Unsupervised Learning

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Machine Learning

Outline

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Unsupervised Learning

- Dataset: $\mathbb{X} = \{\boldsymbol{x}^{(i)}\}_i$, where $\boldsymbol{x}^{(i)}$'s are i.i.d. samples of $\mathbf{x}$
  - No supervision (e.g., labels)

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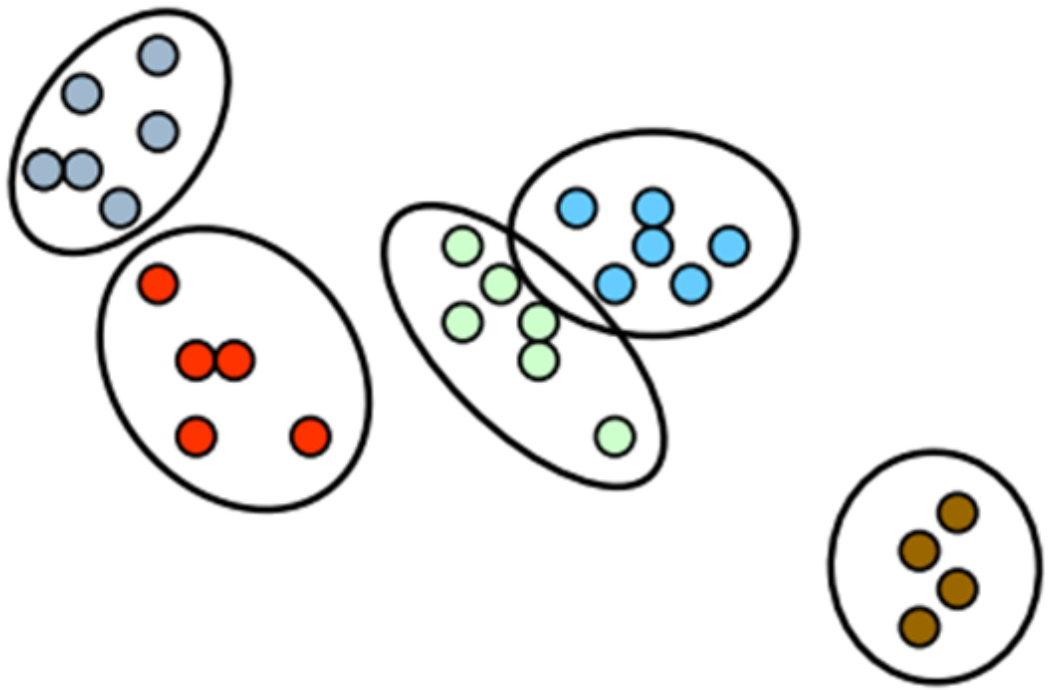
- What can we learn?

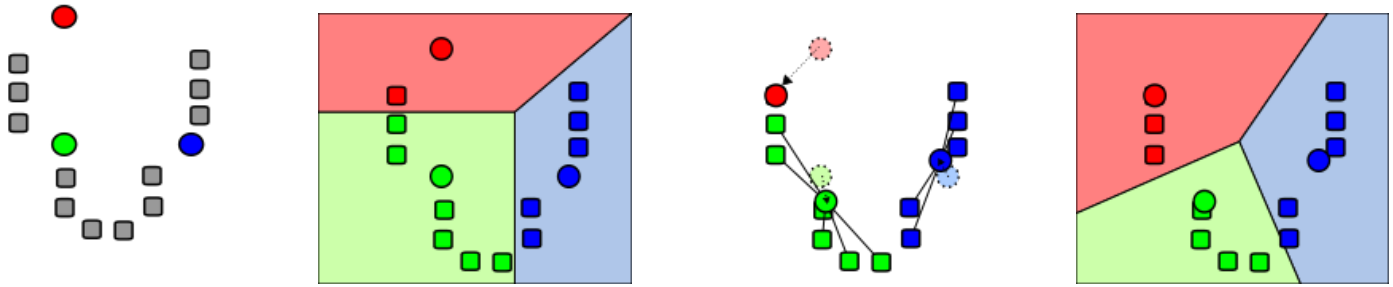
Clustering I

- Goal: to divide $\boldsymbol{x}^{(i)}$'s into K groups/***clusters***
  - Based on some pairwise similarity/distance measure



Clustering II

- K-means algorithm (K fixed):



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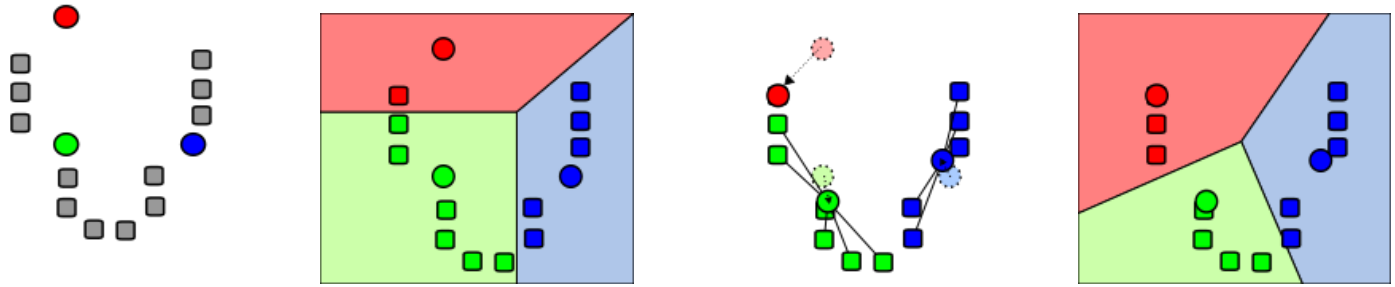
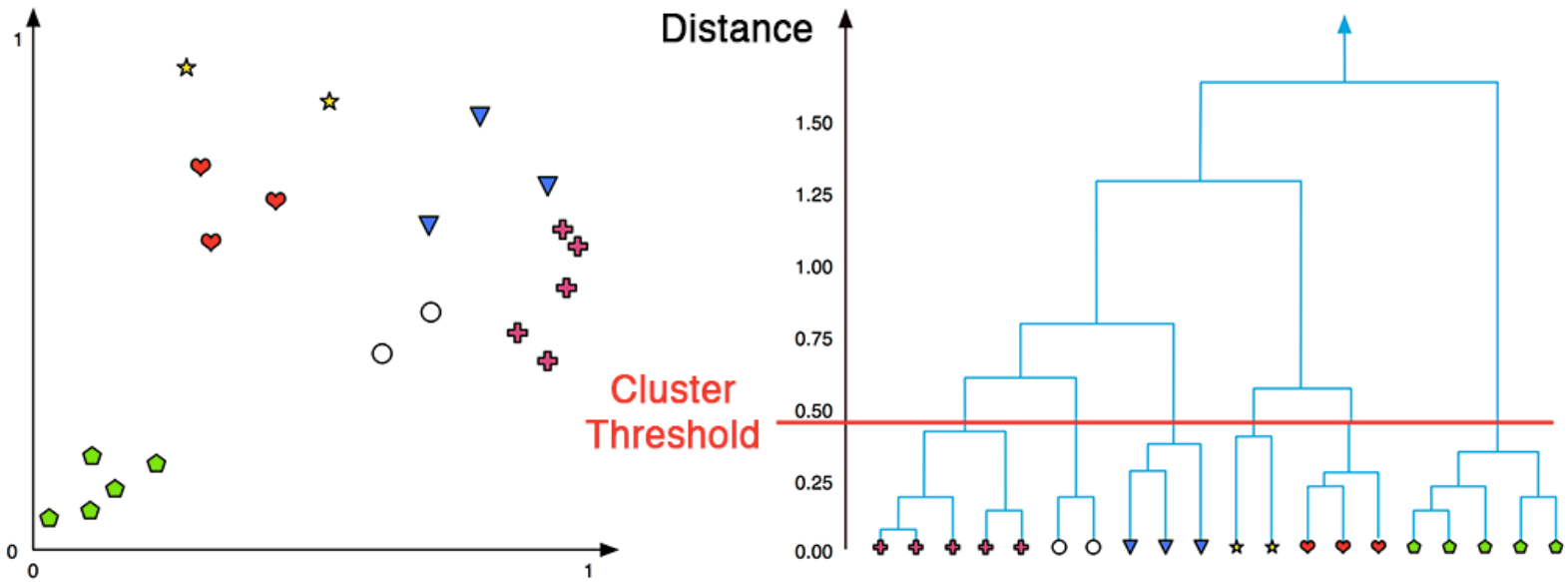


