HW02

Partial Differential Equations and Complex Variables (EE 2021 Summer)

<u>Due date: 09/30 (Thur.)</u> Upload to eeclass.

I. Problems in the Problem Set 11.2 (page 490): **1, 2, 3, 4, 5, 6, 7**

II. Problems in the Problem Set 11.2 (page 490): **10, 11, 24, 28** (and apply the matlab program, graphing the first few partial sums of each of the four series on common axes. Choose the first five or more partial sums until they approximate the given function reasonably well. Compare and comment.)

III. Please convert the F.S. of $\cos(2t) + 3\times \sin(2t)\ into\ the\ form\ of\ X\times \cos(2t + Y)$. What X and Y should be?

We insert these two results into the formula for a_n. The sine terms cancel and so does a factor L^2. This gives

$$a_n = \frac{4k}{n^2\pi^2}\left(2\cos\frac{n\pi}{2} - \cos n\pi - 1\right).$$

Thus,

$$a_2 = -16k/(2^2\pi^2), \qquad a_6 = -16k/(6^2\pi^2), \qquad a_{10} = -16k/(10^2\pi^2), \cdots$$

and $a_n = 0$ if $n \neq 2, 6, 10, 14, \cdots$. Hence the first half-range expansion of $f(x)$ is (Fig. 272a)

$$f(x) = \frac{k}{2} - \frac{16k}{\pi^2}\left(\frac{1}{2^2}\cos\frac{2\pi}{L}x + \frac{1}{6^2}\cos\frac{6\pi}{L}x + \cdots\right).$$

This Fourier cosine series represents the even periodic extension of the given function $f(x)$, of period $2L$.

(b) ***Odd periodic extension.*** Similarly, from (6**) we obtain

$$b_n = \frac{8k}{n^2\pi^2}\sin\frac{n\pi}{2}. \tag{5}$$

Hence the other half-range expansion of $f(x)$ is (Fig. 272b)

$$f(x) = \frac{8k}{\pi^2}\left(\frac{1}{1^2}\sin\frac{\pi}{L}x - \frac{1}{3^2}\sin\frac{3\pi}{L}x + \frac{1}{5^2}\sin\frac{5\pi}{L}x - + \cdots\right).$$

The series represents the odd periodic extension of $f(x)$, of period $2L$.

Basic applications of these results will be shown in Secs. 12.3 and 12.5. ■

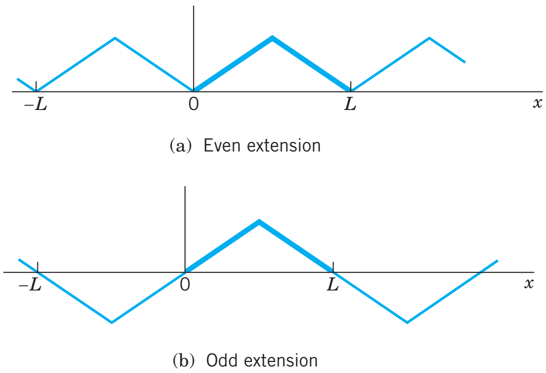


Fig. 272. Periodic extensions of $f(x)$ in Example 6

PROBLEM SET 11.2

1–7 EVEN AND ODD FUNCTIONS

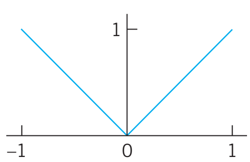
Are the following functions even or odd or neither even nor odd?

1. e^x, $e^{-|x|}$, $x^3 \cos nx$, $x^2 \tan \pi x$, $\sinh x - \cosh x$
2. $\sin^2 x$, $\sin(x^2)$, $\ln x$, $x/(x^2 + 1)$, $x \cot x$
3. Sums and products of even functions
4. Sums and products of odd functions
5. Absolute values of odd functions
6. Product of an odd times an even function
7. Find all functions that are both even and odd.

8–17 FOURIER SERIES FOR PERIOD $p = 2L$

Is the given function even or odd or neither even nor odd? Find its Fourier series. Show details of your work.

8.



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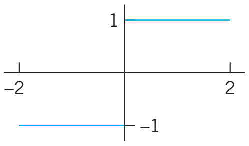

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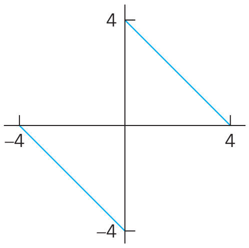
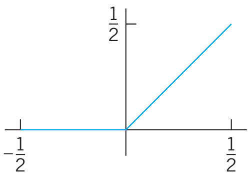


11. $f(x) = x^2 \quad (-1 < x < 1), \quad p = 2$

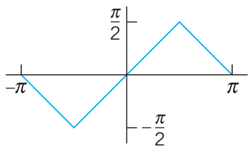
12. $f(x) = 1 - x^2/4 \quad (-2 < x < 2), \quad p = 4$

13.



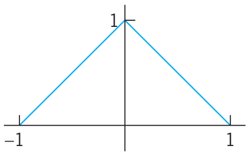
14. $f(x) = \cos \pi x \quad (-\frac{1}{2} < x < \frac{1}{2}), \quad p = 1$

15.



16. $f(x) = x|x| \quad (-1 < x < 1), \quad p = 2$

17.



18. Rectifier. Find the Fourier series of the function obtained by passing the voltage $v(t) = V_0 \cos 100\pi t$ through a half-wave rectifier that clips the negative half-waves.

19. Trigonometric Identities. Show that the familiar identities $\cos^3 x = \frac{3}{4} \cos x + \frac{1}{4} \cos 3x$ and $\sin^3 x = \frac{3}{4} \sin x - \frac{1}{4} \sin 3x$ can be interpreted as Fourier series expansions. Develop $\cos^4 x$.

20. Numeric Values. Using Prob. 11, show that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{1}{6}\pi^2$.

21. CAS PROJECT. Fourier Series of $2L$-Periodic Functions. (a) Write a program for obtaining partial sums of a Fourier series (5).

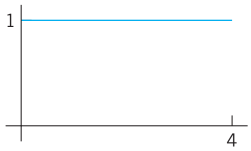
(b) Apply the program to Probs. 8–11, graphing the first few partial sums of each of the four series on common axes. Choose the first five or more partial sums until they approximate the given function reasonably well. Compare and comment.

22. Obtain the Fourier series in Prob. 8 from that in Prob. 17.

23–29 HALF-RANGE EXPANSIONS

Find **(a)** the Fourier cosine series, **(b)** the Fourier sine series. Sketch $f(x)$ and its two periodic extensions. Show the details.

23.



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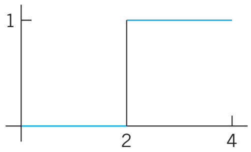

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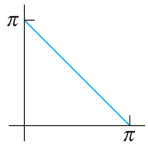

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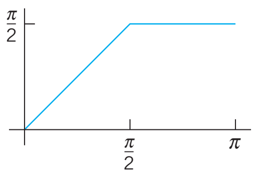

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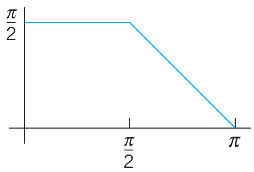

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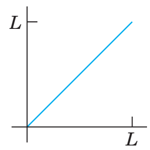

29. $f(x) = \sin x \ (0 < x < \pi)$

30. Obtain the solution to Prob. 26 from that of Prob. 27.