$y_{(p)}' = C_2 + C_3 e^x + (-\sin x) - \cos x - 5e^x - 5xe^x$

$y'' = C_3 e^x - \cos x + \sin x - 5e^x - 5e^x - 5xe^x$

$2\cos x - 5e^x$

$y''' = C_3 e^x + \sin x + \cos x - 5e^x - 5e^x - 5xe^x$

Engineering Math Midterm#2

Class:______________

ID:______________

Name:______________

1. (10%) Solve the general solution of the ODE

   $y''' - y'' = 2\cos x - 5e^x$.

$\frac{-6 \pm \sqrt{36-52}}{2} = \frac{6 \pm 4i}{2} = -3 \pm 2i$

2. (10%) Solve the general solution of the ODE $x^2 y'' + 7xy' + 13y = 0$.

   $y'' + 6y' + 13y = 0$

   $y = C_1 e^{3t} \cos 2t + C_2 e^{3t} \sin 2t$

   $= C_1 x^3 \cos 2\ln x + C_2 x^3 \sin 2\ln|x|$

3. (15%) Solve the general solution of the ODE

   $(2x+1)^2 y'' - (12x+6)y' + 16y = 20$.

4. (15%) Let y_1 and y_2 be two linear independent solutions of a homogeneous equation $y'' + p(x)y' + q(x)y = 0$. Show that every solution of this ODE is a linear combination of y_1 and y_2.

$y' = 2C_1 x - C_1 \ln x^2 - 2C_1 \ln x + C_2$ $\quad - C_1 x \cdot 2\frac{1}{x} \ln x$

$y'' = 2C_1 - \frac{2C_1}{x}\ln x - \frac{2C_1}{x}$

5. (10%) Solve the general solution of the ODE

   $(x^2 - x)y'' - 2xy' + 2y = 0$.

$2x^2 C_1 - 2C_1 x \ln x - 2C_1 x - 2C_1 x + 2C_1 \ln x + 2C_1$

$-4C_1 x^2 -$

$53 \times 4 = 212$, $212 - 16 = 196$

6. (10%) Solve the initial value problem of the ODE

   $y'' - 4y' + 53y = 0,\ y(\pi) = -3,\ y'(\pi) = 3$.

$\frac{4 \pm \sqrt{-196}}{2}$ $\quad 16 -$ $\quad 212 - 16 = 196$ $\quad 14$

$\frac{4 \pm 14i}{2} = 2 \pm 7i$

$C_1 e^{2x}$

$y_1' =$

$y_0'' = 8C_1$

$8(2x+1)^2 C_1 + 8(2x+1)$

$+ 16 C_1 (2x+1)^2 + 16 C_2 (2x+1)^2 \ln|2(x+1)| + 20$

$2x\ln x$

$C_1 x^2 - x\ln x^2 C_1 - C_1 + C_2 x$

$2C_1 \ln x \quad -2C_1$

$2C_1 x - C_1 \ln x^2 - C_1 x \frac{1}{x^2} \cdot 2x + C_2$

$= 2C_1 x - C_1 \ln x^2 - 2C_1 + C_2$

$2C_1 - \frac{2}{x} C_1$

$2C_1 x^2 - 2C_1 x - 2C_1 x + 2C_1$

$-4C_1 x^2 + 2C_1 x \ln x^2 + 4C_1 x - 2C_1 x$

$+ 2C_1 x^2 - 2C_1 x \ln x^2 - 2C_1 + 2C_2 x$

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7. (15%) Solve the general solution of the ODE $y''+y=\tan x$.

(Hint: $\int \sec dx = \ln|\sec x + \tan x| + C$, $\int \csc dx = \ln|\csc x - \cot x| + C$,

$\sin 2x = 2\sin x \cos x$, $\cos 2x = 2\cos^2 x - 1$, $\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$,

$\cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$)

$-C_1 \sin x + C_2 \cos x + \sin x \ln|\sec + \tan| - 1$

$-C_1\cos x - C_2 \sin x + \cos x \ln|\cancel{\sec} + \tan| + \sin x \sec x$

$\frac{\sin}{\cos} = \cancel{\cot}\ \tan$

8. (15%) When solving the particular solution y_p of a 2nd order constant coefficient ODE $y''+ay'+by=ce^{mx}$ by the method of undetermined coefficients, if y_p has the same form as the homogeneous solution y_h of the ODE, show that we can multiply x^n into y_p for solving y_p, where n is the least positive integer such that the repetition between y_h and y_p does not occur.

$v'' + \frac{2 + \frac{-2}{x-1}x}{x} = 0$ $\quad \ln \quad \int \frac{2}{x} + \frac{-2}{x-1}$ $\quad \frac{1-\cos^2 x}{\cos x}$

$t' + (\frac{2}{x} + \frac{-2}{x-1})t = 0$ $\quad e^{\int}$ $\quad = 2\ln x - 2\ln(x-1)$ $\quad \frac{1}{\cos x}$

$= (\frac{x}{x-1})^2$

$t \cdot (\frac{x}{x-1})^2 = C_1$ $\quad -6+9=3$

$t = C_1 \cdot \frac{(x-1)^2}{x^2}$ $\quad 6e^{2x-2\pi}\cos 7x \mp 21e^{2x-2\pi}\sin 7x$

$= C_1 \cdot \frac{x^2-2x+1}{x^2}$ $\quad -\frac{18}{7}e^{2x-2\pi}\sin 7x - 9e^{2x-2\pi}\cos 7x$

$= C_1(1 - \frac{2}{x} + \frac{1}{x^2})$

$= C_1(x - \ln x^2 \mp x^{-1}) + C_2$

$= C_1 x^2 - C_1 x \ln x^2 - C_1 + C_2 x$ $\quad (x^2 - x)$

$2C_1 x - C_1 \ln x^2 - 2C_1 \ln x + C_2$

$= 2C_1 - \frac{2}{x}C_1 \ln x - \frac{2}{x}C_1 +$

$2C_1x^2 - 2xC_1\ln x - 2xC_1 - (2C_1x - \overline{x}$