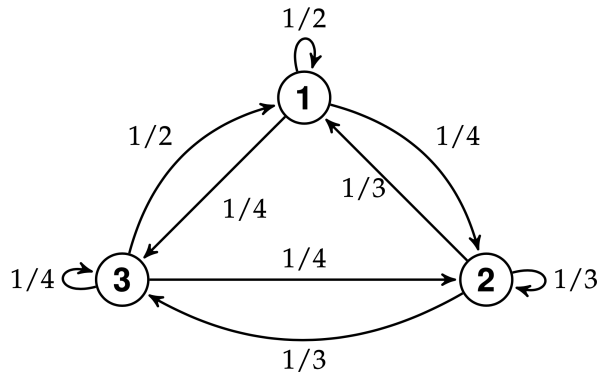# CS5314 Randomized Algorithms

Exam 3 – Session 2
Date: June 30, 2020 (2 hours)

Answer all questions. Total marks = 30 + 70 = 100.

1. Consider a Markov chain $\{X_0, X_1, X_2, \ldots\}$ whose transition diagram is shown in the following figure.

Suppose that

$$\Pr(X_0 = 1) = 1/3, \quad \Pr(X_0 = 2) = 1/2, \quad \Pr(X_0 = 3) = 1/6.$$

   (a) (15%) What is $\Pr(X_1 = 1)$?

   (b) (15%) What is $\Pr((X_0 = 2) \mid (X_1 = 1))$?

2. Suppose that we have a Markov chain with three states, 0, 1, and 2.

   - For state 0, we have probability 0.3 to stay and 0.7 to go to state 1.

   - For state 1, we have probability 0.4 to stay, 0.3 to go to state 2, and probability 0.3 to go to state 0.

   - For state 2, we have probability 0.7 to stay, and 0.3 to go to state 1.

   (a) (10%) Give the transition matrix P for the above Markov chain, where $P_{i,j}$ denotes the probability of moving from state i to state j.

   (b) (10%) Give the graphical representation of the above Markov chain.

   (c) (30%) Argue that the Markov chain is aperiodic and irreducible.

   (d) (20%) Find the stationary probability.