(1) Prove that $A_j = \frac{g(a_j)}{f'(a_j)}$.
(2) Suppose we also know that

$$A_k A_{k+1} > 0,$$

for $k = 1, \cdots, n-1$. Prove that $g(x)$ is of degree $n-1$, and it has $n-1$ distinct real zeros.

9. Suppose $h(x)$ be a real polynomial, $(x-a)^2$ is a factor of $h(x)$, but $(x-a)^3$ is not a factor of $h(x)$. Denote

$$h(x) = (x-a)^2 k(x).$$

Prove that $k(a) \neq 0$, and $k(a) = \frac{f''(a)}{2!}$.

10. Let $p(x) = 6x^2 + \frac{1}{2}$. Find the interval formed by those real numbers b which satisfy the following condition:

$$p(x) + bx \geq 0, \quad \text{for all} \quad x \in [-\frac{1}{2}, \frac{1}{2}].$$

11. Suppose $f(x) \in C^1[0,1]$, $f(x) \in [0,1]$, and $|f'(x)| < 1$ for all $x \in [0,1]$.
(1) Prove that there exists a constant M, $0 \leq M < 1$, such that $|f'(x)| \leq M$ for all $x \in [0,1]$.
(2) Let M be as that in (1). Prove that $|f(x) - f(y)| \leq M|x-y|$ for all $x, y \in [0,1]$.
(3) Let x_0 be a point in $[0,1]$. Since $f(x) \in [0,1]$ for all $x \in [0,1]$, we can define a sequence $(x_n)_{n=0}^{\infty}$ in $[0,1]$ by iteration: $x_1 = f(x_0)$, $x_2 = f(x_1), \cdots, x_{n+1} = f(x_n)$. Prove that the sequence $(x_n)_{n=0}^{\infty}$ is convergent, and if we let $x_* = \lim_{n\to\infty} x_n$, then $f(x_*) = x_*$. [Hint: Apply (2) to prove that the sequence $(x_n)_{n=1}^{\infty}$ is a Cauchy sequence, then apply the continuity of $f(x)$.]

12. Let $f(x)$ is a polynomial of degree n. Is the polynomial $f(x) - x$ a factor of the polynomial $f(f(x)) - x$?