***CHAPTER 1***

**INTRODUCTION TO CORPORATE FINANCE**

**Answers to Concept Questions**

**1.** In the corporate form of ownership, the shareholders are the owners of the firm. The shareholders elect the directors of the corporation, who in turn appoint the firm’s management. This separation of ownership from control in the corporate form of organization is what causes agency problems to exist. Management may act in its own or someone else’s best interests, rather than those of the shareholders. If such events occur, they may contradict the goal of maximizing the share price of the equity of the firm.

**2.** Such organizations frequently pursue social or political missions, so many different goals are conceivable. One goal that is often cited is revenue minimization; i.e., provide whatever goods and services are offered at the lowest possible cost to society. A better approach might be to observe that even a not-for-profit business has equity. Thus, one answer is that the appropriate goal is to maximize the value of the equity.

**3.** Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term *and* long-term. If this is correct, then the statement is false.

**4.** An argument can be made either way. At the one extreme, we could argue that in a market economy, all of these things are priced. There is thus an optimal level of, for example, ethical and/or illegal behavior, and the framework of stock valuation explicitly includes these. At the other extreme, we could argue that these are non-economic phenomena and are best handled through the political process. A classic (and highly relevant) thought question that illustrates this debate goes something like this: “A firm has estimated that the cost of improving the safety of one of its products is $30 million. However, the firm believes that improving the safety of the product will only save $20 million in product liability claims. What should the firm do?”

**5.** The goal will be the same, but the best course of action toward that goal may be different because of differing social, political, and economic institutions.

**6.** The goal of management should be to maximize the share price for the current shareholders. If management believes that it can improve the profitability of the firm so that the share price will exceed $35, then they should fight the offer from the outside company. If management believes that this bidder, or other unidentified bidders, will actually pay more than $35 per share to acquire the company, then they should still fight the offer. However, if the current management cannot increase the value of the firm beyond the bid price, and no other higher bids come in, then management is not acting in the interests of the shareholders by fighting the offer. Since current managers often lose their jobs when the corporation is acquired, poorly monitored managers have an incentive to fight corporate takeovers in situations such as this.

**7.** We would expect agency problems to be less severe in other countries, primarily due to the relatively small percentage of individual ownership. Fewer individual owners should reduce the number of diverse opinions concerning corporate goals. The high percentage of institutional ownership might lead to a higher degree of agreement between owners and managers on decisions concerning risky projects. In addition, institutions may be better able to implement effective monitoring mechanisms on managers than can individual owners, based on the institutions’ deeper resources and experiences with their own management.

**8.** The increase in institutional ownership of stock in the United States and the growing activism of these large shareholder groups may lead to a reduction in agency problems for U.S. corporations and a more efficient market for corporate control. However, this may not always be the case. If the managers of the mutual fund or pension plan are not concerned with the interests of the investors, the agency problem could potentially remain the same, or even increase, since there is the possibility of agency problems between the fund and its investors.

**9.** How much is too much? Who is worth more, Larry Ellison or Tiger Woods? The simplest answer is that there is a market for executives just as there is for all types of labor. Executive compensation is the price that clears the market. The same is true for athletes and performers. Having said that, one aspect of executive compensation deserves comment. A primary reason executive compensation has grown so dramatically is that companies have increasingly moved to stock-based compensation. Such movement is obviously consistent with the attempt to better align stockholder and management interests. In recent years, stock prices have soared, so management has cleaned up. It is sometimes argued that much of this reward is due to rising stock prices in general, not managerial performance. Perhaps in the future, executive compensation will be designed to reward only differential performance, i.e., stock price increases in excess of general market increases.

**10.** Maximizing the current share price is the same as maximizing the future share price at any future period. The value of a share of stock depends on all of the future cash flows of company. Another way to look at this is that, barring large cash payments to shareholders, the expected price of the stock must be higher in the future than it is today. Who would buy a stock for $100 today when the share price in one year is expected to be $80?

***CHAPTER 2***

**ACCOUNTING STATEMENTS, TAXES, AND CASH FLOW**

**Answers to Concepts Review and Critical Thinking Questions**

**1.** True. Every asset can be converted to cash at some price. However, when we are referring to a liquid asset, the added assumption that the asset can be quickly converted to cash at or near market value is important.

**2.** The recognition and matching principles in financial accounting call for revenues, and the costs associated with producing those revenues, to be “booked” when the revenue process is essentially complete, not necessarily when the cash is collected or bills are paid. Note that this way is not necessarily correct; it’s the way accountants have chosen to do it.

**3.** The bottom line number shows the change in the cash balance on the balance sheet. As such, it is not a useful number for analyzing a company.

**4.** The major difference is the treatment of interest expense. The accounting statement of cash flows treats interest as an operating cash flow, while the financial cash flows treat interest as a financing cash flow. The logic of the accounting statement of cash flows is that since interest appears on the income statement, which shows the operations for the period, it is an operating cash flow. In reality, interest is a financing expense, which results from the company’s choice of debt and equity. We will have more to say about this in a later chapter. When comparing the two cash flow statements, the financial statement of cash flows is a more appropriate measure of the company’s performance because of its treatment of interest.

**5.** Market values can never be negative. Imagine a share of stock selling for –$20. This would mean that if you placed an order for 100 shares, you would get the stock along with a check for $2,000. How many shares do you want to buy? More generally, because of corporate and individual bankruptcy laws, net worth for a person or a corporation cannot be negative, implying that liabilities cannot exceed assets in market value.

**6.** For a successful company that is rapidly expanding, for example, capital outlays will be large, possibly leading to negative cash flow from assets. In general, what matters is whether the money is spent wisely, not whether cash flow from assets is positive or negative.

**7.** It’s probably not a good sign for an established company to have negative cash flow from operations, but it would be fairly ordinary for a start-up, so it depends.

**8.** For example, if a company were to become more efficient in inventory management, the amount of inventory needed would decline. The same might be true if the company becomes better at collecting its receivables. In general, anything that leads to a decline in ending NWC relative to beginning would have this effect. Negative net capital spending would mean more long-lived assets were liquidated than purchased.

**9.** If a company raises more money from selling stock than it pays in dividends in a particular period, its cash flow to stockholders will be negative. If a company borrows more than it pays in interest and principal, its cash flow to creditors will be negative.

**10.** The adjustments discussed were purely accounting changes; they had no cash flow or market value consequences unless the new accounting information caused stockholders to revalue the assets.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** To find owners’ equity, we must construct a balance sheet as follows:

Balance Sheet

CA $ 4,300 CL $ 2,900

NFA 24,000 LTD 10,700

OE ??

TA $28,300 TL & OE $28,300

We know that total liabilities and owners’ equity (TL & OE) must equal total assets of $28,300. We also know that TL & OE is equal to current liabilities plus long-term debt plus owners’ equity, so owners’ equity is:

Owners’ equity = $28,300 – 10,700 – 2,900

Owners’ equity = $14,700

And net working capital is current assets minus current liabilities, so:

NWC = Current assets – Current liabilities

NWC = $4,300 – 2,900

NWC = $1,400

**2.** The income statement for the company is:

Income Statement

Sales $473,000

Costs 275,000

Depreciation 42,000

EBIT $156,000

Interest 23,000

EBT $133,000

Taxes 27,930

Net income $105,070

One equation for net income is:

Net income = Dividends + Addition to retained earnings

Rearranging, we get:

Addition to retained earnings = Net income – Dividends

Addition to retained earnings = $105,070 – 25,000

Addition to retained earnings = $80,070

**3.** To find the book value of current assets, we use: NWC = CA – CL. Rearranging to solve for current assets, we get:

Current assets = Net working capital + Current liabilities

Current assets = $850,000 + 2,200,000

Current assets = $3,050,000

The market value of current assets and net fixed assets is given, so:

Book value CA = $3,050,000 Market value CA = $2,700,000

Book value NFA = $4,900,000 Market value NFA = $6,400,000

Book value assets = $7,950,000 Market value assets = $9,100,000

**4.** Taxes = .10($9,525) + .12($38,700 – 9,525) + .22($82,500 – 38,700) + .24($157,500 – 82,500)

+ .32($189,000 – 157,500)

Taxes = $42,169.50

The average tax rate is the total tax paid divided by taxable income, so:

Average tax rate = $42,169.50/$189,000

Average tax rate = .2231, or 22.31%

The marginal tax rate is the tax rate on the next $1 of earnings, so the marginal tax rate is 32 percent.

**5.** To calculate OCF, we first need the income statement:

Income Statement

Sales $22,400

Costs 11,600

Depreciation 2,200

EBIT $8,600

Interest 1,370

Taxable income $7,230

Taxes 1,591

Net income $5,639

OCF = EBIT + Depreciation – Taxes

OCF = $8,600 + 2,200 – 1,591

OCF = $9,209

**6.** Net capital spending = NFAend – NFAbeg + Depreciation

Net capital spending = $1,430,000 – 1,280,000 + 146,000

Net capital spending = $296,000

**7.** The long-term debt account will increase by $30 million, the amount of the new long-term debt issue. Since the company sold 4.5 million new shares of stock with a $1 par value, the common stock account will increase by $4.5 million. The capital surplus account will increase by $53.5 million, the value of the new stock sold above its par value. Since the company had a net income of $7.5 million, and paid $1.7 million in dividends, the addition to retained earnings was $5.8 million, which will increase the accumulated retained earnings account. So, the new long-term debt and stockholders’ equity portion of the balance sheet will be:

|  |  |  |
| --- | --- | --- |
|  | Long-term debt | $ 75,000,000 |
|  | Total long-term debt | $ 75,000,000 |
|  |  |  |
|  | Shareholders’ equity | FCF |
|  | Preferred stock | $ 2,900,000 |
|  | Common stock ($1 par value) | 15,500,000 |
|  | Accumulated retained earnings | 112,800,000 |
|  | Capital surplus | 102,500,000 |
|  | Total equity | $ 233,700,000 |
|  |  |  |
|  | Total liabilities & equity | $ 308,700,000 |

**8.** Cash flow to creditors = Interest paid – Net new borrowing

Cash flow to creditors = $170,000 – (LTDend – LTDbeg)

Cash flow to creditors = $170,000 – ($1,645,000 – 1,565,000)

Cash flow to creditors = $170,000 – 80,000

Cash flow to creditors = $90,000

**9.** Cash flow to stockholders = Dividends paid – Net new equity

Cash flow to stockholders = $335,000 – [(Commonend + APISend) – (Commonbeg + APISbeg)]

Cash flow to stockholders = $335,000 – [($525,000 + 3,750,000) – ($490,000 + 3,400,000)]

Cash flow to stockholders = $335,000 – ($4,275,000 – 3,890,000)

Cash flow to stockholders = –$50,000

Note, APIS is the additional paid-in surplus.

**10.** Cash flow from assets = Cash flow to creditors + Cash flow to stockholders = $90,000 – 50,000

= $40,000

Cash flow from assets = OCF – Change in NWC – Net capital spending

$40,000 = OCF – (–$96,000) – 735,000

Operating cash flow = $40,000 – 96,000 + 735,000

Operating cash flow = $679,000

*Intermediate*

**11.** *a.* The accounting statement of cash flows explains the change in cash during the year. The accounting statement of cash flows will be:

|  |  |  |
| --- | --- | --- |
|  | Statement of cash flows | |
|  | *Operations* |  |
|  | Net income | $129 |
|  | Depreciation | 92 |
|  | Changes in other current assets | (17) |
|  | Change in accounts payable | 17 |
|  | Total cash flow from operations | $221 |
|  |  |  |
|  | *Investing activities* |  |
|  | Acquisition of fixed assets | $(111) |
|  | Total cash flow from investing activities | $(111) |
|  |  |  |
|  | *Financing activities* |  |
|  | Proceeds of long-term debt | $8 |
|  | Dividends | (97) |
|  | Total cash flow from financing activities | ($89) |
|  |  |  |
|  | Change in cash (on balance sheet) | $21 |

*b.* Change in NWC = NWCend – NWCbeg

= (CAend – CLend) – (CAbeg – CLbeg)

= [($84 + 192) – 146] – [($63 + 175) – 129)

= $130 – 109

= $21

*c.* To find the cash flow generated by the firm’s assets, we need the operating cash flow and the capital spending. So, calculating each of these, we find:

|  |  |  |
| --- | --- | --- |
|  | *Operating cash flow* |  |
|  | Net income | $129 |
|  | Depreciation | 92 |
|  | Operating cash flow | $221 |

Note that we can calculate OCF in this manner since there are no taxes.

|  |  |  |
| --- | --- | --- |
|  | *Capital spending* |  |
|  | Ending fixed assets | $417 |
|  | Beginning fixed assets | –398 |
|  | Depreciation | 92 |
|  | Capital spending | $111 |

Now we can calculate the cash flow generated by the firm’s assets, which is:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | *Cash flow from assets* |  |
|  | Operating cash flow | $221 |
|  | Capital spending | –111 |
|  | Change in NWC | –21 |
|  | Cash flow from assets | $ 89 |

**12.** With the information provided, the cash flows from the firm are the capital spending and the change in net working capital, so:

|  |  |  |
| --- | --- | --- |
|  | *Cash flows from the firm* |  |
|  | Capital spending | –$29,000 |
|  | Additions to NWC | –2,400 |
|  | Capital spending and NWC cash flow | –$31,400 |

And the cash flows to the investors of the firm are:

|  |  |  |
| --- | --- | --- |
|  | *Cash flows to investors of the firm* |  |
|  | Sale of long-term debt | –$16,400 |
|  | Sale of common stock | –4,000 |
|  | Dividends paid | 13,100 |
|  | Cash flows to investors of the firm | –$7,300 |

**13.** *a.* The interest expense for the company is the amount of debt times the interest rate on the debt. So, the income statement for the company is:

Income Statement

Sales $865,000

Cost of goods sold 455,000

Selling costs 210,000

Depreciation 105,000

EBIT $ 95,000

Interest 27,200

Taxable income $ 67,800

Taxes 14,238

Net income $ 53,562

*b.* And the operating cash flow is:

OCF = EBIT + Depreciation – Taxes

OCF = $95,000 + 105,000 – 14,238

OCF = $185,762

**14.** To find the OCF, we first calculate net income.

Income Statement

Sales $246,000

Costs 135,000

Other expenses 7,100

Depreciation 19,100

EBIT $84,800

Interest 10,000

Taxable income $74,800

Taxes 18,876

Net income $55,924

Dividends $9,800

Additions to RE $46,124

*a.* OCF = EBIT + Depreciation – Taxes

OCF = $84,800 + 19,100 – 18,876

OCF = $85,024

*b.* CFC = Interest – Net new LTD

CFC = $10,000 – (–$6,800)

CFC = $16,800

Note that the net new long-term debt is negative because the company repaid part of its long-

term debt.

*c.* CFS = Dividends – Net new equity

CFS = $9,800 – 7,900

CFS = $1,900

*d.* We know that CFA = CFC + CFS, so:

CFA = $16,800 + 1,900

CFA = $18,700

CFA is also equal to OCF – Net capital spending – Change in NWC. We already know OCF. Net capital spending is equal to:

Net capital spending = Increase in NFA + Depreciation

Net capital spending = $41,900 + 19,100

Net capital spending = $61,000

Now we can use:

CFA = OCF – Net capital spending – Change in NWC

$18,700 = $85,024 – 61,000 – Change in NWC

Change in NWC = $5,324

This means the company increased its NWC by $5,324.

**15.** The solution to this question works the income statement backwards. Starting at the bottom:

Net income = Dividends + Addition to retained earnings

Net income = $1,720 + 5,300

Net income = $7,020

Now, looking at the income statement:

EBT – (EBT × Tax rate) = Net income

Recognize that EBT × Tax rate is the calculation for taxes. Solving this for EBT yields:

EBT = Net income/(1 – Tax rate)

EBT = $7,020/(1 – .21)

EBT = $8,886.08

Now we can calculate:

EBIT = EBT + Interest

EBIT = $8,886.08 + 2,050

EBIT = $10,936.08

The last step is to use:

EBIT = Sales – Costs – Depreciation

$10,936.08 = $54,000 – 29,500 – Depreciation

Depreciation = $13,563.92

**16.** The market value of shareholders’ equity cannot be negative. A negative market value in this case would imply that the company would pay you to own the stock. The market value of shareholders’ equity can be stated as: Shareholders’ equity = Max [(TA – TL), 0]. So, if TA is $11,900, equity is equal to $1,200, and if TA is $9,400, equity is equal to $0. We should note here that while the market value of equity cannot be negative, the book value of shareholders’ equity can be negative.

