Query Optimization

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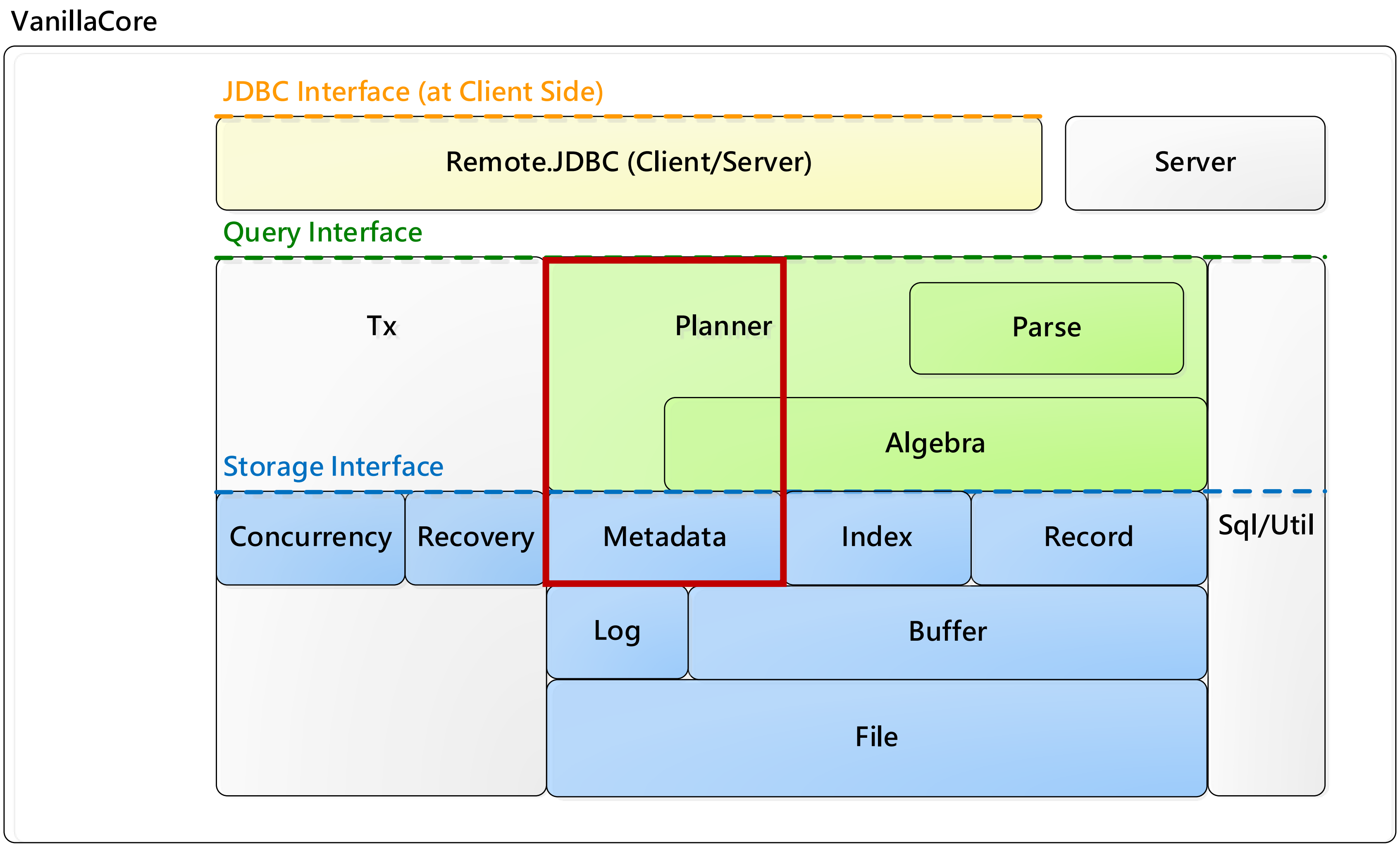
Where Are We?



VanillaCore
JDBC Interface (at Client Side)
Remote.JDBC (Client/Server)
Server
Query Interface
Tx
Planner
Parse
Algebra
Storage Interface
Concurrency
Recovery
Metadata
Index
Record
Sql/Util
Log
Buffer
File

Outline

- Overview
- Cost Estimation
  - Cardinality Estimation
  - Histogram-based Estimation
  - Types of Histograms
- Heuristic Query Optimizer
  - Basic Planner
  - Pushing Select Down
  - Join Ordering
  - Heuristic Query Planner in VanillaCore
- Selinger-Style Query Optimizer

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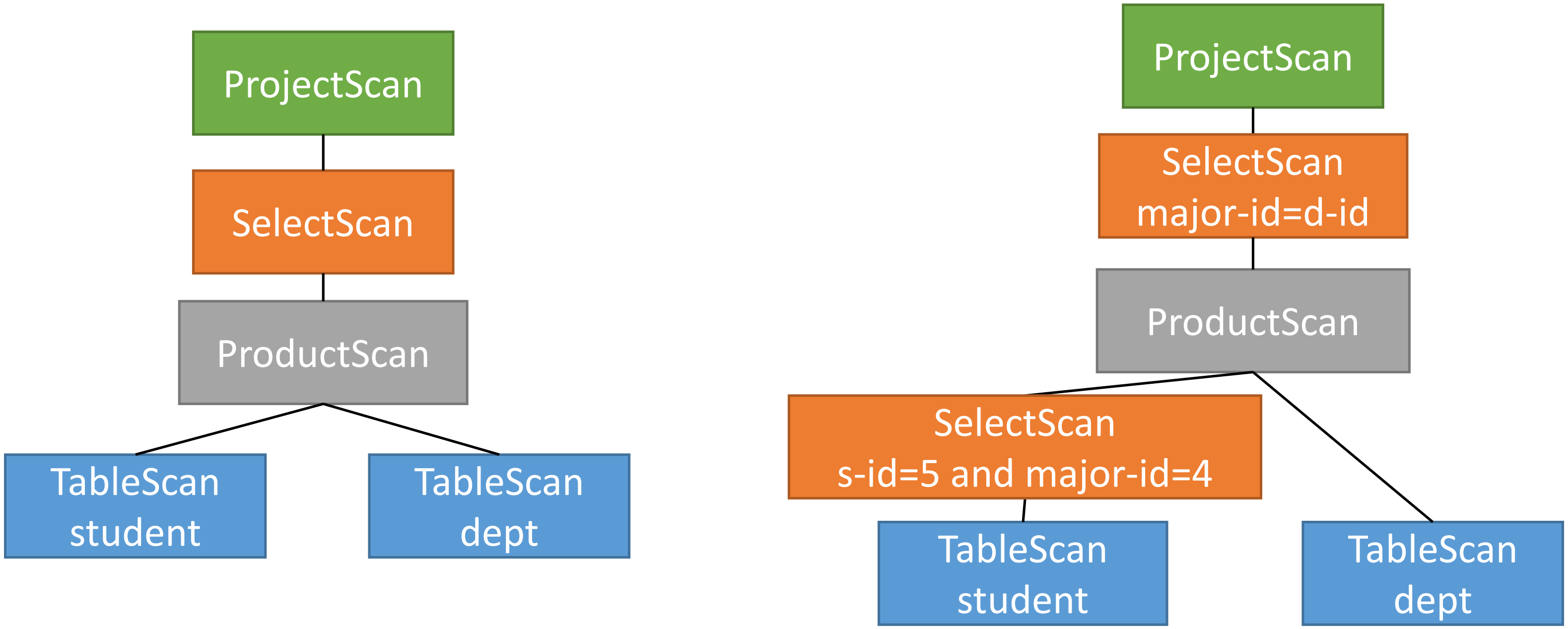
SQL and Relational Algebra

