CALCULUS (MAY 30, 2006)

1. (10) Find (a) the angle, and (b) the parametric equation of the line of intersection of the given planes: $2x + y + z = 4$ and $3x - y - z = 3$

2. (10) Given the four points $\mathbf{A}(2,3,3)$, $\mathbf{B}(4,1,0)$, $\mathbf{C}(-1,2,0)$, $\mathbf{D}(5,4,-2)$, find the equation of the plane passes through $\mathbf{A}, \mathbf{B}$ and is parallel to the line through $\mathbf{C}, \mathbf{D}$.

3. (10) Determine the following limits and give your reason.
(a) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$, (b) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$.

4. (10) Find the distance between the two given lines:
$\mathbf{L}_1 : (1,0,0) + t(\mathbf{j} + \mathbf{k})$ and $\mathbf{L}_2 : (0,1,0) + s(\mathbf{i} - \mathbf{j} + \mathbf{k})$.

5. (10) Find the curvature and the torsion of the helix $(\cos t, \sin t, 2t)$.

6. (10) Find the extrema of $f(x,y,z) = x + 2y + 3z$ on the upper half ball $0 \le z \le \sqrt{1 - x^2 - y^2}$.

7. (10) Suppose $w = f(r)$ and $r = \sqrt{x^2 + y^2 + z^2}$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = \frac{d^2 w}{dr^2} + \frac{2}{r}\frac{dw}{dr}$.

8. (10) Show that the function $f(x,y) = (x^{\frac{1}{3}} + y^{\frac{1}{3}})^3$ is continuous at $(0,0)$, and have all directional derivatives there, but not differentiable there.

9. (10) Find the extrema of $f(x,y,z) = y + z$ subject to the contrains $x + y + z = 1$ and $x^2 + y^2 = 1$.

10. (10) Consider a plane tangent to the surface with the equation $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 1$. Find the sum of the square of the $x-, y-, z-$ intercepts of this plane.