



(i) 答案卷第一張正面為封面。第一張正、反兩面不要寫任何答案。

(ii) 依空格號碼順序在第二張正面寫下所有填充題答案，不要寫計算過程。

(iii) 依計算題之題號順序在第二張反面以後寫下演算過程與答案，每題從新的一頁寫起。

Note: Planck constant  $= 6.63 \times 10^{-34}$  J·s. Bohr radius  $a_0 = 0.0529$  nm.  $1 \text{ eV} = 1.60 \times 10^{-19}$  J. Proton mass  $= 1.67 \times 10^{-27}$  kg. Light speed  $c = 3.00 \times 10^8$  m/s. Energy levels of H atom:  $E_n = -13.6 \text{ eV}/n^2$ .

**Part I. Filling the blank (4 points per blank)**

• The power output of the Sun is  $3.85 \times 10^{26}$  W. The mass of the Sun decreases each second is **[1]** kg.

• Cooper left his daughter Murphy for space exploration when Cooper was 42 and Murphy was 8. When Cooper returned he was only 47, but Murphy was already 88.

(a) Assuming all Cooper did was reaching our nearest star  $\alpha$  Centauri at 4.2 light-years from our Sun and turning back and Cooper's spaceship kept constant speed during the journey, ignoring the gravity of  $\alpha$  Centauri, the speed of Cooper's spaceship is **[2]**  $c$  (in the unit of light speed and 3 significant digits). [Note: no help from warmholes]

(b) If Cooper's spaceship is 8.0 m long in the rest frame and Murphy can measure the length of Cooper's spaceship properly in her inertial frame, what she will get? **[3]** m.

• A black hole ejects two jets in opposite directions. Both jets move at **0.75**  $c$  relative to the black hole. The speed of one jet relative to the other is **[4]**  $c$ .

• The stars appear to have different color. Star 1 appears to glow red, whereas Star 2 looks blue in color. Which star has a higher surface temperature? (A) Star 1, (B) Star 2, (C) both have the same surface temperature, (D) impossible to determine. **[5]**

• The Universe begins from a "Big Bang". The Big Bang emits blackbody radiation and the radiation has cooled down to a temperature of **2.73** K at present time. What is the wavelength at which this radiation has its maximum intensity now? **[6]** mm.

• Jack, in reference frame S, measures two events to be simultaneous. Event A occurs at the point (50.0 m, 0, 0) at the instant 8:20:00 Universal time on June 23, 2015. Event B occurs at the point (150 m, 0, 0) at the same moment. Jill, moving past with a velocity of  $0.800 c \hat{i}$ , also observes the two events. In her reference frame S', which event occurred first? **[7]** and what time interval elapsed between the events? **[8]** sec.

• Five particles are confined in a three dimensional infinite potential well. The possible energy for a particle is  $E_{n_x, n_y, n_z} = \Delta(n_x^2 + n_y^2 + n_z^2)$ , where  $\Delta$  is a constant and  $n_x, n_y, n_z$  are positive integers. Assume that one state can accommodate (容納) one particle only, then the ground state energy of the system is **[9]**  $\Delta$ .

• Find an expression for the normalization constant A for the wave function given by  $\Psi(x) = 0$  for  $|x| > b$  and  $\Psi(x) = A(b^2 - x^2)$  for  $-b \leq x \leq b$ . **[10]**

• A very crude model for an atomic nucleus is a cubical box  $1 \times 10^{-15}$  m on a side. If a proton in such a nucleus made a transition from its first excited state to the ground state and emits a gamma ray. The energy of the gamma ray is **[11]** GeV.

• What is the quantum number for a particle in an infinite square well if the particle's energy is 25 times of the ground-state energy? **[12]**

• In the Compton experiment, the momentum of the incident photon is  $2mc$ , where  $m$  is the rest mass of the electron and  $c$  is the velocity of light. The scattering angle of the photon is  $\phi = 90^\circ$ . (a) The energy of the scattered photon is 13 (b) The magnitude of the momentum of the recoiled electron is 14.

Note: (i) express the answers in terms of  $m$  and  $c$ , (ii) the Compton wavelength shift is  $\Delta\lambda = (h/mc)(1 - \cos\phi)$ .

• Helium (He,  $Z = 2$ ) with one electron removed acts very much like hydrogen, and Bohr model successfully described it. (a) The radius of the ground-state electron orbit is 15 nm. (b) The photon energy emitted in a transition  $n = 2 \rightarrow n = 1$  in this singly ionized helium is 16 eV. (3 significant digits in the answers)

## Part II Problems (10 points per problem)

$$H = \dots \quad k = hf - \phi$$

1.

$$k =$$

The nuclei of the atoms in the bottom half of the periodic table were all produced by "fusion", the collision of the smaller nuclei, during the supernova explosion. This problem roughly models that process. A particle of mass  $m$  moving along the  $x$  axis with a velocity component  $+u$  collides head-on and sticks to a particle of mass  $m/3$  moving along the  $x$  axis with the velocity component  $-u$ . If the final mass is  $M$  and the final velocity is  $v_f$ , (a) Write down the equation for the conservation of momentum, (b) Write down the equation for the conservation of total energy, (c) Derive the final mass  $M$ .

2.

The work function for platinum is 6.35 eV. Ultraviolet light of wavelength 150 nm is incident on the clean surface of a platinum sample. We wish to predict the stopping voltage we will need for electrons ejected from the surface. (a) What is the photon energy of the ultraviolet light? (b) Will these photons have enough energy to eject electrons from platinum? (c) What is the maximum kinetic energy of the ejected photoelectrons? (d) What stopping voltage would be required to arrest the current of photoelectrons?

3.

The wave function of the electron trapped in the one-dimensional infinitely deep potential energy well between  $x = 0$  and  $x = L$  is given by  $\Psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$ . Suppose that the electron is in the state  $n = 2$ .

(a) what is the energy of the electron? (b) what is the probability of finding the electron in the region  $0 \leq x \leq L/4$ ? (c) if the electron jumps from  $n = 2$  state to the ground state by emitting a photon, what is the wavelength of the photon?

4.

The potential energy of a simple harmonic oscillator is given by  $U(x) = \frac{1}{2}m\omega^2 x^2$ . If  $\Psi(x) = Ae^{-Bx^2}$  is a solution of the Schrödinger equation, find (a) the energy  $E$  of the state, and (b) the constant  $B$  in the wave function  $\Psi$ .

$$\frac{n^2 h^2}{8mL^2} = \dots \quad \cos 2\theta = \frac{1}{2} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$1 - \cos^2 \theta = \sin^2 \theta$$

$$\frac{1 - \cos^2 \theta}{2} = \frac{1 + \cos \theta}{2} \Rightarrow \cos \theta = \frac{1}{2}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$