PDE HW1.

No

Date

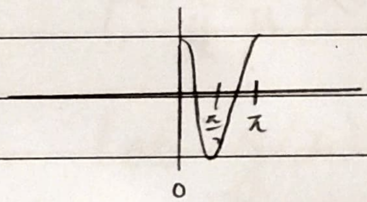
P482 Sect 11.1

1\. Fundamental Period

(a) $\cos x \Rightarrow 2\pi$ #

(b) $\sin x \Rightarrow 2\pi$ #

(c) $\cos 2x \Rightarrow \pi$ #



(d) $\sin 2x \Rightarrow \pi$ #

(e) $\cos \pi x \Rightarrow 2$ #

(f) $\sin \pi x \Rightarrow 2$ #

(g) $\cos 2\pi x \Rightarrow 1$ #

(h) $\sin 2\pi x \Rightarrow 1$ #

2\. Fundamental Period

(a) $\cos nx \Rightarrow \frac{2\pi}{n}$ #

$\frac{2\pi}{n}$

(b) $\sin nx \Rightarrow \frac{2\pi}{n}$ #

(c) $\cos \frac{2\pi x}{L} \Rightarrow L$ #

$$\frac{2\pi}{\frac{2\pi}{L}} = L$$

(d) $\sin \frac{2\pi x}{L} \Rightarrow L$ #

$$\frac{2\pi}{\frac{2\pi}{L}} = L$$

(e) $\cos \frac{2\pi \cdot n x}{L} \Rightarrow \frac{L}{n}$ #

$$\frac{2\pi}{\frac{2\pi \cdot n}{L}} = \frac{L}{n}$$

(f) $\sin \frac{2\pi \cdot n x}{L} \Rightarrow \frac{L}{n}$ #

$$\frac{2\pi}{\frac{2\pi \cdot n}{L}} = \frac{L}{n}$$

3.

let $f(x)$ and $g(x)$ have period p

s.t. $f(x) = f(x+p)$, $g(x) = g(x+p)$

Let $h(x) = af(x) + bg(x)$, $V = \{f, g\}$

thus $h \in V$, $h = af + bg$

$\because h(x) = af(x) + bg(x)$

$\therefore h(x+p) = af(x+p) + bg(x)$

$= af(x) + bg(x)$

$= h(x)$

$\Rightarrow h(x+p) = h(x)$ for $\forall h \in V$

$\Rightarrow$ All function of period p form a vector space. #

No
Date

4.

Let $f(x+p) = f(x)$,

Let $f(x) = \cos x$, $p = 2\pi$, thus

$$f(x) = f(x+2\pi)$$

(a) Let $f(x) = f(x+p)$, f has a fundamental period p

Let $g(x) = f(ax)$; then $f(ax+p) = f(ax)$

$\because f(ax+p) = f(ax) = f(a(x+\frac{p}{a}))$

$\therefore g(x) = g(x+\frac{p}{a})$

EX: $\because f(x) = \cos x$, $\cos x + 2\pi = \cos x$

$\therefore P_f = 2\pi$

$\because f(ax) = \cos ax = g(x)$

$\cos ax + 2\pi = \cos ax = \cos a(x+\frac{2\pi}{a})$

$\therefore g(x) = g(x+\frac{2\pi}{a})$

$P_g = \frac{2\pi}{a} = \frac{P_f}{a}$

$\Rightarrow$ When $a = 2$, $P_g = \pi$ trivially ☆

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No

Date

(b) Let $f(x) = f(x+P_f)$, f has fundamental period P_f

Let $g(x) = f(\frac{x}{b})$, then $f(\frac{x}{b}) = f(\frac{x}{b} + P_f)$

$\because f(\frac{x}{b}) = f(\frac{x}{b} + P_f) = f(\frac{1}{b}(x + bP_f))$

$\therefore g(x) = g(x + bP_f)$, g has fundamental period $P_g = bP_f$ #

EX: Let $f(x) = \cos x$

$\because \cos x = \cos x + 2\pi$

$\therefore P_f = 2\pi$

Let $g(x) = f(\frac{x}{b}) = \cos\frac{x}{b}$

$\because f(\frac{x}{b}) = f(\frac{x}{b} + P_f) = f(\frac{1}{b}(x + bP_f))$

$\therefore g(x) = g(x + bP_f)$, g has fundamental period $P_g = bP_f = b2\pi$

$\Rightarrow$ When $b = 2$, $P_g = 4\pi$ trivially #

No

Date

P482.

