CS2336 Discrete Mathematics

Homework 4
Tutorial: November 30, 2020
Exam 2: December 07, 2020 (10:10–12:30)

Problems marked with * will be explained in the tutorial.

1. A wired equivalent privacy (WEP) key for a wireless fidelity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?

2. (*) Three integers are selected from the integers $\{1, 2, \ldots, 1000\}$. In how many ways can these integers be selected such their sum is divisible by 4?

3. (*) How many arrangements of the letters in MISSISSIPPI have no consecutive S's ?

4. How many bit strings contain exactly five 0s and 14 1s if every 0 must be immediately followed by two 1s?

5. How many ways can n books be placed on k distinguishable shelves

   (a) if the books are indistinguishable copies of the same title?

   (b) if no two books are the same, and the position of the books on the shelves matter?

6. (*) How many ways are there to distribute five distinguishable objects into three indistinguishable boxes so that each of the boxes contains at least one object?

7. (*) How many ways are there to distribute five distinguishable objects into three indistinguishable boxes?

8. How many different terms are there in the expansion $(x_1 + x_2 + \cdots + x_m)^n$ after all terms with identical sets of exponents are added?

9. Suppose that n people are arranged to stand in a line. How many ways are there to choose 3 people from them, such that no one is next to the other within the line?

   (Challenging) What if these n people are arranged to stand in a circle? How many ways are there to choose 3 people from them, such that no one is next to the other within the circle?

10. (*) There are 6 boys and 4 girls. How many ways can they be divided into groups of 2 persons, such that there is no group with two girls?

11. (*, Challenging) Use a combinatorial argument to show that

$$\sum_{k=1}^{n} \binom{n}{k}\binom{n}{k-1} = \binom{2n+2}{n+1}/2 - \binom{2n}{n}.$$

12. Show that if n is an integer then

$$\sum_{k=0}^{n} 3^k \binom{n}{k} = 4^n.$$