CS2336 Discrete Mathematics

Exam 3
January 06, 2020 (10:10–12:30)

Answer all questions. Total marks = 105. Maximum Score = 105/100. Large portion of marks may be deducted for incomplete proofs.

1. (15%) Let A, B, and C be three sets. It is known that the following are true.

   - $|A| = 12$;
   - $|A - C| = 5$;
   - $|B \cup C| = 27$;
   - $|(A \cup B) - C| = 15$;
   - $|(B - C) - A| = 10$.

   Find $|C|$. (Hint: Draw a Venn diagram to help.)

2. Let $\mathbb{Q}^+$ denote the set of positive rational numbers. Consider the function $f : \mathbb{Q}^+ \to \mathbb{Q}^+$ with

   $$f(x) = 2/x.$$

   (a) (10%) Determine if f is one-one. Explain your answer.

   (b) (10%) Determine if f is onto. Explain your answer.

3. (15%) Let $S = \{x, y, z\}$. Construct a binary relation R on S such that R is not reflexive, not symmetric, not antisymmetric, **but** transitive.

   (Note: You may describe R with a directed graph.)

4. (35%)

   (a) (15%) Draw two non-isomorphic simple undirected graphs H_1 and H_2, each with 6 vertices, and the degrees of these vertices are 2, 2, 2, 2, 3, 3, respectively.

   (b) (20%) Show that H_1 and H_2 are non-isomorphic.

5. Let G be a simple undirected graph with 4 vertices. It is known that G and its complement are isomorphic.

   (15%) Draw G.

6. (Challenging)

   It is known that in a class with $N \geq 3$ students, the following conditions hold:

   - For any two students x and y, they are either friends or enemies, but not both.
   - For any two students x and y, there always exists one, and only one, common friend.

   (5%) Show that N is odd, and there is a student who is a friend with all other students.