**17.** Income Statement

Sales $630,000

COGS 465,000

A&S expenses 85,000

Depreciation 135,000

EBIT –$55,000

Interest 70,000

Taxable income –$125,000

Taxes (21%) 0

*a.* Net income –$125,000

*b.* OCF = EBIT + Depreciation – Taxes

OCF = –$55,000 + 135,000 – 0

OCF = $80,000

*c.* Net income was negative because of the tax deductibility of depreciation and interest expense. However, the actual cash flow from operations was positive because depreciation is a non-cash expense and interest is a financing expense, not an operating expense.

**18.** A firm can still pay out dividends if net income is negative; it just has to be sure there is sufficient cash flow to make the dividend payments.

Change in NWC = Net capital spending = Net new equity = 0. (Given)

Cash flow from assets = OCF – Change in NWC – Net capital spending

Cash flow from assets = $80,000 – 0 – 0 = $80,000

Cash flow to stockholders = Dividends – Net new equity

Cash flow to stockholders = $34,000 – 0 = $34,000

Cash flow to creditors = Cash flow from assets – Cash flow to stockholders

Cash flow to creditors = $80,000 – 34,000

Cash flow to creditors = $46,000

Cash flow to creditors is also:

Cash flow to creditors = Interest – Net new LTD

So:

Net new LTD = Interest – Cash flow to creditors

Net new LTD = $70,000 – 46,000

Net new LTD = $24,000

**19.** *a.* The income statement is:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Income Statement | |
|  |  | Sales | $24,360 |
|  |  | Cost of goods sold | 17,600 |
|  |  | Depreciation | 3,400 |
|  |  | EBIT | $ 3,360 |
|  |  | Interest | 860 |
|  |  | Taxable income | $ 2,500 |
|  |  | Taxes | 525 |
|  |  | Net income | $1,975 |

*b.* OCF = EBIT + Depreciation – Taxes

OCF = $3,360 + 3,400 – 525

OCF = $6,235

*c.* Change in NWC = NWCend – NWCbeg

= (CAend – CLend) – (CAbeg – CLbeg)

= ($6,410 – 3,445) – ($5,560 – 3,040)

= $2,965 – 2,520

= $445

Net capital spending = NFAend – NFAbeg + Depreciation

= $21,180 – 18,650 + 3,400

= $5,930

CFA = OCF – Change in NWC – Net capital spending

= $6,235 – 445 – 5,930

= –$140

The cash flow from assets can be positive or negative, since it represents whether the firm raised funds or distributed funds on a net basis. In this problem, even though net income and OCF are positive, the firm invested heavily in both fixed assets and net working capital; it had to raise a net $140 in funds from its stockholders and creditors to make these investments.

*d.* Cash flow to creditors = Interest – Net new LTD

= $860 – 0

= $860

Cash flow to stockholders = Cash flow from assets – Cash flow to creditors

= –$140 – 860

= –$1,000

We can also calculate the cash flow to stockholders as:

Cash flow to stockholders = Dividends – Net new equity

Solving for net new equity, we get:

Net new equity = $1,000 – (–790)

= $1,790

The firm had positive earnings in an accounting sense (NI > 0) and had positive cash flow from operations. The firm invested $445 in new net working capital and $5,930 in new fixed assets. The firm had to raise $140 from its stakeholders to support this new investment. It accomplished this by raising $1,790 in the form of new equity. After paying out $790 of this in the form of dividends to shareholders and $860 in the form of interest to creditors, $140 was left to meet the firm’s cash flow needs for investment.

**20.** *a.* Total assets 2018 = $1,157 + 5,261 = $6,418

Total liabilities 2018 = $481 + 2,856 = $3,337

Owners’ equity 2018 = $6,418 – 3,337 = $3,081

Total assets 2019 = $1,411 + 6,125 = $7,536

Total liabilities 2019 = $534 + 3,256 = $3,790

Owners’ equity 2019 = $7,536 – 3,790 = $3,746

*b.* NWC 2018 = CA – CL = $1,157 – 481 = $676

NWC 2019 = CA – CL = $1,411 – 534 = $877

Change in NWC = NWCend – NWCbeg = $877 – 676 = $201

*c.* We can calculate net capital spending as:

Net capital spending = Net fixed assetsend – Net fixed assetsbeg + Depreciation

Net capital spending = $6,125 – 5,261 + 1,478

Net capital spending = $2,342

So, the company had a net capital spending cash flow of $2,342. We also know that net capital spending is:

Net capital spending = Fixed assets bought – Fixed assets sold

$2,342 = $2,820 – Fixed assets sold

Fixed assets sold = $2,820 – 2,342

Fixed assets sold = $478

To calculate the cash flow from assets, we must first calculate the operating cash flow. The operating cash flow is calculated as follows (we could also prepare a traditional income statement):

EBIT = Sales – Costs – Depreciation

EBIT = $17,688 – 7,118 – 1,478

EBIT = $9,092

EBT = EBIT – Interest

EBT = $9,092 – 392

EBT = $8,700

Taxes = EBT × .22

Taxes = $8,700 × .22

Taxes = $1,914

OCF = EBIT + Depreciation – Taxes

OCF = $9,092 + 1,478 – 1,914

OCF = $8,656

Cash flow from assets = OCF – Change in NWC – Net capital spending

Cash flow from assets = $8,656 – 201 – 2,342

Cash flow from assets = $6,113

*d.* Net new borrowing = LTDend – LTDbeg

Net new borrowing = $3,256 – 2,856

Net new borrowing = $400

Net new borrowing = $400 = Debt issued – Debt retired

Debt retired = $545 – 400

Debt retired = $145

Cash flow to creditors = Interest – Net new LTD

Cash flow to creditors = $392 – 400

Cash flow to creditors = –$8

**21.**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Balance sheet as of Dec. 31, 2018 | | | | | | |
|  | Cash | $4,438 |  |  |  | Accounts payable | $4,661 |
|  | Accounts receivable | 4,874 |  |  |  | Notes payable | 858 |
|  | Inventory | 10,444 |  |  |  | Current liabilities | $5,519 |
|  | Current assets | $19,756 |  |  |  |  |  |
|  |  |  |  |  |  | Long-term debt | $14,537 |
|  | Net fixed assets | $37,211 |  |  |  | Owners' equity | 36,911 |
|  | Total assets | $56,967 |  |  |  | Total liab. & equity | $56,967 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Balance sheet as of Dec. 31, 2019 | | | | | | |
|  | Cash | $5,620 |  |  |  | Accounts payable | $4,520 | |
|  | Accounts receivable | 6,617 |  |  |  | Notes payable | 806 | |
|  | Inventory | 10,733 |  |  |  | Current liabilities | $5,326 | |
|  | Current assets | $22,970 |  |  |  |  |  | |
|  |  |  |  |  |  | Long-term debt | $17,334 | |
|  | Net fixed assets | $39,049 |  |  |  | Owners' equity | 39,359 | |
|  | Total assets | $62,019 |  |  |  | Total liab. & equity | $62,019 | |

2018 Income Statement 2019 Income Statement

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Sales | $8,462.00 |  | Sales | $9,082.00 |
|  | COGS | 2,912.00 |  | COGS | 3,305.00 |
|  | Other expenses | 690.00 |  | Other expenses | 577.00 |
|  | Depreciation | 1,215.00 |  | Depreciation | 1,216.00 |
|  | EBIT | $3,645.00 |  | EBIT | $3,984.00 |
|  | Interest | 567.00 |  | Interest | 652.00 |
|  | EBT | $3,078.00 |  | EBT | $3,332.00 |
|  | Taxes | 646.38 |  | Taxes | 699.72 |
|  | Net income | $2,431.62 |  | Net income | $2,632.28 |
|  |  |  |  |  |  |
|  | Dividends | $1,032.00 |  | Dividends | $1,135.00 |
|  | Additions to RE | 1,399.62 |  | Additions to RE | 1,497.28 |

**22.** OCF = EBIT + Depreciation – Taxes

OCF = $3,984 + 1,216 – 699.72

OCF = $4,500.28

Change in NWC = NWCend – NWCbeg = (CA – CL)end – (CA – CL)beg

Change in NWC = ($22,970 – 5,326) – ($19,756 – 5,519)

Change in NWC = $3,407

Net capital spending = NFAend – NFAbeg + Depreciation

Net capital spending = $39,049 – 37,211 + 1,216

Net capital spending = $3,054

Cash flow from assets = OCF – Change in NWC – Net capital spending

Cash flow from assets = $4,500.28 – 3,407 – 3,054

Cash flow from assets = –$1,960.72

Cash flow to creditors = Interest – Net new LTD

Net new LTD = LTDend – LTDbeg

Cash flow to creditors = $652 – ($17,334 – 14,537)

Cash flow to creditors = –$2,145

Net new equity = Common stockend – Common stockbeg

Common stock + Retained earnings = Total owners’ equity

Net new equity = (OE – RE)end – (OE – RE)beg

Net new equity = OEend – OEbeg + REbeg – REend

REend= REbeg+ Additions to RE

* Net new equity = OEend – OEbeg+ REbeg – (REbeg + Additions to RE)

= OEend – OEbeg – Additions to RE

Net new equity = $39,359 – 36,911 – 1,497.28 = $950.72

Cash flow to stockholders = Dividends – Net new equity

Cash flow to stockholders = $1,135 – 950.72

Cash flow to stockholders = $184.28

As a check, cash flow from assets is –$1,960.72

Cash flow from assets = Cash flow from creditors + Cash flow to stockholders

Cash flow from assets = –$2,145 + 184.28

Cash flow from assets = –$1,960.72

*Challenge*

**23.** We will begin by calculating the operating cash flow. First, we need the EBIT, which can be calculated as:

EBIT = Net income + Current taxes + Deferred taxes + Interest

EBIT = $187 + 81 + 11 + 38

EBIT = $317

Now we can calculate the operating cash flow as:

|  |  |  |
| --- | --- | --- |
|  | *Operating cash flow* |  |
|  | Earnings before interest and taxes | $317 |
|  | Depreciation | 74 |
|  | Current taxes | –81 |
|  | Operating cash flow | $310 |

The cash flow from assets is found in the investing activities portion of the accounting statement of cash flows, so:

|  |  |  |
| --- | --- | --- |
|  | *Cash flow from assets* |  |
|  | Acquisition of fixed assets | $194 |
|  | Sale of fixed assets | –19 |
|  | Capital spending | $175 |

The net working capital cash flows are all found in the operations cash flow section of the accounting statement of cash flows. However, instead of calculating the net working capital cash flows as the change in net working capital, we must calculate each item individually. Doing so, we find:

|  |  |  |
| --- | --- | --- |
|  | *Net working capital cash flow* |  |
|  | Cash | $12 |
|  | Accounts receivable | 12 |
|  | Inventories | –15 |
|  | Accounts payable | –11 |
|  | Accrued expenses | 6 |
|  | Other | –2 |
|  | NWC cash flow | $2 |

Except for the interest expense, the cash flow to creditors is found in the financing activities of the accounting statement of cash flows. The interest expense from the income statement is given, so:

|  |  |  |
| --- | --- | --- |
|  | *Cash flow to creditors* |  |
|  | Interest | $38 |
|  | Retirement of debt | 145 |
|  | Debt service | $183 |
|  | Proceeds from sale of long-term debt | –110 |
|  | Total | $73 |

And we can find the cash flow to stockholders in the financing section of the accounting statement of cash flows. The cash flow to stockholders was:

|  |  |  |
| --- | --- | --- |
|  | *Cash flow to stockholders* |  |
|  | Dividends | $67 |
|  | Repurchase of stock | 32 |
|  | Cash to stockholders | $99 |
|  | Proceeds from new stock issue | –39 |
|  | Total | $60 |

**24.** Net capital spending = NFAend – NFAbeg + Depreciation

= (NFAend – NFAbeg) + (Depreciation + ADbeg) – ADbeg

= (NFAend – NFAbeg)+ ADend – ADbeg

= (NFAend + ADend) – (NFAbeg + ADbeg) = FAend – FAbeg

***CHAPTER 3***

**LONG-TERM FINANCIAL PLANNING AND GROWTH**

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Time trend analysis gives a picture of changes in the company’s financial situation over time. Comparing a firm to itself over time allows the financial manager to evaluate whether some aspects of the firm’s operations, finances, or investment activities have changed. Peer group analysis involves comparing the financial ratios and operating performance of a particular firm to a set of peer group firms in the same industry or line of business. Comparing a firm to its peers allows the financial manager to evaluate whether some aspects of the firm’s operations, finances, or investment activities are out of line with the norm, thereby providing some guidance on appropriate actions to take to adjust these ratios if appropriate. Both allow an investigation into what is different about a company from a financial perspective, but neither method gives an indication of whether the difference is positive or negative. For example, suppose a company’s current ratio is increasing over time. It could mean that the company had been facing liquidity problems in the past and is rectifying those problems, or it could mean the company has become less efficient in managing its current accounts. Similar arguments could be made for a peer group comparison. A company with a current ratio lower than its peers could be more efficient at managing its current accounts, or it could be facing liquidity problems. Neither analysis method tells us whether a ratio is good or bad; both show that something is different, and tell us where to look.

**2.** If a company is growing by opening new stores, then presumably total revenues would be rising. Comparing total sales at two different points in time might be misleading. Same-store sales control for this by only looking at revenues of stores open within a specific period.

**3.** The reason is that, ultimately, sales are the driving force behind a business. A firm’s assets, employees, and, in fact, just about every aspect of its operations and financing exist to directly or indirectly support sales. Put differently, a firm’s future need for things like capital assets, employees, inventory, and financing are determined by its future sales level.

**4.** Two assumptions of the sustainable growth formula are that the company does not want to sell new equity, and that financial policy is fixed. If the company raises outside equity, or increases its debt-equity ratio, it can grow at a higher rate than the sustainable growth rate. Of course, the company could also grow at a faster rate if its profit margin increases, if it changes its dividend policy by increasing the retention ratio, or its total asset turnover increases.

**5.** The sustainable growth rate is greater than 20 percent, because at a 20 percent growth rate the negative EFN indicates that there is excess financing still available. If the firm is 100 percent equity financed, then the sustainable and internal growth rates are equal and the internal growth rate would be greater than 20 percent. However, when the firm has some debt, the internal growth rate is always less than the sustainable growth rate, so it is ambiguous whether the internal growth rate would be greater than or less than 20 percent. If the retention ratio is increased, the firm will have more internal funding sources available, and it will have to take on more debt to keep the debt/equity ratio constant, so the EFN will decline. Conversely, if the retention ratio is decreased, the EFN will rise. If the retention rate is zero, both the internal and sustainable growth rates are zero, and the EFN will rise to the change in total assets.

**6.** Common-size financial statements provide the financial manager with a ratio analysis of the company. The common-size income statement can show, for example, that cost of goods sold as a percentage of sales is increasing. The common-size balance sheet can show a firm’s increasing reliance on debt as a form of financing. Common-size statements of cash flows are not calculated for a simple reason: There is no possible denominator.

**7.** It would reduce the external funds needed. If the company is not operating at full capacity, it would be able to increase sales without a commensurate increase in fixed assets.

**8.** ROE is a better measure of the company’s performance. ROE shows the percentage return earned on shareholder investment. Since the goal of a company is to maximize shareholder wealth, this ratio shows the company’s performance in achieving this goal over the period.

**9.** The EBITD/Assets ratio shows the company’s operating performance before interest, taxes, and depreciation. This ratio would show how a company has controlled costs. While taxes are a cost, and depreciation and amortization can be considered costs, they are not as easily controlled by company management. Conversely, depreciation and amortization can be altered by accounting choices. This ratio only uses costs directly related to operations in the numerator. As such, it gives a better metric to measure management performance over a period than does ROA.

**10.** Long-term liabilities and equity are investments made by investors in the company, either in the form of a loan or ownership. Return on investment is intended to measure the return the company earned from these investments. Return on investment will be higher than the return on assets for a company with current liabilities. To see this, realize that total assets must equal total debt and equity, and total debt and equity is equal to current liabilities plus long-term liabilities plus equity. So, return on investment could be calculated as net income divided by total assets minus current liabilities.

**11.** Presumably not, but, of course, if the product had been *much* less popular, then a similar fate would have awaited due to lack of sales.

**12.** Since customers did not pay until shipment, receivables rose. The firm’s NWC, but not its cash, increased. At the same time, costs were rising faster than cash revenues, so operating cash flow declined. The firm’s capital spending was also rising. Thus, all three components of cash flow from assets were negatively impacted.

**13.** Financing possibly could have been arranged if the company had taken quick enough action. Sometimes it becomes apparent that help is needed only when it is too late, again emphasizing the need for planning.

**14.** All three were important, but the lack of cash or, more generally, financial resources, ultimately spelled doom. An inadequate cash resource is usually cited as the most common cause of small business failure.