- Hierarchical clustering (variable K):



Factorization and Recommendation

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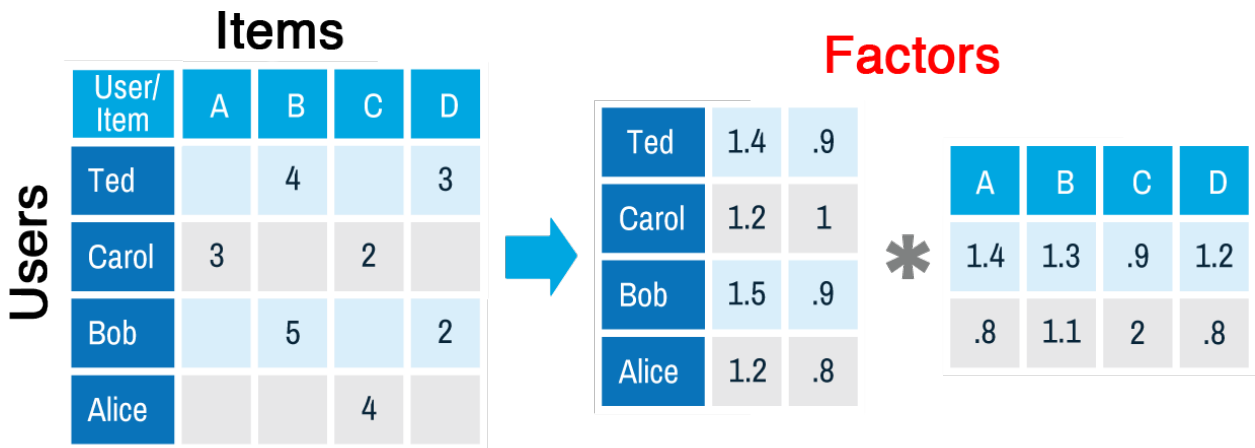

- Non-negative matrix factorization (NMF) [11, 12]:

$$\arg \min_{\boldsymbol{W} \geq \boldsymbol{O}, \boldsymbol{H} \geq \boldsymbol{O}} \|\boldsymbol{X} - \boldsymbol{W}\boldsymbol{H}\|_F$$

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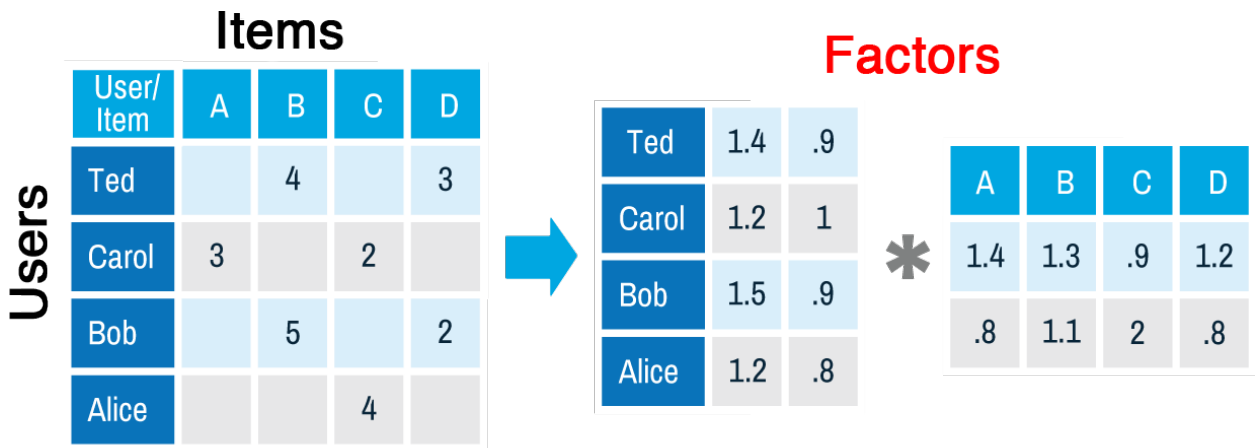

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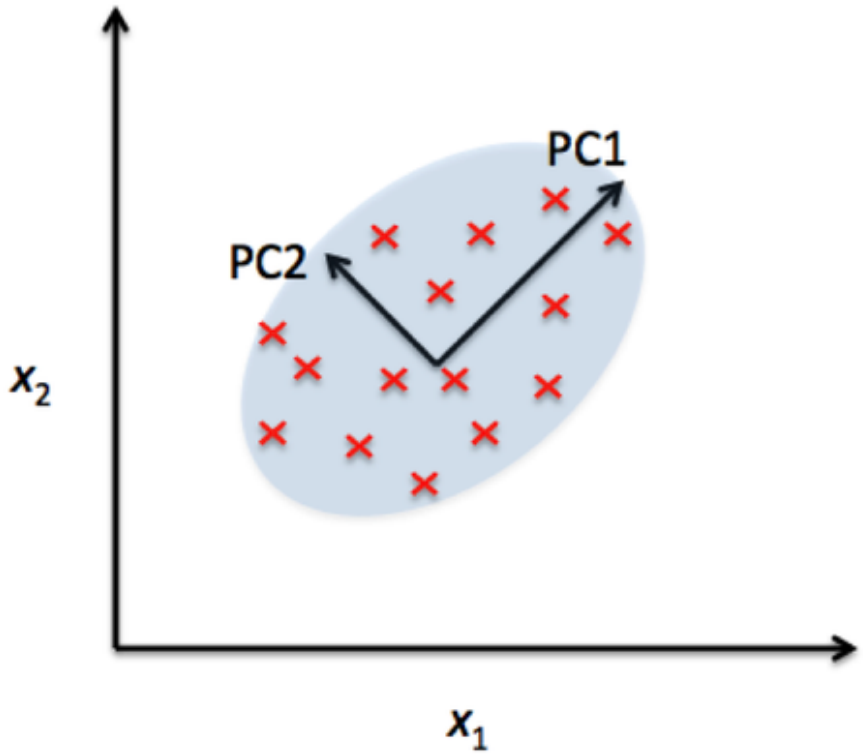
- $\boldsymbol{X}^* = \boldsymbol{W}^* \boldsymbol{H}^*$ a dense matrix and can be used to predict user interests

