(a) reflexive: $x_1+y_1 = x_1+y_1 \Rightarrow (x_1, y_1) R (x_1, y_1)$ #

symmetric: $(x_1, y_1) R (x_2, y_2) \Rightarrow x_1+y_1 = x_2+y_2$

$\Rightarrow x_2+y_2 = x_1+y_1 \Rightarrow (x_2, y_2) R (x_1, y_1)$ #

transitive: $(x_1, y_1) R (x_2, y_2)$ and $(x_2, y_2) R (x_3, y_3)$

$\Rightarrow (x_1+y_1 = x_2+y_2)$ and $(x_2+y_2 = x_3+y_3)$

$\Rightarrow x_1+y_1 = x_3+y_3 \Rightarrow (x_1, y_1) R (x_3, y_3)$ #

reflexive. symmetric. transitive $\Rightarrow R$ is an equivalence relation.

(b) $[(1,3)] = \{(1,3). (2,2). (3,1)\}$

$[(2,4)] = \{(2,4). (3,3). (4,2)\}$

$[(1,1)] = \{(1,1)\}$.

(c) $A = \{(1,1)\} \cup \{(1,2)(2,1)\} \cup \{(1,3).(2,2).(3,1)\}$

$\cup \{(1,4).(2,3).(3,2).(4,1)\}$.

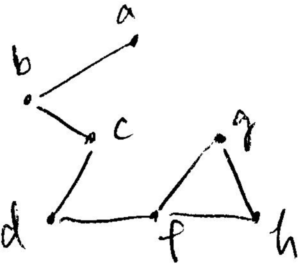
$\cup \{(2,4).(3,3).(4,2)\} \cup \{(3,4).(4,3)\}$

$\cup \{(4,4)\}$.

(a) is a lattice, Because (A, R) has greatest element $\{1,2,3\}$ and least element ϕ, so lub $\{x,y\}$ will not greater than $\{1,2,3\}$ and glb will not less than ϕ, which are in A.

(b) is NOT a lattice. Because glb $\{2,3\} = 1 \notin A$.

(c) is a lattice. Because for $x, y \in Z$. $xRy \Rightarrow x \le y$ the lub $\{x,y\}$ and glb $\{x,y\}$ will alse be integers, which are in Z.



(a) $|E| = 7 \Rightarrow$ # of spanning graph $= 2^7$ ※

(b) $C^3_2 + C^3_3 = 4$ ※

(c) 2^4 ※

	Repeated Edges	Repeated Virtices	
Trail	No	Yes	Open
Circuit	No	Yes	Close
Path	No	No	Open
Cycle	No	No	Close

(a) $2^{\frac{5(5-1)}{2}} = 2^{10}$ ※

(b) $2^{\frac{5\times6}{2}-1} = 2^{14}$ ※

1024

(c) $2^5 \times 3^{10-1} = 2^5 \times 3^9$ ※

(d) 2^5 ※

(e) 1 ※

(f) $\sum_{i=1}^{5} S(5,i) = 1 + \frac{1}{4!}(\overset{1024}{4^5} - 4\times3^5 + \overset{192}{6\times2^5} - 4) + \frac{1}{3!}(3^5 - 3\times2^5 + 3) + \frac{1}{2!}(2^5 - 2) + 1$

$= 1 + 10 + 25 + 15 + 1 = 52$ ※

(g) $\sum_{i=1}^{3} S(3,i) = 1 + \frac{1}{2!}(\overset{8}{2^3} - 2) + 1 = 1 + 3 + 1 = 5$ ※

6. $G=\{V,E\}$. $|V|=n$. $|E|=e$

$\delta=\min_{v\in V}\{deg(v)\}$. $\Delta=\max_{v\in V}\{deg(v)\}$

$n\times\delta \le \sum_{i=1}^{n} deg(v_i) \le n\times\Delta$

$\because \sum_{i=1}^{n} deg(v_i) = 2|E| = 2e$

$\therefore n\delta \le 2e \le n\Delta$

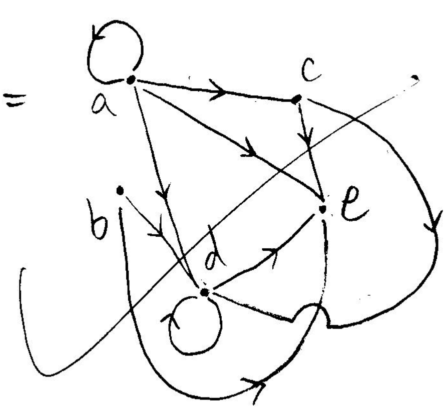
$\Rightarrow \delta \le \frac{2e}{n} \le \Delta$ #

7. $M(R)=\begin{bmatrix} 1&0&1&0&0\\ 0&0&1&0&0\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}$ +10

$M(R^2)=\begin{bmatrix} 1&0&1&0&0\\ 0&0&1&0&0\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}\begin{bmatrix} 1&0&1&0&0\\ 0&0&1&0&0\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}=\begin{bmatrix} 1&0&1&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}$

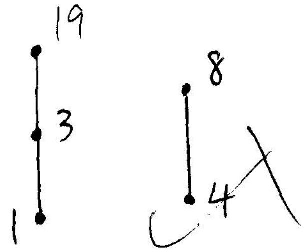
$M(R^3)=\begin{bmatrix} 1&0&1&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}\begin{bmatrix} 1&0&1&0&0\\ 0&0&1&0&0\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}=\begin{bmatrix} 1&0&1&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}=M(R_2)$

$\therefore M(R^{11})=\begin{bmatrix} 1&0&1&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&1&1\\ 0&0&0&0&0 \end{bmatrix}$

$R^{11}=$ 

#

8.



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9. $\mathcal{R}$ is an equivalence relation

$\Rightarrow \mathcal{R}$ is reflexive, symmetric, transitive

(a) $\mathcal{R}$ is reflexive $\Rightarrow (x,x) \in \mathcal{R} \Rightarrow x \mathcal{R} x \Rightarrow x \in [x]$ #

(b) If $[x]=[y]$, by part (a) $x \in [x]$,

so also $x \in [y] \Rightarrow x \mathcal{R} y$ #

(c) Let $w \in [x]$, so $w \mathcal{R} x$,

if $x \mathcal{R} y \Rightarrow w \mathcal{R} y$ (transitive)

$\Rightarrow [x] \subseteq [y]$

Let $t \in [y]$, so $t \mathcal{R} y$

if $x \mathcal{R} y \Rightarrow y \mathcal{R} x$ (symmetric) $\Rightarrow t \mathcal{R} x$ (transitive)

$\Rightarrow [y] \subseteq [x]$

$\therefore [x] = [y]$

(d) If $[x] \neq [y]$ and $[x] \cap [y] \neq \phi$:

Let $v \in [x] \cap [y] \Rightarrow v \mathcal{R} x$ and $v \mathcal{R} y$

$\because v \mathcal{R} x \Rightarrow x \mathcal{R} v$ (symetric)

$\therefore x \mathcal{R} v$ and $v \mathcal{R} y \Rightarrow x \mathcal{R} y$ (transitive)

By part (c), $x \mathcal{R} y \Rightarrow [x]=[y] \longrightarrow$ 與前提相悖

$\therefore [x] \neq [y] \Rightarrow [x] \cap [y] = 0$ #