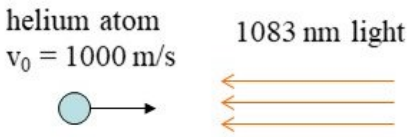
105 學年度第二學期 普通物理 B 期末考試題
Wolfson Ch 29, 33-35 (3rd Ed.) 2017/06/13, ***8:20 am – 09:50 am***

A

(i) 依空格號碼順序在第二張**正面**寫下所有填充題答案，不要寫計算過程。
(ii) 依計算題之順序在第二張**反面**以後寫下演算過程與答案，**每題從新的一頁寫起**。

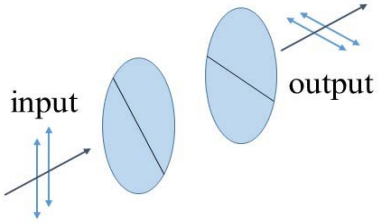
Constants: $1u = 1.67\times10^{-27}$ kg; $\mu_0 = 4\pi\times10^{-7}$ N/A^2; $\varepsilon_0 = 8.85\times10^{-12}$ A^2·s^4/kg·m^3
$h = 6.63\times10^{-34}$ m^2·kg/s; $c = 3\times10^8$ m/s; $m_e = 9.1\times10^{-31}$ kg; $e = 1.6\times10^{-19}$C

Part I. Filling the blank (5 points per blank)

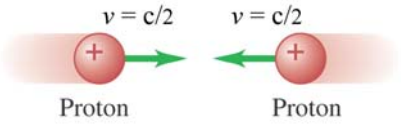


• A helium atom (mass = 4 u) absorbs 1083 nm photons at a rate of 10^6 photons/sec. The atoms initially travels at a speed of 1000 m/s. How long does it take to slow down the atom to zero speed? 【1】 ms (In reality, the atom absorbs and re-emits photons. Since the emitted photons are in all directions, the net momentum they carried away is zero. The emitted photon energy is slightly changed and this ensures energy conservation.)

• Microwave propagates in the *x*-direction at speed *c*. The magnitude of its electric field have the form as: $E = E_p \sin(kx - \omega t)$ and E_p = 30 Volt/m. (a) Find B_p, the amplitude of its magnetic field. 【2】 T (b) What is the average intensity of this wave? 【3】 W/m^2



• One student finds a way to convert vertically polarized light into horizontally polarized light as shown in the right figure. The transmission axis of the first polarizer is set at 45^0 to the vertical direction and the axis of the second polarizer is set horizontally. If the input beam has power P_0, what is the power of the output beam? 【4】 P_0. (Ignore absorption or other losses of the polarizer.)



• Two protons are moving toward each other, each with velocity *c*/2 relative to the laboratory frame. (a) What velocity does one proton measure for the other proton? (What's their relative velocity?) 【5】 *c*. (b) What is the kinetic energy of each proton as measured by an observer at rest in the laboratory frame? The rest energy of one proton is m_pc^2 = 938 MeV. 【6】 MeV (use relativistic energy formula.)

• The followings are four fundamental equations for electromagnetism. (A) $\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$ (B) $\oint \vec{B}\cdot d\vec{A} = 0$ (C) $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ (D) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$. Which equation was modified by James Clerk Maxwell and led to the prediction of electromagnetic wave? 【7】

• The stopping potential in a photoelectric experiment is 2.4 Volt when the illumination radiation has wavelength 365 nm. Determine (a) the work function of the emitting surface 【8】 eV, and (b) the stopping potential for 280-nm radiation on the same surface. 【9】 V

• A beam of aluminum atoms (mass = 27 u) is used to dope a semiconductor chip to set its electrical properties. If the atoms's velocity is known to within 0.2 m/s, how accurately can they be positioned? 【10】nm.

