Assignment 5

Q5, 7, 8

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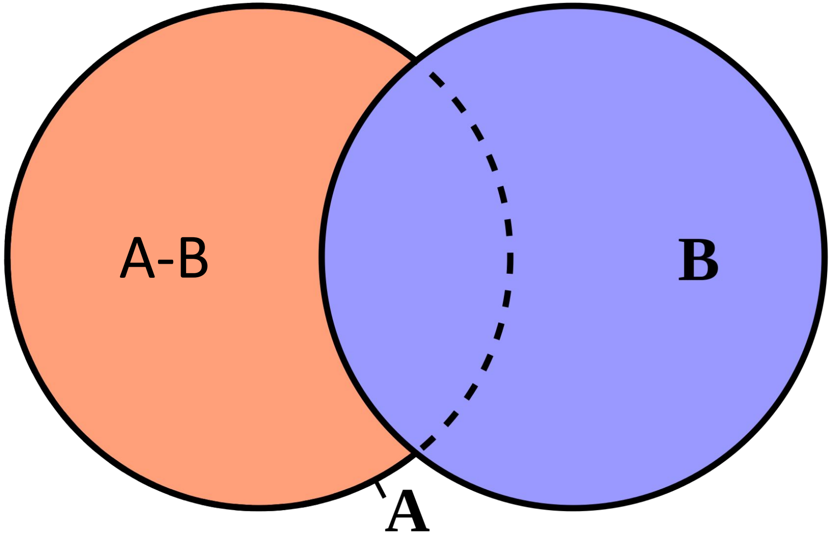
5. (*) For two sets A and B, we use $A - B$ to denote $A \cap \overline{B}$. Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

- It is simple with Venn diagram



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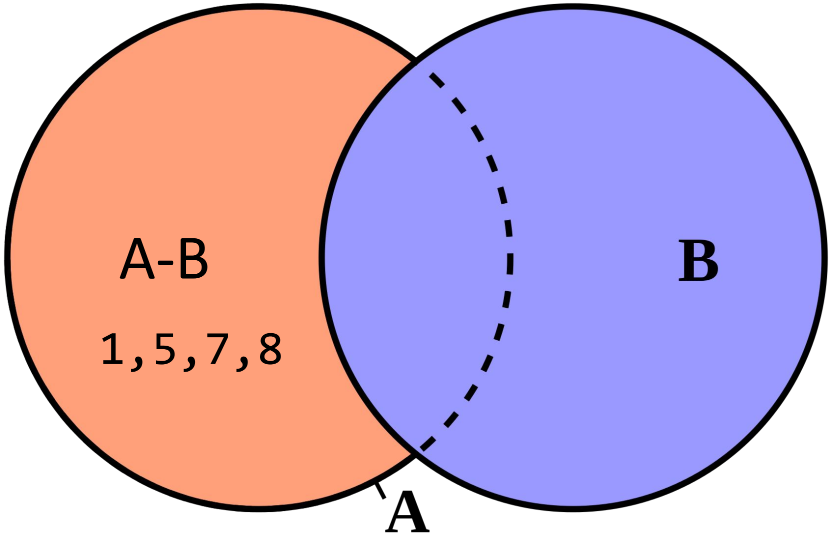

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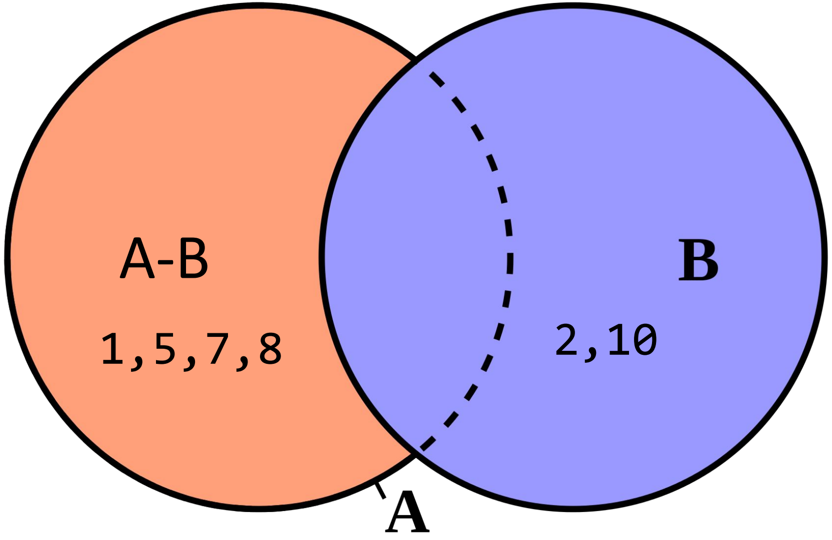

