac{d^2y}{dt^2}+441y=u_0(t)-u_5(t), y(0)=1, y'(0)=1.$

2. (15%)

(a) (5%) Write the integral function of the convolution f*g for the given functions f and g: $f(t)=\cos t$ and $g(t)=e^{-t}$.

(b) (10%) Compute the convolution of f*g.

3. (20%)

(a) (10%) Compute the solution of the initial-value problem

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}+2y=\delta_2(t), y(0)=1, y'(0)=0.$$

(b) (10%) Compute the solution of the initial-value problem

$$\frac{d^2y}{dt^2}+9y=e^{-3t}, y(0)=1, y'(0)=0.$$

4. (20%) Solve the initial-value problem

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=u_2(t)e^{-(t-2)}\cos 3(t-2), y(0)=1, y'(0)=1$$

5. (20%) Compute the Laplace transform of the solution of the initial-value problem.

$$\frac{d^2y}{dt^2}+9y=\cos 3t+\sin 2t, y(0)=1, y'(0)=0.$$

$f(t)$	$F(s)$		$f(t)$	$F(s)$
e^{at}	$\frac{1}{s-a}(s>0)$		t^n	$\frac{n!}{s^{n+1}}(s>0)$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$		$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$		$e^{at}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$		$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$u_a(t)$	$\frac{e^{-as}}{s}(s>0)$		$\delta_a(t)$	e^{-as}
$u_a(t)f(t-a)$	$e^{-as}F(s)$		$e^{at}f(t)$	$F(s-a)$