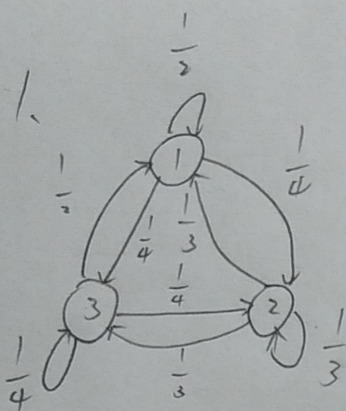
EXAM 3.　　106033233　周聖鋐

1.



(a)

$Pr(X_1=1)$

$= Pr(X_1=1 \cap X_0=1) + Pr(X_1=1 \cap X_0=2)$

$+ Pr(X_1=1 \cap X_0=3)$

$= Pr(X_0=1)\,Pr(X_1=1 \mid X_0=1) + Pr(X_0=2)\,Pr(X_1=1 \mid X_0=2)$

$+ Pr(X_0=3)\,Pr(X_1=1 \mid X_0=3)$

$= \frac{1}{3}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{3} + \frac{1}{6}\times\frac{1}{2} = \frac{1}{6}+\frac{1}{6}+\frac{1}{12} = \frac{2+2+1}{12}$

$= \frac{5}{12}$ #

(b)

$\frac{Pr(X_0=2 \cap X_1=1)}{Pr(X_1=1)} = Pr(X_0=2 \mid X_1=1)$

$= \frac{Pr(X_0=2)\,Pr(X_1=1 \mid X_0=2)}{Pr(X_1=1)} = \frac{\frac{1}{2}\times\frac{1}{3}}{\frac{5}{12}} = \frac{\frac{1}{6}}{\frac{5}{12}} = \frac{\overset{2}{\cancel{12}}}{\cancel{6}\times 5}$

$= \frac{2}{5}$ #

$\frac{1}{3}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{3} + \frac{1}{6}\times\frac{1}{2}$

$= \frac{1}{6}\times 2 + \frac{1}{12} = \frac{4+1}{12} = \frac{5}{12}$

$\frac{1}{6}\times\frac{1}{2} + \frac{1}{3}\times\frac{1}{3} + \frac{1}{3}\times\frac{1}{2}$　　$\frac{\frac{1}{6}}{\frac{5}{12}}$

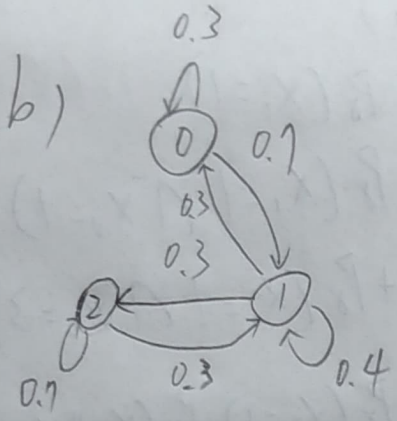
$\frac{1}{12}$　　$\frac{1}{6}$

2.

(a)

	0	1	2
0	0.3	0.7	0
1	0.3	0.4	0.3
2	0	0.3	0.7

(b)



(c)

Irreducible: Since every state (state 0, 1, 2) can reach other 2 state, the graph is strongly connected. Hence the Markov Chain is irreducible. #

Aperiodic: For each state, let $P_{i,j}^{m}$ is the probability from state i to j in m steps

For state 0 $\Rightarrow \gcd\{ m>0, P_{0,0}^{m>0} \} = \gcd\{1,2,3,4,5,6\cdots\} = 1$

For state 1 $\Rightarrow \gcd\{ m>0, P_{1,1}^{m>0} \} = \gcd\{1,2,3,4\cdots\} = 1$

For state 2 $\Rightarrow \gcd\{ m>0, P_{2,2}^{m>0} \} = \gcd\{1,2,3,4\cdots\} = 1$

All states are $d=1$

(d) Note: $Pr(X_{t+h}=k \mid X_t=k) > 0$ for $h \bmod d = 0$, otherwise the prob $= 0$, hence the Markov Chain is aperiodic #

$$[A\ B\ C]\begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0 & 0.3 & 0.7 \end{bmatrix} \Rightarrow \begin{cases} 0.3A + 0.3B = A \\ 0.7A + 0.4B + 0.3C = B \\ 0.3B + 0.7C = C \end{cases}$$

$3B = 7A$

$3B = 3C$

$B = C$

$\Rightarrow \begin{cases} A = 3t \\ B = 7t \\ C = 7t \end{cases}$

$\Rightarrow$ Stationary Distribution:

$$\left[\frac{3}{17}, \frac{7}{17}, \frac{7}{17}\right]$$

A B C #