**15.** Demanding cash up front, increasing prices, subcontracting production, and improving financial resources via new owners or new sources of credit are some of the options. When orders exceed capacity, price increases may be especially beneficial.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** Using the DuPont identity, the ROE is:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (.061)(1.87)(1.35)

ROE = .1540, or 15.40%

**2.** The equity multiplier is:

Equity multiplier = 1 + D/E

Equity multiplier = 1 + .85

Equity multiplier = 1.85

One formula to calculate return on equity is:

ROE = (ROA)(Equity multiplier )

ROE = .073(1.85)

ROE = .1351, or 13.51%

ROE can also be calculated as:

ROE = Net income/Total equity

So, net income is:

Net income = ROE(Total equity)

Net income = .1351($910,000)

Net income = $122,895.50

**3.** This is a multi-step problem involving several ratios. The ratios given are all part of the DuPont Identity. The only DuPont Identity ratio not given is the profit margin. If we know the profit margin, we can find the net income since sales are given. So, we begin with the DuPont Identity:

ROE = .14 = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (Profit margin)(Sales/Total assets)(1 + D/E)

Solving the DuPont Identity for profit margin, we get:

Profit margin = [(ROE)(Total assets)]/[(1 + D/E)(Sales)]

Profit margin = [(.14)($1,520)]/[(1 + 1.35)($3,300)]

Profit margin = .0274

Now that we have the profit margin, we can use this number and the given sales figure to solve for net income:

Profit margin = .0274 = Net income/Sales

Net income = .0274($3,300)

Net income = $90.55

**4.** An increase of sales to $42,112 is an increase of:

Sales increase = ($42,112 – 37,600)/$37,600

Sales increase = .1200, or 12.00%

Assuming costs and assets increase proportionally, the pro forma financial statements will look like this:

Pro forma income statement Pro forma balance sheet

Sales $42,112.00 Assets $ 151,200.00 Debt $ 37,000.00

Costs 29,232.00 Equity 105,151.20

EBIT 12,880.00 Total $ 151,200.00 Total $142,151.20

Taxes (21%) 2,704.80

Net income $10,175.20

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

Dividends = ($2,700/$9,085)($10,175.20)

Dividends = $3,024

The addition to retained earnings is:

Addition to retained earnings = $10,175.20 – 3,024

Addition to retained earnings = $7,151.20

And the new equity balance is:

Equity = $98,000 + 7,151.20

Equity = $105,151.20

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $151,200 – 142,151.20

EFN = $9,048.80

**5.** The maximum percentage sales increase without issuing new equity is the sustainable growth rate. To calculate the sustainable growth rate, we first need to calculate the ROE, which is:

ROE = NI/TE

ROE = $20,066/$88,000

ROE = .2280, or 22.80%

The plowback ratio, *b*, is one minus the payout ratio, so:

*b* = 1 – .30

*b* = .70

Now we can use the sustainable growth rate equation to get:

Sustainable growth rate = (ROE × b)/[1 – (ROE × b)]

Sustainable growth rate = [.2280(.70)]/[1 – .2280(.70)]

Sustainable growth rate = .1899, or 18.99%

So, the maximum dollar increase in sales is:

Maximum increase in sales = $49,000(.1899)

Maximum increase in sales = $9,306.67

**6.** We need to calculate the retention ratio to calculate the sustainable growth rate. The retention ratio is:

*b* = 1 – .20

*b* = .80

Now we can use the sustainable growth rate equation to get:

Sustainable growth rate = (ROE × *b*)/[1 – (ROE × *b*)]

Sustainable growth rate = [.11(.80)]/[1 – .11(.80)]

Sustainable growth rate = .0965, or 9.65%

**7.** We must first calculate the ROE using the DuPont ratio to calculate the sustainable growth rate. The ROE is:

ROE = (PM)(TAT)(EM)

ROE = (.057)(2.65)(1.60)

ROE = .2417, or 24.17%

The plowback ratio is one minus the dividend payout ratio, so:

*b* = 1 – .70

*b* = .30

Now, we can use the sustainable growth rate equation to get:

Sustainable growth rate = (ROE × *b*)/[1 – (ROE × *b*)]

Sustainable growth rate = [.2417(.30)]/[1 – .2417(.30)]

Sustainable growth rate = .0782, or 7.82%

**8.** An increase of sales to $9,462 is an increase of:

Sales increase = ($9,462 – 8,300)/$8,300

Sales increase = .14, or 14%

Assuming costs and assets increase proportionally, the pro forma financial statements will look like this:

Pro forma income statement Pro forma balance sheet

Sales $ 9,462 Assets $ 21,774 Debt $ 8,400

Costs 7,399 Equity 12,763

Net income $ 2,063 Total $ 21,774 Total $ 21,163

If no dividends are paid, the equity account will increase by the net income, so:

Equity = $10,700 + 2,063

Equity = $12,763

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $21,774 – 21,163

EFN = $611

**9.** *a.* First, we need to calculate the current sales and change in sales. The current sales are next year’s sales divided by one plus the growth rate, so:

Current sales = Next year’s sales/(1 + *g*)

Current sales = $320,000,000/(1 + .12)

Current sales = $285,714,286

And the change in sales is:

Change in sales = $320,000,000 – 285,714,286

Change in sales = $34,285,714

We can now complete the current balance sheet. The current assets, fixed assets, and short-term debt are calculated as a percentage of current sales. The long-term debt and par value of stock are given. The plug variable is the addition to retained earnings. So:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Assets |  |  | Liabilities and equity |  |
|  | Current assets | $57,142,857 |  | Short-term debt | $42,857,143 |
|  |  |  |  | Long-term debt | $110,000,000 |
|  |  |  |  |  |  |
|  | Fixed assets | 200,000,000 |  | Common stock | $45,000,000 |
|  |  |  |  | Accumulated retained earnings | 59,285,714 |
|  |  |  |  | Total equity | $104,285,714 |
|  |  |  |  |  |  |
|  | Total assets | $257,142,857 |  | Total liabilities and equity | $257,142,857 |

*b.* We can use the equation from the text to answer this question. The assets/sales and debt/sales are the percentages given in the problem, so:

EFN = × ΔSales – × ΔSales – (PM × Projected sales) × (1 – *d*)



EFN = (.20 + .70) × $34,285,714 – (.15 × $34,285,714) – [(.09 × $320,000,000) × (1 – .30)]

EFN = $5,554,286

*c.* The current assets, fixed assets, and short-term debt will all increase at the same percentage as sales. The long-term debt and common stock will remain constant. The accumulated retained earnings will increase by the addition to retained earnings for the year. We can calculate the addition to retained earnings for the year as:

Net income = Profit margin × Sales

Net income = .09($320,000,000)

Net income = $28,800,000

The addition to retained earnings for the year will be the net income times one minus the dividend payout ratio, which is:

Addition to retained earnings = Net income(1 – *d*)

Addition to retained earnings = $28,800,000(1 – .30)

Addition to retained earnings = $20,160,000

So, the new accumulated retained earnings will be:

Accumulated retained earnings = $59,285,714 + 20,160,000

Accumulated retained earnings = $79,445,714

The pro forma balance sheet will be:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Assets |  |  | Liabilities and equity |  |
|  | Current assets | $64,000,000 |  | Short-term debt | $48,000,000 |
|  |  |  |  | Long-term debt | $110,000,000 |
|  |  |  |  |  |  |
|  | Fixed assets | $224,000,000 |  | Common stock | $45,000,000 |
|  |  |  |  | Accumulated retained earnings | 79,445,714 |
|  |  |  |  | Total equity | $124,445,714 |
|  |  |  |  |  |  |
|  | Total assets | $288,000,000 |  | Total liabilities and equity | $282,445,714 |

The EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $288,000,000 – 282,445,714

EFN = $5,554,286

**10.** *a.* The plowback ratio is one minus the dividend payout ratio, so:

*b* = 1 – .25

*b* = .75

Now, we can use the sustainable growth rate equation to get:

Sustainable growth rate = (ROE × *b*)/[1 – (ROE × *b*)]

Sustainable growth rate = [.121(.75)]/[1 – .121(.75)]

Sustainable growth rate = .0998, or 9.98%

*b.* It is possible for the sustainable growth rate and the actual growth rate to differ. If any of the actual parameters in the sustainable growth rate equation differ from those used to compute the sustainable growth rate, the actual growth rate will differ from the sustainable growth rate. Since the sustainable growth rate includes ROE in the calculation, this also implies that changes in the profit margin, total asset turnover, or equity multiplier will affect the sustainable growth rate.

*c*. The company can increase its growth rate by doing any of the following:

* Increase the debt-to-equity ratio by selling more debt or repurchasing stock.
* Increase the profit margin, most likely by better controlling costs.
* Decrease its total assets/sales ratio; in other words, utilize its assets more efficiently.
* Reduce the dividend payout ratio.

*Intermediate*

**11.** The solution requires substituting two ratios into a third ratio. Rearranging D/TA:

Firm A Firm B

D/TA = .35 D/TA = .45

(TA – E)/TA = .35 (TA – E)/TA = .45

(TA/TA) – (E/TA) = .35 (TA/TA) – (E/TA) = .45

1 – (E/TA) = .35 1 – (E/TA) = .45

E/TA = .65 E/TA = .55

E = .65(TA) E = .55(TA)

Rearranging ROA, we find:

NI/TA = .08 NI/TA = .07

NI = .08(TA) NI = .07(TA)

Since ROE = NI/E, we can substitute the above equations into the ROE formula, which yields:

ROE = .08(TA)/.65(TA) ROE = .07(TA)/.55 (TA)

ROE = .08/.65 ROE = .07/.55

ROE = .1231, or 12.31% ROE = .1273, or 12.73%

**12.** Profit margin = Net income/Sales

Profit margin = –£18,137/£279,386

Profit margin = –.0649, or 6.49%

As long as both net income and sales are measured in the same currency, there is no problem; in fact, except for some market value ratios like EPS and BVPS, none of the financial ratios discussed in the text are measured in terms of currency. This is one reason why financial ratio analysis is widely used in international finance to compare the business operations of firms and/or divisions across national economic borders. The net income in dollars is:

Net income = Profit margin × Sales

Net income = –.0649($359,815)

Net income = –$23,358.24

**13.** *a.* The equation for external funds needed is:

EFN = × ΔSales – × ΔSales – (PM × Projected sales) × (1 – *d*)



where:

Assets/Sales = $24,200,000/$21,860,000 = 1.11

ΔSales = Current sales × Sales growth rate = $21,860,000(.15) = $3,279,000

Short-term debt/Sales = $5,100,000/$21,860,000 = .2333

Profit margin = Net income/Sales = $1,807,500/$21,860,000 = .0827

Projected sales = Current sales × (1 + Sales growth rate) = $21,860,000(1 + .15) = $25,139,000

*d* = Dividends/Net income = $361,500/$1,807,500 = .20

so:

EFN = (1.11 × $3,279,000) – (.2333 × $3,279,000) – (.0827 × $25,139,000) × (1 – .20)

EFN = $1,202,100

*b.* The current assets, fixed assets, and short-term debt will all increase at the same percentage as sales. The long-term debt and common stock will remain constant. The accumulated retained earnings will increase by the addition to retained earnings for the year. We can calculate the addition to retained earnings for the year as:

Net income = Profit margin × Sales

Net income = .0827($25,139,000)

Net income = $2,078,625

The addition to retained earnings for the year will be the net income times one minus the dividend payout ratio, which is:

Addition to retained earnings = Net income(1 – *d*)

Addition to retained earnings = $2,078,625(1 – .20)

Addition to retained earnings = $1,662,900

So, the new accumulated retained earnings will be:

Accumulated retained earnings = $10,200,000 + 1,622,900

Accumulated retained earnings = $11,862,900

The pro forma balance sheet will be:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Assets |  |  | Liabilities and equity |  |
|  | Current assets | $7,935,000 |  | Short-term debt | $5,865,000 |
|  |  |  |  | Long-term debt | $5,800,000 |
|  |  |  |  |  |  |
|  | Fixed assets | 19,895,000 |  | Common stock | $3,100,000 |
|  |  |  |  | Accumulated retained earnings | 11,862,900 |
|  |  |  |  | Total equity | $14,962,900 |
|  |  |  |  |  |  |
|  | Total assets | $27,830,000 |  | Total liabilities and equity | $26,627,900 |

The EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $27,830,000 – 26,627,900

EFN = $1,202,100

*c*. The ROE is:

ROE = Net income/Total equity

ROE = $1,807,500/$13,300,000

ROE = .1359, or 13.59%

And the retention ratio is:

*b* = Retention ratio = Retained earnings/Net income

*b* = $1,446,000/$1,807,500

*b* = .80

Now, we can use the sustainable growth rate equation to get:

Sustainable growth rate = (ROE × *b*)/[1 – (ROE × *b*)]

Sustainable growth rate = [.1359(.80)]/[1 – .1359(.80)]

Sustainable growth rate = .1220, or 12.20%

*d*. The company cannot just cut its dividends to achieve the forecast growth rate. As shown below, even with a zero dividend policy, the EFN will still be $786,375.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Assets |  |  | Liabilities and equity |  |
|  | Current assets | $7,935,000 |  | Short-term debt | $5,865,000 |
|  |  |  |  | Long-term debt | $5,800,000 |
|  |  |  |  |  |  |
|  | Fixed assets | 19,895,000 |  | Common stock | $3,100,000 |
|  |  |  |  | Accumulated retained earnings | 12,278,625 |
|  |  |  |  | Total equity | $15,378,625 |
|  |  |  |  |  |  |
|  | Total assets | $27,830,000 |  | Total liabilities and equity | $27,043,625 |

The EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $27,830,000 – 27,043,625

EFN = $786,375

The company does have several alternatives. It can increase its asset utilization and/or its profit margin. The company could also increase the debt in its capital structure. This will decrease the equity account, thereby increasing ROE.

**14.** This is a multi-step problem involving several ratios. It is often easier to look backward to determine where to start. We need receivables turnover to find days’ sales in receivables. To calculate receivables turnover, we need credit sales, and to find credit sales, we need total sales. Since we are given the profit margin and net income, we can use these to calculate total sales as:

Profit margin = Net income/Sales

.0860 = $386,000/Sales

Sales = $4,488,372

Credit sales are 80 percent of total sales, so:

Credit sales = $4,488,372(.80)

Credit sales = $3,590,698

Now we can find receivables turnover by:

Receivables turnover = Credit sales/Accounts receivable

Receivables turnover = $3,590,698/$191,300

Receivables turnover = 18.77 times

Days’ sales in receivables = 365 days/Receivables turnover

Days’ sales in receivables = 365/18.77

Days’ sales in receivables = 19.45 days

**15.** The solution to this problem requires a number of steps. First, remember that:

Current assets + Net fixed assets = Total assets

So, if we find the current assets and the total assets, we can solve for net fixed assets. Using the numbers given for the current ratio and the current liabilities, we solve for current assets:

Current ratio = Current assets/Current liabilities

Current assets = Current ratio(Current liabilities)

Current assets = 1.29($1,450)

Current assets = $1,870.50

To find the total assets, we must first find the total debt and equity from the information given. So, we find the net income using the profit margin:

Profit margin = Net income/Sales

Net income = Profit margin × Sales

Net income = .081($7,380)

Net income = $597.78

We now use the net income figure as an input into ROE to find the total equity:

ROE = Net income/Total equity

Total equity = Net income/ROE

Total equity = $597.78/.143

Total equity = $4,180.28

Next, we need to find the long-term debt. The long-term debt ratio is:

Long-term debt ratio = .34 = Long-term debt/(Long-term debt + Total equity)

Inverting both sides gives:

1/.34 = (Long-term debt + Total equity)/Long-term debt = 1 + (Total equity/Long-term debt)

Substituting the total equity into the equation and solving for long-term debt gives the following:

1 + ($4,180.28/Long-term debt) = 2.94

Long-term debt = $4,180.28/1.94

Long-term debt = $2,153.48

Now, we can find the total debt of the company:

Total debt = Current liabilities + Long-term debt

Total debt = $1,450 + 2,153.48

Total debt = $3,603.48

And, with the total debt, we can find the total debt & equity, which is equal to total assets:

Total assets = Total debt + Total equity

Total assets = $3,603.48 + 4,180.28

Total assets = $7,783.76

And finally, we are ready to solve the balance sheet identity as:

Net fixed assets = Total assets – Current assets

Net fixed assets = $7,783.76 – 1,870.50

Net fixed assets = $5,913.26

**16.** This problem requires you to work backward through the income statement. First, recognize that Net income = (1 – *T*C)EBT. Plugging in the numbers given and solving for EBT, we get:

EBT = $13,150/(1 – .24)

EBT = $17,302.63

Now, we can add interest to EBT to get EBIT as follows:

EBIT = EBT + Interest paid

EBIT = $17,302.63 + 3,460

EBIT = $20,762.63

To get EBITD (earnings before interest, taxes, and depreciation), the numerator in the cash coverage ratio, add depreciation to EBIT. Note, since there is no amortization in this problem, EBITDA equals EBITD.

