2016 Fall CS 333402 Engineering Mathematics Midterm (I)

Class:_____

ID:_____

Name:_____

1. (10%) Solve the initial value problem of the ODE:

$$yy' = 2x\sec(4y)\ ;\ y(2/3) = \pi/4$$

2. (10%) Solve the general solution of the ODE: $y' = \dfrac{2x+2y}{-3x-3y+7}$

3. (10%) Solve the general solution of the ODE: $y' + \dfrac{2}{x}y = \dfrac{5}{x^2}y^2$

4. (15%) Given a non-exact ODE: $M(x,y)dx + N(x,y)dy = 0$, show that one integrating factor of the ODE is $I(x,y) = I(x-y) = e^{\int f(x-y)d(x-y)}$ when

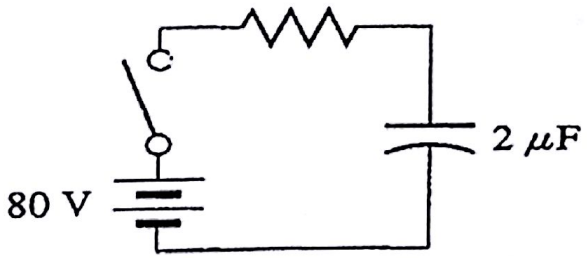
$$\frac{\dfrac{\partial M(x,y)}{\partial y} - \dfrac{\partial N(x,y)}{\partial x}}{N(x,y) + M(x,y)} = f(x-y).$$

5. (15%) Given an ODE: $y' = P(x)y^2 + Q(x)y + R(x)$, with a non-homogeneous solution $S(x)$. Show that the original ODE will become a first-order linear ODE with respect to $z(x)$ if the general solution $y = S(x) + \frac{1}{z(x)}$.

6. (10%) Solve the general solution of the ODE:

$$(xy^2 + 2x^2y^3 + 3xy)dx + (x^2y + 2x^3y^2)dy = 0$$

7. (15%) In the following circuit, the capacitor is initially discharged and R is 100kΩ. How long after the switch is closed will the capacitor voltage be 72 volts? (Here μF denotes 10^{-6} farads.)



8. (15%) A container has Q_0g of sugar dissolved in L liters of water initially. Assume the water containing Q_1g of sugar per liter is flew into the container at a rate of R liters per minute and the well-stirred sugar water is draining from the container at the same rate. Determine the equation of quantity of sugar $Q(t)$ in the container at any time.