Engineering Math Midterm#1

Class:_____

ID:_____

Name:_____

1. (10%) Given that 12 grams of a radioactive element decays to 10 grams in 5 minutes, what is the half-life of this element?

A:19.0089 (minutes)

2. (10%) An object having a temperature of 90 degrees Fahrenheit is placed into an environment kept at 60 degrees. 20 minutes later, the object has cooled to 88 degrees. What will be the temperature of the object after it has been in this environment for 30 minutes? How long will it take for the object to cool to 65 degrees?

A: (a)87.05 degrees (b)519.4 minutes

3. (20%) A woman stands at the junction of two perpendicular roads and her dog is watching her from one of the roads at a distance s feet away. At a given instant the woman starts to walk with constant speed v along the other road, and at the same time the dog begins to run toward the woman with speed $4v$. (a) (10%) Determine the path the dog will take, assuming that it always moves so that it is facing the woman. (b) (10%) Also determine when the dog will eventually catch the woman. (hint: $\int \frac{dx}{\sqrt{1+x^2}} = \ln(x+\sqrt{1+x^2})+c$)

A: (a) $y = \frac{-2}{3}S^{\frac{1}{4}}(S-x)^{\frac{3}{4}} + \frac{2}{5}S^{\frac{-1}{4}}(S-x)^{\frac{5}{4}} + \frac{4s}{15}$ **(b)** $t = \frac{4s}{15v}$

4. (9%) $y' = \frac{x^2 + 2xy - 4y^2}{x^2 - 8xy - 4y^2}$

A: $\ln x - \ln(\frac{y}{x}+1) + \ln(\frac{4y^2}{x^2}+1) = c$

5. (9%) $y' = -e^{-x}y^2 + y + e^x$

A: $y = e^x + \dfrac{1}{-\dfrac{1}{2}e^{-x} + ce^x}$

6. (12%) $(8x^2 + 8xy + 2xy^3 + 2y^4)dx + (6x^2y^2 + 6xy^3)dy = 0$

A: $\phi = 4x^2 + 2xy^3 = c$

7. (7%) $(2xy)dx + (4y + 3x^2)dy = 0; y(1) = 1$

A: $y^4 + x^2y^3 = 2$

8. (8%) $y' = \dfrac{2x - 5y - 9}{-4x + y + 9}$

A: $\ln(x-2) - \ln(\dfrac{y+1}{x-2} - 1) + 2\ln(\dfrac{y+1}{x-2} + 2) = c$

9. (7%) $xy' + y = x\sin x$

A: $y = \dfrac{c + \sin x}{x} - \cos x$

10. (8%) $(ye^{-2x} + y^3)dx - e^{-2x}dy = 0$

A: $\dfrac{1}{y^2} = -\dfrac{1}{2}e^{2x} + ce^{-2x}$