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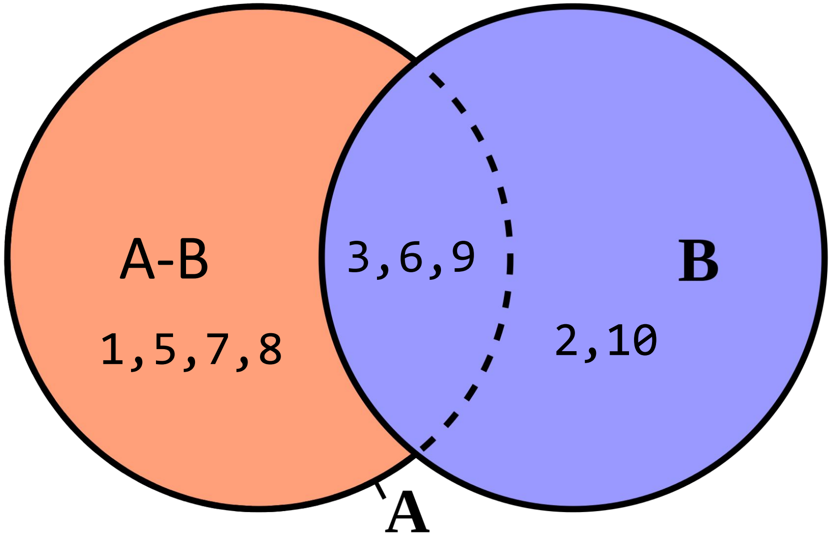
- It is simple with Venn diagram

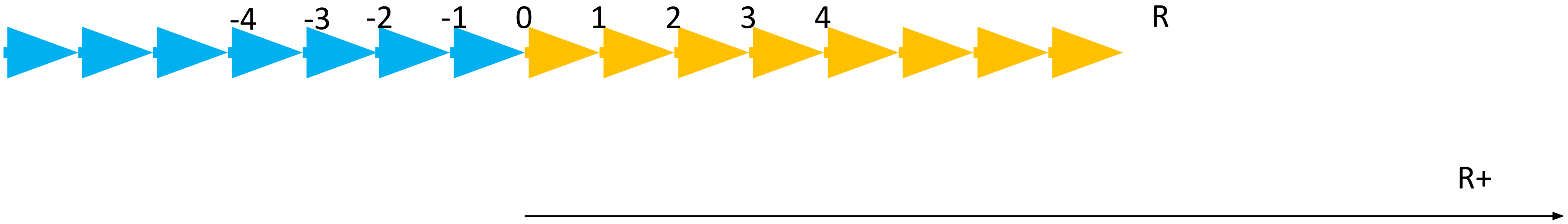
A={1,5,7,8,3,6,9}

B={2,10,3,6,9}



7. (*) Show that the set $\mathbb{R}$ of real numbers and the set $\mathbb{R}^+$ of positive real numbers have the same cardinality. (That is, give a one-to-one correspondence between the items in the two sets.)

- Here give a way to map R / R+



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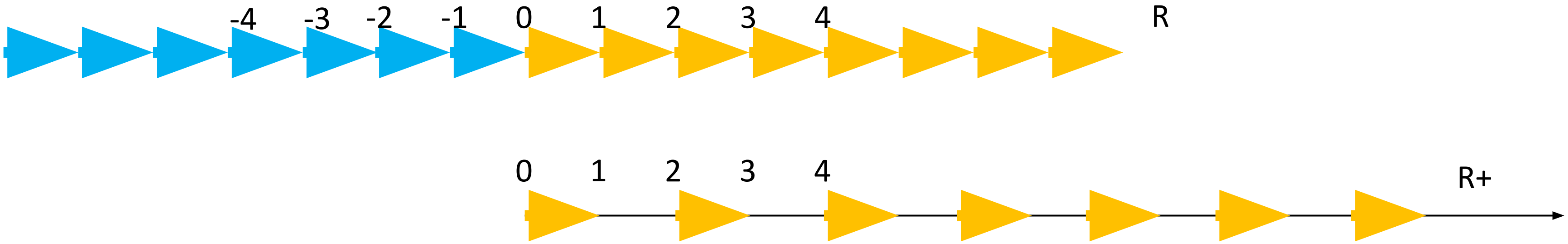

Let x in R

If x = 0, map to 0

If x > 0, map to x + [x]   (floor sign)

7. (*) Show that the set $\mathbb{R}$ of real numbers and the set $\mathbb{R}^+$ of positive real numbers have the same cardinality. (That is, give a one-to-one correspondence between the items in the two sets.)