- A SQL command can be expressed as multiple trees in relational algebra

```
SELECT sname FROM student, dept
WHERE  major-id = d-id AND s-id = 5 AND major-id = 4;
```



Query Optimization

- A good scan tree can be faster than a bad one for orders of magnitude
- Query optimizer:
  1. Generate candidate plan trees
  2. Estimate cost of each corresponding scan tree (not discussed yet)
  3. Pick and open the “best” one to execute query
- Goal (ideally): find the one with least cost
- Goal (in practice): avoid bad trees

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Metric for Cost

- Cost of a query?
- To user: query delay
- Low delay also implies better system throughput

- Typically, I/O delay dominates query delay

Cost Estimation

- For each plan/table p, we estimate **B(p)**
  - #blocks accessed by the corresponding scan

- Usually, estimating B(p) requires more knowledge:
  - **R(p)**: #records output
  - ***Search cost*** (#blocks) of index, if used
  - **V(p,f)**: #distinct values for field f in p

Estimating B(p)

p	B(p)
TablePlan	Actual #blocks cached by StatMgr (via periodic table scanning)
ProjectPlan(c)	B(c)
SelectPlan(c)	B(c)
IndexSelectPlan(t)	IndexSearchCost(R(t), R(p)) + R(p)
ProductPlan(c1, c2)	B(c1) + (R(c1) * B(c2))
IndexJoinPlan(c1, t2)	B(c1) + (R(c1) * IndexSearchCost(R(t2), 1)) + R(p)

- B(c) is evaluated recursively down to the table level

For Any p, We Need to Estimate R(p) and Index Search Cost

- Index Search Cost:
  - `HashIndex.searchCost()`
  - `BTreeIndex.searchCost()`

- Estimating R(p) is called ***cardinality estimation***

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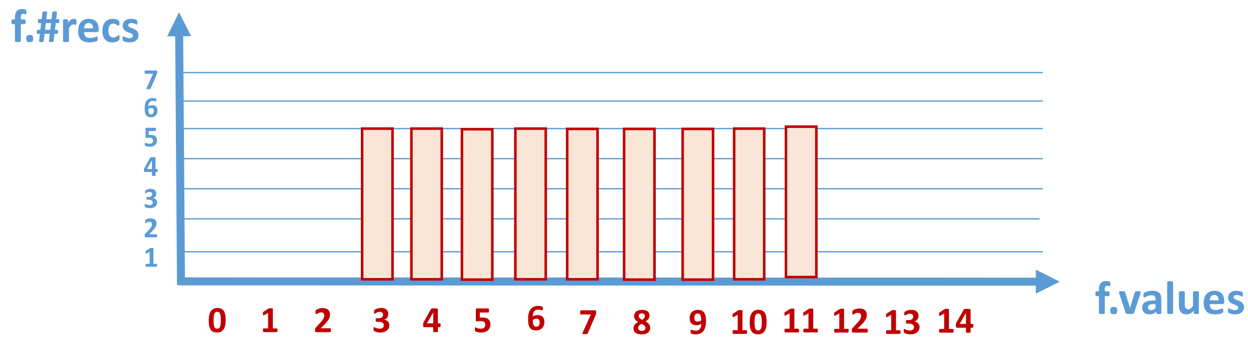
Naïve Approach

- Uniform assumption
  - All values in field appear with the same probability



- Few statistics are enough:

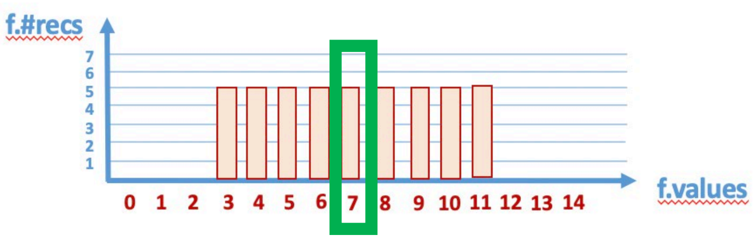
R(c)	**#records** in child plan c
V(c, f)	**#distinct values** in field f in c
Max(c, f)	**Max value** in field f in c
Min(c, f)	**Min value** in field f in c

p = Select(c, f=x)

- R(p)?

R(c)	**#records** in child plan c
V(c, f)	**#distinct values** in field f in c
Max(c, f)	**Max value** in field f in c
Min(c, f)	**Min value** in field f in c

- Selectivity(f=x): $\frac{1}{V(c,f)}$
- R(p): Selectivity(f=x) * R(c)

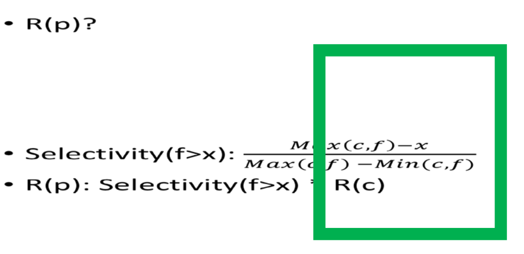


p = Select(c, f>x)

- R(p)?

R(c)	**#records** in child plan c
V(c, f)	**#distinct values** in field f in c
Max(c, f)	**Max value** in field f in c
Min(c, f)	**Min value** in field f in c

- Selectivity(f>x): $\frac{Max(c,f)-x}{Max(c,f)-Min(c,f)}$
- R(p): Selectivity(f>x) * R(c)



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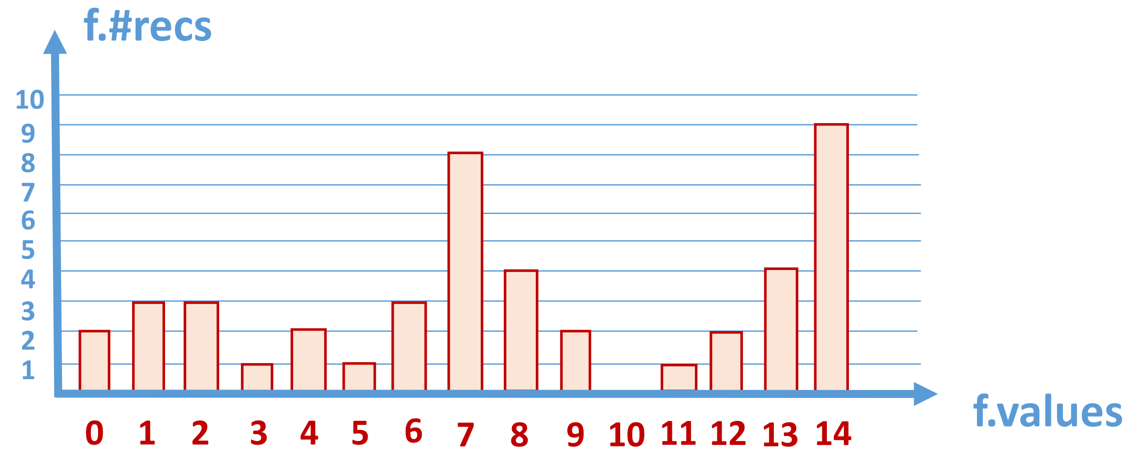
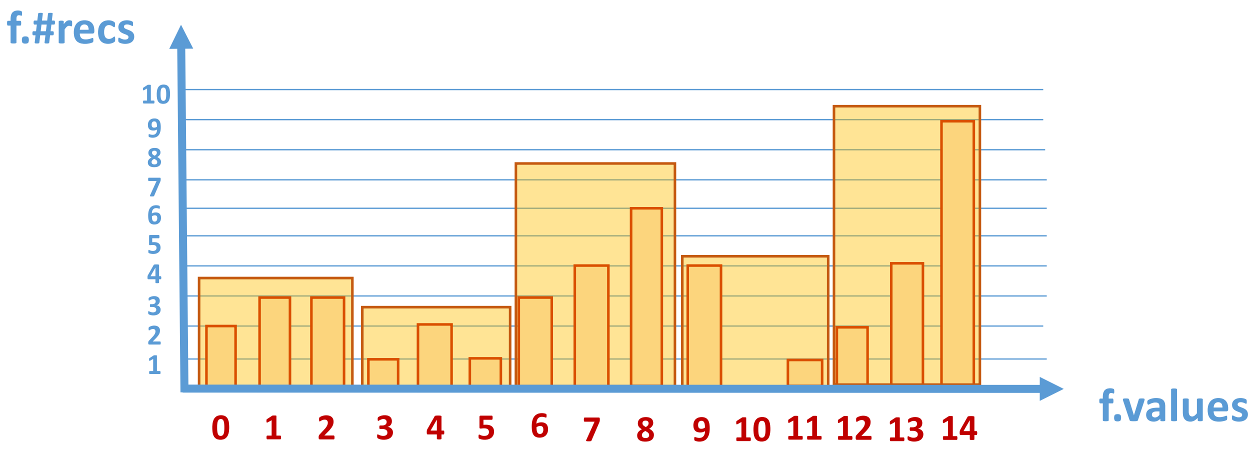
Naïve Estimation is Inaccurate

