2015 Fall CS 333402 Engineering Mathematics Midterm (I)

Class:_____

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1. (7%) Solve the initial value problem of the ODE:

$$xy^2y' = y+1 \; ; \; y(4e^{-3}) = 3$$

2. (10%) Solve the general solution of the ODE: $y' = \dfrac{3x - y - 11}{x + y + 3}$

3. (8%) Solve the general solution of the ODE: $y' = \dfrac{x^3 - y^3}{xy^2}$

4. (10%) Given an ODE $y' + P(x)y = Q(x)$. Show that it has an integrating factor $I(x) = e^{\int P(x)dx}$, and the general solution of the ODE is $y(x) = \dfrac{1}{I(x)}\left[\int Q(x)I(x)dx + c\right]$ where c is arbitrary constant.

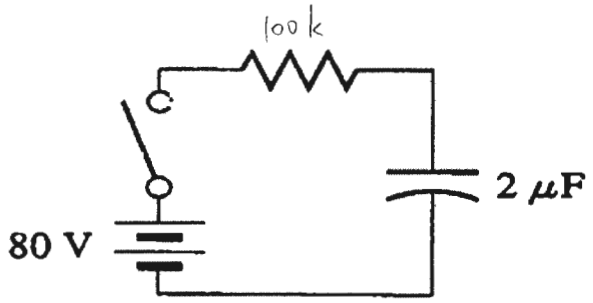
5. (10%) Solve the general solution of the ODE: $y' + 2xy = x^2 + y^2$

6. (10%, 5%) Given an ODE: $(2x^2y^2 + xy)dx + (x^3y + xy)dy = 0$. Please find an integrating factor such that the ODE becomes exact after multiplying the integrating factor into the original ODE. What is the general solution of this ODE?

7. (10%) Given an ODE: $y' = P(x)y^2 + Q(x)y + R(x)$, with a particular solution $S(x)$. Show that the original ODE will become a first-order linear ODE with respect to $z(x)$ if the general solution $y = S(x) + \frac{1}{z(x)}$.

8. (10%) Solve the general solution of the ODE: $x^4 y' = x^3 y - 2y^3$

9. (6%, 6%) In the following circuit, the capacitor is initially discharged and R is 100kΩ. How long after the switch is closed will the capacitor voltage be 50 volts? Determine the current in the resistor at the time. (Here μF denotes 10^{-6} farads.)



$i(t) = \frac{E}{R} e^{\frac{-t}{RC}}$

10. (8%) A cup contains B_0 number of bacteria initially. At the time of 1 hour, the number of bacteria is $3B_0$. If the growth rate of bacteria is proportional to the number of bacteria N(t) presented at time t, determine the time required for the number of bacteria to $20B_0$.

$\frac{dN(t)}{dt} = kN(t)$

$u = x - 2$

$w = \frac{y+5}{x-2}$

$\ln u = \frac{-1}{2}\left(\ln(w+3) + \ln(w-1)_2 + \ln C\right)$