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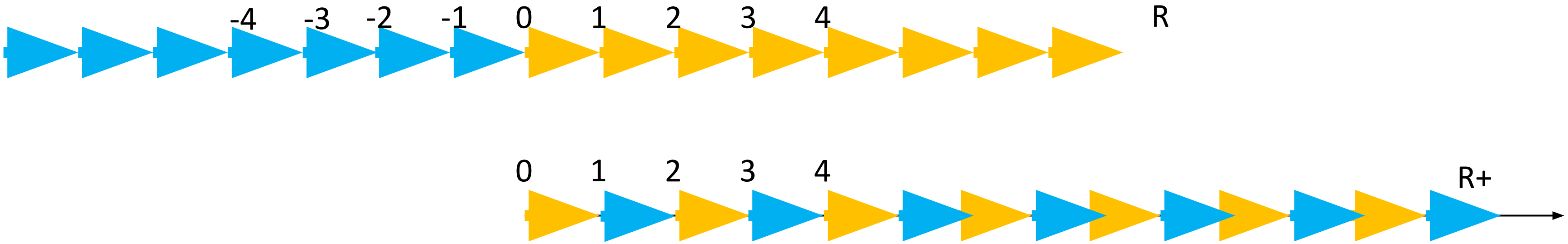
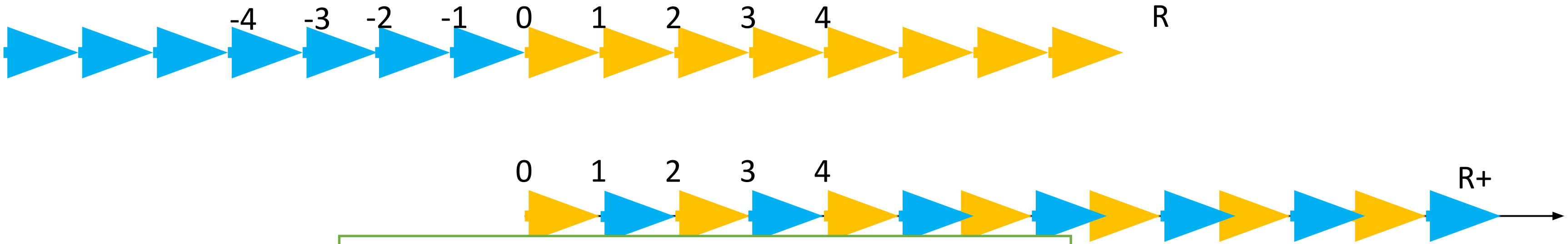


Let x in R

If x = 0, map to 0
If x > 0, map to x + [x] (floor sign)
If x < 0, map to -x-[x] (floor sign)

7. (*) Show that the set $\mathbb{R}$ of real numbers and the set $\mathbb{R}^+$ of positive real numbers have the same cardinality. (That is, give a one-to-one correspondence between the items in the two sets.)

- Here give a way to map R / R+



Because we found R is one one onto R+.
R and R+ have same cardinality.

Let x in R

If $x = 0$, map to 0
If $x > 0$, map to $x + [x]$ (floor sign)
If $x < 0$, map to $-x-[x]$ (floor sign)

Let x in R+

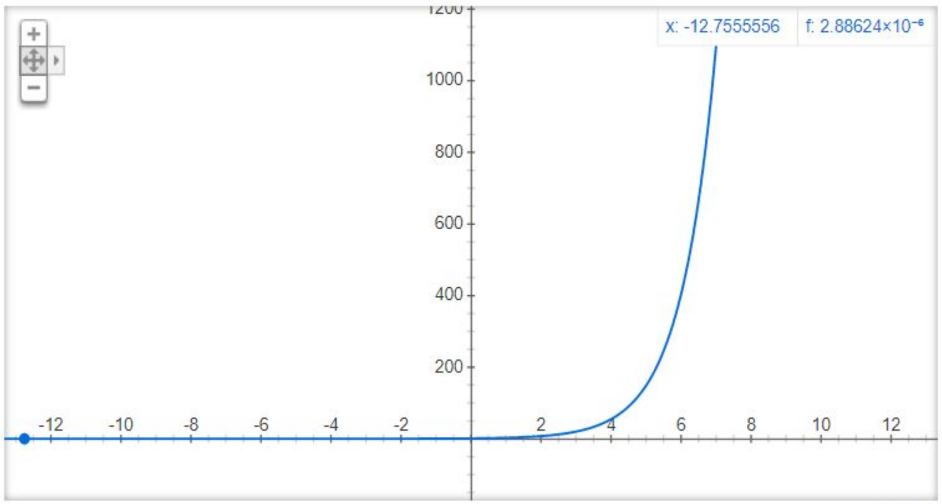
If $x = 0$, map to 0
If $[x]$ is even, map to $x - [x]/2$
If $[x]$ is odd, map to $-x+[x+1]/2$

Another way

$\boldsymbol{x}$ in $\mathbb{R}$ map to $\mathbb{R}$+ by $\boldsymbol{f(x) = e^{x}}$

For any $x \neq y$, $f(x) \neq f(y)$
For any $x \in \mathbb{R}+$, exist $k = \ln x \in \mathbb{R}$ let $f(k) = x$
So $\mathbb{R}+$ is one-one onto $\mathbb{R}$ by f

e^x 的圖表



HW5 Q8

- Peter show that is uncountable by diagonalization technique

$$X = \{ x \mid x \in (0, 1) \text{ and } x \text{ has } k \text{ decimal places and } k \in N \}$$

However, each number in X is a rational number; for instance, 0.33215 = 33215/100000. Thus, $X \subseteq \mathbb{Q}$ (where $\mathbb{Q}$ is countable), which implies X must be countable. So, what's wrong with Peter's proof?

