$\frac{9}{10000}$ $\frac{\frac{1}{10^3}}{1\frac{9}{10}} = \frac{1}{900} \times 27$

Probability (CS 3332)

$f_Y(y) = 1$

$f_X(x) = 1$

Final Exam (June 8, 2016)

Exercise 8.2.24 (15%) Let X and Y be two independent random points from the interval $(0, 1)$. Calculate the probability distribution function of

$$\boxed{\max(X, Y)/\min(X, Y).}$$

$G(t) = F\left(\frac{\max(X,Y)}{\min(X,Y)} \le t\right)$

Example 8.23 (15%) Let the conditional probability density function of X, given that $Y = y$, be

$$f_{X|Y}(x|y) = \frac{x+y}{1+y}e^{-x}, \qquad 0 < x < \infty,\ 0 < y < \infty.$$

$\frac{x+2}{3}e^{-x}$

Find $P(X < 1|Y = 2)$. $= \int_0^1 f_{X|Y}(x|2)\,dx$ $27 \times 0.00\overline{1}$ $0.00\overline{27}$

Exercise 10.3.2 (20%) Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} \sin x \sin y & \text{if } 0 \le x \le \pi/2, 0 \le y \le \pi/2 \\ 0 & \text{otherwise.} \end{cases}$$

$\iint xy \sin x \sin y \, dx\, dy$

Calculate the covariance and the correlation coefficient of X and Y.

Hint. Covariance of X and Y is defined as

$$\mathrm{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = E(XY) - E(X)E(Y)$$

and the correlation coefficient of X and Y is defined as

$$\rho(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y} - \sqrt{E(X)E(Y)}.$$

$\int x \int_0^{\frac{\pi}{2}} \sin y \, dy \, \sin x \, dx$

002347008 Y E(X)

Example 10.22 (15%) What is the expected number of random digits that should be generated to obtain three consecutive zeros? $= \sum_{i=0}^{\infty} XP(X|Y=i)\,P(Y=i)$ $\frac{2}{3}(1-e^{-1}$

Example ~~10.28 (15%)~~ A fisherman catches fish in a large lake with lots of fish, at a Poisson rate of two per hour. If, on a given day, the fisherman spends randomly anywhere between 3 and 8 hours fishing, find the expected value and the variance of the number of fish he catches.

E(X) E(E(N|T))

Hint. $E(N(t)|T=t)$ $\lambda = 2$ $\cancel{E(X|Y=y)} =$ 16 $= 11$

1. $\mathrm{Var}(X) = E[\mathrm{Var}(X|Y)] + \mathrm{Var}(E[X|Y])$
2. Let $N(t)$ denote the number of events that a Poisson process with rate λ produces in an interval of length t. Then the expectation and the variance of $N(t)$ are both equal to λt.
3. Let U be a uniform random variable over an interval (a, b). Then,

$$E[U] = (a+b)/2$$
$$\mathrm{Var}(U) = (b-a)^2/12.$$

$E(N|T=t) = \lambda t$ $E(T) = \frac{a+b}{2}$ $f = \lambda e^{-\lambda t}$ $F(t) = 1 - e^{-\lambda t}$

Exercise 11.2.3 ~~(20%)~~ Let $X_1, X_2, \ldots, X_n$ be independent exponential random variables with identical mean $1/\lambda$. Find the moment-generating function of X_1. Use the moment generating function of X_1 to find the probability distribution function of $X_1 + X_2 + \cdots + X_n$.

$E(X) = E(X)\left[\frac{9}{10} + \frac{9}{100} + \frac{9}{1000}\right] + 3\left[\frac{9}{1000} + \ldots\right]$

$27 \times \frac{10^3}{9}$