CS5314 Randomized Algorithms

Homework 1
Due: April 7, 2020 (before 11:59 p.m.)

1 Part I: Multiple Choice

1. We roll two standard six-sided dice. Compute the following probabilities.

   (a) Pr(the second die is 5 | sum is odd) =? (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$

   (b) Pr(the second die is 4 | sum is 6) =? (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$

2. After lunch one day, Alice suggests to Bob the following method to determine who pays. Alice pulls four six-sided dice from her pocket. These dice are not the standard dice, but have the following numbers on their faces:

   - die A: 1, 1, 1, 5, 5, 5;
   - die B: 3, 3, 3, 3, 3, 3;
   - die C: 0, 0, 4, 4, 4, 4;
   - die D: 2, 2, 2, 2, 6, 6;

   The dice are fair, so each side comes up with equal probability. Alice explains that Alice and Bob will each pick up one of the dice. They will each roll their die, and the one who rolls the lowest number loses and will buy lunch. So as to take no advantage, Alice offers Bob the first choice of the dice.
   In this question, we will see an amazing fact: no matter which die Bob chooses, Alice can always chooses another die such that the probability that Alice wins is greater than 1/2. (In the following questions, we assume that Alice will choose a die such that the probability that Alice wins as high as possible. Please answer A, B, C, or D.)

   (a) Suppose that Bob chooses die A, which die should Alice choose?

   (b) Suppose that Bob chooses die B, which die should Alice choose?

   (c) Suppose that Bob chooses die C, which die should Alice choose?

   (d) Suppose that Bob chooses die D, which die should Alice choose?

2 Part II: Calculation (Express your answer in simplest form)

1. There are four red balls and four blue balls, to be randomly (with equal probability) distributed into two boxes, so that each box contains exactly four balls.[1] What is the probability that balls in the first box are all red?

[1] That is, we repeatedly select a ball to put in either box, with equal probability, until one box has four balls; then, we put the remaining balls in the other box.

2. Three rooks are independently put on a 8×8 chess board, where each rook chooses a square with equal probability. That is, it could happen that two, or even three, rooks are placed at the same square. What is the probability that there are two (or more) rooks attacking each other? *Hint:* What is the physical meaning about one rook attacking another?

3. Two persons are playing a match in which they stop as soon as anyone of them wins 10 games. Suppose that the probability for each person to win any game is $1/2$, independent of other games. What is the probability that they have played exactly 11 games when the match is over?