Communication Test 2 (2010/12/3)

1. Prove the following:

(a) $F^{-1}(\frac{1}{2}(\delta(f-f_o)+\delta(f+f_o)))=\cos(2\pi f_o t)$

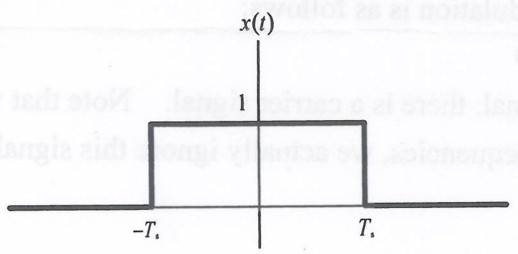
(b) $F(x(t-t_o))=e^{-j2\pi f t_0}X(f)$

(c) $F^{-1}(\frac{1}{2}(X(f-\alpha)+\frac{1}{2}X(f+\alpha))=\cos(2\pi\alpha t)x(t)$

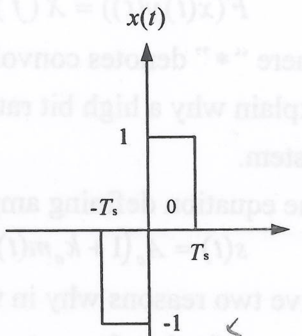
(d) $F(\delta(t))=1$

2. Find the Fourier transforms of the following functions.

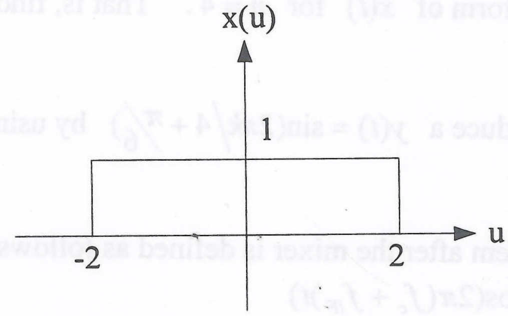
(a)

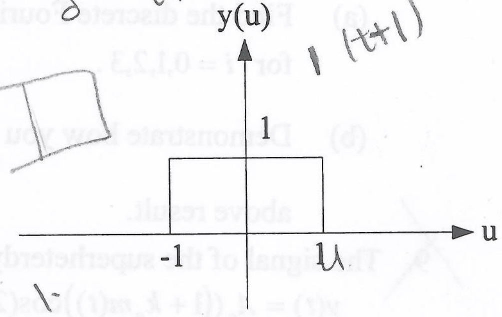


(b)

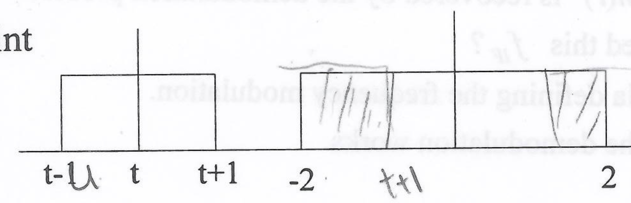


3. Find the convolution of the following functions:





Hint



$4-3t+3+\frac{1}{2}t^2-t+\frac{1}{2}$

$\frac{1}{2}-4+\frac{15}{2}$

$\cos(60^\circ) = \frac{1}{2}$ $\quad \sin 45^\circ = \frac{\sqrt{2}}{2}$

4. Consider $x(t) = \cos(2\pi k/4 + \pi/3)$. Let $a_k = x(k)$ for $k = 0,1,2,3$.

(a) Find the A_i of the discrete Fourier transform for $i = 0,1,2,3$.

(b) Use the following equation to recover $x(t)$ for $t = 0,1,2,3$.

$$x(k) = \frac{1}{n}\left(A_0 + A_{n/2}e^{j\pi k} + 2\sum_{i=1}^{\frac{n}{2}-1} |A_i| \cos\left(\frac{2\pi ik}{n} + \theta_i\right)\right) \text{ for } k = 0,1,2,\cdots,n-1$$

where $\theta_i = \tan^{-1}(b/a)$ if $A_i = a + jb$.

5. Let $F(m(t)) = M(f)$. Show that

$$F(m(t)\cos(2\pi f_c t)) = \frac{1}{2}(M(f - f_c) + M(f + f_c))$$

By using the convolution formula as follows:

$$F(x(t)y(t)) = X(f) * Y(f)$$

where "$*$" denotes convolution.

6. Explain why a high bit rate communication system must also be a wide-band system.

7. The equation defining amplitude modulation is as follows:

$$s(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t)$$

Give two reasons why in the sent signal, there is a carrier signal. Note that when we explain that the system lifts the frequencies, we actually ignore this signal.

8. Suppose we have a signal which is

$$x(t) = \cos(2\pi k/4 + \pi/6).$$

(a) Find the discrete Fourier transform of $x(t)$ for $n = 4$. That is, find A_i, for $i = 0,1,2,3$.

(b) Demonstrate how you can produce a $y(t) = \sin(2\pi k/4 + \pi/6)$ by using the above result.

9. The signal of the superheterdyne system after the mixer is defined as follows:

$$y(t) = A_c((1 + k_a m(t))\cos(2\pi f_c t))\cos(2\pi(f_c + f_{IF})t)$$

(a)Explain how $m(t)$ is recovered by the demodulation process.

(b)Why do we need this f_{IF}?

10. (a) Give the formula defining the frequency modulation.

(b) Describe how the demodulation works.