14.

$$f(x) = x^2, \quad -\pi < x < \pi$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \cos nx \cdot dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \sin nx \cdot dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2\, dx = \frac{1}{2\pi}\cdot\frac{1}{3}x^3\Big|_{-\pi}^{\pi}$$

$\frac{d}{dx}\cos = -\sin$

$$= \frac{1}{6\pi}x^3\Big|_{-\pi}^{\pi} = \frac{1}{6\pi}\left(\pi^3 - (-\pi)^3\right) = \frac{1}{\cancel{6\pi}_{3}}\cdot \cancel{2}\pi^{\cancel{3}\,2}$$

$\frac{d}{dx}\sin = \cos$

$$= \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cdot \cos nx \cdot dx$$

$$= \frac{1}{\pi}\left(x^2\cdot\frac{\sin nx}{n} - \int 2x\cdot\frac{\sin nx}{n}\cdot dx\right)\Big|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[x^2\cdot\frac{\sin nx}{n} - \left(2x\cdot\frac{-\cos nx}{n^2} - \int 2\cdot\frac{-\cos nx}{n^2}\cdot dx\right)\right]\Big|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[x^2\cdot\frac{\sin nx}{n} - \left(2x\cdot\frac{-\cos nx}{n^2} - 2\cdot\frac{-\sin nx}{n^3}\right)\right]\Big|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[x^2 \cdot \frac{\sin nx}{n} + 2x \cdot \frac{\cos nx}{n^2} - 2 \cdot \frac{\sin nx}{n^3} \right]\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\left(\pi^2 \cdot \frac{\sin n\pi}{n} + 2\pi \cdot \frac{\cos n\pi}{n^2} - 2 \cdot \frac{\sin n\pi}{n^3}\right) - \right.$$
$$\left.\left(\pi^2 \cdot \frac{\sin -n\pi}{n} - 2\pi \cdot \frac{\cos -n\pi}{n^2} - 2 \cdot \frac{\sin -n\pi}{n^3}\right)\right]$$

$$= \frac{1}{\pi}\left[\left(\cancel{\pi^2} \cdot \cancel{\frac{\sin n\pi}{n}} + 2\pi \cdot \frac{\cos n\pi}{n^2} - \cancel{2} \cdot \cancel{\frac{\sin n\pi}{n^3}}\right) - \right.$$
$$\left.\left(\cancel{\pi^2} \cdot \cancel{\frac{\sin n\pi}{n}} - 2\pi \cdot \frac{\cos -n\pi}{n^2} - \cancel{2} \cdot \cancel{\frac{\sin n\pi}{n^3}}\right)\right]$$

$$= \frac{1}{\cancel{\pi}}\left[\frac{2\cancel{\pi}}{n^2}(\cos n\pi + \cos -n\pi)\right] = \frac{4}{n^2}\cos n\pi$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cdot \sin nx \cdot dx$$

$$= \frac{1}{\pi}\left(-x^2 \cdot \frac{\cos nx}{n} + \int 2x \cdot \frac{\cos nx}{n} \cdot dx\right)\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-x^2 \cdot \frac{\cos nx}{n} + \left(2x \cdot \frac{\sin nx}{n^2} - \int 2 \cdot \frac{\sin nx}{n^2} dx\right)\right]\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-x^2 \cdot \frac{\cos nx}{n} + \left(2x \cdot \frac{\sin nx}{n^2} + 2 \cdot \frac{\cos nx}{n^3}\right)\right]\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-x^2 \cdot \frac{\cos nx}{n} + 2x \cdot \frac{\sin nx}{n^2} + 2 \cdot \frac{\cos nx}{n^3}\right]\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\left(-\pi^2 \cdot \frac{\cos n\pi}{n} + 2\pi \cdot \frac{\sin n\pi}{n^2} + 2 \cdot \frac{\cos n\pi}{n^3}\right) - \right.$$
$$\left.\left(-\pi^2 \cdot \frac{\cos -n\pi}{n} - 2\pi \cdot \frac{\sin -n\pi}{n^2} + 2 \cdot \frac{\cos -n\pi}{n^3}\right)\right]$$

$$= \frac{1}{\pi}\left[\left(-\cancel{\pi^2} \cdot \cancel{\frac{\cos n\pi}{n}} + 2\pi \cdot \frac{\sin n\pi}{n^2} + \cancel{2} \cdot \cancel{\frac{\cos n\pi}{n^3}}\right) - \right.$$
$$\left.\left(-\cancel{\pi^2} \cdot \cancel{\frac{\cos n\pi}{n}} - 2\pi \cdot \frac{\sin -n\pi}{n^2} + \cancel{2} \cdot \cancel{\frac{\cos n\pi}{n^3}}\right)\right]$$

$$= \frac{1}{\cancel{\pi}}\left[2 \cdot 2\cancel{\pi} \cdot \frac{\sin n\pi}{n}\right] = 0$$

No

Date

$$a_n = \frac{4}{n^2}\cos n\pi = \begin{cases} \frac{4}{n^2}, & n \text{ is even} \\ -\frac{4}{n^2}, & n \text{ is odd} \end{cases}$$

$$\Rightarrow a_0 = \frac{\pi^2}{3}, \quad b_n = 0, \quad a_n = \begin{cases} \frac{4}{n^2}, & n \text{ is even} \\ -\frac{4}{n^2}, & n \text{ is odd} \end{cases}$$

$$\Rightarrow S_5 = \frac{\pi^2}{3} - 4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \frac{1}{4}\cos 4x - \frac{4}{25}\cos 5x$$