

※ 答案紙上需寫下計算過程，否則不予計分。

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1. (20%) Find the Fourier series of the function on the interval.

$$f(x) = x^2 - x + 3, -2 \leq x \leq 2$$

2. (20%) (a) Write the discontinuous points of the functions.
 (b) Write the convergence values for those discontinuous points.
 (c) Find the Fourier series of the function.

$$f(x) = \begin{cases} -1 & \text{for } -4 \leq x < 0 \\ 1 & \text{for } 0 \leq x \leq 4 \end{cases}$$

3. (20%) (a) Write the discontinuous points of the functions.
 (b) Write the convergence values for those discontinuous points.
 (c) Find the Fourier series of the function.

$$f(x) = e^{-|x|} \text{ for } -\pi \leq x \leq \pi$$

4. (20%) Let $f(x) = x^2/2$ for $-\pi \leq x \leq \pi$. Use the Fourier series and evaluate it at an appropriately chosen value of x to sum the series $\sum_{n=1}^{\infty} (-1)^n/n^2$.
 (hint, use the convergence of the Fourier series and $x=0$)

5. (20%) Write the Fourier cosine series of the function on the interval.

$$f(x) = \begin{cases} 0 & \text{for } 0 \leq x < \pi \\ \cos(x) & \text{for } \pi \leq x \leq 2\pi \end{cases}$$

上面用到不定積分公式：

$$\int e^{\alpha x} \cos \beta x dx = \frac{e^{\alpha x} (\alpha \cos \beta x + \beta \sin \beta x)}{\alpha^2 + \beta^2} + C$$

$$\int e^{\alpha x} \sin \beta x dx = \frac{e^{\alpha x} (\alpha \sin \beta x - \beta \cos \beta x)}{\alpha^2 + \beta^2} + C$$