NO.

PDE 10603233 周聖詠

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Practice 1. 2021/10/07

1. $y'' - 8y' + 16 = 3\cos 4t - \sin 5t$

$y(0) = 10, \quad y'(0) = 10$

$\Rightarrow$ Homogeneous : $y'' - 8y' = 0$

Guess $y_h = C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}$

Let $y = e^{\alpha t}$

$y' = \alpha e^{\alpha t} \quad y'' = \alpha^2 e^{\alpha t}$

$\alpha^2 e^{\alpha t} - 8(\alpha e^{\alpha t}) = 0$

$(\alpha^2 - 8\alpha) e^{\alpha t} = 0$

$\alpha^2 - 8\alpha = 0$

$\alpha(\alpha - 8) = 0$

$\alpha = 0, 8$

General Solution $y_h = C_1 e^{0t} + C_2 e^{8t} = C_1 + C_2 e^{8t}$

$4K_1 + 108M$ NO.

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$\Rightarrow$ Nonhomogeneous

$y'' - 8y' = 3\cos 4t - \sin 5t - 16$

Part I : $y'' - 8y' = 3\cos 4t$

Guess $y_{p_1} =$

Let $y = M_1 \cos 4t + K_1 \sin 4t$

$y' = -4M_1 \sin 4t + 4K_1 \cos 4t$

$y'' = -16M \cos 4t - 16K_1 \sin 4t$

Thus

$(-4M_1 \sin 4t + 4K_1 \cos 4t) - 8(-16M_1 \cos 4t - 16K_1 \sin 4t) = 3\cos 4t$

$$\begin{cases} 4K_1 + 108M_1 = 3 \\ -4M_1 + 108K_1 = 0 \end{cases}$$

$M_1 = -\frac{3}{8} \quad K_1 = 0$

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Part II : $y'' - 8y' = -\sin 5t$

Guess $y_{p_2} = M_2 \cos 5t + K_2 \sin 5t$

Let $y = M_2 \cos 5t + K_2 \sin 5t$

$y' = -5M_2 \sin 5t + 5K_2 \cos 5t$

$y'' = -25M_2 \cos 5t - 25K_2 \sin 5t$

$(-5M_2 + 200K_2) \sin 5t +$

$(5K_2 + 200M_2) \cos 5t = -\sin 5t$

$$\begin{cases} -5M_2 + 200K_2 = -1 \\ 5K_2 + 200M_2 = 0 \end{cases}$$

$K_2 = -\frac{1}{8}$, $M_2 = 0$

Part III $y'' - 8y' = -16$

Guess $y_{p_3} = z_2 x^2 + z_1 x + z_0$

$y' = 2z_2 x + z_1$ $y'' = 2z_2$

$(2z_2 x + z_1) - 8(2z_2) = -16$

$z_1 - 16z_2 = -16$

$z_1 = 2$, $z_2 = 0$

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Thus

$$y = y_h + y_{P_1} + y_{P_2} + y_{P_3}$$

$$= C_1 + C_2 e^{8t} - \frac{3}{8}\cos 4t - \frac{1}{8}\sin 5t + 2x$$

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