• A sodium lamp (鈉燈) emits light at the power P = 100 W and at the wavelength λ= 589 nm, and the emission (發光) is uniform in all directions.
(a) What is the rate of photon emission (每秒發射光子數)? 【11】photons/s
(b) What is the rate per square meter at which photons are intercepted by a screen (被螢幕攔截) at a distance of 2.0 m from the lamp? 【12】$m^{-2}s^{-1}$

• What is the wavelength (in nm) of (a) a photon with energy 1.0 eV, 【13】 (b) an electron with kinetic energy 1.0 eV? 【14】

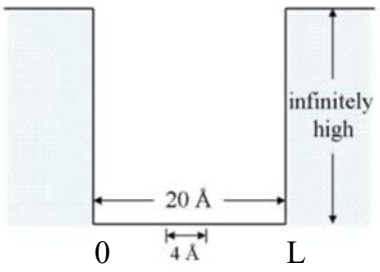
• A tunnel (隧道) is 1200 m long and a spacecraft (太空船) travels at a speed of 0.8 *c*. How much time does it take for the spacecraft to completely pass the tunnel as measured by a passenger in the spacecraft? Neglect (忽略) the length of the spacecraft. 【15】μs

Part II Problems (10 points per problem)

1. (a) Draw a sketch of apparatus for studying the photoelectric effect. Please indicate the names of the instruments. (畫出並簡短說明光電效應實驗裝置的示意圖，該註明的儀器，物理量應標示與說明清楚。)
   (b) Draw a sketch of the result for the stopping potential (maximum kinetic energy K_{max}) vs. light frequency. (畫出光電效應實驗結果，電子最大動能 K_{max} 與照射光頻率 f 的關係圖。)
   (c) Describe the Einstein's theory for the above result. (說明 Einstein 對以上結果提出的假設與解釋。)

2. Bohr assumed an electron orbited a hydrogen nucleus in circular paths.
   (a) What are the angular momentum (穩定軌道的角動量) of the stable orbits in Bohr's model?
   (b) Derive (推導) the radius of the Bohr's hydrogen atom and find the smallest radius for the electron orbit. You need to find the numerical value. (必須算出數值)
   (c) Derive the total energy of the ground state of the hydrogen atom and find the value (數值) for ground state energy.

3. An electron is trapped in a one-dimensional infinite well of width L = 2 nm: U = 0 from 0 to L, U = ∞ elsewhere. Obviously (明顯的) particle can never climb out of the well.



   (a) What is the time independent Schrödinger equation for the electron inside the quantum well?
   (b) Find the energy of the photon that emits when the electron makes a transition from level $n = 4$ to $n = 3$.
   (c) Calculate the de Broglie wavelength of the electron if it is at the third ($n = 3$) state.
   (d) If the electron is at $n = 5$ state, calculate the probability of finding the electron in the 4 Å region of the center (as indicated in the figure). [hint: $\int sin^2 ax \cdot dx = \frac{1}{2}(x - \frac{1}{a} sinax \cdot cosax)$, where a is a constant]

