HW03

Partial Differential Equations and Complex Variables (EE 2021 Summer)

Due date: 10/7 (Thur.) Convert your solutions into pdf and upload to eeclass.

I. Please follow the following instruction, playing the app and picking up the best answer.

II. Review ODE Chaper 2.2, 2.3, 2.4, 2.6, 2.7, 2.8

III. Problems in Problem Set 2.8 (page 91) :**3, 10 (plot with matlab)**

PROBLEM SET 2.8

1. WRITING REPORT. Free and Forced Vibrations. Write a condensed report of 2–3 pages on the most important similarities and differences of free and forced vibrations, with examples of your own. No proofs.

2. Which of Probs. 1–18 in Sec. 2.7 (with x = time t) can be models of mass–spring systems with a harmonic oscillation as steady-state solution?

3–7 STEADY-STATE SOLUTIONS

Find the steady-state motion of the mass–spring system modeled by the ODE. Show the details of your work.

3. $y'' + 6y' + 8y = 42.5 \cos 2t$

4. $y'' + 2.5y' + 10y = -13.6 \sin 4t$

5. $(D^2 + D + 4.25I)y = 22.1 \cos 4.5t$

6. $(D^2 + 4D + 3I)y = \cos t + \frac{1}{3} \cos 3t$

7. $(4D^2 + 12D + 9I)y = 225 - 75 \sin 3t$

8–15 TRANSIENT SOLUTIONS

Find the transient motion of the mass–spring system modeled by the ODE. Show the details of your work.

8. $2y'' + 4y' + 6.5y = 4 \sin 1.5t$

9. $y'' + 3y' + 3.25y = 3 \cos t - 1.5 \sin t$

10. $y'' + 16y = 56 \cos 4t$

11. $(D^2 + 2I)y = \cos \sqrt{2}t + \sin \sqrt{2}t$

12. $(D^2 + 2D + 5I)y = 4 \cos t + 8 \sin t$

13. $(D^2 + I)y = \cos \omega t,\ \omega^2 \neq 1$

14. $(D^2 + I)y = 5e^{-t} \cos t$

15. $(D^2 + 4D + 8I)y = 2 \cos 2t + \sin 2t$

16–20 INITIAL VALUE PROBLEMS

Find the motion of the mass–spring system modeled by the ODE and the initial conditions. Sketch or graph the solution curve. In addition, sketch or graph the curve of $y - y_p$ to see when the system practically reaches the steady state.

16. $y'' + 25y = 24 \sin t, \quad y(0) = 1, \quad y'(0) = 1$

17. $(D^2 + 4I)y = \sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t,$
$y(0) = 0, \quad y'(0) = \frac{3}{35}$

18. $(D^2 + 8D + 17I)y = 474.5 \sin 0.5t, \quad y(0) = -5.4,$
$y'(0) = 9.4$

19. $(D^2 + 2D + 2I)y = e^{-t/2} \sin \frac{1}{2} t, \quad y(0) = 0,$
$y'(0) = 1$

20. $(D^2 + 5I)y = \cos \pi t - \sin \pi t, \quad y(0) = 0, \quad y'(0) = 0$

21. Beats. Derive the formula after (12) from (12). Can we have beats in a damped system?

22. Beats. Solve $y'' + 25y = 99 \cos 4.9t$, $y(0) = 2$, $y'(0) = 0$. How does the graph of the solution change if you change **(a)** $y(0)$, **(b)** the frequency of the driving force?

23. TEAM EXPERIMENT. Practical Resonance.
(a) Derive, in detail, the crucial formula (16).

(b) By considering dC^*/dc show that $C^*(\omega_{max})$ increases as c ($\leqq \sqrt{2mk}$) decreases.

(c) Illustrate practical resonance with an ODE of your own in which you vary c, and sketch or graph corresponding curves as in Fig. 57.

(d) Take your ODE with c fixed and an input of two terms, one with frequency close to the practical resonance frequency and the other not. Discuss and sketch or graph the output.

(e) Give other applications (not in the book) in which resonance is important.

24. Gun barrel. Solve $y'' + y = 1 - t^2/\pi^2$ if $0 \leqq t \leqq \pi$ and 0 if $t \to \infty$; here, $y(0) = 0$, $y'(0) = 0$. This models an undamped system on which a force F acts during some interval of time (see Fig. 59), for instance, the force on a gun barrel when a shell is fired, the barrel being braked by heavy springs (and then damped by a dashpot, which we disregard for simplicity). *Hint:* At π both y and y' must be continuous.

