

(A)

【1】 $E_1 \frac{R_1}{R_2}$	【2】 $\frac{2\kappa_3(\kappa_1 + \kappa_2)}{2\kappa_3 + \kappa_1 + \kappa_2} C_0$	【3】 $50/995 = 0.0503$	【4】 9990
【5】 590	【6】 $\frac{V}{R_2 + R_3}$	【7】 $\frac{V}{R_1}$	【8】 $C(R_1 + R_2 + R_3)$
【9】 $-2z \exp[-(x^2 + z^2)] + \frac{1}{yz^2}$	【10】 (C)(A)(B)	【11】 $5\sigma/(2\epsilon_0), \rightarrow$	【12】 $\sigma/(2\epsilon_0), \rightarrow$
【13】 16	【14】 $2aQE_0(\mathbf{i}-\mathbf{j})$	【15】 $-2m$	

(B)

【1】 $\frac{2\kappa_3(\kappa_1 + \kappa_2)}{2\kappa_3 + \kappa_1 + \kappa_2} C_0$	【2】 $\frac{V}{R_2 + R_3}$	【3】 $\frac{V}{R_1}$	【4】 $C(R_1 + R_2 + R_3)$
【5】 $E_1 \frac{R_1}{R_2}$	【6】 (C)(A)(B)	【7】 $-2z \exp[-(x^2 + z^2)] + \frac{1}{yz^2}$	【8】 $50/995 = 0.0503$
【9】 9990	【10】 590	【11】 $5\sigma/(2\epsilon_0), \rightarrow$	【12】 $\sigma/(2\epsilon_0), \rightarrow$
【13】 $-2m$	【14】 16	【15】 $2aQE_0(\mathbf{i}-\mathbf{j})$	

Part II

(1)

Using Gauss' s Law

While $r < R$

$$2\pi r LE = \frac{1}{\epsilon_0} \int_0^r A r L 2\pi r dr = \frac{2AL\pi}{\epsilon_0} \int_0^r r^2 dr = \frac{2AL\pi}{3\epsilon_0} r^3$$

$$E = \frac{A}{3\epsilon_0} r^2 \dots \dots \dots (a)$$

While $r > R$

$$2\pi r LE = \frac{1}{\epsilon_0} \int_0^R A r L 2\pi r dr = \frac{2AL\pi}{\epsilon_0} \int_0^R r^2 dr = \frac{2AL\pi}{3\epsilon_0} R^3$$

$$E = \frac{A}{3\epsilon_0} \frac{R^3}{r} \dots \dots \dots (b)$$

(2)

$$\rho = \frac{Q}{\left(\frac{4}{3} \pi R^3\right)}$$

For $r < R$

$$4 \pi r^2 E = \frac{1}{\epsilon_0} \int_0^r \rho 4 \pi r^2 dr = \frac{1}{\epsilon_0} \frac{r^3}{R^3} Q$$

$$E = \frac{1}{4 \pi \epsilon_0} \frac{rQ}{R^3} \left(\text{or } k \frac{rQ}{R^3} \right) \dots \dots (a)$$

For $r > R$

$$E = \frac{1}{4 \pi \epsilon_0} \frac{Q}{r^2}$$

$$\begin{aligned} V(r) &= - \int_{\infty}^r E(r) dr = - \left(\int_{\infty}^R E(r) dr + \int_R^r E(r) dr \right) \\ &= - \left(\int_{\infty}^R \frac{1}{4 \pi \epsilon_0} \frac{Q}{r^2} dr + \int_R^r \frac{1}{4 \pi \epsilon_0} \frac{rQ}{R^3} dr \right) \\ &= - \frac{Q}{4 \pi \epsilon_0} \left(\frac{-1}{R} + \frac{r^2 - R^2}{2 R^3} \right) \\ &= \frac{Q}{4 \pi \epsilon_0} \frac{3 R^2 - r^2}{2 R^3} \left(\text{or } \frac{kQ(3 R^2 - r^2)}{2 R^3} \right) \dots \dots \dots (b) \end{aligned}$$

Consider the total energy in the entire space by $u = \epsilon_0 E^2 / 2$

$$\begin{aligned} U &= \int_0^{\infty} \frac{1}{2} \epsilon_0 E(r)^2 4 \pi r^2 dr \\ &= 2 \pi \left(\int_0^R \left(k \frac{rQ}{R^3} \right)^2 r^2 dr + \int_R^{\infty} \left(k \frac{Q}{r^2} \right)^2 r^2 dr \right) \\ &= 2 \pi k^2 \epsilon_0 Q^2 \left(\int_0^R \frac{r^4}{R^6} dr + \int_R^{\infty} \frac{1}{r^2} dr \right) \\ &= \frac{kQ^2}{2} \left(\frac{R^5}{5 R^6} + \frac{1}{R} \right) \\ &= \frac{3 kQ^2}{5 R} \dots \dots \dots (c) \end{aligned}$$

(3)

By Gauss' s Law

$$E L 2 \pi r = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{2 \pi \epsilon_0} \frac{Q}{Lr}$$

$$V_b - V_a = - \int_a^b \frac{1}{2 \pi \epsilon_0} \frac{Q}{Lr} dr$$

$$= - \frac{1}{2 \pi \epsilon_0} \frac{Q}{L} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{2 \pi \epsilon_0 L}{\ln(b/a)} \left(\text{or } \frac{L}{2 k \ln(b/a)} \right) \dots \dots (a)$$

$$C' = \frac{C}{2} 1.5 + \frac{C}{2} 2$$

$$= 1.75 C = \frac{3.5 \pi \epsilon_0 L}{\ln(b/a)} = \frac{0.875 L}{k \ln(b/a)} \dots \dots (b)$$

(4)

(a) S2 open, no current through capacitor, both currents through 3Ω and 5Ω are 4 A. $V_a = 24 - 4 \times 5 = 4$ (V), $V_b = 24 - 4 \times 3 = 12$ (V). Therefore, $V_a - V_b = -8$ (V).

(b) S2 close and steady state, no current through capacitor, Therefore, $V_a - V_b = -8$ (V).

(c) $R_a = 4 \Omega$, $R_b = 8 \Omega$, thus $R^{-1} = (1/4) + (1/8)$, $R = 8/3 \Omega$. Thus $RC = 10 \times 8/3 \mu s = 26.7 \mu s$.