EBITD = EBIT + Depreciation

EBITD = $20,762.63 + 4,380

EBITD = $25,142.63

Now, plug the numbers into the cash coverage ratio and calculate:

Cash coverage ratio = EBITD/Interest

Cash coverage ratio = $25,142.63/$3,460

Cash coverage ratio = 7.27 times

**17.** We can start by multiplying ROE by Total assets/Total assets

ROE =

Rearranging, we get:

ROE =

Next, we can multiply by Sales/Sales, which yields:

ROE =

Rearranging, we get:

ROE =

Next, we can multiply the preceding three factor DuPont equation by EBT/EBT, which yields:

ROE =

We can rearrange as:

ROE =

Finally, multiplying this equation EBIT/EBIT and rearranging yields:

ROE =

ROE =

(1) (2) (3) (4) (5)

The interpretation of each term is as follows:

(1) This is the company's tax burden. This is the proportion of the company's profits retained after paying income taxes.

(2) This is the company’s interest burden. It will be 1.00 for a company with no debt or financial leverage.

(3) This is the company’s operating profit margin. It is the operating profit before interest and taxes per dollar of sales.

(4) This is the company’s operating efficiency as measured by dollar of sales per dollar of total assets.

(5) This is the company’s financial leverage as measured by the equity multiplier.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **18.** | **2018** | **Common**  **size** |  | **2019** | **Common**  **size** | **Common base year** |
| Assets |  |  |  |  |  |  |
| Current assets |  |  |  |  |  |  |
| Cash | $11,459 | 2.90% |  | $14,453 | 3.13% | 1.2613 |
| Accounts receivable | 29,247 | 7.40% |  | 33,304 | 7.21% | 1.1387 |
| Inventory | 52,655 | 13.32% |  | 60,689 | 13.14% | 1.1526 |
| Total | $93,361 | 23.62% |  | $108,446 | 23.48% | 1.1616 |
| Fixed assets |  |  |  |  |  |  |
| Net plant and equipment | 301,978 | 76.38% |  | 353,330 | 76.52% | 1.1701 |
| Total assets | $395,339 | 100% |  | $461,776 | 100% | 1.1681 |
|  |  |  |  |  |  |  |
| Liabilities and Owners’ Equity |  |  |  |  |  |  |
| Current liabilities |  |  |  |  |  |  |
| Accounts payable | $58,483 | 14.79% |  | $66,623 | 14.43% | 1.1392 |
| Notes payable | 24,973 | 6.32% |  | 24,735 | 5.36% | .9905 |
| Total | $83,456 | 21.11% |  | $91,358 | 19.78% | 1.0947 |
| Long-term debt | 34,500 | 8.73% |  | 44,700 | 9.68% | 1.2957 |
| Owners' equity |  |  |  |  |  |  |
| Common stock and paid-in surplus | $54,000 | 13.66% |  | $56,500 | 12.24% | 1.0463 |
| Accumulated retained earnings | 223,383 | 56.50% |  | 269,218 | 58.30% | 1.2052 |
| Total | $277,383 | 70.16% |  | $325,718 | 70.54% | 1.1743 |
| Total liabilities and owners' equity | $395,339 | 100% |  | $461,776 | 100% | 1.1681 |

The common-size balance sheet answers are found by dividing each category by total assets. For example, the cash percentage for 2018 is:

$11,459/$395,339 = .0290, or 2.90%

This means that cash is 2.90 percent of total assets.

The common-base year answers are found by dividing each category value for 2019 by the same category value for 2018. For example, the cash common-base year number is found by:

$14,453/$11,459 = 1.2613

This means the cash balance in 2019 is 1.2613 times as large as the cash balance in 2018.

**19.** To determine full capacity sales, we divide the current sales by the capacity the company is currently

using, so:

Full capacity sales = $530,000/.90

Full capacity sales = $588,889

So, the dollar growth rate in sales is:

Sales growth = $588,889 – 530,000

Sales growth = $58,889

**20.** To find the new level of fixed assets, we need to find the current percentage of fixed assets to full capacity sales. Doing so, we find:

Fixed assets/Full capacity sales = $620,000/$588,889

Fixed assets/Full capacity sales = 1.0528

Next, we calculate the total dollar amount of fixed assets needed at the new sales figure.

Total fixed assets = 1.0528($605,000)

Total fixed assets = $636,962

The new fixed assets necessary is the total fixed assets at the new sales figure minus the current level of fixed assets.

New fixed assets = $636,962 – 620,000

New fixed assets = $16,962

**21.** Assuming costs vary with sales and a 20 percent increase in sales, the pro forma income statement will look like this:

Pro Forma Income Statement

Sales $ 1,069,920

Costs 873,480

Other expenses 21,888

EBIT $ 174,552

Interest 13,400

Taxable income $ 161,152

Taxes (22%) 35,453

Net income $ 125,699

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

Dividends = ($36,224/$103,007)($125,699)

Dividends = $44,204

And the addition to retained earnings will be:

Addition to retained earnings = $125,699 – 44,204

Addition to retained earnings = $81,495

The new retained earnings on the pro forma balance sheet will be:

New retained earnings = $174,730 + 81,495

New retained earnings = $256,225

The pro forma balance sheet will look like this:

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 29,136 Accounts payable $ 78,240

Accounts receivable 44,484 Notes payable 16,320

Inventory 100,080 Total $ 94,560

Total $ 173,700 Long-term debt 155,000

Fixed assets

Net plant and Owners’ equity

equipment 475,800 Common stock and

paid-in surplus $ 130,000

Retained earnings 256,225

Total $ 386,225

Total liabilities and owners’

Total assets $ 649,500 equity $ 635,785

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $649,500 – 635,785

EFN = $13,715

**22.** First, we need to calculate full capacity sales, which is:

Full capacity sales = $891,600/.80

Full capacity sales = $1,114,500

The full capacity ratio at full capacity sales is:

Full capacity ratio = Fixed assets/Full capacity sales

Full capacity ratio = $396,500/$1,114,500

Full capacity ratio = .35576

The fixed assets required at the projected sales figure is the full capacity ratio times the projected sales level:

Total fixed assets = .35576($1,069,920) = $380,640

So, EFN is:

EFN = ($173,700 + 380,640) – $635,785 = –$81,445

Note that this solution assumes that fixed assets are decreased (sold) so the company has a 100 percent fixed asset utilization. If we assume fixed assets are not sold, the answer becomes:

EFN = ($173,700 + 396,500) – $635,785 = –$65,585

**23.** The D/E ratio of the company is:

D/E = ($81,520 + 155,000)/$304,730

D/E = .7762

So the new total debt amount will be:

New total debt = .7762($386,225)

New total debt = $299,773

This is the new total debt for the company. Given that our calculation for EFN is the amount that must be raised externally and does not increase spontaneously with sales, we need to subtract the spontaneous increase in accounts payable. The new level of accounts payable, which is the current accounts payable times the sales growth, will be:

Spontaneous increase in accounts payable = $65,200(.20)

Spontaneous increase in accounts payable = $13,040

This means that $13,040 of the new total debt is not raised externally. So, the debt raised externally, which will be the EFN, is:

EFN = New total debt – (Beginning LTD + Beginning CL + Spontaneous increase in AP)

EFN = $299,773 – ($155,000 + 81,520 + 13,040) = $50,213

The pro forma balance sheet with the new long-term debt will be:

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 29,136 Accounts payable $ 78,240

Accounts receivable 44,484 Notes payable 16,320

Inventory 100,080 Total $ 94,560

Total $ 173,700 Long-term debt 205,213

Fixed assets

Net plant and Owners’ equity

equipment 475,800 Common stock and

paid-in surplus $ 130,000

Retained earnings 256,225

Total $ 386,225

Total liabilities and owners’

Total assets $ 649,500 equity $ 685,998

The funds raised by the debt issue can be put into an excess cash account to make the balance sheet balance. The excess debt will be:

Excess debt = $685,998 – 649,500 = $36,498

To make the balance sheet balance, the company will have to increase its assets. We will put this amount in an account called excess cash, which will give us the following balance sheet:

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 29,136 Accounts payable $ 78,240

Excess cash 36,498 Notes payable 16,320

Accounts receivable 44,484 Total $ 94,560

Inventory 100,080

Total $ 210,198 Long-term debt 205,213

Fixed assets

Net plant and Owners’ equity

equipment 475,800 Common stock and

paid-in surplus $ 130,000

Retained earnings 256,225

Total $ 386,225

Total liabilities and owners’

Total assets $ 685,998 equity $ 685,998

The excess cash has an opportunity cost that we discussed earlier. Increasing fixed assets would also not be a good idea since the company already has enough fixed assets. A likely scenario would be the repurchase of debt and equity in its current capital structure weights. The company’s debt-assets and equity-assets are:

Debt-assets = .7762/(1 + .7762) = .44

Equity-assets = 1/(1 + .7762) = .56

So, the amount of debt and equity needed will be:

Total debt needed = .44($649,500) = $283,824

Equity needed = .56($649,500) = $365,676

So, the repurchases of debt and equity will be:

Debt repurchase = ($94,560 + 205,213) – 283,824 = $15,949

Equity repurchase = $386,225 – 365,676 = $20,549

Assuming all of the debt repurchase is from long-term debt, and the equity repurchase is entirely from the retained earnings, the final pro forma balance sheet will be:

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 29,136 Accounts payable $ 78,240

Accounts receivable 44,484 Notes payable 16,320

Inventory 100,080 Total $ 94,560

Total $ 173,700 Long-term debt 189,264

Fixed assets

Net plant and Owners’ equity

equipment 475,800 Common stock and

paid-in surplus $ 130,000

Retained earnings 235,676

Total $ 365,676

Total liabilities and owners’

Total assets $ 649,500 equity $ 649,500

*Challenge*

**24.** The pro forma income statements for all three growth rates will be:

Pro Forma Income Statement

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | *15 % Sales*  *Growth* | *20% Sales*  *Growth* | *25% Sales*  *Growth* |
|  | Sales | $1,025,340 | $1,069,920 | $1,114,500 |
|  | Costs | 837,085 | 873,480 | 909,875 |
|  | Other expenses | 20,976 | 21,888 | 22,800 |
|  | EBIT | $167,279 | $174,552 | $181,825 |
|  | Interest | 13,400 | 13,400 | 13,400 |
|  | Taxable income | $153,879 | $161,152 | $168,425 |
|  | Taxes (22%) | 33,853 | 35,453 | 37,054 |
|  | Net income | $120,026 | $125,699 | $131,372 |
|  |  |  |  |  |
|  | Dividends | $42,209 | $44,204 | $46,199 |
|  | Add to RE | 77,817 | 81,495 | 85,173 |

We will calculate the EFN for the 15 percent growth rate first. Assuming the payout ratio is constant, the dividends paid will be:

Dividends = ($36,224/$103,007)($120,026)

Dividends = $42,209

And the addition to retained earnings will be:

Addition to retained earnings = $120,026 – 42,209

Addition to retained earnings = $77,817

The new retained earnings on the pro forma balance sheet will be:

New retained earnings = $174,730 + 77,817

New retained earnings = $252,547

The pro forma balance sheet will look like this:

*15% Sales Growth*:

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 27,922 Accounts payable $ 74,980

Accounts receivable 42,631 Notes payable 16,320

Inventory 95,910 Total $ 91,300

Total $ 166,463 Long-term debt $ 155,000

Fixed assets

Net plant and Owners’ equity

equipment 455,975 Common stock and

paid-in surplus $ 130,000

Retained earnings 252,547

Total $ 382,547

Total liabilities and owners’

Total assets $ 622,438 equity $ 628,847

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $622,438 – 628,847

EFN = –$6,409

At a 20 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = ($36,224/$103,007)($125,699)

Dividends = $44,204

And the addition to retained earnings will be:

Addition to retained earnings = $125,699 – 44,204

Addition to retained earnings = $81,495

The new retained earnings on the pro forma balance sheet will be:

New retained earnings = $174,730 + 81,495

New retained earnings = $256,225

The pro forma balance sheet will look like this:

*20% Sales Growth*:

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 29,136 Accounts payable $ 78,240

Accounts receivable 44,484 Notes payable 16,320

Inventory 100,080 Total $ 94,560

Total $ 173,700 Long-term debt 155,000

Fixed assets

Net plant and Owners’ equity

equipment 475,800 Common stock and

paid-in surplus $ 130,000

Retained earnings 256,225

Total $ 386,225

Total liabilities and owners’

Total assets $ 649,500 equity $ 635,785

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $649,500 – 635,785

EFN = $13,715

At a 25 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = ($36,224/$103,007)($131,372)

Dividends = $46,199

And the addition to retained earnings will be:

Addition to retained earnings = $131,372 – 46,199

Addition to retained earnings = $85,173

The new retained earnings on the pro forma balance sheet will be:

New retained earnings = $174,730 + 85,173

New retained earnings = $259,903

The pro forma balance sheet will look like this:

*25% Sales Growth*:

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 30,350 Accounts payable $ 81,500

Accounts receivable 46,338 Notes payable 16,320

Inventory 104,250 Total $ 97,820

Total $ 180,938 Long-term debt $ 155,000

Fixed assets

Net plant and Owners’ equity

equipment 495,625 Common stock and

paid-in surplus $ 130,000

Retained earnings 259,903

Total $ 389,903

Total liabilities and owners’

Total assets $ 676,563 equity $ 642,723

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = $676,563 – 642,723

EFN = $33,840

**25.** The pro forma income statements for all three growth rates will be:

Pro Forma Income Statement

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | *20% Sales*  *Growth* | *30% Sales*  *Growth* | *35% Sales*  *Growth* |
|  | Sales | $1,069,920 | $1,159,080 | $1,203,660 |
|  | Costs | 873,480 | 946,270 | 982,665 |
|  | Other expenses | 21,888 | 23,712 | 24,624 |
|  | EBIT | $174,552 | $189,098 | $196,371 |
|  | Interest | 13,400 | 13,400 | 13,400 |
|  | Taxable income | $161,152 | $175,698 | $182,971 |
|  | Taxes (22%) | 35,453 | 38,654 | 40,254 |
|  | Net income | $125,699 | $137,044 | $142,717 |
|  |  |  |  |  |
|  | Dividends | $44,204 | $48,194 | $50,189 |
|  | Add to RE | 81,495 | 88,851 | 92,529 |

At a 30 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = ($36,224/$103,007)($137,044)

Dividends = $48,194

And the addition to retained earnings will be:

Addition to retained earnings = $137,044 – 48,194

Addition to retained earnings = $88,851

The new retained earnings on the pro forma balance sheet will be:

New addition to retained earnings = $174,730 + 88,851

New addition to retained earnings = $263,581

The new total debt will be:

New total debt = .7762($393,581)

New total debt = $305,482

So, the new long-term debt will be the new total debt minus the new short-term debt, or:

New long-term debt = $305,482 – 101,080

New long-term debt = $204,402

The pro forma balance sheet will look like this:

*Sales growth rate = 30% and debt/equity ratio = .7762:*

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 31,564 Accounts payable $ 84,760

Accounts receivable 48,191 Notes payable 16,320

Inventory 108,420 Total $ 101,080

Total $ 188,175 Long-term debt 204,402

Fixed assets

Net plant and Owners’ equity

equipment 515,450 Common stock and

paid-in surplus $ 130,000

Retained earnings 263,581

Total $ 393,581

Total liabilities and owners’

Total assets $ 703,625 equity $ 699,063

So the excess debt raised is:

Excess debt = $699,063 – 703,625

Excess debt = –$4,562

At a 30 percent growth rate, the firm will need funds in the amount of $4,562 in addition to the external debt already raised. So, the EFN will be:

EFN = $49,402 + 4,562

EFN = $53,964

At a 35 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = ($36,224/$103,007)($142,717)

Dividends = $50,189

And the addition to retained earnings will be:

Addition to retained earnings = $142,717 – 50,189

Addition to retained earnings = $92,529

The new retained earnings on the pro forma balance sheet will be:

New retained earnings = $174,730 + 92,529

New retained earnings = $267,259

The new total debt will be:

New total debt = .7762($397,259)

New total debt = $308,337

So, the new long-term debt will be the new total debt minus the new short-term debt, or:

New long-term debt = $308,337 – 104,340

New long-term debt = $203,997

*Sales growth rate = 35% and debt/equity ratio = .7762:*

Pro Forma Balance Sheet

Assets Liabilities and Owners’ Equity

Current assets Current liabilities

Cash $ 32,778 Accounts payable $ 88,020

Accounts receivable 50,045 Notes payable 16,320

Inventory 112,590 Total $ 104,340

Total $ 195,413 Long-term debt $ 203,997

Fixed assets

Net plant and Owners’ equity

equipment 535,275 Common stock and

paid-in surplus $ 130,000

Retained earnings 267,259

Total $ 397,259

Total liabilities and owners’

Total assets $ 730,688 equity $ 705,596

So the excess debt raised is:

Excess debt = $705,596 – 730,688

Excess debt = –$25,092

At a 35 percent growth rate, the firm will need funds in the amount of $25,092 in addition to the external debt already raised. So, the EFN will be:

EFN = $48,997 + 25,092

EFN = $74,089

**26.** We need the ROE to calculate the sustainable growth rate. The ROE is:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (.042)(1/.80)(1 + .37)

ROE = .0719, or 7.19%

Now, we can use the sustainable growth rate equation to find the retention ratio as:

Sustainable growth rate = (ROE × *b*)/[1 – (ROE × *b*)]

Sustainable growth rate = .11 = [.0719*b*]/[1 – .0719*b*]

*b* = 1.38

This implies the payout ratio is:

Payout ratio = 1 – *b*

Payout ratio = 1 – 1.38

Payout ratio = –.38, or –38%

This answer indicates a dividend payout ratio of negative 38 percent, which is impossible. So, the growth rate is inconsistent with the other constraints. The lowest possible payout rate is 0 (without issuing stock), which corresponds to a retention ratio of 1, or total earnings retention. This problem illustrates a key point we made in the chapter: Sustainable growth analysis forces the user to make internally consistent assumptions.