Dimension Reduction

- Goal: to learn a low dimensional representation $\mathbf{z}$ of $\mathbf{x}$
  - E.g., PCA



Predictive Learning

- Goal: to learn a model that is able to "fill in the blanks"

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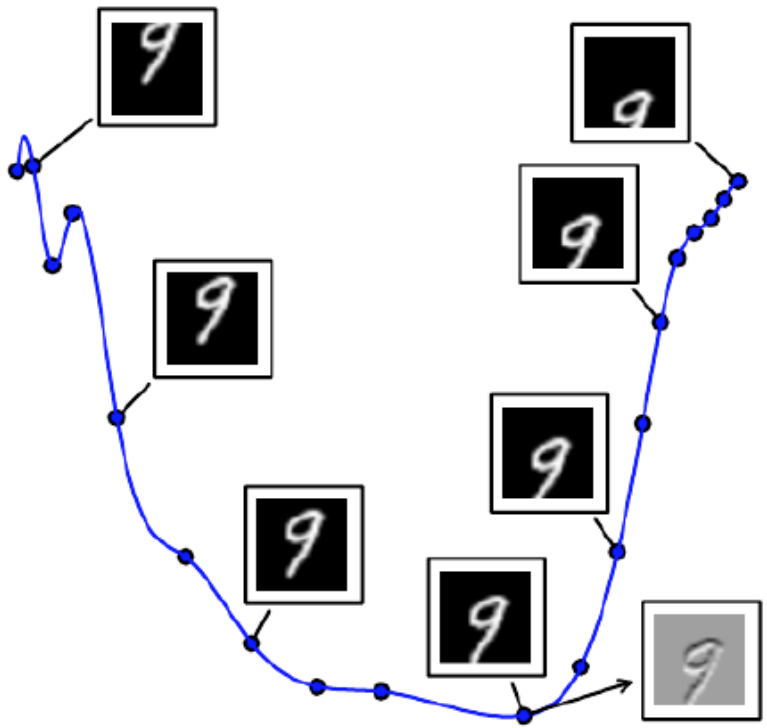
- Links unsupervised tasks with supervised models
- For supervised tasks: more easy data collection
- For unsupervised tasks: better representations of $\mathbf{x}$ and/or $\mathbf{y}$ thanks to DL models

$\boldsymbol{x}^{(i)}$  $\boldsymbol{y}^{(i)}$

Manifold Learning

- Goal: to learn the underlying manifold of $\mathbf{x}$
  - E.g., given a point $\boldsymbol{x}^{(i)}$, output the tangent vector of $\boldsymbol{x}^{(i)}$



Data Synthesis/Generation I

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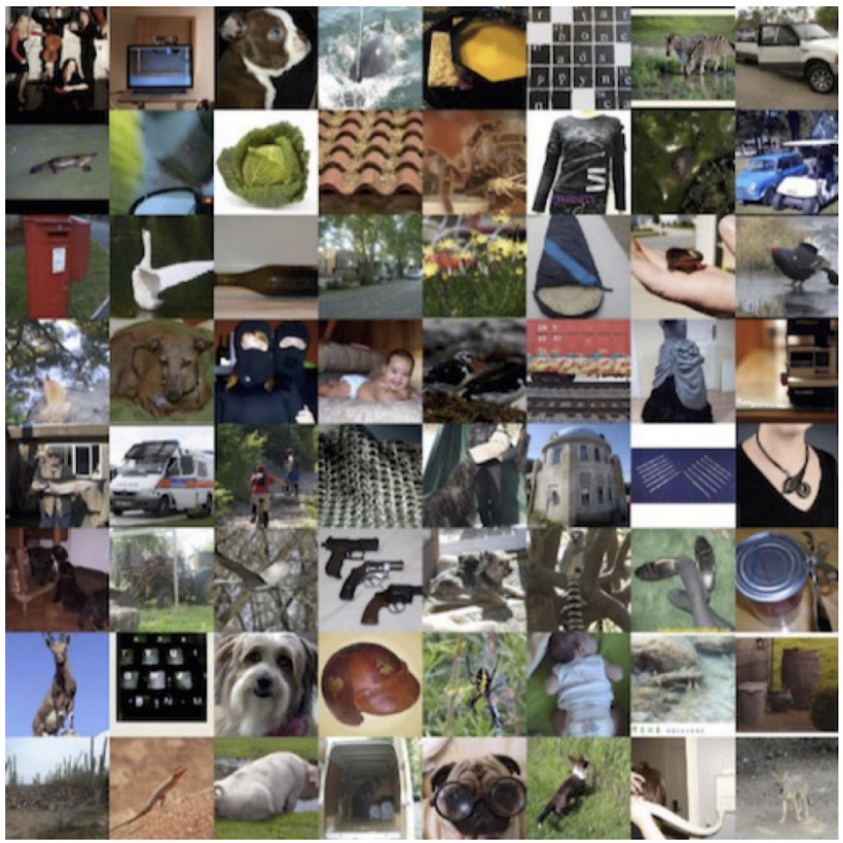
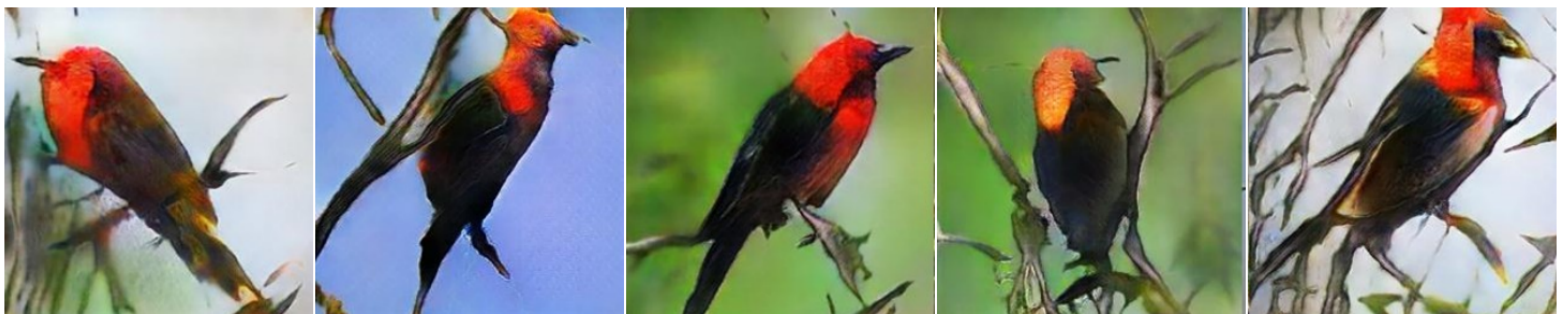
- *Generative adversarial networks* (GANs)



Data Synthesis/Generation II

- Conditional GANs, e.g., text to image synthesis

  "*This bird is completely red with black wings and pointy beak.*"