HW5 Q8

X = { x | x ∈ (0, 1) and x has k decimal places and k ∈ N }

We prove this by contradiction. Assume to the contrary that there is a one-to-one correspondence between items in X and items in $\mathbb{N}$. Then, we can list the items in X one by one, say $x_1, x_2, x_3, \ldots$. Now, consider the number x such that its digit in the first decimal place is different from x_1, its digit in the second decimal place is different from x_2, and in general, its digit in the jth decimal palce is different from x_j for all j. Then, x is not listed by the correspondence, and a contradiction occurs as desired.

- x found by Peter is not in set X

HW5 Q8

X = { x | x ∈ (0, 1) and x has k decimal places and k ∈ N }

x found by Peter is not in set X

- If x in X, let x is k-digit number.
- Consider x_{k+1}, and let k+1 digit in x_{k+1} is 0
  - k+1 digit in x is the inverse of x_{k+1}.
  - But x is k-digit number, k+1 digit in x must be 0.
- So such x not exists.

Assignment 5

Q10,Q11,Q13

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Q10

10. (*) Consider all the five-element subsets of $\{1, 2, 3, \ldots, n\}$. It is known that one quarter of these subsets contain the element 7. What is the value of n?

Q10

of the five-element subsets :C_5^n

of the five-element subsets contain the element 7: C_4^{n-1}

$\Rightarrow C_5^n$ = 4 * C_4^{n-1}

$\Rightarrow$ n*(n-1)*(n-2)*(n-3)*(n-4)/5! = 4*(n-1)*(n-2)*(n-3)*(n-4)/4!

$\Rightarrow$ n = 20

Q11

11. (*) Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

(a) $f(x) = -3x + 4$

(b) $f(x) = 3x^2 + 4$

(c) $f(x) = (x + 1)/(x + 2)$

Q11(a)

- $F(x) = -3x+4$,from R to R

Proof injection:

Let $x , y \in R$ and $F(x) = F(y)$

$\Rightarrow -3x + 4 = -3y + 4$

$\Rightarrow x = y$

$\Rightarrow$ We know that $\forall\, x , y \in R$ (domain) , if $F(x) = F(y)$ then $x = y$

$\Rightarrow$ F is an injection

Q11(a) cont.

- F(x) = -3x+4 ,from R to R

Proof surjection:

let y ∈ R

$\Rightarrow$We can find x = - $\frac{y-4}{3}$ ∈ R such that F(x) = y

$\Rightarrow$We know that ∀ y ∈ R(codomain) ,there is a x ∈ R(domain) such that F(x) = y

$\Rightarrow$F is a surjection

$\Rightarrow$F is injective and surjective , F is a bijection

Q11(b)

- F(x) = $3x^2$ +4 , from R to R

Proof F is not injection:

Let x = 1 , y= - 1

$\Rightarrow$F(x) = 3*1^2+4 = 3*(-1^2)+4 = F(y)

$\Rightarrow$We know that ,$\exists$ x , y $\in$ R (domain) x != y such that F(x) = F(y)

$\Rightarrow$F is not an injection

$\Rightarrow$F is not a bijection

Q11(c)

$F(x) = (x+1)/(x+2)$, from R to R

Obviously , we can find that there is no $y \in R$ (codomain) such that

$F(-2) = y$

$\Rightarrow$ F is not a function .

$\Rightarrow$ F can not be a bijection.

Q13

13. (*) Define $g : \mathbb{Z} \to \mathbb{Z}$, where $g(n) = \lfloor n/2 \rfloor$. Is g a surjection? Is g a bijection?

Q13

g(n) = $\lfloor n/2 \rfloor$,from Z to Z

Proof surjection:

Let y ∈ Z (codomain)

$\Rightarrow$We can find x = 2y such that g(x) = y

$\Rightarrow$F is a surjection

Q13 cont.

$g(n) = \lfloor n/2 \rfloor$,from Z to Z

Proof F is not injection:

Let x = 1,y = 0

$\Rightarrow g(x) = \lfloor 1/2 \rfloor = 0 = g(y)$

$\Rightarrow$g is not an injection

$\Rightarrow$g is not a bijection

HW5 Q15 16 20

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HW5 - Q15

- Transitive property: what if $\nexists y \; x\mathcal{R}y$ for some x?

15. (*) What is wrong with the following argument that attempts to show that if R is a relation on a set S that is both symmetric and transitive, then R is also reflexive?

> "Since xRy implies yRx by the symmetric property,
> xRy and yRx imply xRx by the transitive property,
> thus, xRx is true for each x in S, and so R is reflexive."

A binary relation R on A is said to be symmetric if (a, b) $\in$ R implies (b, a) $\in$ R

A binary relation R on A is said to be transitive if (a, b), (b, c) $\in$ R implies (a, c) $\in$ R

A binary relation R on A is said to be reflexive if (a, a) $\in$ R for every a $\in$ A

HW5 - Q15

15. (*) What is wrong with the following argument that attempts to show that if R is a relation on a set S that is both symmetric and transitive, then R is also reflexive?