        As an aside, we should note that it is possible to have a retention ration greater than 1 if the company issues new stock. However, since the growth rate we are evaluating is perpetual, the company would have to issue stock every year, forever. But, doing so violates our underlying assumption that the sustainable growth rate requires no new equity.

In this case, the maximum sustainable growth rate for this company is:

Maximum sustainable growth rate = (ROE × *b*)/[1 – (ROE × *b*)]

Maximum sustainable growth rate = [.0719(1)]/[1 – .0719(1)]

Maximum sustainable growth rate = .0775, or 7.75%

**27.** We know that EFN is:

EFN = Increase in assets – Addition to retained earnings

The increase in assets is the beginning assets times the growth rate, so:

Increase in assets = A × *g*

The addition to retained earnings next year is the current net income times the retention ratio, times one plus the growth rate, so:

Addition to retained earnings = (NI × *b*)(1 + *g*)

And rearranging the profit margin to solve for net income, we get:

NI = PM(S)

Substituting the last three equations into the EFN equation we started with and rearranging, we get:

EFN = A(*g*) – PM(S)*b*(1 + *g*)

EFN = A(*g*) – PM(S)*b* – [PM(S)*b*]*g*

EFN = –PM(S)*b* + [A – PM(S)*b*]*g*

**28.** We start with the EFN equation we derived in Problem 27 and set it equal to zero:

EFN = 0 = –PM(S)*b* + [A – PM(S)*b*]*g*

Substituting the rearranged profit margin equation into the internal growth rate equation, we have:

Internal growth rate = [PM(S)*b*]/[A – PM(S)*b*]

Since:

ROA = NI/A

ROA = PM(S)/A

We can substitute this into the internal growth rate equation and divide both the numerator and denominator by A. This gives:

Internal growth rate = {[PM(S)*b*]/A}/{[A – PM(S)*b*]/A}

Internal growth rate = *b*(ROA)/[1 – *b*(ROA)]

To derive the sustainable growth rate, we must realize that to maintain a constant D/E ratio with no external equity financing, EFN must equal the addition to retained earnings times the D/E ratio:

EFN = (D/E)[PM(S)*b*(1 + *g*)]

EFN = A(*g*) – PM(S)*b*(1 + *g*)

Solving for g and then dividing both the numerator and denominator by A:

Sustainable growth rate = PM(S)*b*(1 + D/E)/[A – PM(S)*b*(1 + D/E )]

Sustainable growth rate = [ROA(1 + D/E)*b*]/[1 – ROA(1 + D/E)*b*]

Sustainable growth rate = *b*(ROE)/[1 – *b*(ROE)]

**29.** In the following derivations,the subscript “E” refers to end of period numbers, and the subscript “B” refers to beginning of period numbers. TE is total equity and TA is total assets.

For the sustainable growth rate*:*

Sustainable growth rate = (ROEE × *b*)/(1 – ROEE × *b*)

Sustainable growth rate = (NI/TEE × *b*)/(1 – NI/TEE × *b*)

We multiply this equation by:

(TEE/TEE)

Sustainable growth rate = (NI/TEE × *b*)/(1 – NI/TEE × *b*) × (TEE/TEE)

Sustainable growth rate = (NI × *b*)/(TEE – NI × *b*)

Recognize that the denominator is equal to beginning of period equity, that is:

(TEE – NI × *b*) = TEB

Substituting this into the previous equation, we get:

Sustainable rate = (NI × *b*)/TEB

Which is equivalent to:

Sustainable rate = (NI/TEB) × *b*

Since ROEB = NI/TEB

The sustainable growth rate equation is:

Sustainable growth rate = ROEB × *b*

For the internal growth rate:

Internal growth rate = (ROAE × *b*)/(1 – ROAE × *b*)

Internal growth rate = (NI/TAE × *b*)/(1 – NI/TAE × *b*)

We multiply this equation by:

(TAE/TAE)

Internal growth rate = (NI/TAE × *b*)/[(1 – NI/TAE × *b*) × (TAE/TAE)]

Internal growth rate = (NI × *b*)/(TAE – NI × *b*)

Recognize that the denominator is equal to beginning of period assets, that is:

(TAE – NI × *b*) = TAB

Substituting this into the previous equation, we get:

Internal growth rate = (NI × *b*)/TAB

Which is equivalent to:

Internal growth rate = (NI/TAB) × *b*

Since ROAB = NI/TAB

The internal growth rate equation is:

Internal growth rate = ROAB × *b*

**30.** Since the company issued no new equity, shareholders’ equity increased by retained earnings. Retained earnings for the year were:

Retained earnings = NI – Dividends

Retained earnings = $80,000 – 44,000

Retained earnings = $36,000

So, the equity at the end of the year was:

Ending equity = $260,000 + 36,000

Ending equity = $296,000

The ROE based on the end of period equity is:

ROE = $80,000/$296,000

ROE = .2703, or 27.03%

The plowback ratio is:

Plowback ratio = Addition to retained earnings/NI

Plowback ratio = $36,000/$80,000

Plowback ratio = .45, or 45%

Using the equation presented in the text for the sustainable growth rate, we get:

Sustainable growth rate = (ROE × *b*)/[1 – (ROE × *b*)]

Sustainable growth rate = [.2703(.45)]/[1 – .2703(.45)]

Sustainable growth rate = .1385, or 13.85%

The ROE based on the beginning of period equity is

ROE = $80,000/$260,000

ROE = .3077, or 30.77%

Using the shortened equation for the sustainable growth rate and the beginning of period ROE, we get:

Sustainable growth rate = ROE × *b*

Sustainable growth rate = .3077 × .45

Sustainable growth rate = .1385, or 13.85%

Using the shortened equation for the sustainable growth rate and the end of period ROE, we get:

Sustainable growth rate = ROE × *b*

Sustainable growth rate = .2703 × .45

Sustainable growth rate = .1216, or 12.16%

Using the end of period ROE in the shortened sustainable growth rate results in a growth rate that is too low. This will always occur whenever the equity increases. If equity increases, the ROE based on end of period equity is lower than the ROE based on the beginning of period equity. The ROE (and sustainable growth rate) in the abbreviated equation is based on equity that did not exist when the net income was earned.

***CHAPTER 4***

**DISCOUNTED CASH FLOW VALUATION**

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Assuming positive cash flows and interest rates, the future value increases and the present value decreases.

**2.** Assuming positive cash flows and interest rates, the present value will fall and the future value will rise.

**3.** The better deal is the one with equal installments.

**4.** Yes, they should. APRs generally don’t provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.

**5.** A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue.

**6.** It’s a reflection of the time value of money. TMCC gets to use the $24,099 immediately. If TMCC uses it wisely, it will be worth more than $100,000 in thirty years.

**7.** This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it is to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a “call” feature. Such features are discussed at length in a later chapter.

**8.** The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $100,000? Thus, our answer does depend on who is making the promise to repay.

**9.** The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.

**10.** The price would be higher because, as time passes, the price of the security will tend to rise toward $100,000. This rise is a reflection of the time value of money. As time passes, the time until receipt of the $100,000 grows shorter, and the present value rises. In 2019, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC’s financial position could deteriorate. Either event would tend to depress the security’s price.

**Solutions to Questions and Problems**

*NOTE: All-end-of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The time line for the cash flows is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $7,800 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

The simple interest per year is:

$7,800 × .077 = $600.60

So, after 10 years, you will have:

$600.60 × 10 = $6,006 in interest.

The total balance will be $7,800 + 6,006 = $13,806

With compound interest, we use the future value formula:

FV = PV(1 + *r*)*t*

FV = $7,800(1.077)10 = $16,377.65

The difference is:

$16,377.65 – 13,806 = $2,571.65

**2.** To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

*a.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,250 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $1,250(1.05)10 = $2,036.12

*b.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,250 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $1,250(1.10)10 = $3,242.18

*c.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,250 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $1,250(1.05)20 = $3,316.62

*d.* Because interest compounds on the interest already earned, the interest earned in part *c* is more than twice the interest earned in part *a*. With compound interest, future values grow exponentially.

**3.** To find the PV of a lump sum, we use:

PV = FV/(1 + *r)t*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 6 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $13,827 | |

PV = $13,827/(1.07)6 = $9,213.51

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 11 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $43,852 | |

PV = $43,852/(1.15)11 = $9,425.69

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 19 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $725,380 | |

PV = $725,380/(1.11)19 = $99,868.60

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 29 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $590,710 | |

PV = $590,710/(1.18)29 = $4,861.79

**4.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV/PV)1/*t* – 1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 4 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$189 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $287 | |

FV = $287 = $189(1 + *r*)4; *r* = ($287/$189)1/4 – 1 = .1101, or 11.01%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 8 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$410 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $887 | |

FV = $887 = $410(1 + *r*)8; *r* = ($887/$410)1/8 – 1 = .1013, or 10.13%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | | |  | |  | |  | |  | |  | |  | |  | |  | | 14 | |
|  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$51,700 | | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $152,184 | |

FV = $152,184 = $51,700(1 + *r*)14; *r* = ($152,184/$51,700)1/14 – 1 = .0802, or 8.02%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | | |  | |  | |  | |  | |  | |  | |  | |  | | 27 | |
|  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$21,400 | | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $538,600 | |

FV = $538,600 = $21,400(1 + *r*)27; *r* = ($538,600/$21,400)1/27 – 1 = .1269, or 12.69%

**5.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *t*, we get:

*t* = ln(FV/PV)/ln(1 + *r*)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | ? | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$625 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,104 | |

FV = $1,104 = $625(1.07)*t*; *t* = ln($1,104/$625)/ln 1.07 = 8.41 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | ? | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$810 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $5,275 | |

FV = $5,275 = $810(1.112)*t*; *t* = ln($5,275/$810)/ln 1.12 = 16.53 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | | |  | |  | |  | |  | |  | |  | |  | |  | | ? | |
|  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$16,500 | | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $245,830 | |

FV = $245,830 = $16,500(1.17)*t*; *t* = ln($245,830/$16,500)/ln 1.17 = 17.21 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | | |  | |  | |  | |  | |  | |  | |  | |  | | ? | |
|  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$21,500 | | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $215,000 | |

FV = $215,000 = $21,500(1.08)*t*; *t* = ln($215,000/$21,500)/ln 1.08 = 29.92 years

**6.** To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *t*, we get:

*t* = ln(FV/PV)/ln(1 + *r*)

The length of time to double your money is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | ? | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $2 | |

FV = $2 = $1(1.0625)*t*

*t* = ln 2/ln 1.0625 = 11.43 years

The length of time to quadruple your money is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | ? | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $4 | |

FV = $4 = $1(1.0625)*t*

*t* = ln 4/ln 1.0625

*t* = 22.87 years

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

**7.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | 20 | | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | –$425,000,000 | | |

To find the PV of a lump sum, we use:

PV = FV/(1 + *r)t*

PV = $425,000,000/(1.059)20

PV = $135,042,269.46

**8.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 4 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1,680,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,100,000 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV/PV)1/*t* – 1

*r* = ($1,100,000/$1,680,000)1/3 – 1

*r* = –.1317, or –13.17%

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

**9.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $75 | | $75 | | $75 | | $75 | | $75 | | $75 | | $75 | | $75 | | $75 | |

A consol is a perpetuity.To find the PV of a perpetuity, we use the equation:

PV = *C*/*r*

PV = $75/.031

PV = $2,419.35

**10.** To find the future value with continuous compounding, we use the equation:

FV = PV*ert*

*a.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 9 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $2,350*e*.12(9) = $6,516.74

*b.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 5 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $2,350*e*.08(5) = $3,452.92

*c.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 17 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $2,350*e*.05(17) = $5,386.24

*d.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,350 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV *=* $2,350*e*.09(10)= $5,563.30

**11.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | |  | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $795 | | $945 | | $1,325 | | $1,860 | |  | |  | |  | |  | |  | |  | |

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

PV = FV/(1 + *r*)*t*

PV@10% = $795/1.10 + $945/1.102 + $1,325/1.103 + $1,860/1.104 = $3,769.62

PV@18% = $795/1.18 + $945/1.182 + $1,325/1.183 + $1,860/1.184 = $3,118.22

PV@24% = $795/1.24 + $945/1.242 + $1,325/1.243 + $1,860/1.244 = $2,737.40

**12.** The times lines are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $4,350 | | $4,350 | | $4,350 | | $4,350 | | $4,350 | | $4,350 | | $4,350 | | $4,350 | | $4,350 | |  | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $6,900 | | $6,900 | | $6,900 | | $6,900 | | $6,900 | |  | |  | |  | |  | |  | |

To find the PVA, we use the equation:

PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

At an interest rate of 5 percent:

X@5%: PVA = $4,350{[1 – (1/1.05)9]/.05} = $30,919.02

Y@5%: PVA = $6,900{[1 – (1/1.05)5]/.05} = $29,873.39

And at an interest rate of 22 percent:

X@22%: PVA = $4,350{[1 – (1/1.22)9]/.22} = $16,470.34

Y@22%: PVA = $6,900{[1 – (1/1.22)5]/.22} = $19,759.11

Notice that the PV of Cash flow X has a greater PV than Cash flow Y at an interest rate of 5 percent, but a lower PV at an interest rate of 22 percent. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At a higher interest rate, these bigger cash flows earlier are more important since the cost of waiting (the interest rate) is so much greater.

**13.** To find the PVA, we use the equation:

PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 15 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | |

PVA@15 years: PVA = $5,200{[1 – (1/1.07)15]/.07} = $47,361.15

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 40 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | |

PVA@40 years: PVA = $5,200{[1 – (1/1.07)40]/.07} = $69,324.89

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 75 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | |

PVA@75 years: PVA = $5,200{[1 – (1/1.07)75]/.07} = $73,821.07

To find the PV of a perpetuity, we use the equation:

PV = *C*/*r*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | |

PV = $5,200/.07

PV = $74,285.71

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75-year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $464.65.