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Self-Supervised Learning

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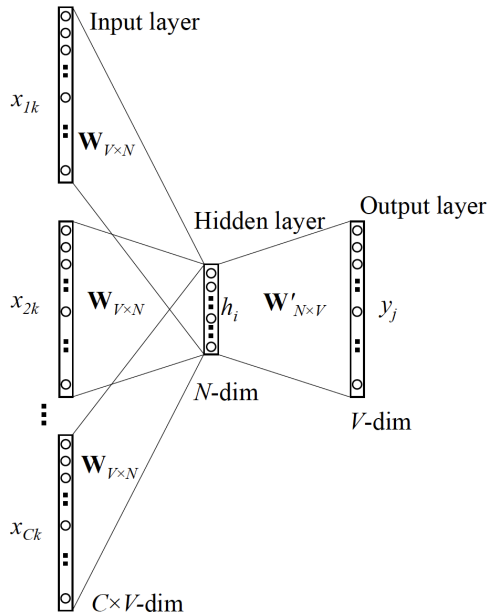
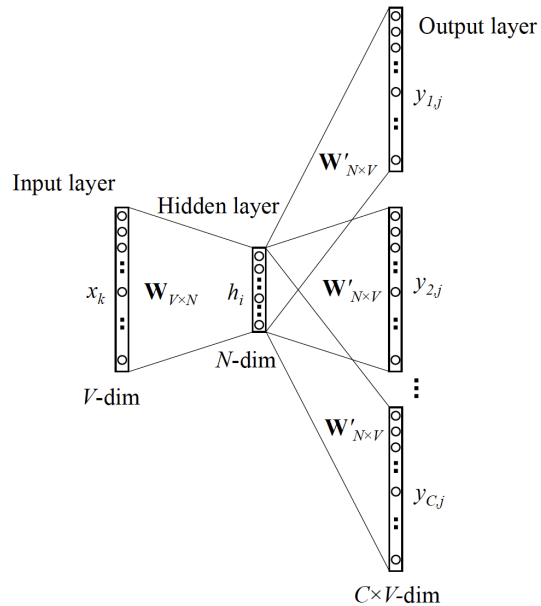


CBOW



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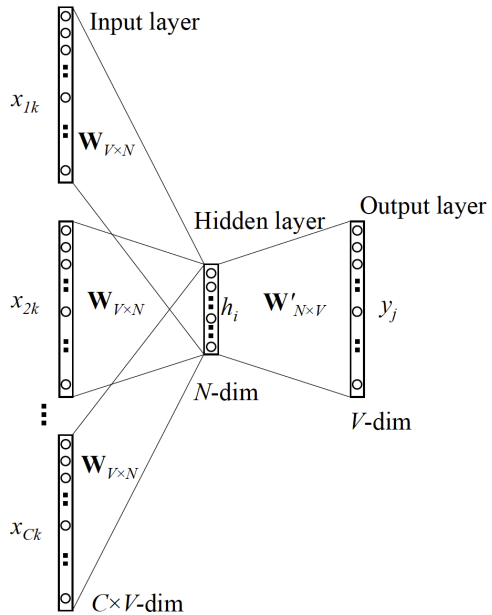

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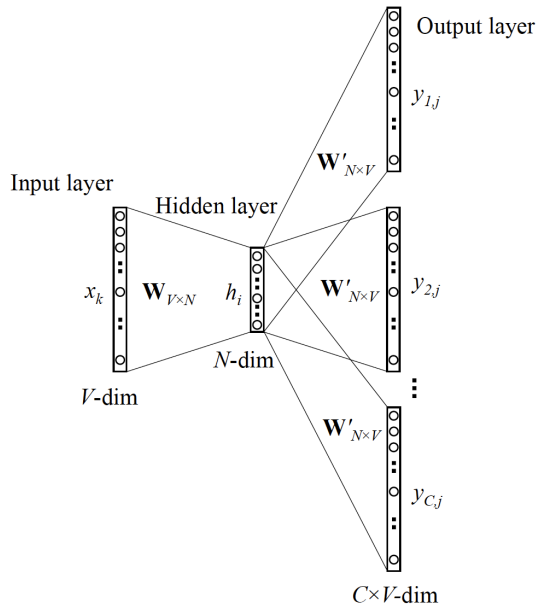

Skip Gram

- Latent representation $\mathbf{h}$ encodes the ***semantics*** of a word
  - No need for synonym dictionary; big data tell that already

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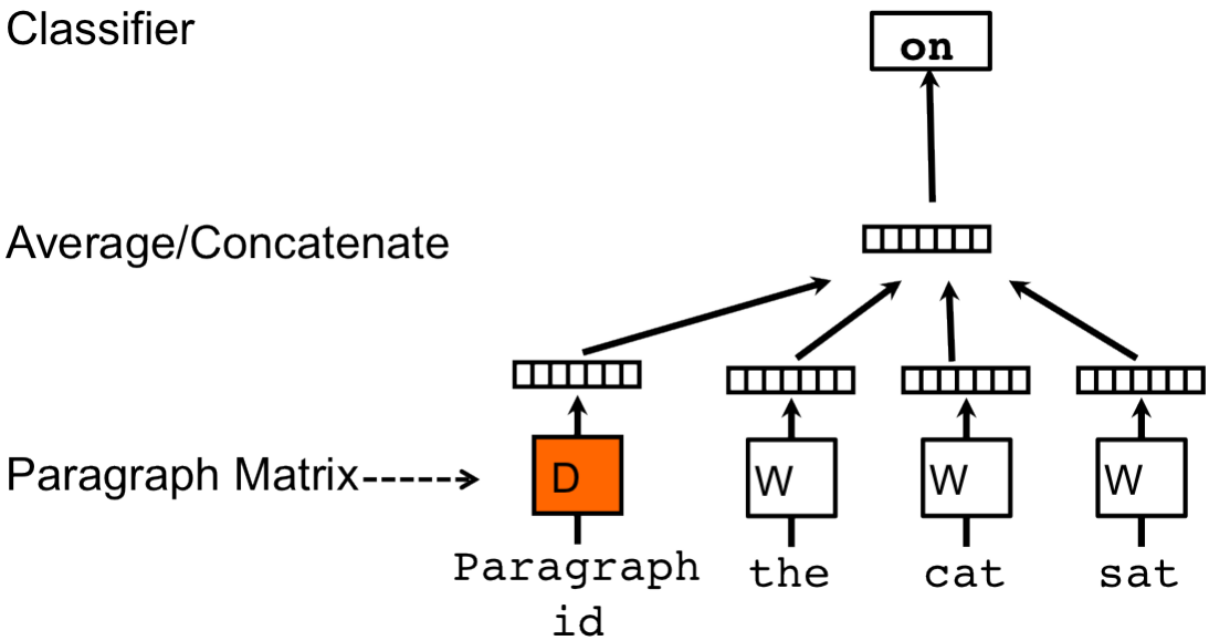
- Doc2vec [9]: to capture the ***context*** not explained by words



Filling Images

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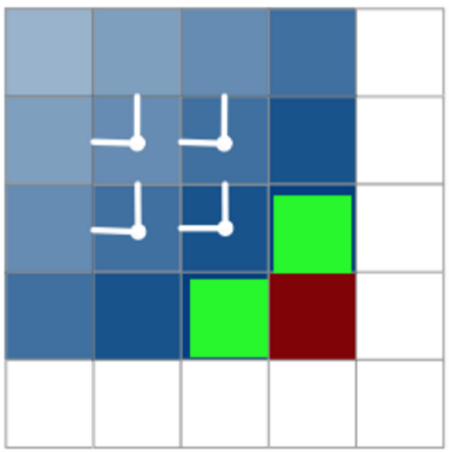
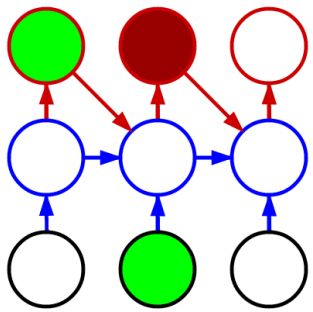
- PixelRNN [24], PixelCNN [23]





