CS2336 Discrete Mathematics

Homework 6
Tutorial: January 04, 2021
Exam 3: Januray 11, 2021 (2.5 hours)

Problems marked with * will be explained in the tutorial.

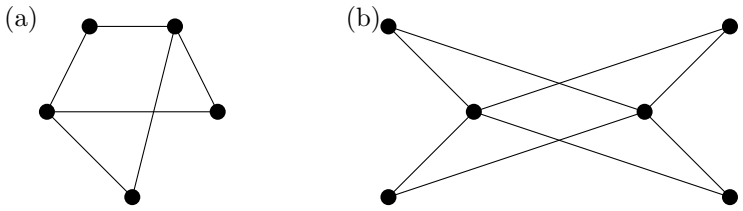
1. Let G be a graph with v vertices and e edges. Let M be the maximum degree of the vertices of G, and let m be the minimum degree of the vertices of G. Show that

   (a) $2e/v \geq m$

   (b) $2e/v \leq M$

2. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

3. For each of the following graphs, determine whether it is bipartite.



4. If G is a graph, the *complement* of G, denoted by $\overline{G}$, is a graph with the same vertex set, such that an edge e exists in $\overline{G}$ if and only if e does not exist in G.

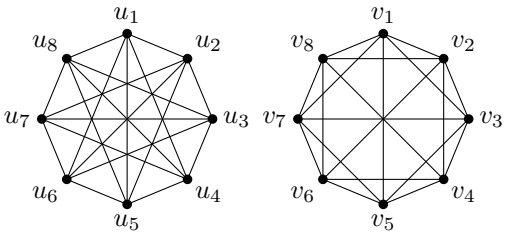
   Suppose it is known that a simple graph G has 15 edges and its complement graph $\overline{G}$ has 13 edges. How many vertices does G have?

5. Show that if G is a simple graph with n vertices, then the union of G and $\overline{G}$ is K_n.

6. (*) Two graphs G and H are *isomorphic* if there is a one-to-one correspondence f between the vertices of G and the vertices of H, such that u, v are adjacent in G if and only if $f(u), f(v)$ are adjacent in H. (See Lecture Note 13, pages 26 and 27 for some examples.)

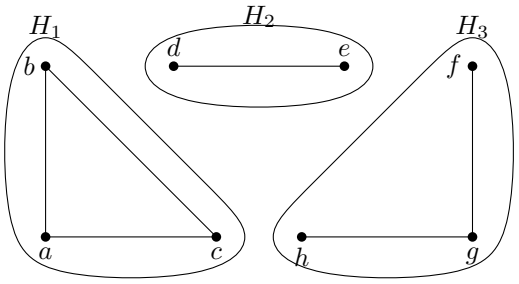
   Suppose that G and H are isomorphic simple graphs. Show that their complementary graphs $\overline{G}$ and $\overline{H}$ are also isomorphic.

7. (*) Determine whether the following two graphs are isomorphic.



Hint: Q6.

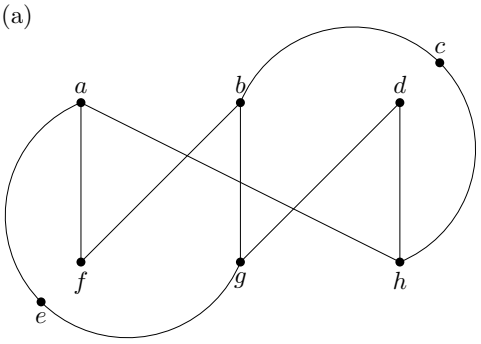
8. (*) An undirected graph is called *connected* if there is a path between every pair of distinct vertices of the graph, and a *connected component* of graph G is a maximal connected subgraph of G. For example, in the following graph, there are 3 connected components H_1, H_2, and H_3.



Suppose that a planar graph has k connected components, e edges, and v vertices. Also suppose that the plane is divided into r regions by a planar representation of the graph. Find a formula for r in terms of e, v, and k.

9. (*) Determine whether the following graphs is homeomorphic to $K_{3,3}$. (The definition of "homeomorphic" will be discussed on Dec 30, 2020.)

(a)



(b)