[CS 2336]Discrete Mathematics: Autumn 2009

1^{st} exam (close book)
Examination Date: October 26, 2009
Time: 13:10-15:00

1. Let $S = \{5, 8, 11, ..., 56, 59\}$. How many elements must we select from S to ensure that there will be at least two whose sum is 70? **(10%)** 12

2. Use the law of logic to verify each of the following logical equivalences.

   a) $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$ **(5%)**

   b) $\neg[\neg[(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow (q \wedge r)$ **(5%)**

3. Let f be a function $f : A \to B$.

   a) f is invertible if it is one-to-one and onto.

      i What does it suffice to show? **(4%)**

      ii Prove it. **(4%)**

   b) f is invertible only if it is one-to-one and onto.

      i What does it suffice to show? **(4%)**

      ii Prove it. **(4%)**

4. Please answer the following questions:

   a) Let $m(x, y)$ denote the open statement "x is a multiple of y," where the universe for each of the variables x, y comprises all integers. Determine the truth value of each of the following statements; if a quantified statement is false, provide an explanation or a counterexample. **(8%)**

      i) $m(8, 3)$
      ii) $m(9, 3)$
      iii) $\forall y\ m(y, 1)$
      iv) $\forall x\ m(0, x)$
      v) $\forall x\ m(x, x)$
      vi) $\forall y\ \exists x\ m(y, x)$
      vii) $\exists y\ \forall x\ m(y, x)$
      viii) $\forall x\ \forall y\ [(m(x, y) \wedge m(y, x)) \to (x = y)]$

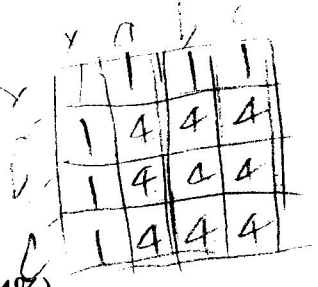
   b) Determine which of the eight statements in part (a) will change in truth value if the universe for each of the variables x, y were restricted to just the positive integers. **(2%)** IV V VIII

5. Let $A = \{1, 2, 3, 4\}$, $B = \{2, 7\}$, and $C = \{3, 4, 7\}$. Determine:

   a) $A \times B$. **(2%)**

   b) $A \cup (B \times C)$. **(2%)**

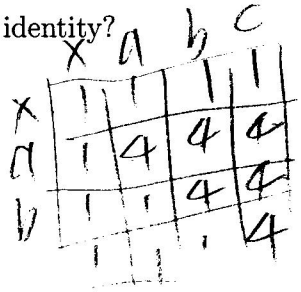
   c) $(A \times C) \cap (B \times C)$. **(2%)**



6. Let $A = \{x, a, b, c\}$. Please answer the following questions:

   a) How many closed binary operations on A have x as the identity? **(4%)**

   b) How many commutative closed binary operations on A have x as the identity? **(4%)**



7. Let p, q, r, s denote the following statements:

   p: I finish writing my discrete mathematics homework before dinner.
   q: I will play WOW games tonight.
   r: Today is Friday.
   s: All Assignments are finished.

   a) Finishing discrete mathematics homework before dinner is necessary for my playing WOW games tonight. **(5%)**
   b) All assignments are fininshed and today is Friday are sifficient for me to play WOW games tonight. **(5%)**

8. Answer the following questions.

   a) In how many ways can 85085 be factored into three factors, each greater than 1, if the order of the factors is not relevant? **(5%)**
   b) In how many ways can one factor 85085 into two or more factors where each factor is greater than 1 and the order of the factors is to be taken into consideration? **(5%)**

9. Use inference rule to establish the validity of the following arguments: (don't use truth table)

   a) $[p \wedge (\neg q \rightarrow \neg p) \wedge (\neg q \vee r)] \rightarrow r$ **(10%)**

   b) **(10%)**

$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ p \vee s \\ t \rightarrow q \\ \neg s \\ \hline \therefore \ \neg r \rightarrow \neg t \end{array}$$