More

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- Better representations/predictions without the need for labels

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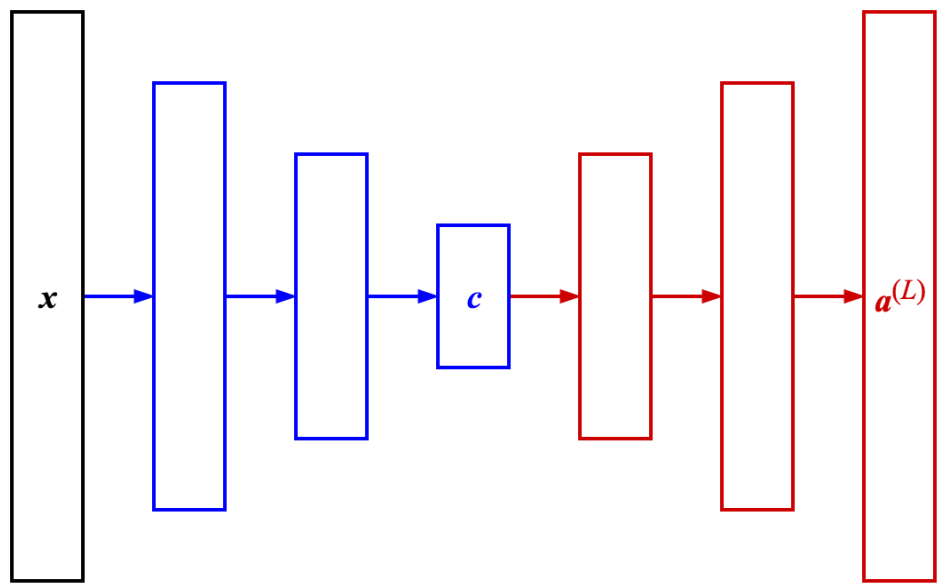
Autoencoders I

- ***Encoder***: to learn a low dimensional representation $\mathbf{c}$ (called ***code***) of input $\mathbf{x}$
- ***Decoder***: to reconstruct $\mathbf{x}$ from $\mathbf{c}$



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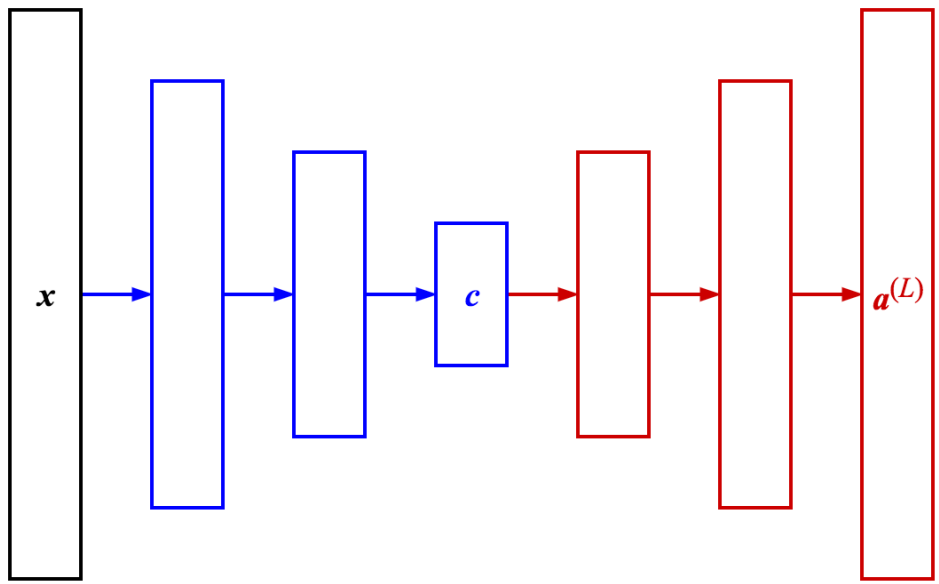

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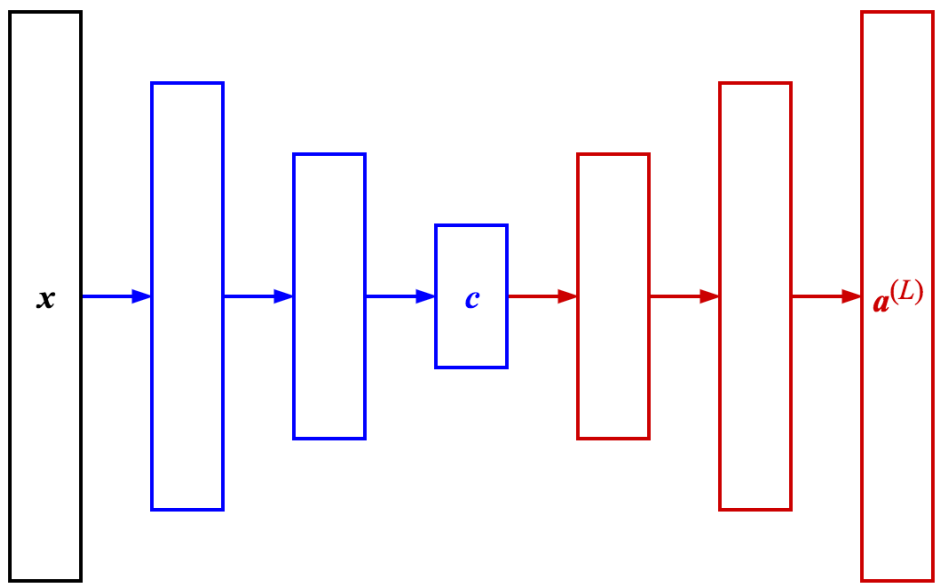