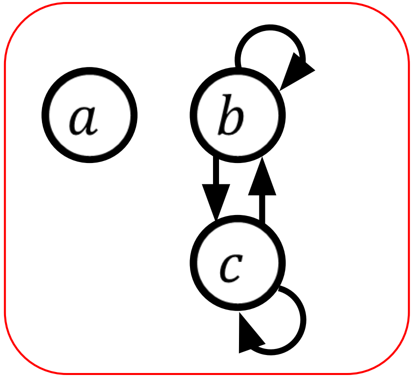
> "Since xRy implies yRx by the symmetric property,
> xRy and yRx imply xRx by the transitive property,
> thus, xRx is true for each x in S, and so R is reflexive."

- Consider some relation on a set S $= \{a, b, c\}$:
- It's symmetric

  $b\mathcal{R}c \to c\mathcal{R}b$, $c\mathcal{R}b \to b\mathcal{R}c$
- It's transitive

  $(b\mathcal{R}c \land c\mathcal{R}b) \to b\mathcal{R}b$

  $(c\mathcal{R}b \land b\mathcal{R}c) \to c\mathcal{R}c$



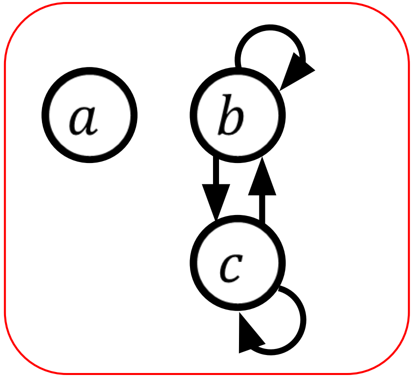
HW5 - Q15

- However, it’s not reflexive.

A binary relation R on A is said to be symmetric if $(a, b) \in R$ implies $(b, a) \in R$

A binary relation R on A is said to be transitive if $(a, b), (b, c) \in R$ implies $(a, c) \in R$

A binary relation R on A is said to be reflexive if $(a, a) \in R$ for $\boxed{\text{every}}$ $a \in A$

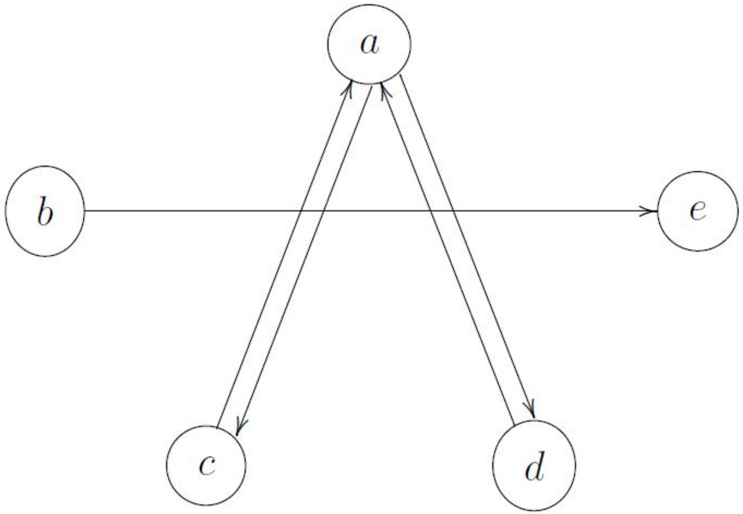


HW5 – Q16

16. (*)

(a) Find the reflexive closure of R as represented by the following directed graph.

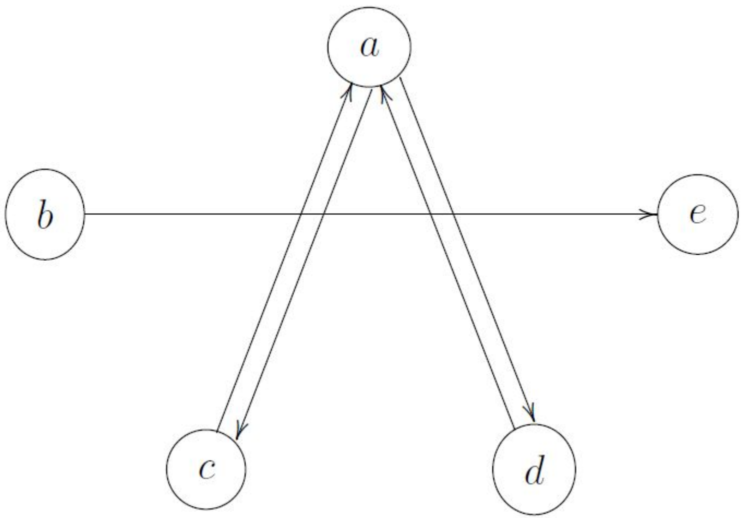
(b) Find the transitive closure of R as represented by the following directed graph.