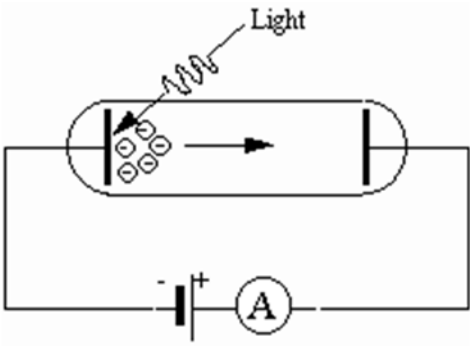
Part I Answer Sheet

A【1】	10.9 ms (0.0109 s)	= B【9】		B【1】	1/4
A【2】	10^{-7} Tesla	= B【12】		B【2】	D
A【3】	1.19W/m^2	= B【10】		B【3】	12 nm (or 11.7)
A【4】	1/4	= B【1】		B【4】	2.96×10^{20}
A【5】	0.8 = 4/5	= B【11】		B【5】	5.89×10^{18}/m^2 s
A【6】	145 MeV (141 OK)	= B【13】		B【6】	3
A【7】	D	= B【2】		B【7】	1.0
A【8】	1.0	= B【7】		B【8】	3.43
A【9】	3.43	= B【8】		B【9】	10.9 ms (0.0109 s)
A【10】	12 nm (or 11.7)	= B【3】		B【10】	1.19W/m^2
A【11】	2.96×10^{20}	= B【4】		B【11】	0.8 = 4/5
A【12】	5.89×10^{18}/m^2 s	= B【5】		B【12】	10^{-7} Tesla
A【13】	1240 nm	= B【14】		B【13】	145 MeV (141 OK)
A【14】	1.23 nm	= B【15】		B【14】	1240 nm
A【15】	3	= B【6】		B【15】	1.23 nm

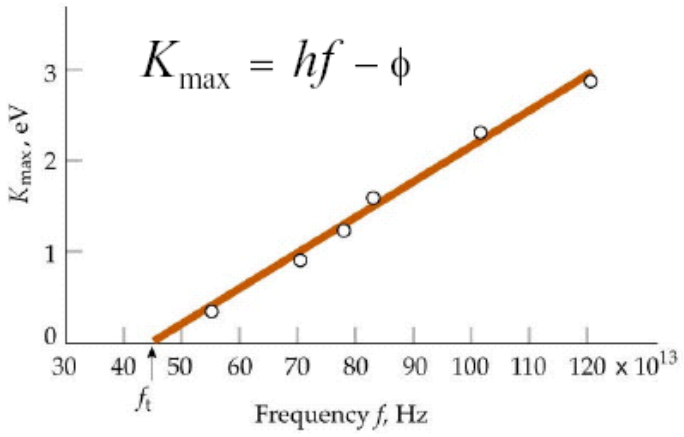
Part II Answer Sheet

【A1 = B2 】(4, 3, 3 points)

a.



b



c. $K_{max} = hf - \phi$ where ϕ is the work function for the material.

【A2 = B3 】(3, 3, 4 points)

$$(a)\, L_n = m_e v_n r_n = n\frac{h}{2\pi} = n\hbar$$

$$(b)\, F = k_e\frac{e^2}{r_n^2} = \frac{m_e v_n^2}{r_n} \Rightarrow r_n = n^2 \underbrace{\frac{\hbar^2}{m_e^2 k_e e^2}}_{a_0} \Rightarrow a_0 = \frac{\hbar^2}{m_e k_e e^2} = 0.053\ \text{nm}$$

$$(c) \Rightarrow E_n = K + U = \frac{1}{2}m_e v_n^2 - k_e\frac{e^2}{r_n} = -\frac{1}{2}k_e\frac{e^2}{r_n} = -\underbrace{\left(\frac{k_e e^2}{2a_0}\right)}_{E_0=13.6}\frac{1}{n^2} = -\frac{13.6}{n^2}$$

in (c), many people directly write -13.6eV/n^2 without derivation, in this case, take off 2pts.

【A3 = B1 】(3, 2, 2, 3 points)

(a)

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad or \quad \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

(b)

$$E_n = \frac{h^2}{8mL^2}n^2$$

$$E_{mn} = \frac{h^2}{8mL^2}(m^2 - n^2) = 1.06 \times 10^{-19} J = 0.66 eV$$

(c)

$$\lambda = \frac{2L}{n} = \frac{40}{3} = 13.3\ \text{Å} = 1.33\ nm$$

(d) $\psi = \sqrt{\frac{2}{L}}\sin\frac{5\pi}{L}x$

$$\text{probability} = \int_{8\text{Å}}^{12\text{Å}} \frac{2}{L}\sin^2\frac{5\pi x}{L} = \frac{1}{5}$$

(or plot the probability vs position using the concept of standing wave, and directly find it is 1/5)