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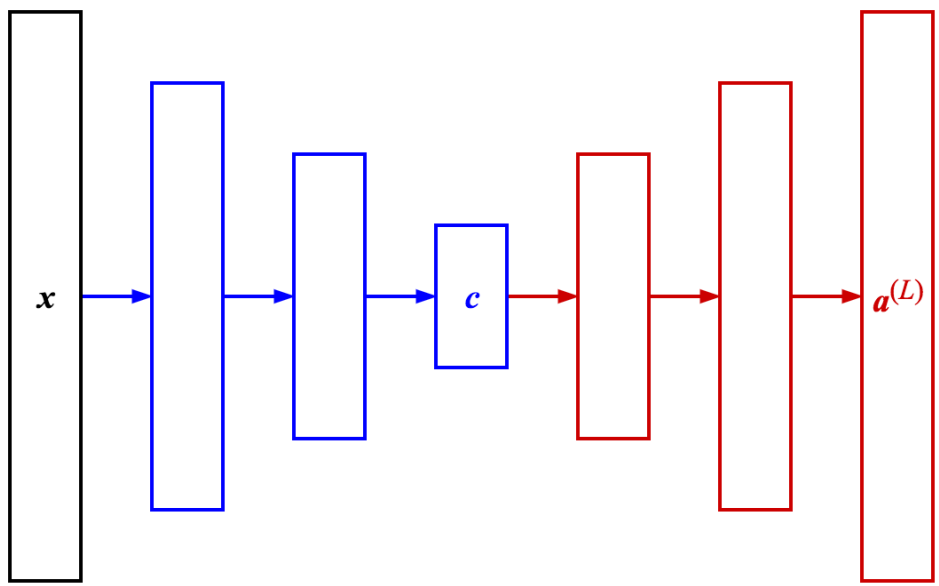
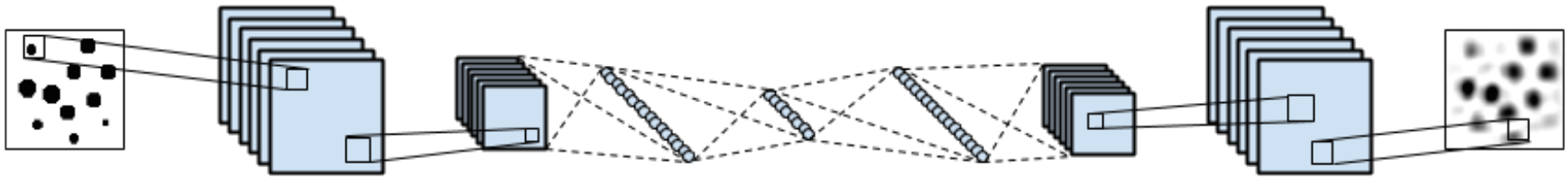


Convolutional Autoencoders

- Convolution + deconvolution layers:



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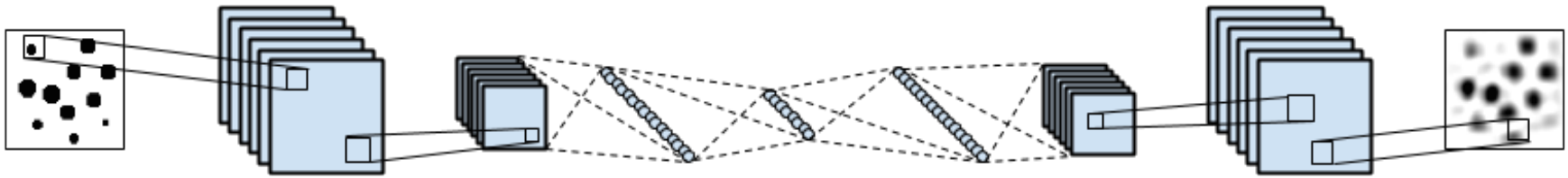

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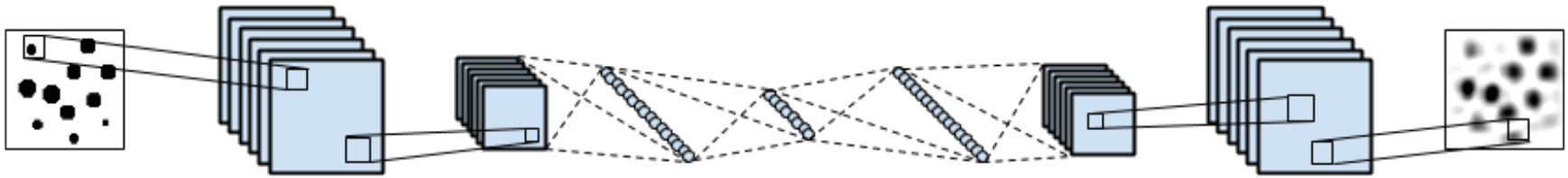

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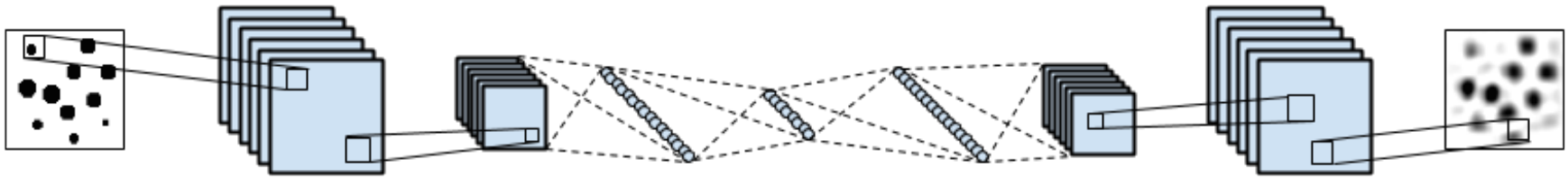

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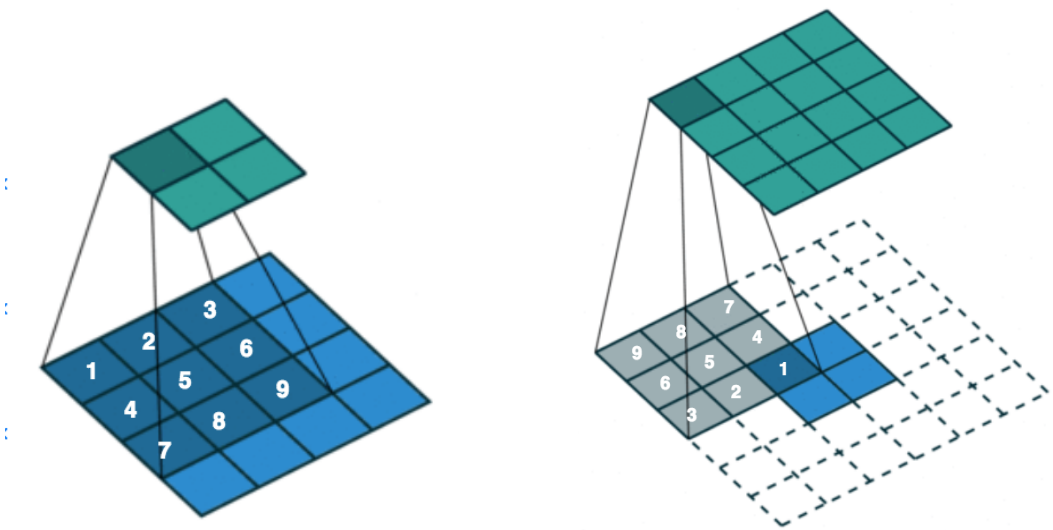
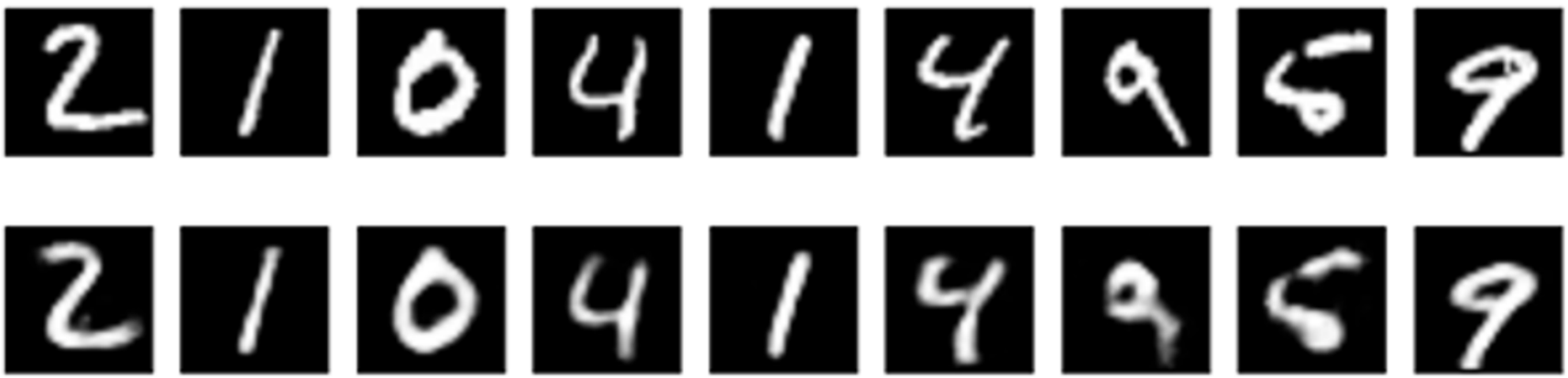