**14.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | |

This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

PV = *C*/*r*

PV = $15,000/.038

PV = $394,736.84

To find the interest rate that equates the perpetuity cash flows with the PV of the cash flows, we can use the PV of a perpetuity equation:

PV = *C*/*r*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$325,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | | $15,000 | |

$325,000 = $15,000/*r*

We can now solve for the interest rate as follows:

*r* = $15,000/$325,000

*r* = .0462, or 4.62%

**15.** For discrete compounding, to find the EAR, we use the equation:

EAR = [1 + (APR/*m*)]*m* – 1

EAR = [1 + (.071/4)]4 – 1 = .0729, or 7.29%

EAR = [1 + (.132/12)]12 – 1 = .1403, or 14.03%

EAR = [1 + (.089/365)]365 – 1 = .0931, or 9.31%

To find the EAR with continuous compounding, we use the equation:

EAR = *er* – 1

EAR = *e*.081 – 1 = .0844, or 8.44%

**16.** Here, we are given the EAR and need to find the APR. Using the equation for discrete compounding:

EAR = [1 + (APR/*m*)]*m* – 1

We can now solve for the APR. Doing so, we get:

APR = *m*[(1 + EAR)1/*m* – 1]

EAR = .101 = [1 + (APR/2)]2 – 1 APR = 2[(1.101)1/2 – 1] = .0986, or 9.86%

EAR = .174 = [1 + (APR/12)]12 – 1 APR = 12[(1.174)1/12 – 1] = .1615, or 16.15%

EAR = .086 = [1 + (APR/52)]52 – 1 APR = 52[(1.086)1/52 – 1] = .0826, or 8.26%

Solving the continuous compounding EAR equation:

EAR = *er* – 1

We get:

APR = ln(1 + EAR)

APR = ln(1 + .113)

APR = .1071, or 10.71%

**17.** For discrete compounding, to find the EAR, we use the equation:

EAR = [1 + (APR/*m*)]*m* – 1

So, for each bank, the EAR is:

First National: EAR = [1 + (.114/12)]12 – 1 = .1201, or 12.01%

First United: EAR = [1 + (.116/2)]2 – 1 = .1194, or 11.94%

A higher APR does not necessarily mean a higher EAR. The number of compounding periods within a year will also affect the EAR.

**18.** The cost of a case of wine is 10 percent less than the cost of 12 individual bottles, so the cost of a case will be:

Cost of case = (12)($10)(1 – .10)

Cost of case = $108

Now, we need to find the interest rate. The cash flows are an annuity due, so:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 12 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$108  $10 | | $10 | | $10 | | $10 | | $10 | | $10 | | $10 | | $10 | | $10 | | $10 | |

PVA = (1 + *r)C*({1 – [1/(1 + *r*)]*t*}/*r*)

$108 = (1 + *r*)$10({1 – [1/(1 + *r*)12]/*r*)

Solving for the interest rate, we get:

*r* = .0198, or 1.98% per week

So, the APR of this investment is:

APR = .0198(52)

APR = 1.0277, or 102.77%

And the EAR is:

EAR = (1 + .0198)52 – 1

EAR = 1.7668, or 176.68%

The analysis appears to be correct. He really can earn about 177 percent buying wine by the case. The only question left is this: Can you really find a fine bottle of Bordeaux for $10?

**19.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ? | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$16,450 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | |

Here, we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

PVA = *C*({1 – [1/(1 + *r*)]*t*}/*r*)

$16,450 = $400{[1 – (1/1.011)*t*]/.011}

Now, we solve for *t*:

1/1.011*t* = 1 – [($16,450)(.011)/($400)]

1.011*t* = 1/.5476 = 1.8261

*t* = ln 1.8261/ln 1.011

*t* = 55.04 months

**20.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 1 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $3 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $4 | |

Here, we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

FV = PV(1 + *r*)

$4 = $3(1 + *r*)

*r* = 4/3 – 1 = 33.33% per week

The interest rate is 33.33% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

APR = (52)33.33%

APR = 1,733.33%

And using the equation to find the EAR:

EAR = [1 + (APR/*m*)]*m* – 1

EAR = [1 + .3333]52 – 1

EAR = 313,916,515.69%

*Intermediate*

**21.** To find the FV of a lump sum with discrete compounding, we use:

FV = PV(1 + *r*)*t*

*a.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 11 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $1,000(1.089)11 = $2,554.50

*b.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 22 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $1,000(1 + .089/2)22 = $2,606.07

*c.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 132 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $1,000(1 + .089/12)132 = $2,652.19

*d.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 11 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

To find the future value with continuous compounding, we use the equation:

FV = PV*ert*

FV = $1,000*e*.089(11) = $2,661.79

*e.* The future value increases when the compounding period is shorter because interest is earned on previously accrued interest. The shorter the compounding period, the more frequently interest is earned, and the greater the future value, assuming the same stated interest rate.

**22.** The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest paid by First Simple Bank over 10 years will be:

.053(10) = .53

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of $1, or:

(1 + *r*)10

Setting the two equal, we get:

(.053)(10) = (1 + *r*)10 – 1

*r* = 1.531/10 – 1

*r* = .0434, or 4.34%

**23.** Although the stock and bond accounts have different interest rates, we can draw one time line, but we need to remember to apply different interest rates. The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | | **...** |  | |  | | 360 | | 361 | | **…** |  | | 660 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Stock | | $850 | | $850 | | $850 | | $850 | | $850 | | *C* | | *C* | | *C* | |
| Bond | | $350 | | $350 | |  | $350 | | $350 | | $350 | |  |

We need to find the annuity payment in retirement. Our retirement savings end at the same time the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock account: FVA = $850[{[1 + (.10/12) ]360 – 1}/(.10/12)] = $1,921,414.74

Bond account: FVA = $350[{[1 + (.06/12) ]360 – 1}/(.06/12)] = $351,580.26

So, the total amount saved at retirement is:

$1,921,414.74 + 351,580.26 = $2,272,995.00

Solving for the withdrawal amount in retirement using the PVA equation gives us:

PVA = $2,272,995 = *C*[1 – {1/[1 + (.07/12)]300}/(.07/12)]

*C* = $2,272,995/141.4869

*C* = $16,065.06 withdrawal per month

**24.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 4 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $4 | |

Since we are looking to quadruple our money, the PV and FV are irrelevant as long as the FV is four times as large as the PV. The number of periods is four, the number of quarters per year. So:

FV = $4 = $1(1 + *r*)(12/3)

*r* = .4142, or 41.42%

**25.** Here, we need to find the interest rate for two possible investments. Each investment is a lump sum, so:

G:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 6 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$65,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $125,000 | |

PV = $65,000 = $125,000/(1 + *r*)6

(1 + *r*)6 = $125,000/$65,000

*r* = 1.9231/6 – 1

*r* = .1151, or 11.51%

H:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$65,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $205,000 | |

PV = $65,000 = $205,000/(1 + *r*)10

(1 + *r*)10 = $205,000/$65,000

*r* = 3.1541/10 – 1

*r* = .1217, or 12.17%

**26.** This is a growing perpetuity. The present value of a growing perpetuity is:

PV = C/(*r* – g)

PV = $175,000/(.097 – .038)

PV = $2,966,101.69

It is important to recognize that when dealing with annuities or perpetuities, the present value equation calculates the present value one period before the first payment. In this case, since the first payment is in two years, we have calculated the present value one year from now. To find the value today, we discount this value as a lump sum. Doing so, we find the value of the cash flow stream today is:

PV = FV/(1 + *r*)*t*

PV = $2,966,101.69/(1 + .097)1

PV = $2,703,830.17

**27.** The dividend payments are made quarterly, so we must use the quarterly interest rate. The quarterly interest rate is:

Quarterly rate = Stated rate/4

Quarterly rate = .038/4

Quarterly rate = .0095

The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $2.25 | | $2.25 | | $2.25 | | $2.25 | | $2.25 | | $2.25 | | $2.25 | | $2.25 | | $2.25 | |

Using the present value equation for a perpetuity, we find the value today of the dividends paid must be:

PV = C/*r*

PV = $2.25/.0095

PV = $236.84

**28.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | **…** |  | | 30 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | |

We can use the PVA annuity equation to answer this question. The annuity has 28 payments, not 27 payments. Since there is a payment made in Year 3, the annuity actually begins in Year 2. So, the value of the annuity in Year 2 is:

PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

PVA = $7,300({1 – [1/(1 + .07)]28}/.07)

PVA = $88,600.91

This is the value of the annuity one period before the first payment, or Year 2. So, the value of the cash flows today is:

PV = FV/(1 + *r*)*t*

PV = $88,600.91/(1 + .07)2

PV = $77,387.47

**29.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | **…** |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | | $750 | | $750 | | $750 | | $750 | |

We need to find the present value of an annuity. Using the PVA equation, and the 11 percent interest rate, we get:

PVA = *C*({1 – [1/(1 + *r)*]*t*}/*r*)

PVA = $750({1 – [1/(1 + .11)]15}/.11)

PVA = $5,393.15

This is the value of the annuity in Year 5, one period before the first payment. Finding the value of this amount today, we find:

PV = FV/(1 + *r*)*t*

PV = $5,393.15/(1 + .08)5

PV = $3,670.49

**30.** The amount borrowed is the value of the home times one minus the down payment, or:

Amount borrowed = $725,000(1 – .20)

Amount borrowed = $580,000

The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $580,000 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |

The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

PVA = $580,000 = *C*({1 – [1/(1 + .054/12)]360}/(.054/12))

*C* = $3,256.88

Now, at Year 8 (Month 96), we need to find the PV of the payments which have not been made. The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 96 | | 97 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $3,256.88 | | $3,256.88 | | $3,256.88 | | $3,256.88 | | $3,256.88 | | $3,256.88 | | $3,256.88 | | $3,256.88 | | $3,256.88 | |

The balloon payment will be:

PVA = $3,256.88({1 – [1/(1 + .054/12)]22(12)}/(.054/12))

PVA = $502,540.87

**31.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 12 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $12,400 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

Here, we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts.After the first six months, the balance will be:

FV = $12,400[1 + (.0199/12)]6 = $12,523.89

This is the balance in six months. The FV in another six months will be:

FV = $12,523.89[1 + (.18/12)]6 = $13,694.17

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

Interest = $13,694.17 – 12,400 = $1,294.17

**32.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$2,750,000 | | $273,000 | | $273,000 | | $273,000 | | $273,000 | | $273,000 | | $273,000 | | $273,000 | | $273,000 | | $273,000 | |

The company would be indifferent at the interest rate that makes the present value of the cash flows equal to the cost today. Since the cash flows are a perpetuity, we can use the PV of a perpetuity equation. Doing so, we find:

PV = *C*/*r*

$2,750,000 = $273,000/*r*

*r* = $273,000/$2,750,000

*r* = .0993, or 9.93%

**33.** The company will accept the project if the present value of the increased cash flows is greater than the cost. The cash flows are a growing perpetuity, so the present value is:

PV = *C*{[1/(*r* – *g*)] – [1/(*r* – *g*)] × [(1 + *g*)/(1 + *r*)]*t*}

PV = $41,000{[1/(.10 – .04)] – [1/(.10 – .04)] × [(1 + .04)/(1 + .10)]5}

PV = $167,112.08

The company should accept the project since the cost is less than the increased cash flows.

**34.** Since your salary grows at 3.7 percent per year, your salary next year will be:

Next year’s salary = $74,500(1 + .037)

Next year’s salary = $77,256.50

This means your deposit next year will be:

Next year’s deposit = $77,256.50(.05)

Next year’s deposit = $3,862.83

Since your salary grows at 3.7 percent, your deposit will also grow at 3.7 percent. We can use the present value of a growing annuity equation to find the value of your deposits today. Doing so, we find:

PV = *C*{[1/(*r* – *g*)] – [1/(*r* – *g*)] × [(1 + *g*)/(1 + *r*)]*t*}

PV = $3,862.83{[1/(.094 – .037)] – [1/(.094 – .037)] × [(1 + .037)/(1 + .094)]40}

PV = $59,798.32

Now, we can find the future value of this lump sum in 40 years. We find:

FV = PV(1 + *r*)*t*

FV = $59,798.32(1 + .094)40

FV = $2,174,612.53

This is the value of your savings in 40 years.

**35.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $4,700 | | $4,700 | | $4,700 | | $4,700 | | $4,700 | | $4,700 | | $4,700 | | $4,700 | | $4,700 | |

The relationship between the PVA and the interest rate is:

PVA falls as *r* increases, and PVA rises as *r* decreases.

FVA rises as *r* increases, and FVA falls as *r* decreases.

The present values of $4,700 per year for 20 years at the various interest rates given are:

PVA@10% = $4,700{[1 – (1/1.10)20]/.10} = $40,013.75

PVA@5% = $4,700{[1 – (1/1.05)20]/.05} = $58,572.39

PVA@15% = $4,700{[1 – (1/1.15)20]/.15} = $29,418.86

**36.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ? | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | –$40,000 | |
|  | | $350 | | $350 | | $350 | | $350 | |  | $350 | | $350 | | $350 | | $350 | | $350 | |

Here, we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

FVA = $40,000 = $350[{[1 + (.10/12)]*t* – 1}/(.10/12)]

Solving for *t*, we get:

1.00833*t* = 1 + [($40,000)(.10/12)/$350]

*t* = ln 1.95238/ln 1.00833

*t* = 80.62 payments

**37.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$88,000 | | $1,725 | | $1,725 | | $1,725 | | $1,725 | | $1,725 | | $1,725 | | $1,725 | | $1,725 | | $1,725 | |

Here, we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

PVA = $88,000 = $1,725[{1 – [1/(1 + *r*)]60}/*r*]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

*r* = .548%

The APR is the periodic interest rate times the number of periods in the year, so:

APR = 12(.548%)

APR = 6.58%

**38.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $1,025 | | $1,025 | | $1,025 | | $1,025 | | $1,025 | | $1,025 | | $1,025 | | $1,025 | | $1,025 | |

The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $1,025 monthly payments is:

PVA = $1,025[(1 – {1/[1 + (.048/12)]}360)/(.048/12)] = $195,362.62

The monthly payments of $1,025 will amount to a principal payment of $195,362.62. The amount of principal you will still owe is:

$275,000 – 195,362.62 = $79,637.38

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $79,637.38 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal, will be:

Balloon payment = $79,637.38[1 + (.048/12)]360

Balloon payment = $335,161.06

**39.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | |  | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$6,700 | | $1,400 | | ? | | $2,300 | | $2,700 | |  | |  | |  | |  | |  | |  | |

We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and

subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know is:

PV of Year 1 CF: $1,400/1.071 = $1,307.19

PV of Year 3 CF: $2,300/1.0713 = $1,872.23

PV of Year 4 CF: $2,700/1.0714 = $2,052.13

So, the PV of the missing CF is:

$6,700 – 1,307.19 – 1,872.23 – 2,052.13 = $1,468.44

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

$1,468.44(1.071)2 = $1,684.37

**40.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1M | | $1.335M | | $1.67M | | $2.005M | | $2.34M | | $2.675M | | $3.01M | | $3.345M | | $3.68M | | $4.015M | | $4.35M | |

To solve this problem, we need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of $1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

$1,000,000 + $1,335,000/1.058 + $1,670,000/1.0582 + $2,005,000/1.0583 + $2,340,000/1.0584 + $2,675,000/1.0585 + $3,010,000/1.0586 + $3,345,000/1.0587 + $3,680,000/1.0588

+ $4,015,000/1.0589 + $4,350,000/1.05810

PV = $20,969,067.06

**41.** Here, we are finding the interest rate for an annuity cash flow.We are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

Amount borrowed = .80($5,500,000) = $4,400,000

The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$4,400,000 | | $26,500 | | $26,500 | | $26,500 | | $26,500 | | $26,500 | | $26,500 | | $26,500 | | $26,500 | | $26,500 | |

Using the PVA equation:

PVA = $4,400,000 = $26,500[{1 – [1/(1 + *r*)]360}/*r*]

Unfortunately, this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

*r* = .504%

The APR is the monthly interest rate times the number of months in the year, so:

APR = 12(.504%)

APR = 6.04%

And the EAR is:

EAR = (1 + .00504)12 – 1

EAR = .0621, or 6.21%

**42.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 3 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $165,000 | |

The profit the firm earns is the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

PV = $165,000/1.133

PV = $114,353.28

And the firm’s profit is:

Profit = $114,353.28 – 103,000

Profit = $11,353.28

To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 3 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$103,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $165,000 | |

$103,000 = $165,000/( 1 + *r*)3

*r* =($165,000/$103,000)1/3 – 1

*r* = .1701, or 17.01%

**43.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | | **…** |  | | 5 | | 6 | |  | | **…** |  | | 25 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | | $8,500 | | $8,500 | | $8,500 | | $8,500 | |

We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 20 payments, so the PV of the annuity is:

PVA = $8,500{[1 – (1/1.067)20]/.067}

PVA = $92,187.54

Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at *t* = 5. To find the value today, we find the PV of this lump sum. The value today is:

PV = $92,187.54/1.0675

PV = $66,657.67

**44.** The time line for the annuity is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 180 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | $1,940 | | $1,940 | | $1,940 | | $1,940 | | $1,940 | | $1,940 | | $1,940 | | $1,940 | | $1,940 | |

This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:

PVA2 = $1,940[{1 – 1/[1 + (.06/12)]96}/(.06/12)]

PVA2 = $147,624.72

Note that this is the PV of this annuity exactly seven years from today. Now, we can discount this lump sum to today as well as finding the PV of the annuity for the first seven years. The value of this cash flow today is:

