Calculus I Final Exam. B

2015 年 1 月 6 號

Note:There is no multiple choice question.

1. Define the function $F(x) : (0, \infty) \to \mathbb{R}$, by $F(x) = \int_1^{e^{x^2}} \frac{1}{t} dt$. Can you say $F(x)$ is one-to-one, onto, one-to-one and onto, or not the both?

2. $y = f(x) = \frac{x^3+1}{x+1}$, Find the increasing interval.

3. Calculate $F(x) = \int_0^{\frac{\pi}{3}} sec^5(x)tan^5(x)dx$.

4. $F(x) = \int_0^{x^2} (2t^2 - 3)dt$. Find the local extreme points of $F'(x)$.

5. Calculate $F(x) = \int_0^1 x^3\sqrt{1-x^2}dx$.

6. Find the volume by revolving about x-axis the region bounded by the graphs $y = |x| + 1$and $y = 2x^2$.

7. $f(x) = sin(x^2) + \frac{2}{5}x^5$, $x \in [2, 4]$. Find its absolute extreme points.

8. "**If $f'(x) = 0$, $\forall x \in (a, b)$, then f is a constant on $[a, b]$.**"
Is there any mistake in the above statement? If there is one, correct it.

9. $f(x) = tan[e^{ln(x^4+2)}]$, $g(x) = ln|e^x + sin(x^4)|$. Find $f'(x)$ and $g'(x)$.

10. $f(x) = 2x^6 - 9x^5 + 5x^4 + 30x^3 - 60x^2$. Find its points of reflection.

11. "**Suppose c is a critical point of $f(x)$, if $\exists\delta > 0$, such that $f'(x) > 0, \forall x \in (c - \delta, c)$, and $f'(x) < 0$, $\forall x \in (c, c + \delta)$, then c is a local maximum point.** "
Do we need to add more conditions in order to make the above statement correct?
If we do, pick up one in the answer sheet.

12. Calculate $\int_1^{e^2} \frac{(1+lnx)^3}{x} dx$.

13. Find the volume by revolving about y-axis the region bounded by the graphs $y = x^4 - 1$ and $y = x - 1$.

14. [**True or False**]"If the function f is one-to-one and onto,$f(a) = b$, and $f'(a)$ exists. Then $(f^{-1})'(b)$ exists."

15. 老師游泳的自由式怎麼游 (此題為填充題)

Ans options B

1. $\frac{2}{15}$
2. $\frac{1}{15}$
3. $\frac{-1}{15}$
4. $\frac{-2}{15}$
5. Add "f is continuous at c.".
6. Add "f is decreasing at on $[c, c+\delta)$".
7. Do not have to add.
8. Add "f is increasing at on $(c-\delta, c)$.".
9. change (a, b) to $[a, b)$
10. change $[a, b]$ to (a, b)
11. change $[a, b]$ to $(a, b]$
12. change (a, b) to $(a, b]$
13. $\frac{81}{4}$
14. $\frac{13}{2}$
15. $\frac{65}{4}$
16. 20
17. $x = 2$ is the absolute minimum, $x = 4$ is the absolute maximum.
18. $x = x = \sqrt{\frac{3\pi}{2}}$ is the absolute minimum, $x = 4$ is the absolute maximum.
19. $x = 4$ is the absolute minimum, $x = 2$ is the absolute maximum.
20. $x = 2$ is the absolute minimum, $x = \sqrt{\frac{3\pi}{2}}$ is the absolute maximum.
21. $x = \sqrt[4]{\frac{3}{10}}$ is a local minimum, $x = -\sqrt[4]{\frac{3}{10}}$ is a local maximum.
22. $x = 0$ is a local minimum, $x = \sqrt[4]{\frac{2}{3}}$is a local maximum.
23. $x = -\sqrt[4]{\frac{2}{3}}$ is a local minimum, $x = 0$ is a local maximum.
24. $x = -\sqrt[4]{\frac{3}{10}}$ is a local minimum, $x = \sqrt[4]{\frac{3}{10}}$ is a local maximum.
25. -1, 1 and 2
26. -1 and 0
27. -1 and 2
28. -1, 0 and 2
29. **True**
30. **False**
31. $f'(x) = sec^2(x^4+2)$
    $g'(x) = \frac{e^x+4x^3cos(x^4)}{e^x+sin(x^4)}$
32. $f'(x) = 4x^3sec^2(x^4+2)$
    $g'(x) = \frac{e^x+4x^3sin(x^4)}{e^x+cos(x^4)}$
33. $f'(x) = 4x^3sec^2(x^4+2)$
    $g'(x) = \frac{e^x+4x^3cos(x^4)}{e^x+sin(x^4)}$
34. $f'(x) = csc^2(x^4+2)$
    $g'(x) = \frac{e^x+4x^3sin(x^4)}{e^x+sin(x^4)}$
35. $(-\infty, -1]$ and $[\frac{1}{2}, +\infty)$
36. $(-\infty, -1]$
37. $[\frac{1}{2}, +\infty)$
38. $[-1, \frac{1}{2}]$
39. $\frac{10}{3}\pi$
40. $\frac{47}{15}\pi$
41. $\frac{5}{3}\pi$
42. $\frac{46}{15}\pi$
43. $\frac{2\pi}{9}$
44. $\frac{3\pi}{10}$
45. $\frac{17\pi}{45}$
46. $\frac{3\pi}{5}$
47. one-to-one and onto.
48. only one-to-one.
49. only onto.
50. neither one-to-one nor onto.
51. $\frac{8408}{315}$
52. $\frac{6408}{315}$
53. $\frac{7408}{315}$
54. $\frac{9408}{315}$
55. There is no any correct answer in this option sheet.