Codes & Reconstructed $\mathbf{x}$

- A 32-bit code can roughly represents a 32×32 (1024 dimensional) MNIST image



Manifolds I

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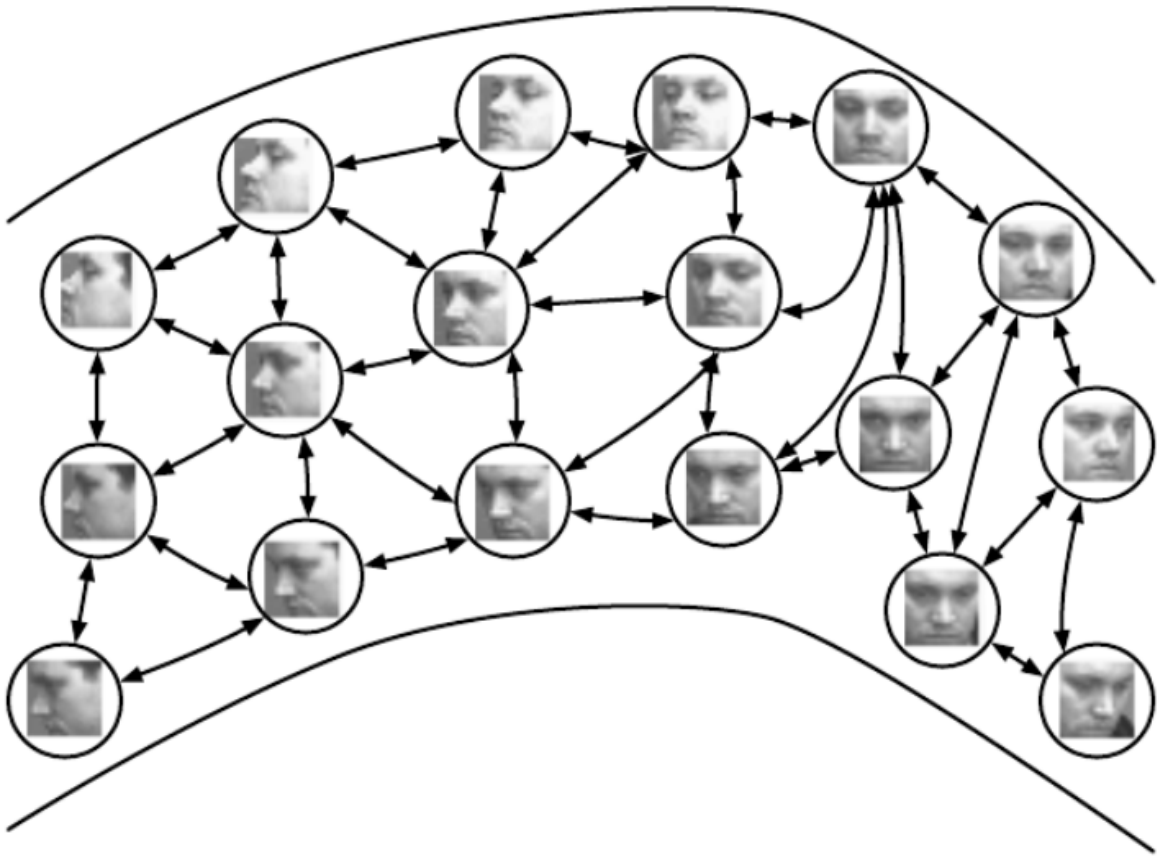
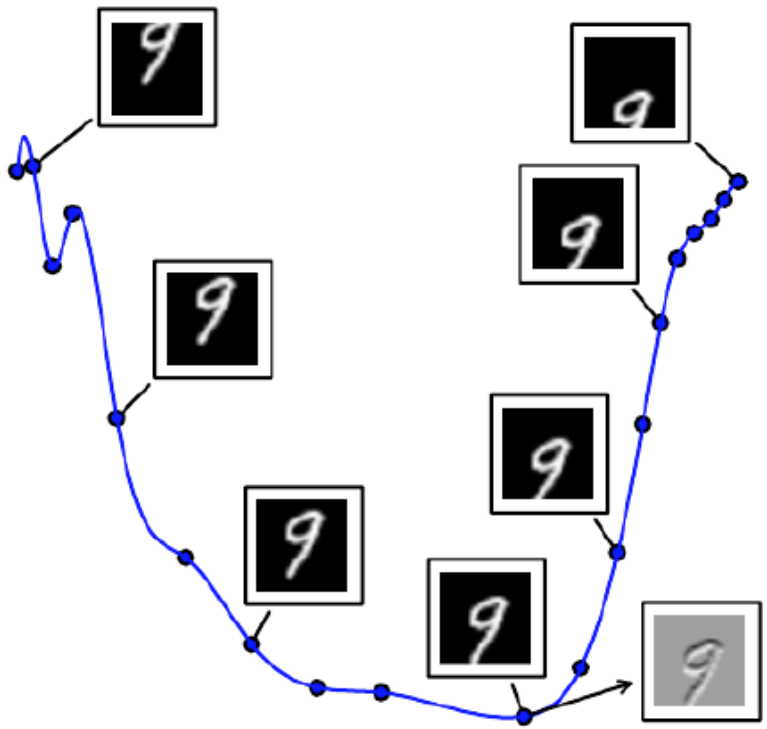
- A manifold is a topological space that are ***linear locally***



Manifolds II

- For each point $\boldsymbol{x}$ on a manifold, we have its ***tangent space*** spanned by ***tangent vectors***
  - Local directions specify how one can change $\boldsymbol{x}$ infinitesimally while staying on the manifold



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- Contractive autoencoder [19]: regularizes the code $\mathbf{c}$ such that it is invariant to local changes of $\mathbf{x}$:

$$\Omega(\mathbf{c}) = \sum_n \left\| \frac{\partial \boldsymbol{c}^{(n)}}{\partial \boldsymbol{x}^{(n)}} \right\|_F^2$$