PV = $147,624.72/[1 + (.11/12)]84 + $1,940[{1 – 1/[1 + (.11/12)]84}/(.11/12)]

PV = $181,893.99

**45.** The time line for the annuity is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 180 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | $1,175 | | $1,175 | | $1,175 | | $1,175 | | $1,175 | | $1,175 | | $1,175 | | $1,175 | | $1,175 | |
|  | |  | |  | |  | |  | |  |  | |  | |  | |  | | FV | |

Here, we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate and payments. First, we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

FVA = $1,175[{[ 1 + (.064/12)]180 – 1}/(.064/12)]

FVA = $353,610.97

Now, we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

FV = $353,610.97 = PV*e*.07(15)

PV = $353,610.97*e–*1.05

PV = $123,741.83

**46.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | **…** | 7 | | **…** | 14 | | 15 | |  | | **…** |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | | PV | |  | | $2,350 | | $2,350 | | $2,350 | | $2,350 | |

To find the value of the perpetuity at *t* = 7, we first need to use the PV of a perpetuity equation. Using this equation we find:

PV = $2,350/.063

PV = $37,301.59

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | | **…** |  | | 7 | |  | |  | | **…** |  | | 14 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | | PV | |  | |  | |  | | $37,301.59 | |

Remember that the PV of a perpetuity (and annuity) equation gives the PV one period before the first payment, so, this is the value of the perpetuity at *t* = 14. To find the value at *t* = 7, we find the PV of this lump sum as:

PV = $37,301.59/1.0637

PV = $24,321.73

**47.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 12 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$26,000 | | $2,519.83 | | $2,519.83 | | $2,519.83 | | $2,519.83 | | $2,519.83 | | $2,519.83 | | $2,519.83 | | $2,519.83 | | $2,519.83 | |

To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

PVA = $26,000 = $2,519.83{(1 – [1/(1 + *r*)]12 )/*r*}

Again, we cannot solve this equation for *r*, so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

*r* = 2.403% per month

So the APR is:

APR = 12(2.403%)

APR = 28.84%

And the EAR is:

EAR = (1.02403)12 – 1

EAR = 32.97%

**48.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | **…** |  | | **…** | 18 | | 19 | |  | | **…** |  | | 28 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | | $5,230 | | $5,230 | | $5,230 | | $5,230 | |

The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

Monthly rate = .10/12 = .0083

To get the semiannual interest rate we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

Semiannual rate = 1.00836 – 1 = .0511, or 5.11%

We can now use this rate to find the PV of the annuity. The PV of the annuity is:

PVA @ *t* = 9: $5,230{[1 – (1/1.0511)10]/.0511} = $40,178.89

Note that this is the value one period (six months) before the first payment, so it is the value at *t* = 9. So, the value at the various times the question asked for uses this value 9 years from now.

PV @ *t* = 5: $40,178.89/1.05118 = $26,977.40

Note that you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

EAR = (1 + .0083)12 – 1 = .1047, or 10.47%

So, we can find the PV at *t* = 5 using the following method as well:

PV @ *t* = 5: $40,178.89/1.10474 = $26,977.40

The value of the annuity at the other times in the problem is:

PV @ *t* = 3: $40,178.89/1.051112 = $22,105.54

PV @ *t* = 3: $40,178.89/1.10476 = $22,105.54

PV @ *t* = 0: $40,178.89/1.051118 = $16,396.55

PV @ *t* = 0: $40,178.89/1.10479 = $16,396.55

**49.** *a.*  The time line for the ordinary annuity is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $13,250 | | $13,250 | | $13,250 | | $13,250 | | $13,250 | |  | |  | |  | |  | |

If the payments are in the form of an ordinary annuity, the present value will be:

PVA = *C*({1 – [1/(1 + *r*)*t*]}/*r*))

PVA = $13,250[{1 – [1/(1 + .078)]5}/.078]

PVA = $53,183.45

The time line for the annuity due is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| $13,250 | | $13,250 | | $13,250 | | $13,250 | | $13,250 | |  | |  | |  | |  | |  | |

If the payments are an annuity due, the present value will be:

PVAdue = (1 + *r*) PVA

PVAdue = (1 + .078)$53,183.45

PVAdue = $57,331.76

*b.* The time line for the ordinary annuity is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | | FV | |  | |  | |  | |  | |
|  | | $13,250 | | $13,250 | | $13,250 | | $13,250 | | $13,250 | |  | |  | |  | |  | |

We can find the future value of the ordinary annuity as:

FVA = *C*{[(1 + *r*)*t* – 1]/*r*}

FVA = $13,250{[(1 + .078)5 – 1]/.078}

FVA = $77,423.06

The time line for the annuity due is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $13,250 | | $13,250 | | $13,250 | | $13,250 | | $13,250 | | FV | |  | |  | |  | |  | |

If the payments are an annuity due, the future value will be:

FVAdue = (1 + *r*) FVA

FVAdue = (1 + .075)$77,423.06

FVAdue = $83,462.06

*c.* Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an ordinary due will always be higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.

**50.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | | 59 | | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$84,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| *C* | | *C* | | *C* | | *C* | | *C* | |  | *C* | | *C* | | *C* | | *C* | |  | |

We need to use the PVA due equation, that is:

PVAdue = (1 + *r*)PVA

Using this equation:

PVAdue = $84,000 = [1 + (.0608/12)] × *C*[{1 – 1/[1 + (.0608/12)]60}/(.0608/12)

*C* = $1,618.88

Notice, to find the payment for the PVA due we compound the payment for an ordinary annuity forward one period.

*Challenge*

**51.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | | 23 | | 24 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$3,350 | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| *C* | | *C* | | *C* | | *C* | | *C* | |  | *C* | | *C* | | *C* | | *C* | |  | |

The monthly interest rate is the annual interest rate divided by 12, or:

Monthly interest rate = .107/12

Monthly interest rate = .00892

Now we can set the present value of the lease payments equal to the cost of the equipment, or $3,350. The lease payments are in the form of an annuity due, so:

PVAdue = (1 + *r*)*C*({1 – [1/(1 + *r*)]*t*}/*r*)

$3,350 = (1 + .00892)*C*({1 – [1/(1 + .00892)]24}/.00892)

*C* = $154.29

**52.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | **…** |  | |  | | 15 | | 16 | | 17 | | 18 | | 19 | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | | $72,000 | | $72,000 | | $72,000 | | $72,000 | |  | |  | |
|  | |  | |  |  | |  | |  | |  | | $72,000 | | $72,000 | | $72,000 | | $72,000 | |
|  | | *C* | |  | *C* | | *C* | | *C* | |  | |  | |  | |  | |  | |

First, we will calculate the present value of the college expenses for each child. The expenses are an annuity, so the present value of the college expenses is:

PVA = *C*({1 – [1/(1 + *r*)]*t*}/*r*)

PVA = $72,000({1 – [1/(1 + .079)]4}/.079)

PVA = $239,004.91

This is the cost of each child’s college expenses one year before they enter college. So, the cost of the oldest child’s college expenses today will be:

PV = FV/(1 + *r*)*t*

PV = $239,004.91/(1 + .079)14

PV = $82,434.04

And the cost of the youngest child’s college expenses today will be:

PV = FV/(1 + *r*)*t*

PV = $239,004.91/(1 + .079)16

PV = $70,804.96

Therefore, the total cost today of your children’s college expenses is:

Cost today = $82,434.04 + 70,804.96

Cost today = $153,239.01

This is the present value of your annual savings, which are an annuity. So, the amount you must save each year will be:

PVA = *C*({1 – [1/(1 + *r*)]*t* }/*r* )

$153,239.01 = *C*({1 – [1/(1 + .079)]15}/.079)

*C* = $17,793.68

**53.** The salary is a growing annuity, so we use the equation for the present value of a growing annuity. The salary growth rate is 3.8 percent and the discount rate is 7.1 percent, so the value of the salary offer today is:

PV = *C* {[1/(*r* – *g*)] – [1/(*r* – *g*)] × [(1 + *g*)/(1 + *r*)]*t*}

PV = $71,000{[1/(.071 – .038)] – [1/(.071 – .038)] × [(1 + .038)/(1 + .071)]25}

PV = $1,167,636.64

The yearly bonuses are 10 percent of the annual salary. This means that next year’s bonus will be:

Next year’s bonus = .10($71,000)

Next year’s bonus = $7,100

Since the salary grows at 3.8 percent, the bonus will grow at 3.8 percent as well. Using the growing annuity equation, with a 3.8 percent growth rate and a 7.1 percent discount rate, the present value of the annual bonuses is:

PV = *C* {[1/(*r* – *g*)] – [1/(*r* – *g*)] × [(1 + *g*)/(1 + *r*)]*t*}

PV = $7,100{[1/(.071 – .038)] – [1/(.071 – .038)] × [(1 + .038)/(1 + .071)]25}

PV = $116,763.66

Notice the present value of the bonus is 10 percent of the present value of the salary. The present value of the bonus will always be the same percentage of the present value of the salary as the bonus percentage. So, the total value of the offer is:

PV = PV(Salary) + PV(Bonus) + Bonus paid today

PV = $1,167,636.64 + 116,763.66 + 10,000

PV = $1,294,400.30

**54.** Here, we need to compare two options. In order to do so, we must get the value of the two cash flow streams to the same time, so we will find the value of each today. We must also make sure to use the aftertax cash flows, since they are more relevant. For Option A, the aftertax cash flows are:

Aftertax cash flows = Pretax cash flows(1 – tax rate)

Aftertax cash flows = $250,000(1 – .28)

Aftertax cash flows = $180,000

So, the cash flows are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | | 30 | | 31 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| $180,000 | | $180,000 | | $180,000 | | $180,000 | | $180,000 | |  | $180,000 | | $180,000 | | $180,000 | | $180,000 | |  | |

The aftertax cash flows from Option A are in the form of an annuity due, so the present value of the cash flow today is:

PVAdue = (1 + *r*)*C*({1 – [1/(1 + *r)*]*t*}/*r*)

PVAdue = (1 + .0585)$180,000({1 – [1/(1 + .0585)]31}/.0585)

PVAdue = $2,697,950.16

For Option B, the aftertax cash flows are:

Aftertax cash flows = Pretax cash flows(1 – tax rate)

Aftertax cash flows = $200,000(1 – .28)

Aftertax cash flows = $144,000

The cash flows are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | | 29 | | 30 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| $530,000 | | $144,000 | | $144,000 | | $144,000 | | $144,000 | |  | $144,000 | | $144,000 | | $144,000 | | $144,000 | | $144,000 | |

The aftertax cash flows from Option B are an ordinary annuity, plus the cash flow today, so the present value is:

PV = *C*({1 – [1/(1 + *r*)]*t*}/*r*) + CF0

PV = $144,000{1 – [1/(1 + .0585)]30}/.0585) + $530,000

PV = $2,544,360.13

You should choose Option A because it has a higher present value on an aftertax basis.

**55.** We need to find the first payment into the retirement account. The present value of the desired amount at retirement is:

PV = FV/(1 + *r*)*t*

PV = $2,500,000/(1 + .094)30

PV = $168,818.62

This is the value today. Since the savings are in the form of a growing annuity, we can use the growing annuity equation and solve for the payment. Doing so, we get:

PV = *C* {[1/(*r* – *g*)] – [1/(*r* – *g*)] × [(1 + *g*)/(1 + *r*)]*t*}

$168,818.62 = *C*{[1/(.094 – .03)] – [1/(.094 – .03)] × [(1 + .03)/(1 + .094)]30}

*C* = $12,922.47

This is the amount you need to save next year. So, the percentage of your salary is:

Percentage of salary = $12,922.47/$86,000

Percentage of salary = .1503, or 15.03%

Note that this is the percentage of your salary you must save each year. Since your salary is increasing at 3 percent, and the savings are increasing at 3 percent, the percentage of salary will remain constant.

**56.** Since she put $1,500 down, the amount borrowed will be:

Amount borrowed = $17,000 – 1,500

Amount borrowed = $15,500

So, the monthly payments will be:

PVA = *C*({1 – [1/(1 + *r*)]*t*}/*r*)

$15,500 = *C*[{1 – [1/(1 + .068/12)]60}/(.068/12)]

*C* = $305.46

The amount remaining on the loan is the present value of the remaining payments. Since the first payment was made on October 1, 2018, and she made a payment on October 1, 2020, there are 35 payments remaining, with the first payment due immediately. So, we can find the present value of the remaining 34 payments after November 1, 2020, and add the payment made on this date. So the remaining principal owed on the loan is:

PV = *C*({1 – [1/(1 + *r*)]*t*}/*r*) + C0

PV = $305.46[{1 – [1/(1 + .068/12)]34}/(.068/12)]

*C* = $9,422.19

She must also pay a one percent prepayment penalty and the payment is due on November 1, 2020, so the total amount of the payment is:

Total payment = Balloon amount(1 + Prepayment penalty) + Current payment

Total payment = $9,422.19(1 + .01) + $305.46

Total payment = $9,821.87

**57.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | **…** | 120 | |  | | **…** |  | | 360 | | 361 | | **…** | 600 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | –$1,800 | | –$1,800 | |  | |  | |  | | $17,500 | | $17,500 | |
|  | |  | |  | $350,000 | | *C* | |  | *C* | | *C* | |  | |  | $1,500,000 | |

The cash flows for this problem occur monthly, and the interest rate given is the EAR. Since the cash flows occur monthly, we must get the effective monthly rate. One way to do this is to find the APR based on monthly compounding, and then divide by 12. So, the pre-retirement APR is:

EAR = .11 = [1 + (APR/12)]12 – 1; APR = 12(1.111/12 – 1) = .1048, or 10.48%

And the post-retirement APR is:

EAR = .08 = [1 + (APR/12)]12 – 1; APR = 12(1.081/12 – 1) = .0772, or 7.72%

First, we will calculate how much he needs at retirement. The amount needed at retirement is the PV of the monthly spending plus the PV of the inheritance. The PV of these two cash flows is:

PVA = $17,500{1 – [1/(1 + .0772/12)12(20)]}/(.0772/12)

PVA = $2,136,360.28

PV = $1,500,000/(1 + .08)20

PV = $321,822.31

So, at retirement, he needs:

$2,136,360.28 + 321,822.31 = $2,458,182.59

He will be saving $1,800 per month for the next 10 years until he purchases the cabin. The value of his savings after 10 years will be:

FVA = $1,800[{[ 1 + (.1048/12)]12(10) – 1}/(.1048/12)]

FVA = $379,062.59

After he purchases the cabin, the amount he will have left is:

$379,062.59 – 350,000 = $29,062.59

He still has 20 years until retirement. When he is ready to retire, this amount will have grown to:

FV = $29,062.59[1 + (.1048/12)]12(20)

FV = $234,311.67

So, when he is ready to retire, based on his current savings, he will be short:

$2,458,182.59 – 234,311.67 = $2,223,870.92

This amount is the FV of the monthly savings he must make between Years 10 and 30. So, finding the annuity payment using the FVA equation, we find his monthly savings will need to be:

FVA = $2,223,870.92 = *C*[{[ 1 + (.1048/12)]12(20) – 1}/(.1048/12)]

*C* = $2,750.46

**58.** To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing option is the PV of the lease payments, plus the $2,400. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 36 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $2,400 | | $580 | | $580 | | $580 | | $580 | | $580 | | $580 | | $580 | | $580 | | $580 | |

PV = $2,400 + $580{1 – [1/(1 + .06/12)12(3)]}/(.06/12)

PV = $21,465.19

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 36 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $37,000 | |  | |  | |  | |  | |  | |  | |  | |  | | –$22,000 | |

PV = $22,000/[1 + (.06/12)]12(3)

PV = $18,384.19

The PV of the decision to purchase is:

$37,000 – 18,384.19 = $18,615.81

In this case, it is cheaper to buy the car than to lease it since the PV of the leasing cash flows is lower. To find the break-even resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

$37,000 – PV of resale price = $21,465.19

PV of resale price = $15,534.81

The resale price that would make the PV of the lease versus buy decision equal is the FV of this value, so:

Break-even resale price = $15,534.81[1 + (.06/12)]12(3)

Break-even resale price = $18,590.21

**59.** To find the quarterly salary for the player, we first need to find the PV of the current contract. The cash flows for the contract are annual, and we are given a daily interest rate. We need to find the EAR so the interest compounding is the same as the timing of the cash flows. The EAR is:

EAR = [1 + (.057/365)]365 – 1

EAR = .0587, or 5.87%

The PV of the current contract offer is the sum of the PV of the cash flows. So, the PV is:

PV = $7,900,000 + $4,300,000/1.0587 + $4,900,000/1.05872 + $5,700,000/1.05873

+ $6,700,000/1.05874 + $7,300,000/1.05875 + $8,400,000/1.05876

PV = $37,929,060.53

The player wants the contract value to be increased by $3,500,000, so the PV of the new contract will be:

PV = $37,929,060.53 + 3,500,000

PV = $41,429,060.53

The player has also requested a signing bonus payable today in the amount of $10 million. We can subtract this amount from the PV of the new contract. The remaining amount will be the PV of the future quarterly paychecks.

