PDE HW 3

106033233 周聖諺

P91.

3. $y''+6y'+8y=42.5\cos 2t$

$\Rightarrow$ Homogeneous

$y''+6y'+8y=0$

Guess $y_h = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t}$

Let $y = e^{\alpha t}$

$\alpha^2 e^{\alpha t} + 6\alpha e^{\alpha t} + 8e^{\alpha t} = 0$

$(\alpha^2+6\alpha+8)e^{\alpha t} = 0$

$(\alpha+4)(6+2)e^{\alpha t} = 0$

$\alpha = -4, -2$

$\Rightarrow y_h = c_1 e^{-4t} + c_2 e^{-2t}$

$\Rightarrow$ Nonhomogeneous

$y''+6y'+8y=42.5\cos 2t$

Guess $y_p = K\cos 2t + M\sin 2t$

$y' = -2K\sin 2t + 2M\cdot\cos 2t$

$y'' = -4K\cdot\cos 2t - 4M\cdot\sin 2t$

$\frac{85}{80} = \frac{17}{16}$

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$(-4K\cos 2t - 4M\sin 2t) + 6(-2K\sin 2t + 2M\cos 2t)$

$+ 8(K\cdot\cos 2t + M\sin 2t) = 42.5\cos 2t$

$(-4K + 12M + 8K)\cos 2t + (-4M - 12K + 8M)\sin 2t$

$= 42.5\cos 2t$

$18 \times \frac{4.9}{2}$ $= 18 \times 24.5$ $=$

$\begin{cases} 4K + 12M = 42.5 \\ -12K + 4M = 0 \end{cases}$, $12M = 36K$

$\begin{array}{r} 49 \\ +36 \\ \hline 85 \end{array}$

$4K + 36K = 42.5$

$40K = 42.5$

$M = 3K$

$K = \frac{17}{16}$, $M = \frac{51}{16}$, $y_p = \frac{17}{16}\cos 2t + \frac{51}{16}\sin 2t$

$\Rightarrow$ Steady State Solution

$y_p = \frac{17}{16}\cos 2t + \frac{51}{16}\sin 2t$ #

Transient Solution:

$y = c_1 e^{-4t} +$

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P92.

10.

$$y'' + 16y = 56\cos 4t$$

$\Rightarrow$ Homogeneous

$$y'' + 16y = 0$$

Guess $y_h = e^{\alpha t}(A\cos wt + B\sin wt)$

Let $y = e^{\alpha t}$

$$y' = \alpha e^{\alpha t} \qquad y'' = \alpha^2 e^{\alpha t}$$

$$\alpha^2 e^{\alpha t} + 16 e^{\alpha t} = 0$$

$$\alpha^2 + 16 = 0$$

$$\alpha = \pm\sqrt{-16} = \pm 4i$$

Thus $y = e^{4i}, e^{-4i}$

$$y_h = A\cos 4t + B\sin 4t$$

$\Rightarrow$ Nonhomogeneous

$$y'' + 16y = 56\cos 4t$$

Guess $y_p = M\cos 4t + K\sin 4t$

Let $y = M\cos 4t + K\sin 4t$

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$$y' = -4M \sin 4t + 4K \cos 4t$$

$$y'' = -16M \cos 4t - 16K \sin 4t$$

$$(-16M \cos 4t - 16K \sin 4t) + 16(M \cos 4t + K \sin 4t) = 56 \cos 4t$$

$\Rightarrow$ There is no solution for M, K

Guess $y_p = e^{\alpha t}(A \cos 4t + B \sin 4t)$

Let $y = e^{\alpha t}(A \cos 4t + B \sin 4t)$

$$= e^{\alpha t} A \cos 4t + e^{\alpha t} B \sin 4t$$

$$y' = \alpha e^{\alpha t} A \cos 4t - 4e^{\alpha t} A \sin 4t + \alpha e^{\alpha t} B \sin 4t + 4e^{\alpha t} B \cos 4t$$

$$y'' = \alpha^2 e^{\alpha t} A \cos 4t - 4\alpha e^{\alpha t} A \sin 4t - 4\alpha e^{\alpha t} A \sin 4t - 16 e^{\alpha t} A \cos 4t + \alpha^2 e^{\alpha t} B \sin 4t + 4\alpha e^{\alpha t} B \cos 4t + 4\alpha e^{\alpha t} B \cos 4t - 16 e^{\alpha t} B \sin 4t$$

$$= (\alpha^2 e^{\alpha t} A - 16 e^{\alpha t} A + 4\alpha e^{\alpha t} B + 4\alpha e^{\alpha t} B) \cos 4t + (-4\alpha e^{\alpha t} A - 4\alpha e^{\alpha t} A + \alpha^2 e^{\alpha t} B - 16 e^{\alpha t} B) \sin 4t$$

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$(\alpha^2 A - 16A + 4\alpha B + 4\alpha B) \cdot e^{\alpha t} \cos 4t$
$+ (-4\alpha A - 4\alpha A + \alpha^2 B - 16B) \cdot e^{\alpha t} \sin 4t$
$+ 16 e^{\alpha t} A \cdot \cos 4t + 16 e^{\alpha t} B \cdot \sin 4t$
$= 56 \cos 4t$

Thus, $\alpha = 0$

$$\begin{cases} (\alpha^2 A - 16A + 8\alpha B) + 16A = 56 \\ (\alpha^2 B - 16B - 8\alpha A) + 16B = 0 \end{cases}$$

$$\begin{cases} \alpha^2 A + 8\alpha B = 56 \\ \alpha^2 B - 8\alpha A = 0 \end{cases}$$

$\Rightarrow$ There is no solution for A, B, α

Guess $y_p = t(A \cos 4t + B \sin 4t)$

Let $y = t(A \cos 4t + B \sin 4t)$

$y' = (A \cos 4t + B \sin 4t) + t(-4A \sin 4t + 4B \cos 4t)$

$y'' = (-4A \sin 4t + 4B \cos 4t) + (-4A \sin 4t + 4B \cos 4t) + t(-16A \cos 4t - 16B \sin 4t)$

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$(4B+4B-16At)\cos 4t + (-4A-4A-16Bt)$
$\sin 4t + 16(At\cos 4t + Bt\sin 4t) = 56\cos 4t$

$(8B - \cancel{16At} + \cancel{16At})\cos 4t + (-8A - \cancel{16Bt} + \cancel{16Bt})$
$\sin 4t = 56\cos 4t$

$$8B\cos 4t - 8A\sin 4t = 56\cos 4t$$

$$\begin{cases} 8B = 56 \\ -8A = 0 \end{cases} \Rightarrow A = 0,\ B = 7$$

$$y_p = 7t\sin 4t$$

$\Rightarrow$ Thus, transient solution:

$$y = y_h + y_p = A\cos 4t + B\sin 4t + 7t\sin 4t$$

#

Chooses

1. B
2. A
3. C
4. C
5. B
6. B