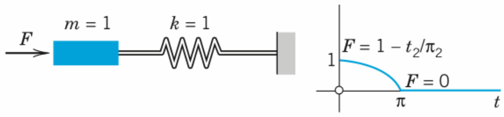


Fig. 59. Problem 24

25. CAS EXPERIMENT. Undamped Vibrations.
(a) Solve the initial value problem $y'' + y = \cos \omega t$, $\omega^2 \neq 1$, $y(0) = 0$, $y'(0) = 0$. Show that the solution can be written

$$y(t) = \frac{2}{1 - \omega^2} \sin\left[\tfrac{1}{2}(1 + \omega)t\right] \sin\left[\tfrac{1}{2}(1 - \omega)t\right].$$

(b) Experiment with the solution by changing ω to see the change of the curves from those for small ω (>0) to beats, to resonance, and to large values of ω (see Fig. 60).

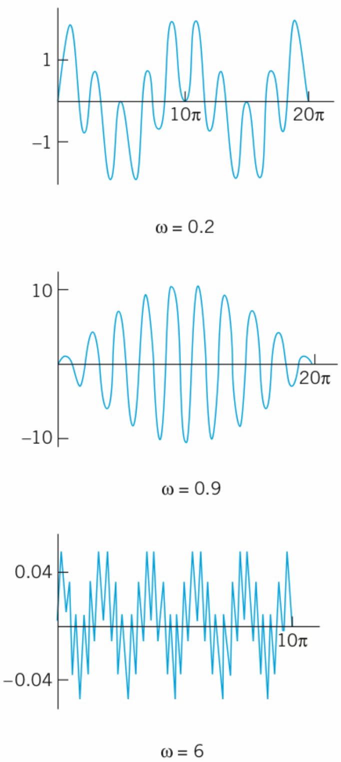


Fig. 60. Typical solution curves in CAS Experiment 25

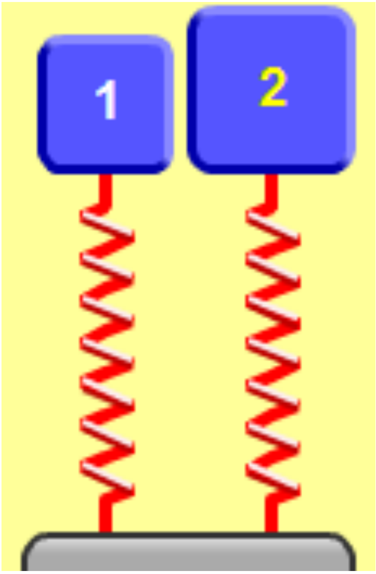
Resonance

https://phet.colorado.edu/en/simulation/resonance

Learning Goals: Students will be able to:

- Identify/explain the variables that affect the natural frequency of a mass-spring system.
- Explain the distinction between transient and steady-state behavior in a driven system.
- Identify which variables affect the duration of the transient behavior.
- Recognize the phase relationship between the driving frequency and the natural frequency, especially how the phase is different above and below resonance.
- Give examples the application of real-world systems to which the understanding of resonance should be applied and explain why.

1. **Which system will have the lower resonant frequency?**

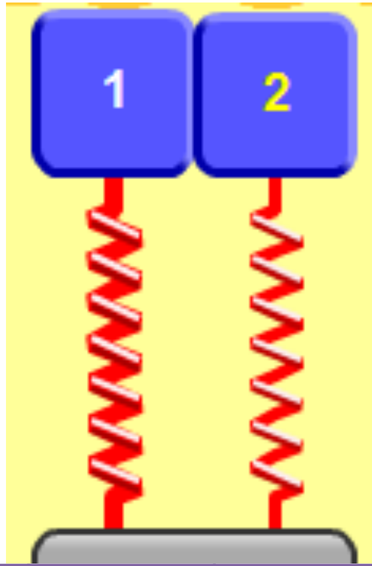


B

Mass (kg)	2.5	5.0
Spring constant (N/m)	100	100
Driver Amplitude (cm)	2.0	4.0

A) 1   B) 2 C) Same frequency

2. **Which system will have the higher resonant frequency?**

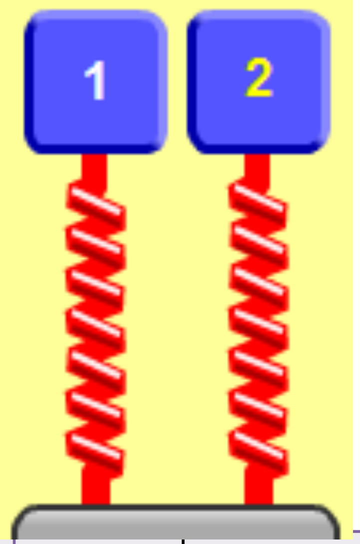


A

Mass (kg)	**5.0**	**5.0**
Spring constant (N/m)	**200**	**100**
Driver Amplitude (cm)	**2.0**	**4.0**

A) 1   B) 2 C) Same frequency.

