Final Exam (35%)

CS3330 Scientific Computing, Instructor: Cheng-Hsin Hsu

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7:00 p.m. – , Jan. 12th, 2016

- **Please create a single latex document, write your solution (no need to copy the questions, but please clearly mark the question numbers *in order*) into it. That is, all the answers should go to the same .tex file (NOT 1 tex file per question).**
- **Please typeset your .tex file, and submit both (and only) your .tex and .pdf files before you leave the classroom. No partial credits will be given to students who fail to submit his/her .tex and .pdf files. I will only grade the .pdf file.**
- **Remember to put your name and student ID on the first page.**
- **Always remember to include both your source code and your results (whatever produced by your code), such as text outputs and figures. Important, no partial credits will be given if figures are not included in your PDF file.**
- **You are allowed (actually encouraged) to search online for tips.**
- **You are not allowed to copy and paste source codes from the Internet. Furthermore, you cannot exchange (online/offline) messages with any of your peers during the exam. These are considered as academic dishonesty, which automatically leads to zero point. Furthermore, we will have no choice but report this incident to the university.**

1) (5%) Please answer the following questions.
   a) Matlab stands for two English words. What are the two words?
   b) Who creates the original Matlab?
   c) What is the programming language used to implement the original Matlab?
   d) Currently, which numerical linear algebra package is used by Matlab?
   e) Does Matlab support symbolic computing? How?
2) (5%) Google and read the concept of the Bezier curve online, e.g., on Wikipedia.
   a) (1%) Summarize its definition.

$Y = \pm\sqrt{-a^2+2aX+c^2X^2} + b$

5, 1.8, 1.6, 1.4, 1.4, 1.6, 1.8

b) (1%) Plot 5 randomly generated data points on the x-y plane and connect them with straight lines.

c) (1%) Label the data points as p_1, p_2, p_3, p_4, and p_5.

d) (2%) Draw a 4th-order Bezier curve on top of the above 5 data points

3) (5%) This is to recreate a 3D plot:

a) (1%) Plot the function $\sin(x)/x$ over the interval $[-\pi, \pi]$.

b) (1%) Plot the function of $|\sin(x)/x|$ over the interval $[-4\pi, 4\pi]$.

c) (3%) Write a Matlab script to reconstruct the 3D plot in Fig. 3.

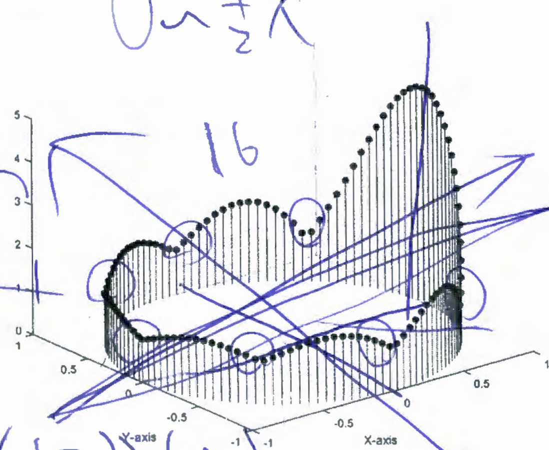
$2 \rightarrow 4$

$16 \times 6 + 32 = 16 \times 8$

$0 \sim \frac{1}{2}\pi$

16

7×4



2(floor(abs(theta)/ $\frac{1}{8}\pi$))+1)

0 1 2 3 4 5 6 7

0 1 1 1 1 1 1 1

2

$-\frac{1}{8}\pi$

$\frac{7}{8}\pi$

$= -0.39$

Fig. 1. Target 3D plot.

4) (5%) A true-color image (such as a jpg file) of size $m \times n$ can be represented as a 3-dimensional array of size $m \times n \times 3$, where each layer (or page) is the pixel intensity of R (red), G (green), and B (blue), respectively. For certain image processing applications, you need to reshape the 3-dimensional matrix into a 2-dimensional matrix of size $3 \times (mn)$, where each column is the RGB intensity vector of a pixel. (For instance, the first column is the RGB intensity of pixel (1,1), the second column is the RGB intensity of pixel (2,1), and so on.)

a) (2%) Write a Matlab script to read a jpg file (any file you can find online), convert it into a $3 \times (mn)$ matrix. Please include your Matlab source code in your PDF. Please also dump the first 10 columns of your new matrix.

$2a=\alpha$ $\quad a=\frac{\alpha}{2}$ $\quad c^2=\beta+\frac{\alpha^2}{4}$

$-a^2+c^2=\beta$ $\quad -\frac{\alpha^2}{4}+c^2=\beta$ $\quad c=\sqrt{\beta+\frac{\alpha^2}{4}}$

b) (3%) RGB is a popular color space, but is not *THE* color space. Many applications use YUV color space instead, where Y is the luminance (brightness) component, U and V are chrominance (color) components. There are different RGB to YUV conversion functions, please use the following functions to convert your 2-dimensional RGB matrix into YUV matrix: $Y = 0.299R + 0.587G + 0.114B$, $U = -0.147R - 0.289G + 0.436B$, and $V = 0.615R - 0.515G - 0.1B$, where R, G, B, Y, U, and V values are normalized to $[0, 255]$. Then plot the Y, U, and V components of your jpg file (same as what you used above) as grayscale images. Include your image in the PDF file. Note that there should be three images in your PDF, corresponding to the three components. $K=256$

m $\quad n$

5) (5%) If we use 256 colors to represent an indexed-color image of 640×480, what is the compression ratios of the following cases when compared with a 8-bit full-color image of the same size? Why?

$-\frac{b^2}{a^2}=\alpha$

$a^2=-\frac{b^2}{\alpha}$

a) (1%) Use pixels for k-means clustering $\quad 1\le k\le m\times n$ $\quad q=240$

b) (2%) Use 2×2 blocks for k-means clustering $\quad 1\le k\le \frac{1}{4}m\times n$

c) (2%) Use $q \times q$ blocks for k-means clustering assuming q is a factor of both 480 and 640. $\quad 1\le k\le \frac{1}{q^2}m\times n$

6) (5%) Please transform the following nonlinear models into linear ones. Please make sure you express the original nonlinear parameters in terms of the linear parameters after transformation. Remember to show your work.

$x=a+r\cos(t)$

$y=b+r\sin(t)$

a) (1%) $(x - a)^2 + (y - b)^2 = c^2$. $\quad x^2-2ax+a^2+y^2-2by+b^2=c^2$

b) (2%) $y = \frac{ax}{x^2+b^2}$. $\quad x^2+y^2-2ax-2by=c^2-a^2-b^2$

c) (2%) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. $\quad y^2=-x^2$

7) (5%)

a) (2%) Create a sine wave of 4-second duration, 16000-Hz sample rate, with a frequency linearly varying from 0 to 800 Hz.

$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$

b) (2%) Plot a 2-D figure of the first 10,000 samples with time (in seconds) as the x-axis, and amplitude (between $[-1, 1]$) as the y-axis.

c) (1%) Assume that you use y to store the amplitude, what will happen if you play $-y$ instead of y?

$y^2=b^2-\frac{b^2}{a^2}x^2$ $\quad \frac{0:4\times16000-1}{16000}$ $\quad d=\frac{1}{Fs}$

before = m×n×3×8

$\cancel{x^2y+b^2y=ax}$

$\frac{1}{y}=\frac{x^2+b^2}{ax}=\frac{1}{a}x+\frac{b^2}{a}\frac{1}{x}$ $\quad$ after = m×n×log2(k)+k*3*8