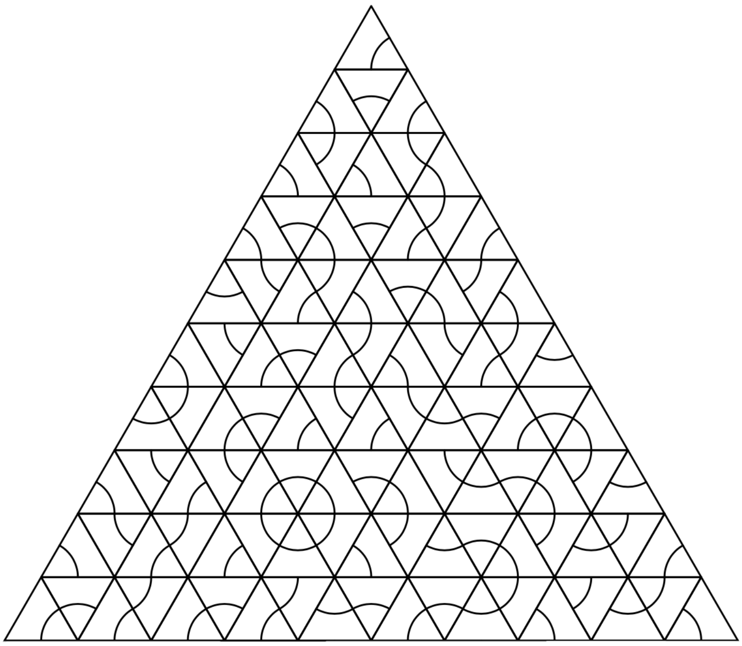
CS5314 Randomized Algorithms

Homework 2
Due: 11:59 pm, April 21, 2020 (through iLMS)

1. An equilateral triangle is tiled with n^2 smaller congruent equilateral triangles such that there are n smaller triangles along each side of the original triangle. For each of these smaller equilateral triangles, we randomly choose a vertex v of the triangle and draw an arc with v as the center connecting the midpoints of the two sides of the triangle.

   The case when $n = 10$ is shown as follows.

   Find the expected number of full circles formed, in terms of n.

   Hint: Design appropriate indicators, and use linearity of expectation.

2. A deck of n playing cards, which contains five jokers, is well-shuffled. The cards are turned up one by one from the top until the third joker appears. What is the expected number of cards to be turned up?

   Hint: Let X be the expected number of cards to be turned up. What is the relationship between $\Pr(X = k)$ and $\Pr(X = n - k + 1)$?

3. A lost tourist arrives at a point with 4 roads. There are no signs on the roads. The first road brings him back to the same point after 2 hours of walk. The second road leads to the city after 3 hours of walk. The third road brings him back to the same point after 4 hours of walk. The last road leads to the city after 5 hours of walk.

   Assuming that the tourist chooses a road equally likely at all times. (That is, a road may be chosen again and again.) What is the mean time until the tourist arrives to the city?

4. We roll a fair 6-sided die over and over again.

   (a) What is the expected number of rolls until a 6 turns up twice in a row (i.e., a 6 followed by a 6)?

   (A) 24 (B) 30 (C) 36 (D) 42 (E) 48

   (b) What is the expected number of rolls until the sequence 65 appears (i.e., a 6 followed by a 5)?

   (A) 24 (B) 30 (C) 36 (D) 42 (E) 48

   Remark: For Q4, there seems to be no easy way. Use brute-force calculation (or perhaps, memoryless property to help).

5. Suppose that 14 boys and 6 girls line up in a row. Let N be the number of places in the row where a boy and a girl are standing next to each other.

   For example, for the row BGBBGBGBBBGBGBBBGBBB we have $N = 12$.

   The expected value of N is closest to

   (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

   Hint: Design appropriate indicators, and use linearity of expectation.