Final Examination on Algorithms

Teacher: Biing-Feng Wang

June 26, 2007

Problem 1: (10%, Problems selected from midterm examination-Part I)

(1) (5%) Prove that $9n^3 + 41n^2 = O(n^4)$ according to the definition of O.

(2) (5%) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = T(\lfloor 2n/3 \rfloor) + 1$ for $n \geq 2$ and $T(1) = 1$.

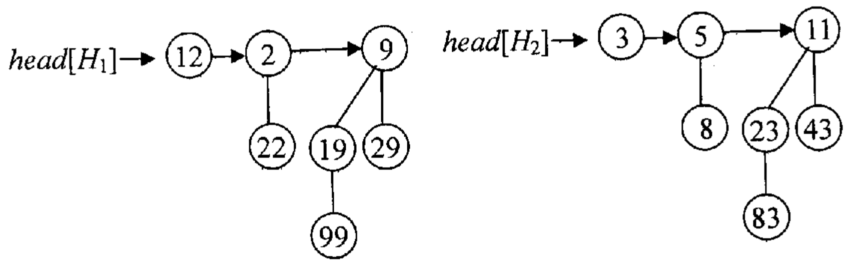
Problem 2: (15%, Problems selected from midterm examination-Part II)

(1) (7%) Let $X_1, X_2, \ldots, X_k$ be k arrays. Each X_i contains an arbitrary sequence of integers within the range $[1, n^{\log\log n}]$, where n is the total number of elements in all the lists. Give an efficient algorithm to sort each X_i. What is the time complexity?

(2) (8%) Suppose you have one machine and a set of n jobs $a_1, a_2, \ldots, a_n$ to process on that machine. Each job a_j has a processing time t_j, a profit p_j, and a deadline d_j. The machine can process only one job at a time, and job a_j must run uninterruptedly for t_j consecutive time units. If job a_j is completed by its deadline d_j, you receive a profit p_j, but if it is completed after its deadline, you receive a profit of 0. Give an algorithm to find the schedule that obtains the maximum amount of profit, assuming that all processing times are integers between 1 and n^3. What's the running time?

Problem 3: (10%, Mergeable heaps)

(1) (5%) Use the following example to explain how two binomial heaps are united.



(2) (5%) What is the time complexity for uniting two heaps? Explanation is necessary.

Problem 4: (13%, Minimum spanning trees)

(1) (8%) Write down Prim's minimum spanning tree algorithm.

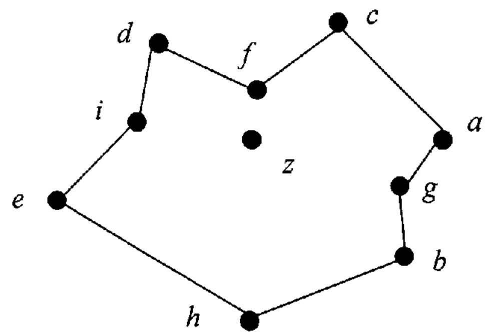
(2) (5%) If binomial heap is used, what is the running time of Prim's algorithm? Explanation is necessary.

Problem 5: (13%, Elementary graph algorithms)

(1) (8%) Give an efficient algorithm for topological sort.

(2) (5%) What is the time complexity?

Problem 6: (10%, Convex hulls) A simple polygon P is *star-shaped* if there exists a point z within P such that for all point q within P the line segment $\overline{zq}$ lies entirely within P. Such a vertex z is called a *locus* of P. Let P be a star-shaped simple polygon specified by its n vertices in counterclockwise order. Show that if a locus z of P is given, the convex hull of the vertices of P can be computed in $O(n)$ time.



A star-shaped simple polygon P = (h, b, g, a, c, f, d, i, e) and a locus z.

Problem 7: (14%, Maximum flow)

(1) (8%) Give an efficient algorithm for finding a maximum matching of a bipartite graph.

(2) (6%) What is the time complexity? Explanation is necessary.

Problem 8: (15%, The subset sum problem)

(1) (8%) Give an approximation scheme that finds a good approximation to the *largest value not larger than t* that is a sum of some subset of the given input list. No explanation is necessary.

(2) (7%) Modify your answer in (1) to find a good approximation to the *smallest value not less than t* that is a sum of some subset of the given input list. No explanation is necessary.

Bonus: (10%) This problem is to verify whether you had done homework by yourself. Please answer either of the following. Proofs and analyses are unnecessary. (If you answer both, only the one getting less score will be counted.)

(a) Give an efficient algorithm to find a minimum path cover of a directed acyclic graph. What is the time complexity?

(b) Give an efficient algorithm that finds an optimal vertex cover for a tree in linear time.