Problems for Exam 2

1. Let Let $A \in R^{n\times n}$ have eigenvalues $1, 3, 5, \cdots, (2n-1)$. What is the trace of A? What is $det(A)$?

2. Let $H \in R^{4\times 4}$ be any Householder matrix, $\mathbf{x} = [-1, 1, -3, 5]^t$, what is $\|H\mathbf{x}\|_2$?

3. Let $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, Find $Sup_\theta\{\|R_\theta\|_2\}$ and $Sup_\theta\{\|R_\theta\|_1\}$.

4. Let $q > 1$, define

$$y_q = \frac{1}{q + \frac{1}{q+\cdots}}$$

What is y_q? In particular, evaluate y_2 and y_6.

5. Let $H \in R^{n\times n}$ be a Householder matrix, show that $det(H) = -1$.

6. Prove that $I - AB$ has the same eigenvalues as $I - BA$ if either A or B is nonsingular.

7. Give an algorithm based on Newton method to approximate the root of $f(x) = 0.5e^{x/3} - 5x^2 + 3\sin(\pi x) = 0$ located in $[0,\ 1]$ which is accurate to within 10^{-5}. Does your algorithm guarantee finding the root in $[0,\ 1]$? Explain.

8. Given

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 1 & -3 \end{bmatrix}.$$

(a) Find the characteristic polynomials of A and B, respectively.

(b) What are the eigenvalues of matrix A?

(c) Write down a spectrum decomposition of matrix A.

(d) What are the singular values of matrix B?

(e) Evaluate $\|A\|_1 + \|A\|_2 + \|B\|_2$.

(f) Evaluate $det(e^A \cdot e^B)$.