- In the real world, values in a field are seldom uniform distributed
- p = Select(c, f=14)
- Estimated R(p) = $\frac{1}{15}$ * R(c) = 3
- Actually, R(p) = 9



Histogram

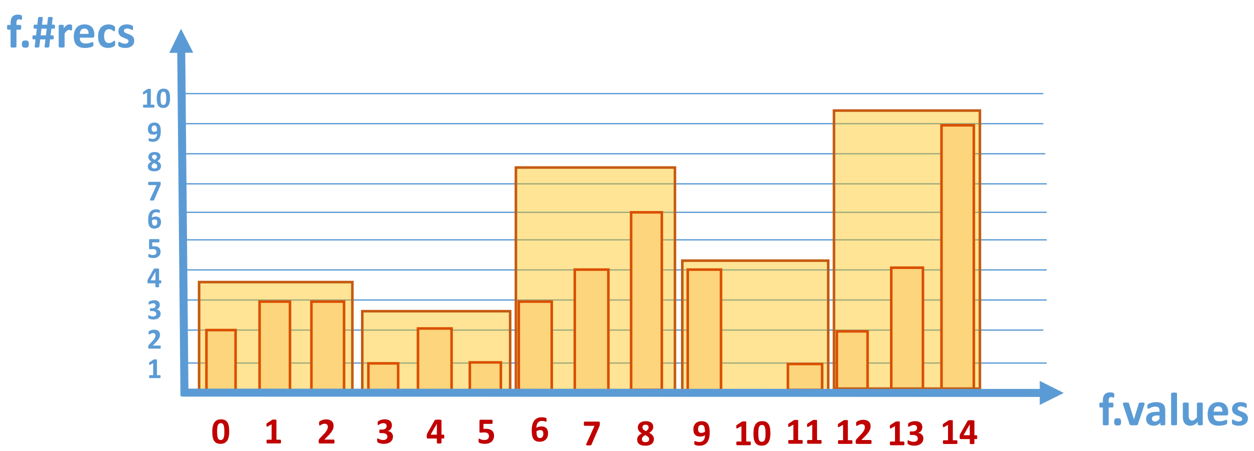
- Approximates value distribution in every field
- Partitions field values into a set of ***buckets***



- More #buckets, more accurate approximation
  - Tradeoff between accurate and storage cost

Buckets

- Each bucket b collects statistics of a value range
  - Assumes ***uniform distribution of records and values*** in b



- R(p, f, b): #records
- V(p, f, b): #distinct values
- Range(p, f, b): value range

Cardinality Estimation

- Not matter what p is, we have

$$R(p) = \sum_{b \in p.hist.buckets(f)} R(p, f, b)$$

  for any f

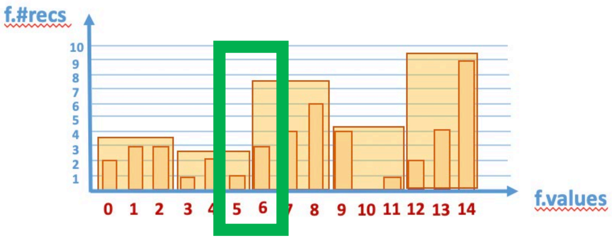
- Problem: how to construct the histogram?

Range Selection (1/2)

- p = Select(c, f in Range)
- For each bucket b ***in f***:
  - Selectivity = $\frac{|Range(c,f,b) \cap Range|}{|Range(c,f,b)|}$
  - R(p,f,b) = R(c,f,b) * selectivity
  - V(p,f,b) = V(c,f,b) * selectivity
  - Range(p,f,b) = Range(c,f,b) ∩ Range
- Assumptions:
  - #Records in a bucket are uniformly distributed
  - Values in a bucket are uniformly distributed

Given ∀f,b:

R(c, f, b)
V(c, f, b)
Range(c, f, b)



Range Selection (2/2)

- p = Select(c, f in Range)
- For each bucket b in ***f' ≠ f***:
  - Reduction = $\frac{\sum_b R(p,f,b)}{R(c)}$
  - R(p,f',b) = R(c,f',b) * Reduction
  - V(p,f',b) = min(V(c,f',b), R(p,f',b))
  - Range(p,f',b) = Range(c,f',b)
- Assumptions:
  - Values in different fields are independent with each other

Given ∀f,b:

R(c, f, b)
V(c, f, b)
Range(c, f, b)

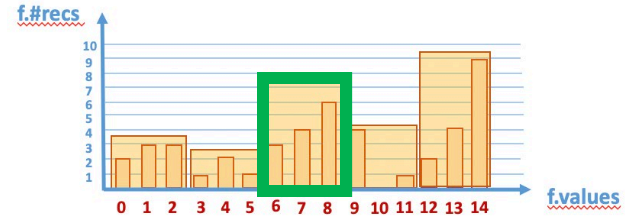
Product

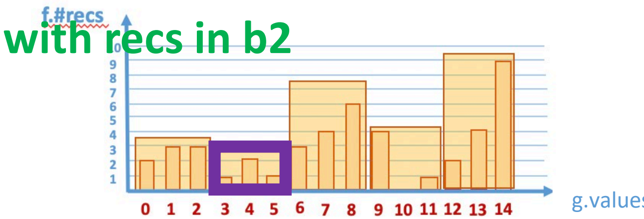
- p = Product(c1, c2)
- For each (b,f) in c1:
  - R(p,f,b) = R(c1,f,b) * R(c2)
  - V(p,f,b) = V(c1,f,b)
  - Range(p,f,b) = Range(c1,f,b)
- For each (b,f) in c2:
  - R(p,f,b) = R(c2,f,b) * R(c1)
  - V(p,f,b) = V(c2,f,b)
  - Range(p,f,b) = Range(c2,f,b)

Given ∀f,b:

R(c1, f, b)
V(c1, f, b)
Range(c1, f, b)
R(c2, f, b)
V(c2, f, b)
Range(c2, f, b)

Join Selection (1/2)





- p = Select(c, f=g)
- For each bucket **b1** in f and **b2** in g:
  - If Range(c,f,b1) ∩ Range(c,f,b2)=Ø, discard and b1 nd b2
  - minV = min(V(c,f,b1), V(c,f,b2))
  - R1 = R(c,f,b1) * $\frac{minV}{V(c,f,b1)} * \frac{1}{V(c,g,b2)} * \frac{R(c,g,b2)}{\mathrm{R}(c)}$
  - R2 = R(c,g,b2) * $\frac{minV}{V(c,g,b2)} * \frac{1}{V(c,f,b1)} * \frac{R(c,f,b1)}{\mathrm{R}(c)}$
  - R(p,f,b1) = R(p,g,b2) = min(R1, R2)
  - V(p,f,b1) = V(p,g,b2) = min(V(c,f,b1), V(c,g,b2))
  - Range(p,f,b1) = Range(p,g,b2) = Range(c,f,b1) ∩ Range(c,f,b2)

