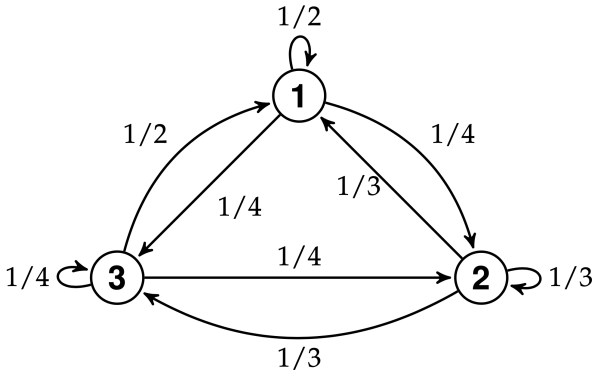
CS5314 Randomized Algorithms

Exam 3
Date: June 23, 2020 (2 hours)

Answer all questions. Total marks = 30 + 70 = 100.

1. Consider a Markov chain $\{X_0, X_1, X_2, \ldots\}$ whose transition diagram is shown in the following figure.



Suppose that

$$\Pr(X_0 = 1) = 1/2, \quad \Pr(X_0 = 2) = 1/4, \quad \Pr(X_0 = 3) = 1/4.$$

(a) (15%) What is $\Pr(X_1 = 2)$?

(b) (15%) What is $\Pr((X_1 = 2) \cap (X_2 = 3) \cap (X_3 = 2))$?

2. Suppose that we have a Markov chain with three states, 0, 1, and 2.

- For state 0, we have probability 0.2 to stay and 0.8 to go to state 1.
- For state 1, we have probability 0.2 to stay, 0.3 to go to state 2, and probability 0.5 to go to state 3.
- For state 2, we have probability 0.4 to stay, and 0.6 to go to state 1.

(a) (10%) Give the transition matrix P for the above Markov chain, where $P_{i,j}$ denotes the probability of moving from state i to state j.

(b) (10%) Give the graphical representation of the above Markov chain.

(c) (30%) Argue that the Markov chain is aperiodic and irreducible.

(d) (20%) Find the stationary probability.