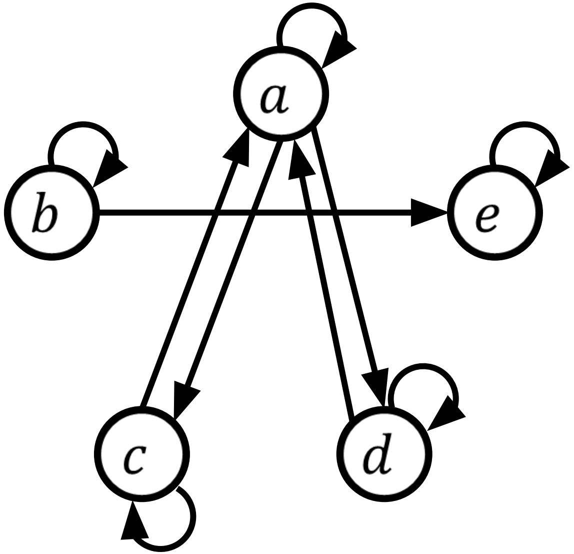


HW5 – Q16 (a)

A binary relation R on A is said to be reflexive if $(a, a) \in R$ for every $a \in A$

- Find the reflexive closure – **note** that it also contains the original graph.

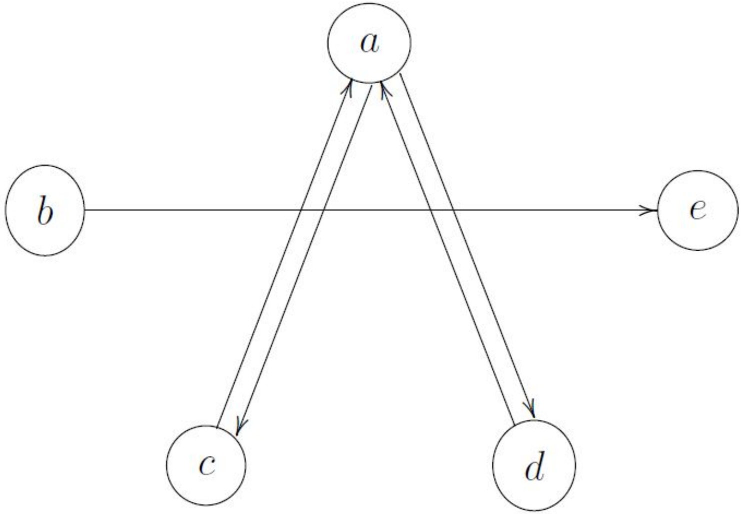


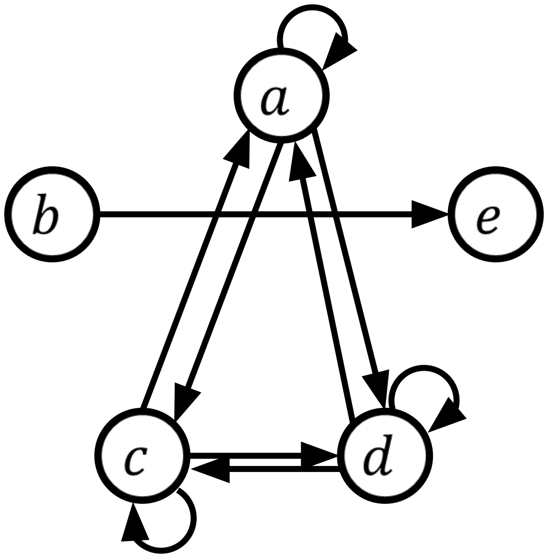


HW5 – Q16 (a)

A binary relation R on A is said to be transitive if $(a, b), (b, c) \in R$ implies $(a, c) \in R$

- Find the transitive closure – **note** that it also contains the original graph.





HW5 – Q20 (a)

- Let $A = \{0,1,2,3,4\}$ and $B = \{1,3,5,7,9\}$.
- If the relation $P: aPb \Leftrightarrow a \equiv b(mod\ 4)$

mod 4 remainder	A	B
0	0, 4	
1	1	1, 5, 9
2	2	
3	3	3, 7

- Then, $P = \{(1,1), (1,5), (1,9), (3,3), (3,7)\}$
- Note that for a relation aPb, $a \in A\ and\ b \in B$.

HW5 – Q20 (b)

- Let $A = \{0,1,2,3,4\}$ and $B = \{1,3,5,7,9\}$.
- If the relation $P: aPb \Leftrightarrow 2\ divides\ (a+b)$

(i) a = odd && b = odd
(ii) a = even && b = even

	A	B
odd	1, 3	1, 3, 5, 7, 9
even	0, 2, 4	

- Then, $P = \{(1,1),(1,3),(1,5),(1,7),(1,9),(3,1),(3,3),(3,5),(3,7),(3,9)\}$

HW5 – Q20 (c)

- Let $A = \{0,1,2,3,4\}$ and $B = \{1,3,5,7,9\}$.
- If the relation $P: aPb \Leftrightarrow a = b \; or \; a - 1 = b$
- Then, $P = \{(1,1),(2,1),(3,3),(4,3)\}$

Homework 6

Question 6,7

洪嘉陽

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Q6

- Definition: Two graphs G and H are ***isomorphic*** if there is a one-to-one correspondence f between the vertices of G and the vertices of H, such that u, v are adjacent in G if and only if $f(u), f(v)$ are adjacent in H.
- Suppose that G and H are ***isomorphic*** simple graphs.
- Show that their complementary graphs $\bar{G}$ and $\bar{H}$ are also *isomorphic*.

