CALCULUS AUG. 15, 2003

1. (10) Give an example that both f and g are continuous functions on $[-1,1]$, but there is no $c \in (-1,1)$ such that $\int_{-1}^{1} f(t)g(t)dt = f(c)\int_{-1}^{1} g(t)dt$.
(Hint: g must takes both positive and negative value)

2. (15) Find the area of the region bounded by three curves
$y = 6 - x^2, y = x(x < 0), y = -x(x > 0)$.

3. (15) The region bounded by $y = \sqrt{x}$, x-axis for $0 < x < 4$ is revolved about the line $y = 2$. Find the volume of the solid it generated.

4. (15) The upper half of the disc $x^2 + y^2 \leq 4$ is revolved about the line $x = 2$. Find the volume of the solid it generated.

5. (15) The ellipse $b^2x^2 + a^2y^2 = a^2b^2$ enclosed a region of area πab.
Find the centroid of the upper half of the region.

6. (15) Let f be a one to one twice differentiable function and $g = f^{-1}$. Show that

$$g''(x) = -\frac{f''[g(x)]}{(f'[g(x)])^3}.$$

7. (15) Let $E_n = 1 + \cdots + \frac{1}{n} - \ln n$, show that $\frac{1}{2} < E_n < 1$ for all n.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

CALCULUS AUG. 26, 2003

1. (10) Find the derivatives of the following functions : (a) $(\tan x)^{\cos x}$. (b) $\sec(\tan^{-1} \ln x)$.

2. (60) Evaluate the following integrals :
(a) $\int \sec^5 x dx$, (b) $\int \frac{dx}{(x^2+1)^3}$, (c) $\int \frac{\sqrt{x^2+9}}{x^2} dx$,
(d) $\int \cos(\ln x) dx$, (e) $\int \frac{dx}{x^4-1}$, (f) $\int \frac{dx}{2\sin x + \cos x}$.

3. (10) Find the centroid of the region bounded by $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 1$, x-axis and y-axis.

4. (10) Find the volume of the solid generated by revolving the region bounded by $y = \cosh x, y = 1$ for $0 \le x \le 1$ about x-axis.

5. (10) Find the area of the region bounded by $y = \tan^{-1} x$, x-axis for $0 \le x \le 1$.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

CALCULUS AUG. 8, 2003

1. (15) Show that $\frac{37}{60} < \int_1^2 \frac{dx}{x} < \frac{47}{60}$.

2. (15) Evaluate the integral $\int_0^4 f(x)dx$ for $f(x) = 2x + 1$ if $0 \le x \le 1$ and $f(x) = 4 - x$ if $1 < x \le 4$.

3. (15) Sketch the region bounded by $y = 8 - x^2$ and $y = x^2$ and find its area.

4. (15) Find function f such that $f''(x) = \cos x, f'(0) = 2, f(0) = 1$.

5. (40) Evaluate the following integrals :
(a) $\int_0^1 \frac{x+3}{\sqrt{x+1}}dx$, (b) $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sqrt{1+\tan x}}dx$.
(c) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos x + \sin^5 x)dx$. (d) $\int_1^3 [2 + (x-1)^7]dx$.

微積分109 # Calculus Exam. I. 07/29/03.

1. 24分
a. $\lim\limits_{t \to 2} \dfrac{\sqrt{1+\sqrt{2+t}} - \sqrt{3}}{t-2} = ?$

b. $\dfrac{d}{dx}\sqrt{x+\sqrt{x+\sqrt{x}}} = ?$

c. $\dfrac{d}{dx}\left[\sec^3(\cos x^2)\right] = ?$

d. $\dfrac{\tan(xy)}{x+y} = \dfrac{1}{\pi}$, $\left.\dfrac{dy}{dx}\right|_{(x,y)=(\frac{\pi}{2},\frac{\pi}{2})} = ?$

2. 10分 If $f(x) \le M$, $\forall x \in \mathbb{R}$ and if $\lim\limits_{x\to 0} f(x) = L$, prove that $L \le M$.