Match rate with recs in b2

Match rate with recs not in b2

- Assumptions:
  - Values in bucket are uniformly distributed
  - All values in the range having smaller number of values appear in the range having larger number of values
  - Values in different fields are independent with each other

Join Selection (2/2)

- p = Select(c, f=g)
- For each bucket b in ***f’ ∉ {f, g}***:
  - Reduction = $\frac{\sum_b R(p,f,b)}{R(c)}$
  - R(p,f’,b) = R(c,f’,b) * Reduction
  - V(p,f’,b) = min(V(c,f’,b), R(p,f’,b))
  - Range(p,f’,b) = Range(c,f’,b)
- Assumptions:
  - Values in different fields are independent with each other

Cost Estimation in VanillaCore

- B(p): `p.blocksAccessed()`
- Histogram-based cardinality estimation:
  - R(p): `p.histogram().recordsOutput()`
  - V(p,f): `p.histogram().distinctVaues(f)`
- Each plan builds its own histogram in constructor
- Important utility methods to trace:
  - `SelectPlan.constantRangeHistorgram()`
  - `ProductPlan.productHistogram()`
  - `SelectPlan.joinFieldHistogram()`
  - `AbstractJointPlan.joinHistogram()`

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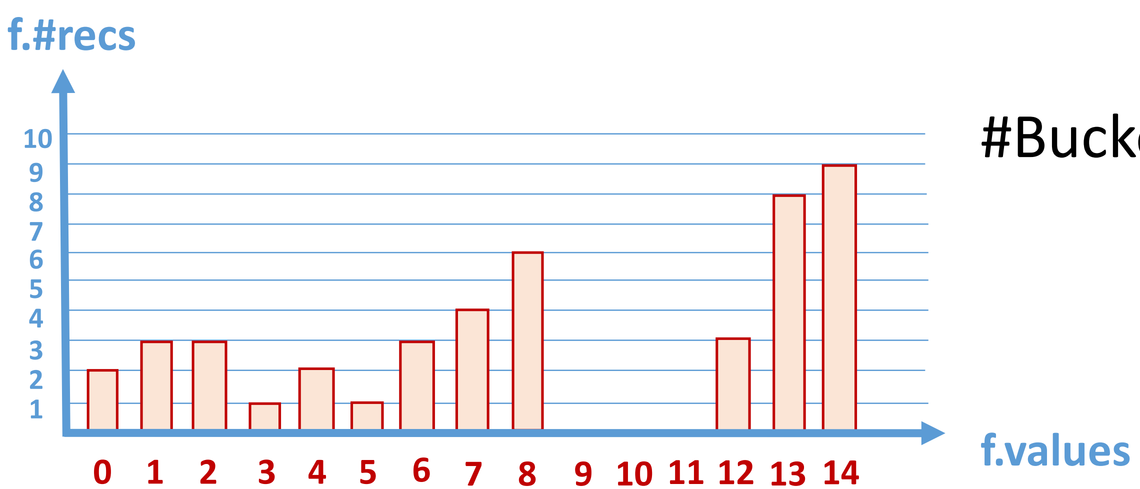
Table Histogram at Lowest-Level

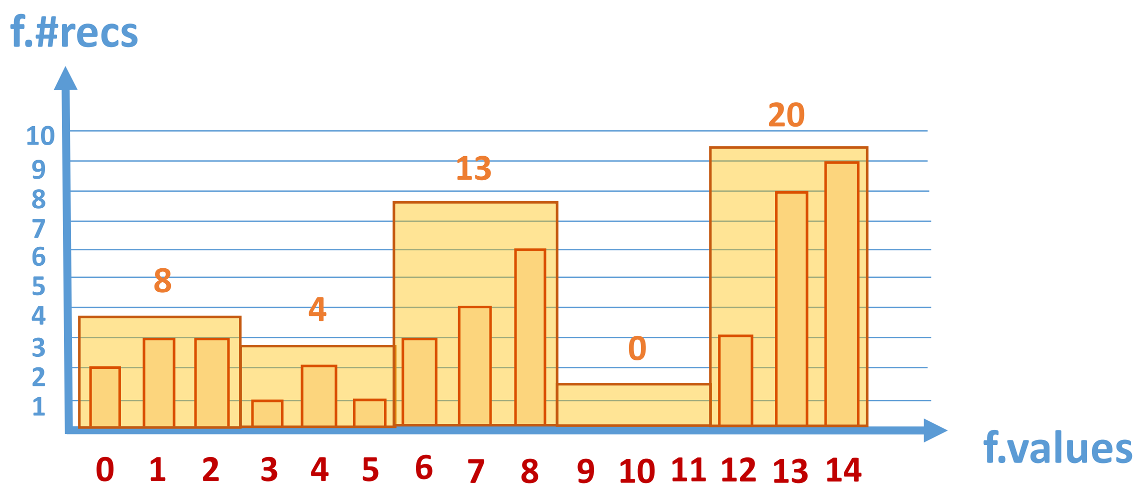
- Data structure that approximates value distribution
- Partitions field values into a set of ***buckets***
- Each bucket b collects statistics of a value range
  - Assumes ***uniform distribution of records and values*** in b
- Given a fixed #buckets, how to decide bucket ranges?



#Buckets = 5

Equi-Width Histogram

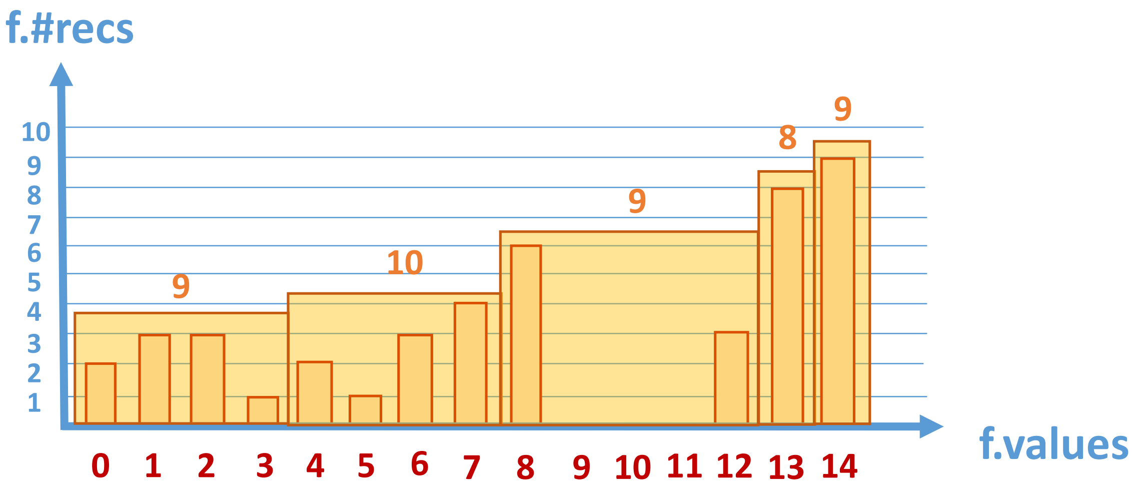
- Partition strategy: all buckets have the same range
- $|\text{Range}(b)| = \dfrac{Max(p,f) - Min(p,f) + 1}{\#Buckets}$



