**Exam on Engineering Mathematics    Jan. 15, 2010**

1. (20%) (a) Draw the graph of the signal $f(t) = u_0(t) - u_1(t)$

   (b) Using the Laplace transform of the solution of the initial-value problem..

   $\frac{d^2y}{dt^2} + 25y = f(t)$ to find the solution with y(0)=1, y'(0)=0.

   $\cos 5(t-1)$

   $= \cos$

   (c) What is the steady state solution?

2. (10%) Compute the convolution f and g for the given functions f and g.

   $f(t) = \sin t$ and $g(t) = u_2(t) - \delta_2(t)$.

3 (20% ) (a) Compute the solution of the initial-value problem

$$\frac{d^2y}{dt^2} + 4y = \delta_{100}(t), y(0) = 1, y'(0) = 0.$$

(b) What is the solution when t < 100?

(c) What is the steady state solution ?

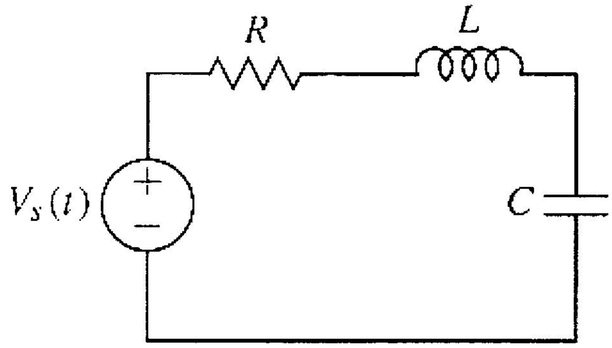
4. ( 20%) Solve the initial-value problem

$$\frac{dy^2}{dt^2} = -y + u_5(t)\sin(t-5), y(0) = 1, y'(0) = -1.$$

5. (10%) Solve the initial-value problem $\frac{dy^2}{dt^2} = -y + \sin(t), y(0) = 1, y'(0) = 0.$

6. (20%) Solve the initial-value problem (20%)Assume that v(t) is the voltage across the capacitor. Given R = 4000 ohms, C =0.25 x $10^{-6}$ Farads and $V_s(t) = u_0(t) - u_{1000}(t)$, L = 1.6 Henrys. (a) Solve v(t) for the R-L-C circuit. For v(0)=1 and v'(0)=0.

(b) What is the solution for t <1000?




(c) What is the steady state solution ?

| $y(t) = \mathcal{L}^{-1}[Y]$ | $Y(s) = \mathcal{L}[y]$ | $y(t) = \mathcal{L}^{-1}[Y]$ | $Y(s) = \mathcal{L}[y]$ |
|---|---|---|---|
| $y(t) = e^{at}$ | $Y(s) = \dfrac{1}{s-a} \quad (s > a)$ | $y(t) = t^n$ | $Y(s) = \dfrac{n!}{s^{n+1}} \quad (s > 0)$ |
| $y(t) = \sin \omega t$ | $Y(s) = \dfrac{\omega}{s^2 + \omega^2}$ | $y(t) = \cos \omega t$ | $Y(s) = \dfrac{s}{s^2 + \omega^2}$ |
| $y(t) = e^{at} \sin \omega t$ | $Y(s) = \dfrac{\omega}{(s-a)^2 + \omega^2}$ | $y(t) = e^{at} \cos \omega t$ | $Y(s) = \dfrac{s-a}{(s-a)^2 + \omega^2}$ |
| $y(t) = t \sin \omega t$ | $Y(s) = \dfrac{2\omega s}{(s^2 + \omega^2)^2}$ | $y(t) = t \cos \omega t$ | $Y(s) = \dfrac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ |
| $y(t) = u_a(t)$ | $Y(s) = \dfrac{e^{-as}}{s} \quad (s > 0)$ | $y(t) = \delta_a(t)$ | $Y(s) = e^{-as}$ |

$$\mathcal{L}[u_a(t)y(t-a)] = e^{-as}\mathcal{L}[y] = e^{-as}Y(s)$$

$$\mathcal{L}^{-1}[e^{-as}Y] = u_a(t)y(t-a)$$

$$\mathcal{L}[e^{at}y(t)] = Y(s-a)$$

$$\mathcal{L}^{-1}[Y(s-a)] = e^{at}\mathcal{L}^{-1}[Y] = e^{at}y(t)$$