CS2336 Discrete Mathematics

Homework 2
Tutorial: October 26, 2020
Exam 1: November 2, 2020 (10:10 – 12:30)

Problems marked with * will be explained in the tutorial.

1. (*) Show that: If n is perfect square, then $n + 2$ is not a perfect square.
2. (*) Show that any odd integer is the difference of two squares.
3. Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.
4. Show that for any real number x, $x^2 - 3x + 2 > 0$ if and only if $x < 1$ or $x > 2$.
5. For each of the following statements, provide an indirect proof by stating and proving the contrapositive of the given statement.
   (a) For all integers m and n, if mn is odd, then m, n are both odd.
   (b) For all integers m and n, if $m + n$ is even, then m, n are both even or both odd.
6. Use "prove by cases" to show the following results:
   (a) If n is a natural number, then $n^2 + n + 3$ is odd.
   (b) If a and b are real numbers, $|a - b| = |b - a|$
7. (*) Show that $x^5 - x^4 + x^3 - x^2 + x - 1 = 0$ has an integral root.
8. (Lecture Review) Prove that when a white square and a black square are removed from an 8×8 chessboard, you can tile the remaining squares of the checkerboard using dominoes.

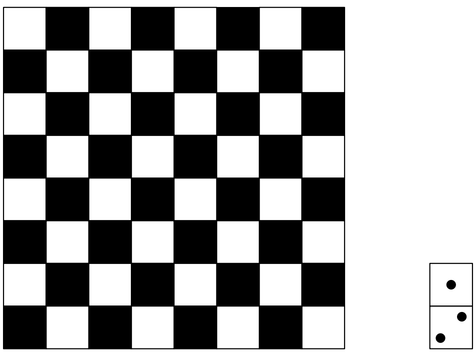

Figure 1: A checkerboard and a domino piece.

9. (*, Challenging) Let α be an angle such that $\alpha = \tan^{-1}(1/3) + \tan^{-1}(1/2)$ and $0 \leq \alpha < 2\pi$. Show that $\alpha = \pi/4$ without using a calculator.
10. (*) Prove or disprove the following:

$$\text{If } p_1, p_2, \ldots, p_n \text{ are the } n \text{ smallest primes, then } k = p_1 p_2 \cdots p_n + 1 \text{ is prime.}$$

11. (*, Challenging) Consider the equation $z^{13} - z^2 - 15015 = 0$.

   (a) Show that the equation does not have any integral root.

   Hint: Show that for any z that satisfies the above equation, (i) z cannot be an odd number, and (ii) z cannot be an even number.

   (b) Show that the equation does not have any rational root.