- Problem: some buckets may be wasted

Equi-Depth Histogram

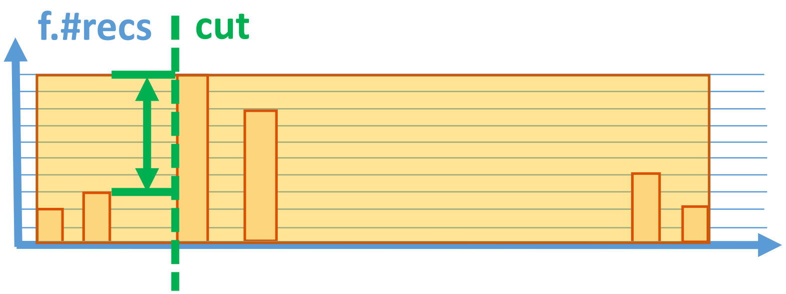
- Partition strategy: all buckets have the same #recs
- Depth = $\dfrac{R(p)}{\#Buckets}$



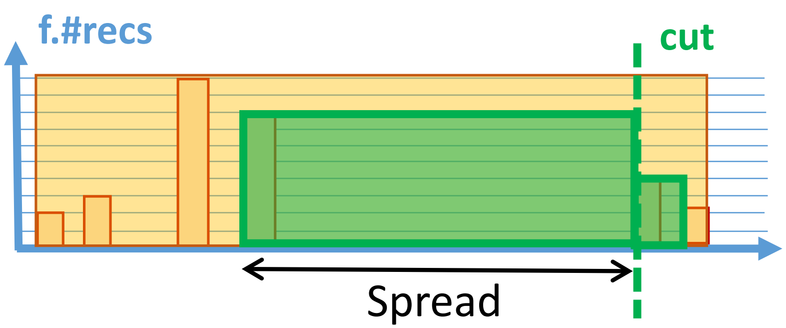
- Problem: records/values in a bucket may ***not*** be uniformly distributed

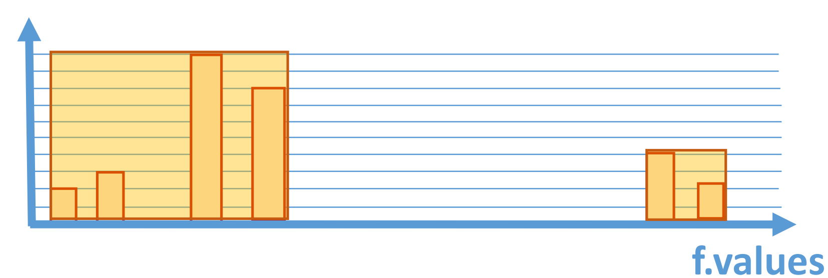
Max-Diff Histogram

- Partition strategy: split buckets at values with max. diff in #rec (MaxDiff(F)) or area (MaxDiff(A)):
    1. #recs: uniform #records in each bucket



    2. Area: uniform #records ***and values*** in each bucket





Histogram in VanillaCore

- Table histograms are statistics metadata
  - org.vanilladb.core.storage.metadata.statistics
- Accessed (by TablePlan) via StatMgr.getTableStatInfo()

```
Histogram
---
---
+ Histogram()
+ Histogram(fldnames : Set<String>)
~ Histogram(dists : Map<String, Collection<Bucket>>)
+ Histogram(hist : Histogram)
+ fields() : Set<String>
+ buckets(fldname : String) : Collection<Bucket>
+ addField(fldname : String)
+ addBucket(fldname : String, bkt : Bucket)
+ setBuckets(fldname : String, bkts : Collection<Bucket>)
+ recordsOutput() : double
+ distinctValues(fldname : String) : double
+ toString() : String
+ toString(int) : String
```

```
Bucket
---
---
+ Bucket(valrange : ConstantRange, freq : double, distvals
: double)
+ Bucket(valrange : ConstantRange, freq : double, distvals
: double, pcts : Percentiles)
+ valueRange() : ConstantRange
+ frequency() : double
+ distinctValues() : double
+ distinctValues(range : ConstantRange) : double
+ valuePercentiles() : Percentiles
+ toString() : String
+ toString(int) : String
```

Building Histogram (1/2)

- When system starts up:
- StatMgr:
  - Scans table and calls SampledHistogramBuilder.sample()
  - When done, calls SampledHistogramBuilder.newMaxDiffHistogram()
- Histogram types:
  - MaxDiff(A) : when field value is numeric
  - MaxDiff(F) : otherwise

Building Histogram (2/2)

- At runtime:
- StatMgr tacks #recs updated for each table
  - QueryPlanner calls StatMgr.countRecordUpdates() after executing modify/insert/delete queries
- Rebuilds histogram ***in background*** when StatMgr.getTableStatInfo() is called
  - If #recs updated > threshold (e.g., 100)
- StatisticsRefreshTask:
  - Scans table and calls SampledHistogramBuilder.sample()
  - When done, calls SampledHistogramBuilder.newMaxDiffHistogram()

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  2. Estimate cost of each corresponding scan tree
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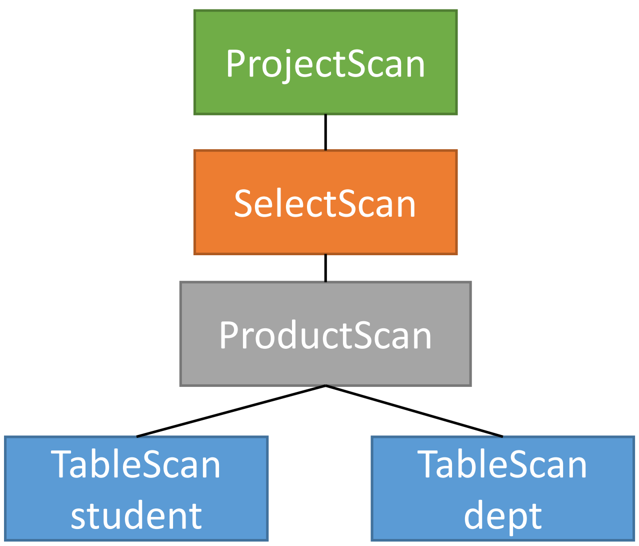
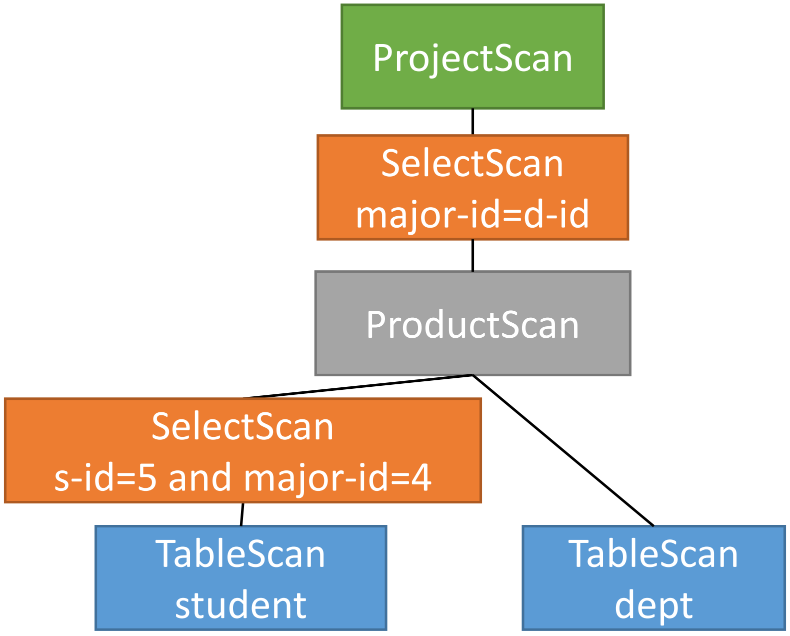


