$S = x_p \cdot t^2$ $\quad 0 \quad 2t$

$z' - (2Py + Q)z = -R$ $\qquad \dfrac{dy}{dx} = \dfrac{-2x}{y}$

Engineering Math Midterm#2

Class:______________

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$x' = c(200 - x)x$

$\Rightarrow x' = \underbrace{-cx^2}_{P} + \underbrace{200cx}_{Q} + \underbrace{200c}_{R}$

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$Q \cdot \frac{1}{ct}\left(200 - \frac{1}{ct}\right) = \frac{-1}{ct^2} + \frac{200}{t}$

1. (8%) Suppose a person carrying a virus returns to an isolated group of 200 persons. Assume that the day rate at which the virus spreads out is proportional to the multiplication of the number of infected persons and the number of non-infected persons. It is observed that the number of infected persons is 15 after 5 days. Determine the number of infected persons after 12 days.

$(5) = 15 \qquad x'(5) = c\,15 \times 185$

$(12) = ?$

$x = \frac{1}{t} + \frac{1}{c}\frac{-1}{t^2} = c\left(200 - \frac{1}{t}\right)\left(\frac{1}{t}\right)$

$\to x' = \frac{-1}{t^2} \qquad = \frac{-1}{t^2} + \frac{200}{t}$ $\quad Q(0) = Q_0$

2. (12%) A container has Q_0g of sugar dissolved in 120 liters of water. Assume the water containing 0.3g of sugar per liter is flew into the container at a rate of r liters per minute and the well-stirred sugar water is draining from the container at the same rate. Determine the quantity of sugar Q(t) in the container at any time. Also find the limiting quantity QL that is present after a very long time.

$\frac{dQ}{dt} = \cancel{0.3 \times 120} \times r - \frac{Q}{120} \times r \Rightarrow Q' = 36 - \frac{r}{120}Q$

3. (10%) Let y_1 and y_2 be two solutions of a homogeneous equation $y'' + p(x)y' + q(x)y = 0$. Show the Wronskian of y_1 and y_2 is expressed as $W(y_1, y_2) = y_1(x)y_2'(x) - y_1'(x)y_2(x) = c_1 e^{\int -p(x)dx}$.

$W' + p(x)W = 0$

$\Rightarrow y_1'' + py_1' + y_1 = 0 \qquad y_1''y_2 + py_1'y_2 + y_1y_2 = 0$

$y_2'' + py_2' + y_2 = 0 \quad -) \quad y_2''y_1 + py_2'y_1 + y_1y_2 = 0$

$(y_1''y_2 - y_2''y_1) + p(y_2'y_1 - y_1'y_2) = 0 \quad \Rightarrow W =$

$\Rightarrow \frac{d}{dx}\left(We^{\int +p\,dx}\right) = 0$

$\Rightarrow We^{\int p(x)dx} = C_1$

$(xy)p^2 + (y^2 - xy - 2x^2)p + (2x^2 - 2xy) = 0$

$(xp + (y - x))(yp + 2x)$

4. (8%) Solve the general solution of the ODE

$$x^2 y \cancel{(y')^2} + (xy^2 - x^2 y - 2x^3)\cancel{y'} + (2x^3 - 2x^2 y) = 0.$$

$y_p = e^{x} = u$

$(p + yx^2)(xyp + (2x - 2)) = 2x \cdot x^2(x - y)$ $\quad y = uv$

$[p + (x - y)][p + 2x^2]$

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5. (12%) Solve the general solution of the ODE $y''+y=(x-1)\cos x$

$y_1 = 1$

$y_2 = e^{-x}$

$$\begin{bmatrix} 1 & e^{-x} \\ 0 & -e^{-x} \end{bmatrix} \begin{bmatrix} \phi_1' \\ \phi_2' \end{bmatrix} = \begin{bmatrix} 0 \\ (x-1)\cos x \end{bmatrix}$$

$\phi_1' = \dfrac{(x-1)\cos x \cancel{e^{-x}}}{\cancel{e^{-x}}}$

$\phi_2' = \dfrac{(x-1)\cos x}{-e^{-x}}$

6. (15%) Solve the general solution of the ODE $x^2y''-2xy'+2y=x^2+2$

$x(A\cos x + B\sin x) + (C\cos x + B\sin x)$

$y_p = (Ax^2)(B\cos x + C\sin x)$

$\Rightarrow y_p' = (Ax^2)(-B\sin x + C\cos x)$

$+ 2Ax(B\cos x + C\sin x)$

7. (15%) Solve the general solution of the ODE $x^2y''-4xy'+6y=2(\ln x)^2$;

$x>0$.

$x^2(-A\sin x + B\cos x)$

$y_p'' = (Ax^2)(-B\cos x - C\sin x)$

$+ 2Ax(-B\sin x + C\cos x)$

$+ 2Ax(-B\sin x + C\cos x)$

$v'' + \dfrac{2u' + Pu}{u} v' = \dfrac{R}{u}$

$+ 2A(-B\cos x - C\sin x)$

8. (15%) Solve the general solution of the ODE

$2xy''+(1-4x)y'+(2x-1)y=e^x$ (Hint: $\boxed{y=e^x}$ is a homogeneous solution)

u

$y'' + \underbrace{\left(\frac{1}{2x} - 2\right)}_{P} y' + \underbrace{\left(1 - \frac{1}{2x}\right)}_{Q} y = \underbrace{\frac{e^x}{2x}}_{R}$

$\Rightarrow (-2A$

9. (15%) Solve the general solution of the ODE $2y''-4y'+2y=\dfrac{4e^x}{2x-1}$

$y_1' \Rightarrow x(-A\sin x + B\cos x)$

$\cancel{+(A\cos x + B\sin x)} + \left[(A+C)\cos x + (B+D)\sin x\right]$

$\cancel{+(C\cos x + D\sin x)}$

$y_p'' \Rightarrow x(-A\cos x - B\sin x)$

$+ (-A\sin x + B\cos x)$

$+ [(-A-C)\sin x + (B+D)\cos x]$

$= 4Ax(-B\sin x + C\cos x)$

$+ 2A(-B\cos x - C\sin x)$

$= (4AC)x\cos x$

$-4ABx\sin x$

$2AB\cos x$

$-2AC\sin x$