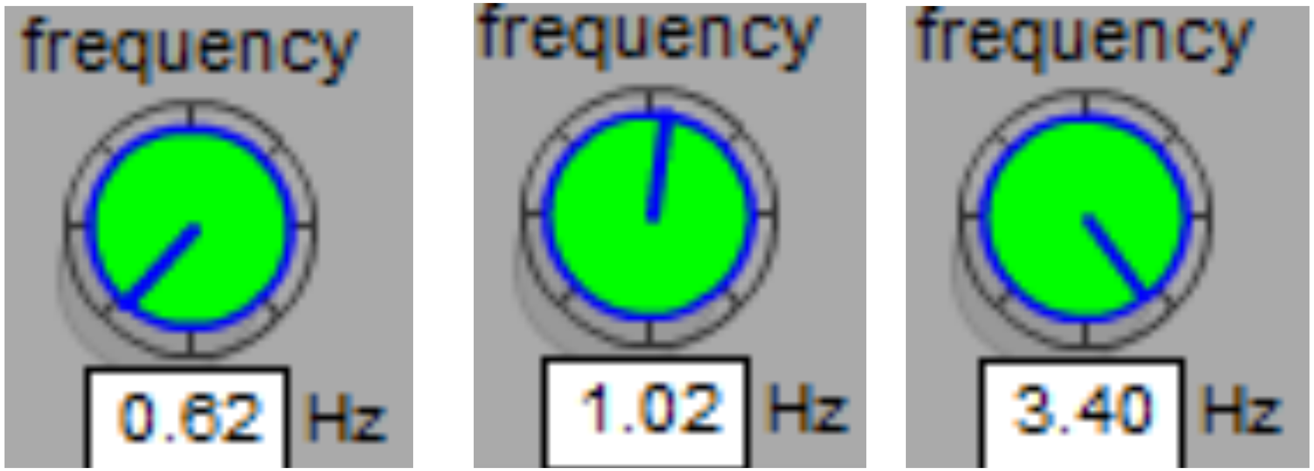
3. **Which system will have the lower resonant frequency?**

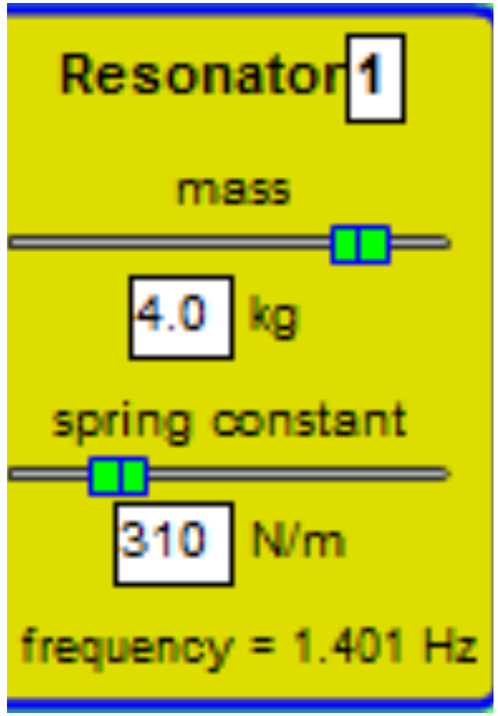


Mass (kg)	3.0	3.0
Spring constant (N/m)	400	400
Driver Amplitude (cm)	0.5	1.5

A) 1 B) 2 C) Same frequency.

4. **If the frequency f of the driver is not the same as the resonant frequency, which statement is most accurate?**





The steady-state amplitude is ..

a) smallest at the highest driver f.

b) largest at the highest driver f.

c) is largest at driver f nearest the resonant frequency.

d) is independent of driver f.

C

5. **Transient behavior will last longer when**

A) the damping constant is decreased.
B) the driving amplitude is increased.
C) Both of these.
D) None of these. The transient behavior is independent of the damping and the amplitude.

B

6. **Two resonators are driven at the same driving frequency and amplitude. One resonator has a resonant frequency 2 Hz below the driving frequency. The other has a resonant frequency 2 Hz above the driving frequency.**

Which resonator has the smallest steady-state amplitude?

A) The lower frequency resonator.
B) The higher frequency resonator.
C) Both have the same steady-state amplitude.

B