# Calculus I Final Exam C 卷

2014/06/17(二)

$x^2+2$

$2y^2+1$

**Note: There is NO any multiple choice question.**

1. (6%) The function $f(x,y)=\frac{1}{3}x^3+2x+\frac{2}{3}y^3+y$ has no local maximum points and no local minimum points on $R^2$. Give your reasons.

2. (6%) $u=f(x,y,z)$, $x=\sin(t^2+s^2)$, $y=ts+s^2$, and $z=e^{s+t^2}$, find $\frac{\partial u}{\partial t}$ and $\frac{\partial u}{\partial s}$.

3. (6%) Who is the greatest handsome guy in our class ?

4. (10%) $f(x,y)=x^2-2xy+2y$, find the absolute maximum points and the absolute minimum points on $D=\{(x,y)\mid 0\le x\le 2,\ 0\le y\le 3x\}$.

5. (6%) Compute the definite integral $\int_0^\infty \frac{e^{-3x}-e^{-x}}{x}dx$.

6. (6%) Find the directional derivative of $f(x,y)=x^3-2xy^2+3y$ at $(1,2)$ in the direction $\mathbf{u}=(3,4)$.

7. (6%) Find the interval of the convergence of $\sum_{k=0}^{\infty}\frac{1}{k3^k}(x-2)^k$.

8. (6%) What's the Taylor's series of $f(x)=x^2\cos x$ at 0.

9. (6%) Does this series

$$\sum_{k=2}^{\infty}(-1)^k\frac{1}{k\ln k[\ln\ln k]} \to 0$$

converge absolutely, converge conditionally, or diverge ?

$\boxed{\cos x}$

$\to \sin x$

$-\cos x$

10. (6%) $u=f(x,y)$, $x=\ln(\sin t^2)$, and $y=\sqrt{1+t^2}$, find $\frac{du}{dt}$.

11. (6%) Compute the indefinite integral $\int\frac{\sec^5 x-1}{\sec x-1}dx$.

12. (6%) Differentiate $f(x)=\sec^{-1}(\sqrt{x^2+1})$ with respect to $x$.

13. (6%) Compute $\iiint_\Omega z\sqrt{x^2+y^2+z^2}\,dxdydz$, where

$[\ln(\ln k)]$

$$\Omega=\{(x,y,z)\mid 0\le x\le\sqrt{9-y^2},\ 0\le y\le 3,\ 0\le z\le\sqrt{9-(x^2+y^2)}\}.$$

14. (6%) Compute $\iint_\Omega (x^2+y^2)dxdy$, where

$$\Omega=\{(r,\theta)\mid r^2=\cos 2\theta,\ 0\le\theta\le\frac{\pi}{4}\}.$$

15. (6%) Calculate $\lim_{x\to\infty} xe^{-x^2}\int_0^x e^{t^2}dt$.

$2\cos^2 2\theta$

16. (6%) Pick up the mistake in the following statement.

If $f$ has the first partial derivatives on $D\subseteq R^3$, then

$$\nabla f(\mathrm{x}_0)=\frac{\partial f}{\partial x}(\mathrm{x}_0)\vec{i}+\frac{\partial f}{\partial y}(\mathrm{x}_0)\vec{j}+\frac{\partial f}{\partial z}(\mathrm{x}_0)\vec{k}.\ \mathrm{x}_0\in D.$$

cont

$\left(\frac{\sqrt{2}}{2}\right)^3$ $\frac{2\sqrt{2}}{8}$