3. 18分 $f(x) = \begin{cases} x^3 \cos \frac{1}{x^2}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

a. find $f'(x)$.

b. Is f' continuous at $x=0$? why?

4. 12分 $f(x) = \sin x + |x|$, $x \in [-3\pi, 3\pi]$.

Sketch the graph of f and indicate critical points and inflection points.

5. 15分 $f(x) = x^3 - 3x + \alpha$, $\alpha \in \mathbb{R}$.

a. Show that $f(x)$ has at most one zero in $[-1, 1]$.

b. Determine the values of α s.t. $f(x)$ has one zero in $[-1, 1]$.

6. 10分 Suppose f is diff. on $(0, \infty)$ with $f'(x) = \frac{1}{x}$ and $f(1) = 0$.

Prove that $\forall a \in \mathbb{R}$, $f(ax) = f(a) + f(x)$. (Consider $\frac{d}{dx} f(ax)$)

7. Suppose f is continuous on $[a, b]$ and f' is decreasing on (a, b).

12分 a. Prove that $g(x) \stackrel{\text{def.}}{=} f(x) - L(x) > 0$ on (a, b), where $L(x) = \dfrac{f(b)-f(a)}{b-a}(x-a) + f(a)$ is the function of the secant line between $(a, f(a))$ & $(b, f(b))$, note: $g(a) = g(b) = 0$.

8分 b. Use a. to prove that $\sin\left(\frac{a+b}{2}\right) \ge \dfrac{\sin a + \sin b}{2}$ for $a, b \in [0, \pi]$.

Quiz 2. ?/18/03

1.

15分 a. $\lim_{x\to 0} \frac{\sin^2 x^3}{x^2} = ?$ why?

15分 b. $\lim_{x\to \frac{\pi}{2}} (f(x)\cos x) = 1$, $\lim_{x\to\frac{\pi}{2}} f(x) = ?$ why?

15分 c. $\lim_{x\to 0} \left[\sin x^2 \left(\sin\frac{1}{x} - \cos\frac{1}{x}\right)\right] = ?$ why?

2. $f(x) = \dfrac{x^5 - 9x^2 + 3}{x^2+1}$

30分

Prove that ① $\exists\ a \in [0,1]$ s.t. $f(a) = a^2$.

② $\exists\ c, d \in \mathbb{R}$ s.t. f maps $[0,1]$ <u>onto</u> $[c,d]$.

3. $f(x) = \begin{cases} x^3, & x \le 1 \\ mx+b, & x > 1 \end{cases}$ $m, b \in \mathbb{R}$.

30分

a. find all possible (m,b) s.t. f is continuous at 1 with reasons.

b. find all possible (m,b) s.t. f is differentiable at 1 with reasons.

Quiz 1

07/11/2003.

1. Use ε-δ to prove that

15分 a. $\lim_{x\to 2} x^3 = 8$

15分 b. $\lim_{x\to 2} x^{1/3} = 2^{1/3}$

15分 c. $\lim_{x\to 2} x^{10/3} = 2^{10/3}$ (use a, b)

2.

12分 a. $f(x) = [x] = n$ if $n \le x < n+1$ n integer.

$\lim_{x\to 2.1} f(x) = ?$ why?

12分 b. If $\lim_{x\to 0} \frac{f(x)}{\sin x} = 7$, $\lim_{x\to 0} f(x) = ?$ why?

12分 c. If $\lim_{x\to 3} g(x) = -2 = g(3)$, $\lim_{x\to 3} \sqrt{g^2(x) + 12} = ?$

why?

3.

20分 If $\lim_{x\to c} g(x) = L$ and f is continuous at L,

(i.e. $\lim_{y\to L} f(y) = f(L)$)

prove that $\lim_{x\to c} f\circ g(x) = f(L)$.