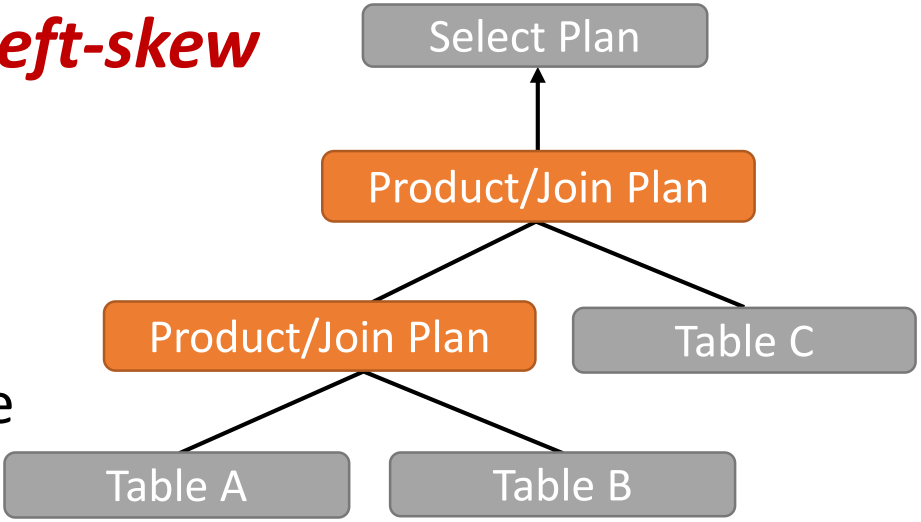


In Reality…

- Generating all candidate plan trees are too costly
  - #trees with n products/joins = Catalan number:

$$\frac{1}{n+1}\binom{2n}{n}$$

- Compromise: consider ***left-skew*** candidate trees only
- Query planner's goal
  - Avoiding bad trees
  - Not finding the best tree



Why Left-Skew Trees Only?

- Tend to be better than plans of other shapes
- Because many join algorithms scan right child c2 multiple times
- Normally, we don’t want c2 to be a complex subtree

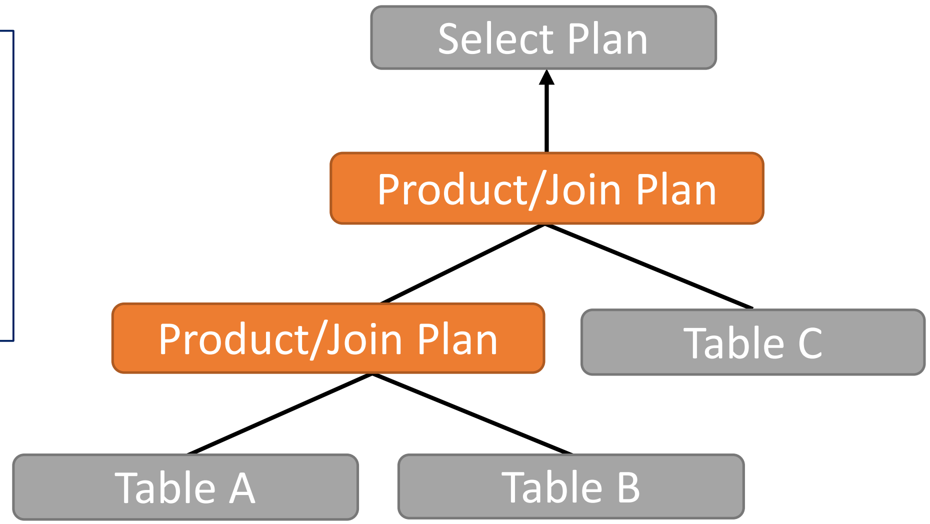
BasicQueryPlanner

```java
public Plan createPlan(QueryData data, Transaction tx) {
	// Step 1: Create a plan for each mentioned table or view
	List<Plan> plans = new ArrayList<Plan>();
	for (String tblname : data.tables()) {
		String viewdef = VanillaDb.catalogMgr().getViewDef(tblname, tx);
		if (viewdef != null)
			plans.add(VanillaDb.newPlanner().createQueryPlan(viewdef, tx));
		else
			plans.add(new TablePlan(tblname, tx));
	}
	// Step 2: Create the product of all table plans
	Plan p = plans.remove(0);
	for (Plan nextplan : plans)
		p = new ProductPlan(p, nextplan);
	// Step 3: Add a selection plan for the predicate
		p = new SelectPlan(p, data.pred());
	// Step 4: Add a group-by plan if specified
		if (data.groupFields() != null) {
			p = new GroupByPlan(p, data.groupFields(), data.aggregationFn(), tx);
	}
	// Step 5: Project onto the specified fields
	p = new ProjectPlan(p, data.projectFields());
	// Step 6: Add a sort plan if specified
	if (data.sortFields() != null)
		p = new SortPlan(p, data.sortFields(), data.sortDirections(), tx);
	// Step 7: Add a explain plan if the query is explain statement
	if (data.isExplain())
		p = new ExplainPlan(p);
	return p;
}
```

- Product/join order follows what’s written in SQL

Cost & Bettlenecks

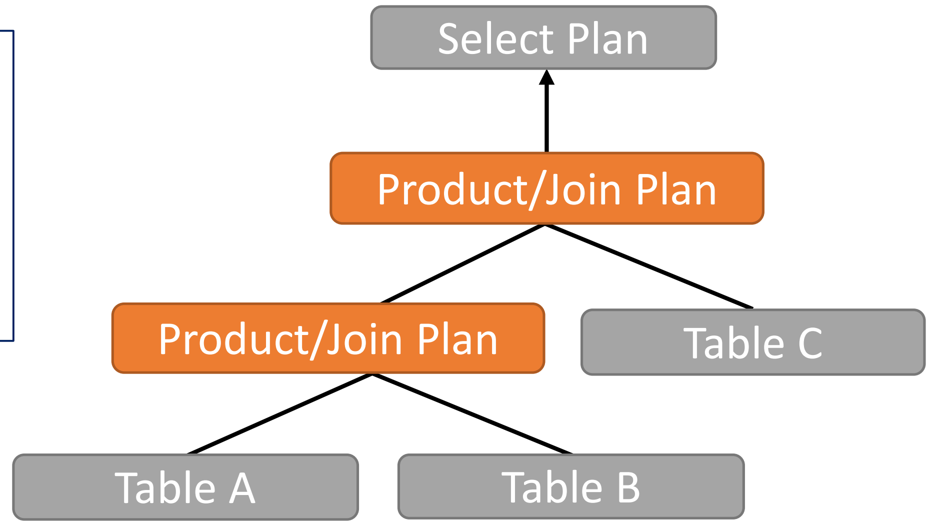
SELECT	A.c1, B.c2, C.c3
FROM	A, B, C
WHERE	A.aid = C.aid
AND	B.bid = C.bid
AND	A.c2 = xxx



- B(root) dominated by ***#recs*** of product/join ops
  - B(Product(c1, c2)) = B(c1) + (**R(c1)** * B(c2))
  - B(IndexJoin(c1, c2)) = B(c1) + (**R(c1)** * SearchCost(…)) + …
  - B(Select(c)) = B(c)

Optimizations

```
SELECT   A.c1, B.c2, C.c3
FROM     A, B, C
WHERE    A.aid = C.aid
AND      B.bid = C.bid
AND      A.c2 = xxx
```

