The 4th Exam on Linear Algebra Dec. 20th, 2004

(1) (15%) Given the basis $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$ for R^3, use the Gram-schmit process to obtain the orthonormal basis.

(2) (15%) Consider the vector C[-1,1] with inner product defined by

$$< f, g >= \int_{-1}^{1} f(x)g(x)dx .$$

Find an orthonormal basis for the subspace spanned by 1, x, x^2.

3. (15%) For the following matrix, determine a basis for each of the subspace $R(A^T)$, N(A), R(A) and $N(A^T)$.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

4. (10%) Let S be the subspace of R^4 spanned by $x_1 = (1,0,-2,1)^T$ and $x_2 = (1,0,3,-2)^T$. Find a basis for $S^{\perp}$.

5. (10%) Find the least squares solution to the following system

$$\begin{aligned} -x_1 + x_2 &= 10 \\ 2x_1 + x_2 &= 5 \\ x_1 - 2x_2 &= 20 \end{aligned} .$$

6. (15%) Find the best least squares fit by a linear function to the data

x	-1	0	1	2
y	0	1	3	9

7. (10%) Given $x = (1,1,1,1)^T$ .and $y = (8,2,2,0)^T$

(a) determine the angle Θ between x and y. (b) Find the vector projection p of x onto y.

8. (10%) let $\{u_1, u_2, u_3\}$ be an orthonormal bais for an inner product space V and let

$u = u_1 + 2u_2 + 3u_3$ and $v = u_1 + 7u_3$

Determine the value of each of the following

(a) <u,v>

(b) ||u|| and ||v||

-2, 1, 1, -2

3, -2, 1, 3

1, 3, 15