Q6 solution

- u and v are adjacent in $\bar{G}$
- $\Leftrightarrow u$ and v are not adjacent in G

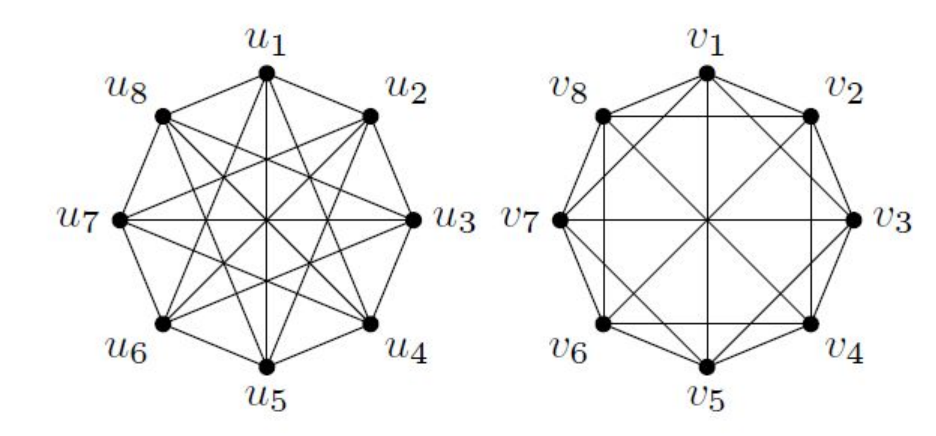
  (By definition of complementary graph)
- $\Leftrightarrow f(u)$ and $f(v)$ are not adjacent in H

  (By definition of isomorphic)
- $\Leftrightarrow f(u)$ and $f(v)$ are adjacent in $\overline{H}$

  (By definition of complementary graph)

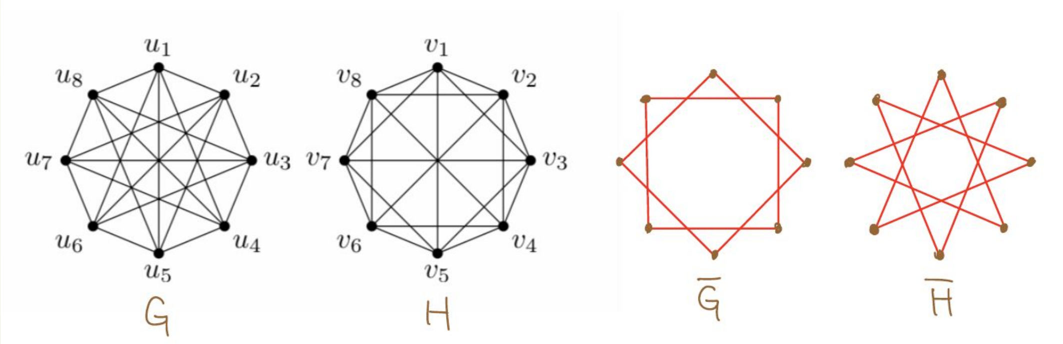
Q7

- Determine whether the following two graphs are isomorphic.



Q7 solution

- Take complementary to two graph, say $\bar{G}$ and $\overline{H}$
- Since $\bar{G}$ has two connected components, and $\overline{H}$ has only one connected component.
- $\bar{G}$ and $\overline{H}$ are not isomorphic.
- By Q6, G and H are not isomorphic.



Assignment 6

Q8/Q9

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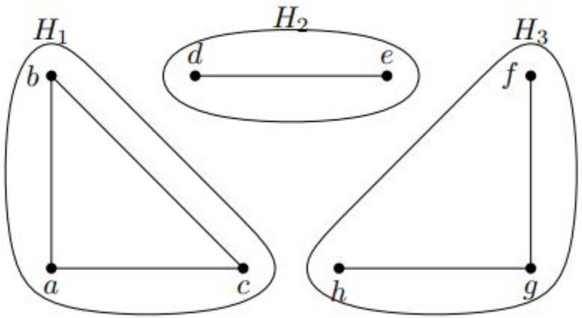
Q8

(*) An undirected graph is called *connected* if there is a path between every pair of distinct vertices of the graph, and a *connected component* of graph G is a maximal connected subgraph of G. For example, in the following graph, there are 3 connected components H_1, H_2, and H_3.