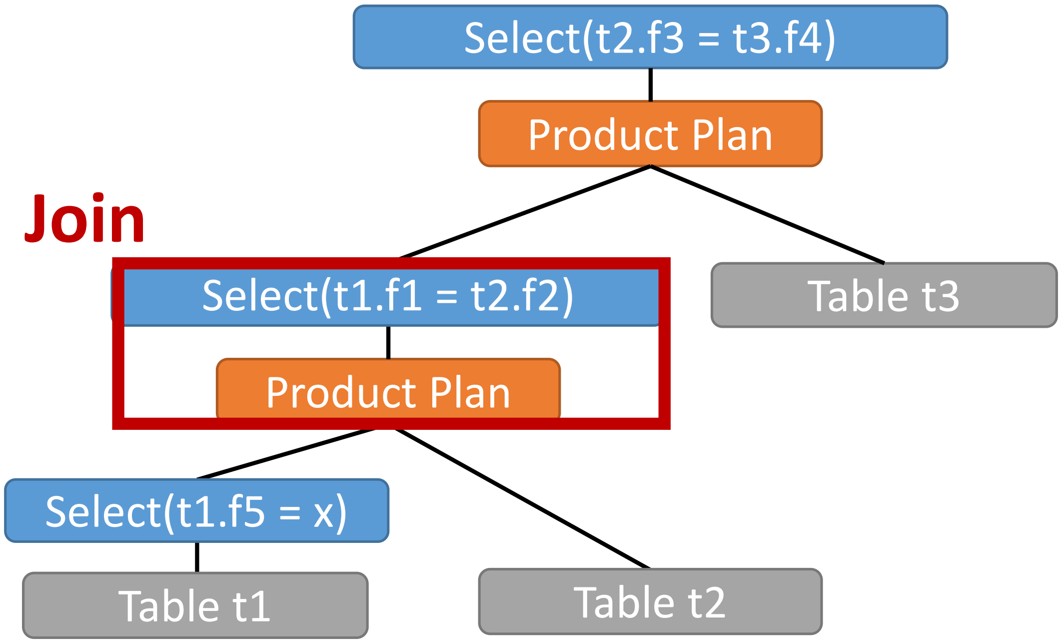


- Goal $\downarrow$B(root) reduced to $\downarrow$R(c1) and $\downarrow$R(c2)
- Heuristics:
  - Pushing Select ops down
  - Greedy Join ordering

Pushing Select Ops Down

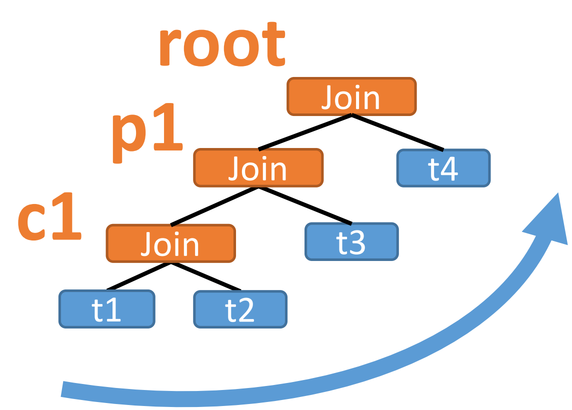
- Execute Select ops as early as possible
- ↓R(c1) and ↓R(c2) of each product/join op

SELECT	*
FROM	t1, t2, t3
WHERE	t1.f1 = t2.f2
AND	t2.f3 = t3.f4
AND	t1.f5 = x



Greedy Join Ordering

- B(root) = **B(p1)** + (**R(p1)** * …) + …
  - ↓ B(root) implies ↓(p1)
- **B(p1)** = **B(c1)** + (**R(c1)** * …) + …
  - ↓ B(root) also implies ↓(c1)
- **…**
- B(root) ∝ R(p1) + R(c1) + …



- ***Greedy Join ordering***: repeatedly add table to the "trunk" that result in lowest R(trunk)

HeuristicPlanner in VanillaCore

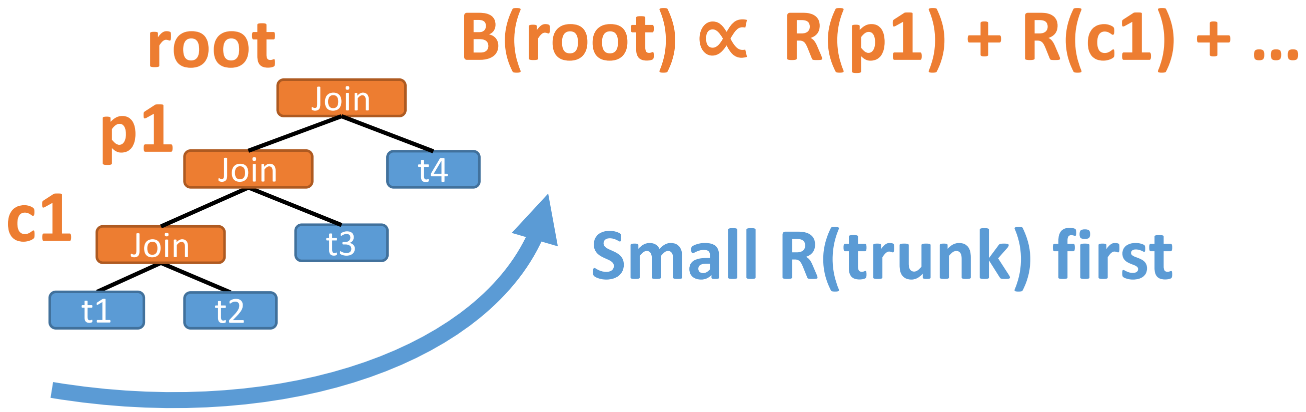
```
public Plan createPlan(QueryData data, Transaction tx) {
    // Step 1: Create a TablePlanner object for each mentioned table/view
    int id = 0;
    for (String tbl : data.tables()) {
        String viewdef = VanillaDb.catalogMgr().getViewDef(tbl, tx);
        if (viewdef != null)
            views.add(VanillaDb.newPlanner().createQueryPlan(viewdef, tx));
        else {
            TablePlanner tp = new TablePlanner(tbl, data.pred(), tx, id);
            tablePlanners.add(tp);
        }
        id += 1;
    }
    // Step 2: Choose the lowest-size plan to begin the trunk of join
    Plan trunk = getLowestSelectPlan();
    // Step 3: Repeatedly add a plan to the join trunk
    while (!tablePlanners.isEmpty() || !views.isEmpty()) {
        Plan p = getLowestJoinPlan(trunk);
        if (p != null)
            trunk = p;
        else
            // no applicable join
            trunk = getLowestProductPlan(trunk);
    }
    // Step 4: Add a group by plan if specified
    // Step 5. Project on the field names
    // Step 6: Add a sort plan if specified
    // Step 7: Add a explain plan if the query is explain statement
}
```

Feasible Select ops applied

Outline

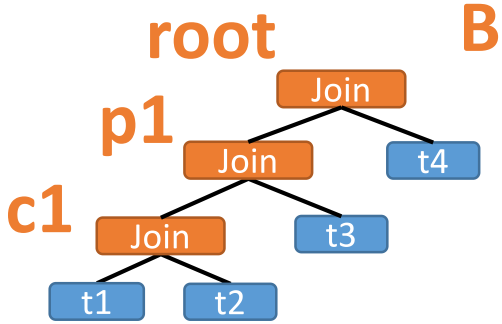
- Overview
- Cost Estimation
  - Cardinality Estimation
  - Histogram-based Estimation
  - Types of Histograms
- Heuristic Query Optimizer
  - Basic Planner
  - Pushing Select Down
  - Join Ordering
  - Heuristic Query Planner in VanillaCore
- Selinger-Style Query Optimizer

Why not HeuristicPlanner?



