HW2

1. Prove the identity of each of the following Boolean equations, using algebraic manipulation:

   (a) $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C = 1$

$$
\begin{aligned}
\text{(a)}\quad & \bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C \\
&= \bar{A}B + AB + \bar{B}\bar{C} + \bar{B}C \\
&= B(\bar{A}+A) + \bar{B}(\bar{C}+C) \\
&= B + \bar{B} \\
&= 1
\end{aligned}
$$

   (b) $Y + \bar{X}Z + X\bar{Y} = X + Y + Z$

(b)

$$
\begin{aligned}
& Y + \bar{X}Z + X\bar{Y} \\
&= Y + \bar{Y}X + \bar{X}Z \\
&= X + Y + \bar{X}Z \\
&= Y + X + \bar{X}Z \\
&= Y + X + Z
\end{aligned}
$$

note: $A + \bar{A}B = (A + AB) + \bar{A}B$
$= A + AB + \bar{A}B$
$= A + B$

2. Simplify the following Boolean expressions to expressions containing a minimum number of literals:

   (a) $\overline{(A+B+C)} \cdot \overline{ABC}$

$$
\begin{aligned}
\text{(a)}\quad & \left(\overline{A+B+C}\right) \cdot \overline{ABC} \\
&= \left(\overline{(A+B+C) + ABC}\right) \\
&= \left(\overline{(A+ABC) + B + C}\right) \\
&= \left(\overline{A+B+C}\right)
\end{aligned}
$$

note: $A + A(BC)$
$= A$

(b) $AB\bar{C} + AC$

$$AB\bar{C} + AC = AB\bar{C} + A\bar{B}C + ABC = AB\bar{C} + ABC + A\bar{B}C + ABC$$
$$= A(B\bar{C} + BC) + A(\bar{B}C + BC) = AB + AC = A(B + C)$$

3. Using DeMorgan's theorem, express the function $F = \bar{A}BC + A\bar{C} + \bar{A}B$

(a) with only OR and complement operations.

F = A'BC+AC'+A'B
(F')' = ((A'BC+AC'+A'B)')'
= ((A'BC)'(AC')'(A'B)')'
= ((A+B'+C')(A'+C)(A+B'))'
= (A+B'+C')'+(A'+C)'+(A+B')'

(b) with only AND and complement operations.

$$F = \bar{A}BC + A\bar{C} + \bar{A}B = \overline{\overline{\bar{A}BC + A\bar{C} + \bar{A}B}} = \overline{\overline{\bar{A}BC} \cdot \overline{A\bar{C}} \cdot \overline{\bar{A}B}}$$

4. Obtain the truth table of the following functions, and express each function in sum-of minterms and product-of-maxterms form:

(a) $(X + YZ)(Z + YX)$

4. (a) $f = (X+YZ)(Z+YX)$

truth table

X	Y	Z	(X+YZ)	(Z+YX)	f
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

sum-of-minterms

$f = \bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$
$= m_3 + m_5 + m_6 + m_7$
$= \Sigma(3,5,6,7)$

product-of-maxterms

$f = (X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+Y+Z)$
$= M_0 \cdot M_1 \cdot M_2 \cdot M_4$
$= \Pi(0,1,2,4)$

(b) $W\bar{X}Y + W\bar{X}Z + WX\bar{Z} + XY$

4(b)

$f = W\bar{X}Y + W\bar{X}Z + WX\bar{Z} + XY$

W	X	Y	Z	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

sum-of-minterms

$f = \bar{W}XY\bar{Z} + \bar{W}XYZ + W\bar{X}\bar{Y}Z + W\bar{X}Y\bar{Z} + W\bar{X}YZ + WX\bar{Y}\bar{Z} + WXY\bar{Z} + WXYZ$

$= m_6 + m_7 + m_9 + m_{10} + m_{11} + m_{12} + m_{14} + m_{15}$

$= \Sigma(6,7,9,10,11,12,14,15)$

product-of-maxterms

$f = (W+X+Y+Z)(W+X+Y+\bar{Z})(W+X+\bar{Y}+Z)(W+X+\bar{Y}+\bar{Z})(W+\bar{X}+Y+Z)(W+\bar{X}+Y+\bar{Z})(\bar{W}+X+Y+Z)(\bar{W}+\bar{X}+Y+\bar{Z})$

$= M_0 \cdot M_1 \cdot M_2 \cdot M_3 \cdot M_4 \cdot M_5 \cdot M_8 \cdot M_{13}$

$= \Pi(0,1,2,3,4,5,8,13)$

5. For the Boolean functions E and F, as given in the following truth table:

X	Y	Z	E	F
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

(a) Express E and F in sum-of-minterms and product-of-maxterms algebraic form

sum-of-minterms

$E = x' \cdot y' \cdot z' + x' \cdot y \cdot z' + x \cdot y' \cdot z' + x \cdot y \cdot z$

$F = x' \cdot y' \cdot z + x' \cdot y \cdot z + x \cdot y' \cdot z' + x \cdot y \cdot z'$

product-of-maxterms

$E = (x'+y'+z)(x'+y+z)(x+y'+z)(x+y+z')$

$F = (x'+y'+z')(x'+y+z')(x+y'+z)(x+y+z)$

(b) Draw the logic diagram of E and F with sum-of-minterm