陳朝欽 線代 Test 2

Test 2 for CS2334

November 8, 2004

10

(5 pts) 1. Mark ○ if the statement is *true*, and mark × otherwise.

(○)(a) Let $R^+ = \{x|\ x > 0\}$. For $\forall\ x, y \in R^+$ and $\alpha \in R^+$, the addition $\oplus$ and scalar multiplication $\odot$ are defined as $x \oplus y = xy$ and $\alpha \odot x = x^\alpha$, respectively. Under these definitions, R^+ is a vector space over R^+.

(×)(b) For $\forall\ \mathbf{x} = [x_1, x_2]^t$, $\mathbf{y} = [y_1, y_2]^t$ in R^2 and $\alpha \in R$, the vector addition $\oplus$ and scalar multiplication $\odot$ are defined as $\mathbf{x} \oplus \mathbf{y} = [x_1 + y_1 + 1, x_2 + y_2 + 1]^t$ and $\alpha \odot \mathbf{x} = [\alpha x_1, \alpha x_2]^t$, respectively. Under these definitions, R^2 is a vector space over R.

(×)(c) If $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n$ span R^n, then they are linearly independent.

(×)(d) If $V = span(\mathbf{v}_1, \mathbf{v}_2, \ \cdots, \mathbf{v}_n)$, then $\{\mathbf{v}_1, \mathbf{v}_2, \ \cdots, \mathbf{v}_n\}$ is a basis for V.

(×)(e) Let $A \in R^{m\times n}$, then $R(A) \subset R^m$ and $Null(A) \subset R^n$.

(○)(f) Let $L:\ R^n \ \rightarrow\ R^n$ be a linear transform defined by $L(\mathbf{x}) = A\mathbf{x}$, where $A \in R^{n\times n}$, then $Ker(L) = \mathbf{0}$ iff A is nonsingular.

(×)(g) An affine transform is a linear transform.

9

(15 pts) 2. Answer each of the following questions.

(A) Express $\mathbf{x} = [6, 3, 1]$ as a linear combination of $\mathbf{u} = [1, 1, 1]$, $\mathbf{v} = [1, 1, 0]$, $\mathbf{w} = [1, 0, 0]$.

$\overset{1}{\mathbf{u}}$, $\overset{2}{\mathbf{v}}$, $\overset{3}{\mathbf{w}}$

$X = u + 2V + 3W$

(B) Prove or disaprove that $\{[3, 1, -4]^t,\ [2, 5, 6]^t,\ [1, 4, 8]^t\}$ is a basis for R^3.

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ -4 & 6 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ 2 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & \frac{13}{3} & \frac{11}{3} \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -\frac{9}{13} \\ 0 & \frac{13}{3} & \frac{11}{3} \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & \frac{13}{3} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$\therefore$ is a basis for R^3

(C) The rank of A is ~~3~~, where

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{bmatrix}$$

(D) $dim(Null(A))$ = 2, where A is as defined in **(C)**.

(E) Find a basis for $R(A)$, where A is as defined in **(C)**.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(F) Let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in R^3$ and let $L : R^3 \to R^2$ be a linear transformation such that $L(\mathbf{x}) = [1, 0]^t$, $L(\mathbf{y}) = [0, 1]^t$, $L(\mathbf{z}) = [1, -1]^t$. Then $L(2\mathbf{x} - 3\mathbf{y} + 4\mathbf{z})$ = $[6, -7]^t$

(G) Let $L : R^3 \to R^2$ be a linear transformation such that $L([1, 0, 0]^t) = [1, 1]^t$, $L([0, 1, 0]^t) = [1, -1]^t$, $L([0, 0, 1]^t) = [1, 0]^t$. Then $Ker(L)$ = ~~2~~

(H) Let $H \in R^{2\times 2}$ be a Householder matrix defined by $H = I - 2\mathbf{u}\mathbf{u}^t$, where $\mathbf{u} = [1/2, \sqrt{3}/2]^t$. Then H^tH = $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$