Suppose that a planar graph has k connected components, e edges, and v vertices. Also suppose that the plane is divided into r regions by a planar representation of the graph. Find a formula for r in terms of e, v, and k.

Q8

Suppose k connected component is $H_1 \sim H_k$, each component H_i have v_i vertices, e_i edges and r_i regions.

Solution 1

Obviously, each connected component satisfy Euler's planer formula. If we sum these k equation, we will count infinite region k times. So we count infinite region after summation.

Q8

$$H_1 : v_1 + (r_1 - 1) = e_1 + 1$$

$$H_2 : v_2 + (r_2 - 1) = e_2 + 1$$

…

$$H_k : v_k + (r_k - 1) = e_k + 1$$

Sum all equation : $G : v + (r - 1) = e + k$, $\left(\sum_{i=1}^{k}(r_i - 1) = r - 1\right)$

Add infinite region : $G : r = e - v + k + 1$

Q8

Solution 2

We can "connect" these k component by adding $k-1$ edges, so it will construct a connected graph G', and edges $e' = e + (k-1)$, vertices $v' = v$, regions $r' = r$.

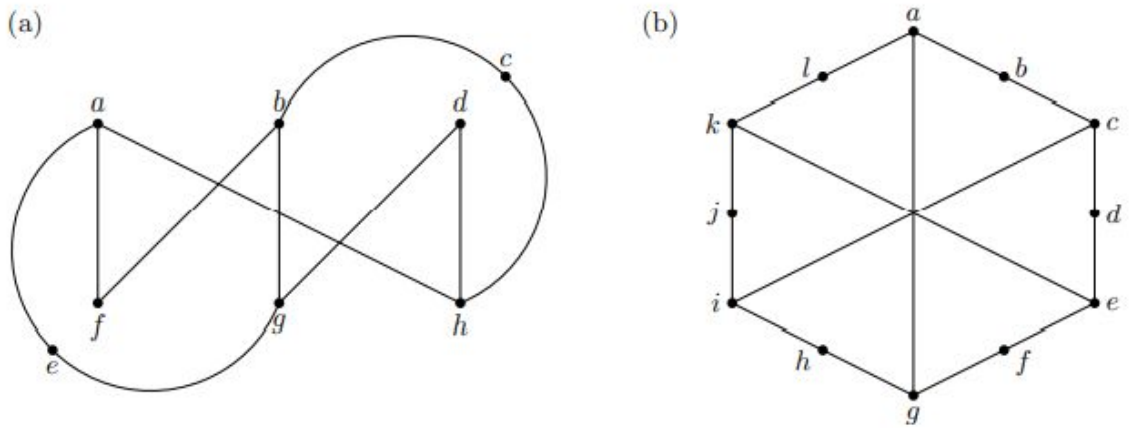
G' satisfy Euler's planer formula: $v' + r' = e' + 2$, so we have the equation of G.

$G: v + r = e + (k-1) + 2 = e + k + 1.$

$G: r = e - v + k + 1.$

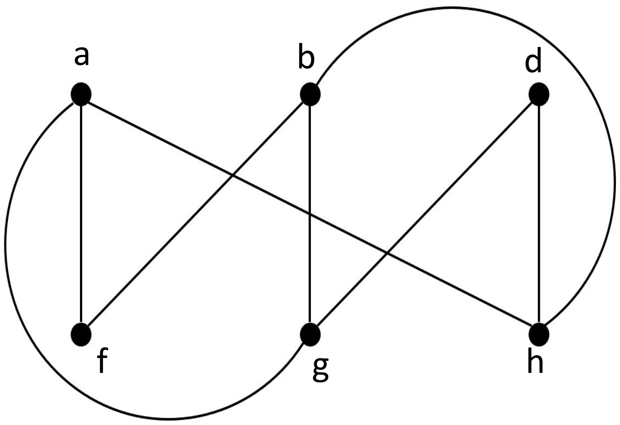
Q9

(*) Determine whether the following graphs is homeomorphic to $K_{3,3}$. (The definition of "homeomorphic" will be discussed on Dec 30, 2020.)

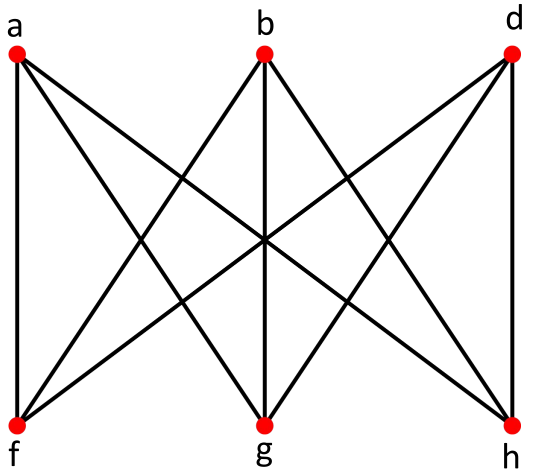


Q9

(a)



8 edges



$k_{3,3}$, 9 edges

Q9

(b)