二上 (張)

Exam on Differential Equation Dec. 12, 2014

1. (20%)

   For the following system, $\frac{d^2y}{dt^2}+16y=0$, y(0)=1, y'(0)=0,

   (a) (10%)Find the solution for y(t) and y'(t).

   (b) (10%) Assume that $Y(t)=\begin{bmatrix} y(t) \\ \frac{y(y)}{dt} \end{bmatrix}=\begin{bmatrix} y(t) \\ y'(t) \end{bmatrix}$, find the matrix A for the matrix form, and solve the solution of Y(t).

2. (10%)

   For the following system,

   $\frac{dY}{dt}=\begin{bmatrix} -5 & 0 \\ 1 & -3 \end{bmatrix}Y$, with the initial condition $Y_0=(1,0)$

   Find the solution with the given initial value.

3. (15%)

   For the second-order equation $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=0$, with the initial conditions

   y(0)=1, $v(0)=1, v=\frac{dy}{dt}$. Find y(t) and v(t).

4. (20%)

   For the following system,

   $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+40y=e^{-t}$. y(0)=1, y'(0)=0,

   (a) (10%) Find the free response of the system.

   (b) (10%) Find the forced response of the system.

5. (20%)

   For the following system,

   $\frac{d^2y}{dt^2}+100y=\cos 10t$.

   (a) (10%) Compute the solution with y(0)=1, y'(0)=0,

   (b) (5%) Draw y(t) vs. t for the initial valued solution.

   (c) (5%) Find the steady state solution of the system.

6. (15%)

   For the $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=e^{-2t}$, with the initial conditions y(0)=1, $y'(0)=1$, find