The Third Exam on Linear Algebra Nov. 29th, 2004

(1) (10%) Let $b_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $b_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $b_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and let L be the linear transformation from

R^2 into R^3 defined by $L(x) = x_1 b_1 + x_2 b_2 + (x_1 + x_2) b_3$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Find the matrix A

representing L with respect to the bases $[e_1, e_2]$ into $[b_1, b_2, b_3]$.

(2) (10%) Let L be the linear transformation from R^3 into R^3 defined by

$L(x) = (2x_1 - x_2 - x_3, 2x_2 - x_1 - x_3, 2x_3 - x_1 - x_2)^T$

(a) Determine the standard representation A of L and (b) use it to find $L([1,1,1]^T)$.

3.(20%) Let L be the linear operator mapping from R^3 into R^3 defined by L(x) = Ax, where

$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$ and let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

(a) Find the transition matrix V corresponding to the change of basis from $[v_1, v_2, v_3]$ to $[e_1, e_2, e_3]$ and

(b) Use it to determine matrix B representing L with respect to $[v_1, v_2, v_3]$.

4. (20%) Let L be the operator on P_3 defined by L(p(x)) = xp'(x)+ p''(x)

$a_1 \cancel{x} + a_2($

(a) Find the matrix A representing L with respect to $[1, x, x^2]$.

(b) Find the matrix B representing L with respect to $[1, x, 1 + x^2]$.

(c) Find the matrix S such that $B = S^{-1}AS$.

$L(p(x)) =$

(d) If $p(x) = a_0 + a_1(x) + a_2(1 + x^2)$, calculate $L^2(p(x))$.

5. (20%)Let L be the linear operator mapping from R^3 into R^2

L: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \to \begin{bmatrix} x_2 + x_1 \\ x_3 + x_2 \end{bmatrix}$

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(a) Find Ker(L).

(b) Find L(R^3).

6. (20%) Prove whether the following transformation is linear or not.

(a)L: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ → $[x_1, x_2, x_1^2 + x_2^2]^T$ (b))L: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ → $[x_1, x_2, x_1 + 2x_2]^T$