- Assumption: ↓R(c1) implies ↓R(p1))
- May ***not*** be true: match rate matters
- Exhaustively searching the best join order?
  - Cost: **O(n!)** for n joins (e.g., 8! = 40320)

Selinger-Style Optimizer



$B(root) \propto R(p1) + R(c1) + \ldots$

- Consider the best trees after 1, 2, 3, … joins
- Observation: if R(**t3 ⋈ t1 ⋈ t2**) <= R(**t2 ⋈ t3 ⋈ t1**), then R(**t3 ⋈ t1 ⋈ t2** ⋈ t4) <= R(**t2 ⋈ t3 ⋈ t1** ⋈ t4)
- We can use ***dynamic programming*** to avoid repeating computations

Selinger Optimizer Example (1/6)

- Here are 3 relations to join: A, B, C
- Step 1: compute the cost (R) of each relation's cheapest plan

1-Set	Best Plan	R
{A}	Index Select Plan	10
{B}	Table Plan	30
{C}	Select Plan	20

Selinger Optimizer Example (2/6)

- Step 2: compute the cost of 2-way joins reusing 1-way cost just cached
- R(A⋈B) = R(B⋈A), so we just keep one

1-Set	Best Plan	R
{A}	Index Select Plan	10
{B}	Table Plan	30
{C}	Select Plan	20

2-Set	Best Plan	R
{A,B}	A⋈B	159
{A,C}	A⋈C	98
{B,C}	B⋈C	77

Selinger Optimizer Example (3/6)

- Here are 3 relations to join – A, B, C
- Step 2
  - Compute the cost of 2-way join by estimating all permutation using the single relation cost just cached
  - Ex. {A, B} =
    - Compare {A}B Cost: 159
    - Compare {B}A Cost: 189

Because the R(AB), R(BA) is the same, we can only keep the least cost one

Sub plan	Best Plan	Cost
{A}	Index Select Plan	10
{B}	Table Plan	30
{C}	Select Plan	20

Sub plan	Best Plan	Cost
{A, B}	$A \bowtie B$	159

Selinger Optimizer Example (4/6)

- Here are 3 relations to join – A, B, C
- Step 2
  - Compute the cost of 2-way join by estimating all permutation using the single relation cost just cached
  - Ex. {A, B} =
    - Compare {A}B   Cost: 159
    - Compare {B}A   Cost: 189

Sub plan	Best Plan	Cost
{A}	Index Select Plan	10
{B}	Table Plan	30
{C}	Select Plan	20

Sub plan	Best Plan	Cost
{A, B}	A⋈B	159
{A, C}	C⋈A	98
{B, C}	C⋈B	77

Selinger Optimizer Example (5/6)

- Here are 3 relations to join – A, B, C
- Step 3
  - Compute the cost of 3-way join by estimating all left-deep tree permutation using the step2's record
  - Ex. {A, B, C} =
    - Compare ({A, B})C Cost: 259
    - Compare ({B, C})A Cost: 111
    - Compare ({C, A})B Cost: 100

Sub plan	Best Plan	Cost
{A, B}	A⋈B	159
{A, C}	C⋈A	98
{B, C}	C⋈B	77

Sub plan	Best Plan	Cost

Selinger Optimizer Example (6/6)

- Here are 3 relations to join – A, B, C
- Step 3
  - Compute the cost of 3-way join by estimating all left-deep tree permutation using the step2's record
  - Ex. {A, B, C} =
    - Compare ({A, B})C Cost: 259
    - Compare ({B, C})A Cost: 111
    - Compare ({C, A})B Cost: 100

Sub plan	Best Plan	Cost
{A, B}	A⋈B	159
{A, C}	C⋈A	98
{B, C}	C⋈B	77

Sub plan	Best Plan	Cost
{A, B, C}	C⋈A⋈B	100

```
private Plan getAllCombination(Plan viewTrunk) {
        long finalKey = 0;

        // for layer = 1, use select down strategy to construct
        for (TablePlanner tp: tablePlanners) {
                Plan bestPlan = null;
                if (viewTrunk != null) {
                        bestPlan = tp.makeJoinPlan(viewTrunk);
                        if (bestPlan == null)
                        bestPlan = tp.makeProductPlan(viewTrunk);
                }
                else
                   bestPlan = tp.makeSelectPlan();

                AccessPath ap = new AccessPath(tp, bestPlan);
                lookupTbl.put(ap.getAPId(), ap);

                // compute final access path id
                finalKey += ap.getAPId();
        }

        .
        .
        .

}
```

```java
// construct all combination layer by layer
for (int layer = 2; layer <= tablePlanners.size(); layer++) {
	Set<Long> keySet = new HashSet<Long>(lookupTbl.keySet());

	for (TablePlanner rightOne: tablePlanners) {
		for (Long key: keySet) {
			AccessPath leftTrunk = lookupTbl.get(key);

			// cannot join with table which (layer-1) combination already included
			if (leftTrunk.isUsed(rightOne.getId()))
			    continue;

			// do join
			Plan bestPlan = rightOne.makeJoinPlan(leftTrunk.getPlan());
			if (bestPlan == null)
			    bestPlan = rightOne.makeProductPlan(leftTrunk.getPlan());

			AccessPath candidate = new AccessPath(leftTrunk, rightOne, bestPlan);
			AccessPath ap = lookupTbl.get(candidate.getAPId());

			// there is no access path contains this combination
			if (ap == null) {
			    lookupTbl.put(candidate.getAPId(), candidate);
			}
			// check whether new access path is better than previous
			else {
				if (candidate.getCost() < ap.getCost())
				    lookupTbl.put(candidate.getAPId(), candidate);
			}
		}
	}
	// remove the elements belong to layer-1
	// because in the next layer we only need this layer's combination
	for (Long key: keySet)
	    lookupTbl.remove(key);
}

return lookupTbl.get(finalKey).getPlan();
```

- Iterate all table planners to join with all existing (layer-1) combination to construct this layer

```java
public class AccessPath {
	private Plan p;
	private AccessPathId apId;
	private long cost = 0;
	private ArrayList<Integer> tblUsed = new ArrayList<Integer>();

	public class AccessPathId {
		long id;

		AccessPathId(TablePlanner tp) {
		    this.id = (long) Math.pow(2,tp.getId());
		}

		AccessPathId(AccessPath ap, TablePlanner tp) {
		    this.id = ap.getAPId()+(long) Math.pow(2,tp.getId());
		}
		public long getID() {
		    return id;
		}
	}

	public AccessPath (TablePlanner newTp, Plan p) {
		this.p = p;
		this.tblUsed.add(newTp.getId());
		this.apId = new AccessPathId(newTp);
		this.cost = p.recordsOutput();
	}
	public AccessPath (AccessPath preAp, TablePlanner newTp, Plan p) {
		this.p = p;
		this.tblUsed.addAll(preAp.getTblUsed());
		this.tblUsed.add(newTp.getId());
		this.apId = new AccessPathId(preAp, newTp);

		// approximate cost = previous cost + new cost
		this.cost = preAp.getCost() + p.recordsOutput();
	}
}
```

- Using **sum of pow(2, tp.id)** to represent the combination of tables in this access path
- Using **pow(2, tp.id)** to avoid problems with different combinations but with the same apID
- Then we can use apID as the key of the lookup table

```java
public class AccessPath {
	private Plan p;
	private AccessPathId apId;
	private long cost = 0;
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	public class AccessPathId {
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		AccessPathId(AccessPath ap, TablePlanner tp) {
		    this.id = ap.getAPId()+(long) Math.pow(2,tp.getId());
		}
		public long getID() {
		    return id;
		}
	}


	public AccessPath (TablePlanner newTp, Plan p) {
		this.p = p;
		this.tblUsed.add(newTp.getId());
		this.apId = new AccessPathId(newTp);
		this.cost = p.recordsOutput();
	}
	public AccessPath (AccessPath preAp, TablePlanner newTp, Plan p) {
		this.p = p;
		this.tblUsed.addAll(preAp.getTblUsed());
		this.tblUsed.add(newTp.getId());
		this.apId = new AccessPathId(preAp, newTp);

		// approximate cost = previous cost + new cost
		this.cost = preAp.getCost() + p.recordsOutput();

	}
}
```

- Using **sum of pow(2, tp.id)** to represent the combination of tables in this access path
- Using **pow(2, tp.id)** to avoid problems with different combinations but with the same apID
- Then we can use apID as the key of the lookup table

- Approximate B(root) using R(p1) + R(c1)…

Reference

- https://db.inf.uni-tuebingen.de/staticfiles/teaching/ws1011/db2/db2-selectivity.pdf
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