$41,429,060.53 – 10,000,000 = $31,429,060.53

To find the quarterly payments, first realize that the interest rate we need is the effective quarterly rate. Using the daily interest rate, we can find the quarterly interest rate using the EAR equation, with the number of days being 91.25, the number of days in a quarter (365/4). The effective quarterly rate is:

Effective quarterly rate = [1 + (.057/365)]91.25 – 1

Effective quarterly rate = .01435 or 1.435%

Now, we have the interest rate, the length of the annuity, and the PV. Using the PVA equation and solving for the payment, we get:

PVA = $31,429,060.53 = *C*{[1 – (1/1.01435)24]/.01435}

*C* = $1,557,264.39

**60.** The time line for the cash flows is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 1 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$16,720 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $20,000 | |

To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the $20,000 you must repay in one year, and the $16,720 you borrow today. The interest rate of the loan is:

$20,000 = $16,720(1 + *r*)

*r* = ($20,000/$16,720) – 1

*r* = .1962, or 19.62%

Because of the discount, you only get the use of $16,720, and the interest you pay on that amount is 19.62 percent, not 16.4 percent.

**61.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | –24 | | –23 | | **…** | –12 | | –11 | | **…** | 0 | | 1 | |  | | **…** | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | $3,250 | | $3,250 | | $3,583.33 | | $3,583.33 | | $3,916.67 | | $3,916.67 | | $3,916.67 | |
|  | |  | |  |  | |  | |  | $150,000 | |  | |  | |  |  | |
|  | |  | |  |  | |  | |  | $25,000 | |  | |  | |  |  | |

Here, we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate, so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

APR = 12[(1.074)1/12 – 1]

APR = .0716, or 7.16%

To find the value today of the back pay from two years ago, we will find the FV of the annuity (salary), and then find the FV of the lump sum value of the salary. Doing so gives us:

FV = ($39,000/12)[{[ 1 + (.0716/12)]12 – 1}/(.0716/12)](1 + .074)

FV = $43,288.32

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year’s back pay:

FVA = ($43,000/12)[{[ 1 + (.0716/12)]12 – 1}/(.0716/12)]

FVA = $44,439.62

Next, we find the value today of the five year’s future salary:

PVA = ($47,000/12){[{1 – {1/[1 + (.0716/12)]12(5)}]/(.0716/12)}

PVA = $197,046.10

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

Award = $43,288.32 + 44,439.62 + 197,046.10 + 150,000 + 25,000

Award = $459,774.05

As the plaintiff, you would prefer a lower rate. In this problem, we are calculating both the PV and FV of annuities. A lower rate will decrease the FVA, but increase the PVA. So, by using a lower rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time period, this is the more important cash flow to the plaintiff.

**62.** To find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

Loan repayment amount = $10,000(1.08)

Loan repayment amount = $10,800

The amount you will receive today is the principal amount of the loan times one minus the points.

Amount received = $10,000(1 – .03)

Amount received = $9,700

So, the time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 9 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$9,700 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $10,800 | |

Now, we find the interest rate for this PV and FV.

$10,800 = $9,700(1 + *r*)

*r* = ($10,800/$9,700) – 1

*r* = .1134, or 11.34%

With a quoted interest rate of 11 percent and two points, the EAR is:

Loan repayment amount = $10,000(1.11)

Loan repayment amount = $11,100

Amount received = $10,000(1 – .02)

Amount received = $9,800

$11,100 = $9,800(1 + *r*)

*r* = ($11,100/$9,800) – 1

*r* = .1327, or 13.27%

The effective rate is not affected by the loan amount, since it drops out when solving for *r*.

**63.** First, we will find the APR and EAR for the loan with the refundable fee. Remember, we need to use the actual cash flows of the loan to find the interest rate. With the $2,900 application fee, you will need to borrow $302,900 to have $300,000 after deducting the fee. The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $302,900 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |

Solving for the payment under these circumstances, we get:

PVA = $302,900 = *C*{[1 – 1/(.053/12)360]/(.053/12)}

*C* = $1,682.02

We can now use this amount in the PVA equation with the original amount we wished to borrow, $300,000.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$300,000 | | $1,682.02 | | $1,682.02 | | $1,682.02 | | $1,682.02 | | $1,682.02 | | $1,682.02 | | $1,682.02 | | $1,682.02 | | $1,682.02 | |

Solving for *r*, we find:

PVA = $300,000 = $1,682.02[{1 – [1/(1 + *r*)]360}/*r*]

Solving for *r* with a spreadsheet, on a financial calculator, or by trial and error, gives:

*r* = .4489% per month

APR = 12(.4489%)

APR = 5.39%

EAR = (1 + .004489)12 – 1

EAR = .0552, or 5.52%

With the nonrefundable fee, the APR of the loan is the quoted APR since the fee is not considered part of the loan. So:

APR = 5.30%

EAR = [1 + (.053/12)]12 – 1

EAR = .0543, or 5.43%

**64.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 36 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1,000 | | $46.11 | | $46.11 | | $46.11 | | $46.11 | | $46.11 | | $46.11 | | $46.11 | | $46.11 | | $46.11 | |

Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is $1,000, and the payments are $46.11 per month for three years, so the interest rate on the loan is:

PVA = $1,000 = $46.11[{1 – [1/(1 + *r*)]36}/*r*]

Solving for *r* with a spreadsheet, on a financial calculator, or by trial and error, gives:

*r* = 3.04% per month

APR = 12(3.04%)

APR = 36.54%

EAR = (1 + .03040)12 – 1

EAR = .4332, or 43.32%

It’s called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

**65.** We will calculate the number of periods necessary to repay the balance with no fee first. We need to use the PVA equation and solve for the number of payments.

Without fee and annual rate = 18.6%:

PVA = $12,000 = $250{[1 – (1/1.0155)*t*]/.0155} where .0155 = .186/12

Solving for *t*, we get:

*t* = ln{1/[1 – ($12,000/$250)(.0155)]}/ln(1.0155)

*t* = ln 3.90625/ln 1.0155

*t* = 88.59 months

Without fee and annual rate = 8.2%:

PVA = $12,000 = $250{[1 – (1/1.006833)*t* ]/.006833} where .006833 = .082/12

Solving for *t*, we get:

*t* = ln{1/[1 – ($12,000/$250)(.006833)]}/ln(1.006833)

*t* = ln 1.4881/ln 1.006833

*t* = 58.37 months

You will pay off your account:

88.59 – 58.37 = 30.22 months quicker

Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 8.20%:

PVA = $12,240 = $250{[1 – (1/1.006833)*t* ]/.006833} where .006833 = .082/12

Solving for *t*, we get:

*t* = ln{1/[1 – ($12,240/$250)(.006833)]}/ln(1.006833)

*t* = ln 1.5028/ln 1.006833

*t* = 59.81 months

You will pay off your account:

88.59 – 59.81 = 28.78 months quicker

**66.** We need to find the FV of the premiums to compare with the cash payment promised at age 65. We

have to find the value of the premiums at Year 6 first since the interest rate changes at that time. So:

FV1 = $500(1.11)5 = $842.53

FV2 = $600(1.11)4 = $910.84

FV3 = $700(1.11)3 = $957.34

FV4 = $800(1.11)2 = $985.68

FV5 = $900(1.11)1 = $999.00

Value at Year 6 = $842.53 + 910.84 + 957.34 + 985.68 + 999 + 1,000

Value at Year 6 = $5,695.39

Finding the FV of this lump sum at the child’s 65th birthday:

FV = $5,695.39(1.07)59

FV = $308,437.08

The policy is not worth buying; the future value of the policy is $308,437.08, but the policy contract will pay off $300,000. The premiums are worth $8,437.08 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

PV = $500/1.11 + $600/1.112 + $700/1.113 + $800/1.114 + $900/1.115 + $1,000/1.116

PV = $3,044.99

And the value today of the $300,000 at age 65 is:

PV = $300,000/1.0759

PV = $5,539.60

PV = $5,539.60/1.116

PV = $2,961.70

The premiums still have the higher cash flow. At Year 0, the difference is $83.29. When you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest $83.29, the difference in the cash flows at time zero, for 6 years at an 11 percent interest rate, and then for 59 years at a 7 percent interest rate. How much will it be worth? Without doing calculations, you know it will be worth $8,437.08, the difference in the cash flows at Year 65!

**67.** Since the payments occur at six month intervals, we need to get the effective six-month interest rate. We can calculate the daily interest rate since we have an APR compounded daily, so the effective six-month interest rate is:

Effective six-month rate = (1 + Daily rate)180 – 1

Effective six-month rate = (1 + .09/360)180 – 1

Effective six-month rate = .0460, or 4.60%

Now, we can use the PVA equation to find the present value of the semi-annual payments. Doing so, we find:

PVA = *C*({1 – [1/(1 + *r*)]*t* }/*r* )

PVA = $1,900,000({1 – [1/(1 + .0460]40 }/.0460)

PVA = $34,458,785.87

This is the value six months from today, which is one period (six months) prior to the first payment. So, the value today is:

PV = $34,458,785.87/(1 + .0460)

PV = $32,942,697.79

This means the total value of the lottery winnings today is:

Value of winnings today = $32,942,697.79 + 5,500,000

Value of winnings today = $38,442,697.79

You should not take the offer since the value of the offer is less than the present value of the payments.

**68.** Here, we need to find the interest rate that makes the PVA, the college costs, equal to the FVA, the savings. The PV of the college costs is:

PVA = $20,000[{1 – [1/(1 + *r*)]4}/*r*]

And the FV of the savings is:

FVA = $9,000{[(1 + *r*)6 – 1]/*r*}

Setting these two equations equal to each other, we get:

$20,000[{1 – [1/(1 + *r*)]4}/*r*] = $9,000{[(1 + *r*)6 – 1]/*r*}

Reducing the equation gives us:

9(1 + *r*)10 – 29(1 + *r*)4 + 20 = 0

Now, we need to find the root of this equation. We can solve using trial and error, a root-solving calculator routine, or a spreadsheet. Using a spreadsheet, we find:

*r* = 8.07%

**69.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | … |  | | 10 | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | –$25,000 | | –$25,000 | | –$25,000 | | –$25,000 | | –$25,000 | | –$25,000 | | –$25,000 | | –$25,000 | |
|  | | $51,000 | |  | $51,000 | | $51,000 | |  |  | |  | |  | |  | |  | |

Here, we need to find the interest rate that makes us indifferent between an annuity and a perpetuity. To solve this problem, we need to find the PV of the two options and set them equal to each other. The PV of the perpetuity is:

PV = $25,000/*r*

And the PV of the annuity is:

PVA = $51,000[{1 – [1/(1 + *r*)]10}/*r*]

Setting them equal and solving for *r*, we get:

$25,000/*r =* $51,000[{1 – [1/(1 + *r*)]10}/*r*]

$25,000/$51,000 = 1 – [1/(1 + *r*)]10

.49021/10 = 1/(1 + *r*)

*r* = 1/.50981/10 – 1

*r* = .0697, or 6.97%

**70.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | | 3 | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | $50,000 | |  | | $50,000 | |  | | $50,000 | |  | | $50,000 | |  | | $50,000 | |

The cash flows in this problem occur every two years, so we need to find the effective two year rate. One way to find the effective two year rate is to use an equation similar to the EAR, except use the number of days in two years as the exponent. (We use the number of days in two years since it is daily compounding; if monthly compounding was assumed, we would use the number of months in two years.) So, the effective two-year interest rate is:

Effective 2-year rate = [1 + (.09/365)]365(2) – 1

Effective 2-year rate = .1972, or 19.72%

We can use this interest rate to find the PV of the perpetuity. Doing so, we find:

PV = $50,000/.1972

PV = $253,561.53

This is an important point: Remember that the PV equation for a perpetuity (and an ordinary annuity) tells you the PV one period before the first cash flow. In this problem, since the cash flows are two years apart, we have found the value of the perpetuity one period (two years) before the first payment, which is one year ago. We need to compound this value for one year to find the value today. The value of the cash flows today is:

FV = $253,561.53(1 + .09/365)365

FV = $277,437.42

The second part of the question assumes the perpetuity cash flows begin in four years. In this case, when we use the PV of a perpetuity equation, we find the value of the perpetuity two years from today. So, the value of these cash flows today is:

PV = $253,561.53/(1 + .09/365)2(365)

PV = $211,797.09

**71.** To solve for the PVA due:

PVA =



PVAdue =



PVAdue =



PVAdue = (1 + *r*)PVA

And the FVA due is:

FVA = *C* + C(1 + *r*) + *C*(1 + *r*)2 + …. + *C*(1 + *r*)*t* – 1

FVAdue = *C*(1 + *r*) + C(1 + *r*)2 + …. + *C*(1 + *r*)*t*

FVAdue = (1 + *r*)[*C* + C(1 + *r*) + …. + *C*(1 + *r*)*t* – 1]

FVAdue = (1 + *r*)FVA

**72.** *a.* The APR is the interest rate per week times 52 weeks in a year, so:

APR = 52(7.1%)

APR = 369.20%

EAR = (1 + .071)52 – 1

EAR = 34.4040, or 3,440.40%

*b.* In a discount loan, the amount you receive is lowered by the discount, and you repay the full principal. With a discount of 7.1 percent, you would receive $9.29 for every $10 in principal, so the weekly interest rate would be:

$10 = $9.29(1 + *r*)

*r* = ($10/$9.29) – 1

*r* = .0764, or 7.64%

Note the dollar amount we use is irrelevant. In other words, we could use $.929 and $1, $92.90 and $100, or any other combination and we would get the same interest rate. Now we can find the APR and the EAR:

APR = 52(7.64%)

APR = 397.42%

EAR = (1 + .0764)52 – 1

EAR = 45.0450, or 4,504.50%

*c.* Using the cash flows from the loan, we have the PVA and the annuity payments and need to find the interest rate, so:

PVA = $68.43 = $25[{1 – [1/(1 + *r*)]4}/*r*]

Using a spreadsheet, trial and error, or a financial calculator, we find:

*r* = 17.11% per week

APR = 52(17.11%)

APR = 889.82%

EAR = 1.171152 – 1

EAR = 3,690.8773, or 369,087.73%

**73.** To answer this, we can diagram the perpetuity cash flows, which are: (Note, the subscripts are only to differentiate when the cash flows begin. The cash flows are all the same amount.)

…..

*C*3

*C*2 *C*2

*C*1 *C*1 *C*1



Thus, each of the increased cash flows is a perpetuity in itself. So, we can write the cash flows stream as:

*C*1/*r* *C*2/*r* *C*3/*r* *C*4/*r* ….

 

So, we can write the cash flows as the present value of a perpetuity with a perpetuity payment of:

*C*2/*r* *C*3/*r* *C*4/*r* ….



The present value of this perpetuity is:

PV = (*C*/*r*)/*r* = *C*/*r*2

So, the present value equation of a perpetuity that increases by *C* each period is:

PV = *C*/*r* + *C*/*r*2

**74.** Since it is only an approximation, we know the Rule of 72 is exact for only one interest rate. Using the basic future value equation for an amount that doubles in value and solving for *t*, we find:

FV = PV(1 + *r*)*t*

$2 = $1(1 + *r*)*t*

ln(2) = *t* ln(1 + *r*)

*t* = ln(2)/ln(1 + *r*)

We also know the Rule of 72 approximation is:

*t* = 72/*r*

We can set these two equations equal to each other and solve for *r*. We also need to remember that the exact future value equation uses decimals, so the equation becomes:

.72/*r* = ln(2)/ln(1 + *r*)

0 = (.72/*r*)/[ln(2)/ln(1 + *r*)]

It is not possible to solve this equation directly for *r*, but using Solver, we find the interest rate for which the Rule of 72 is exact is 7.846894 percent.

**75.** We are only concerned with the time it takes money to double, so the dollar amounts are irrelevant. So, we can write the future value of a lump sum with continuously compounded interest as:

$2 = $1*ert*

2 = *ert*

*rt* = ln(2)

*rt* = .693147

*t* = .693147/*r*

Since we are using percentage interest rates while the equation uses decimal form, to make the equation correct with percentages, we can multiply by 100

*t* = 69.3147/*r*