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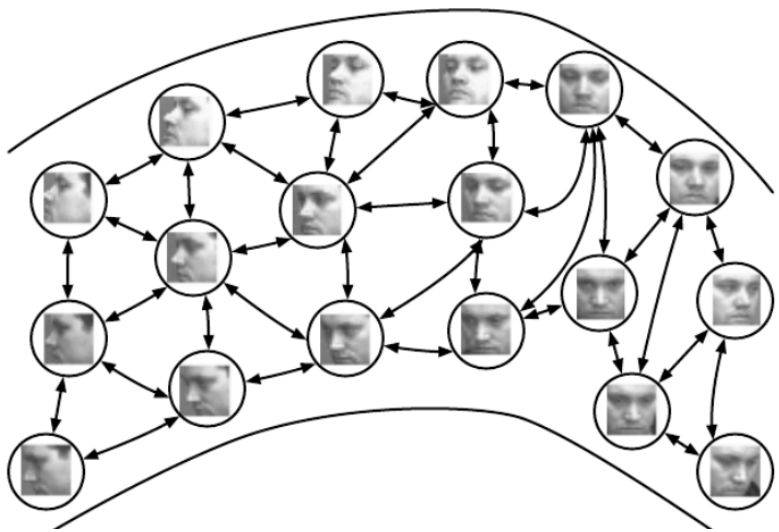
  - $\partial \boldsymbol{c}^{(n)} / \partial \boldsymbol{x}^{(n)}$ is a Jacobian matrix
- Prevents the encoder from changing $\mathbf{c}$ easily for changes of $\mathbf{x}$
  - Except for the changes of $\mathbf{x}$ that follows tangent vectors



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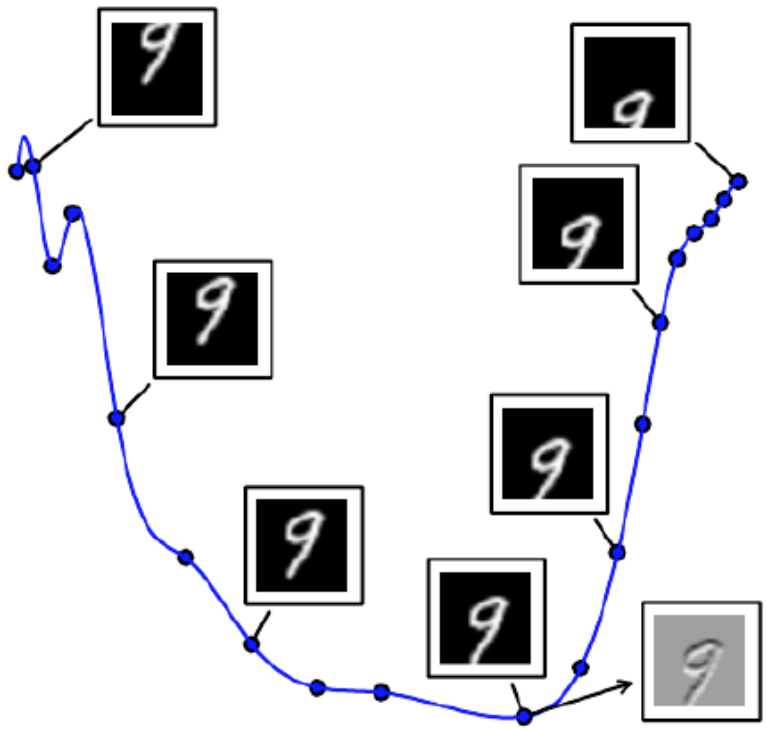
  - Encoder: to encode $\mathbf{x}$ ***with*** random noises
  - Decoder: to reconstruct $\mathbf{x}$ ***without*** noises

Getting Tangent Vectors I

- The code $\mathbf{c}$ represents a coordinate on a low dimensional manifold
  - E.g., the blue line
- How to get the tangent vectors of a given $\boldsymbol{c}$?



Getting Tangent Vectors II

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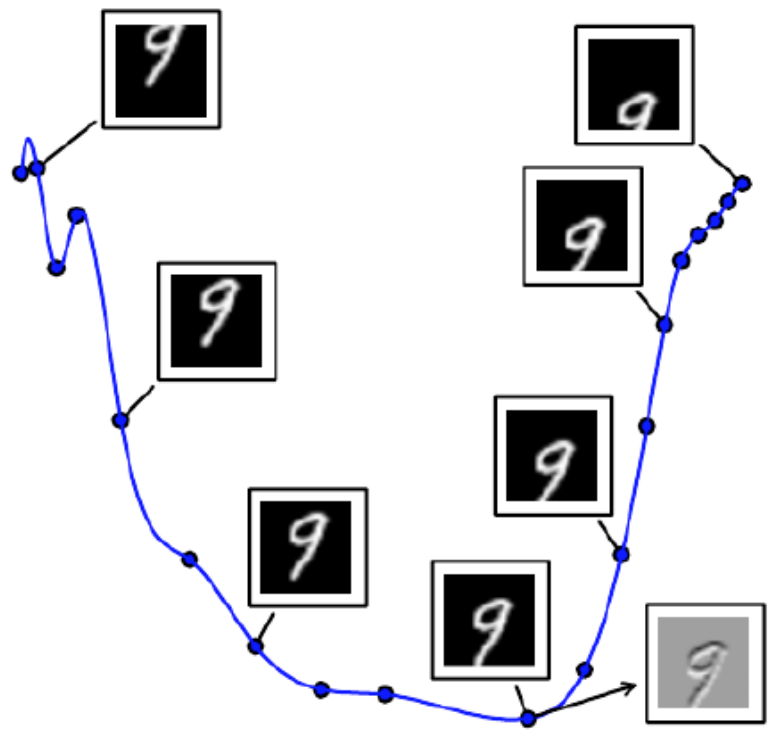

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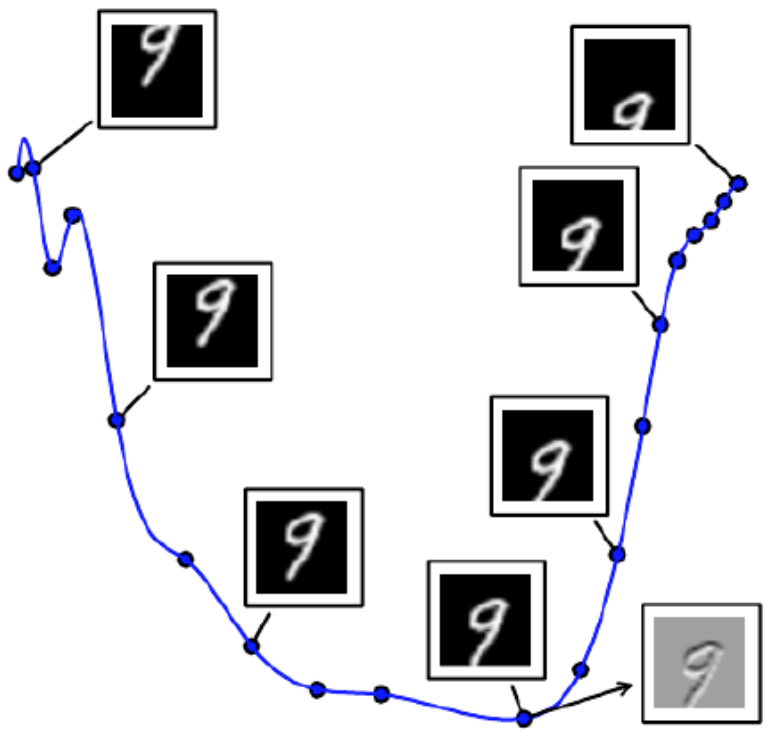

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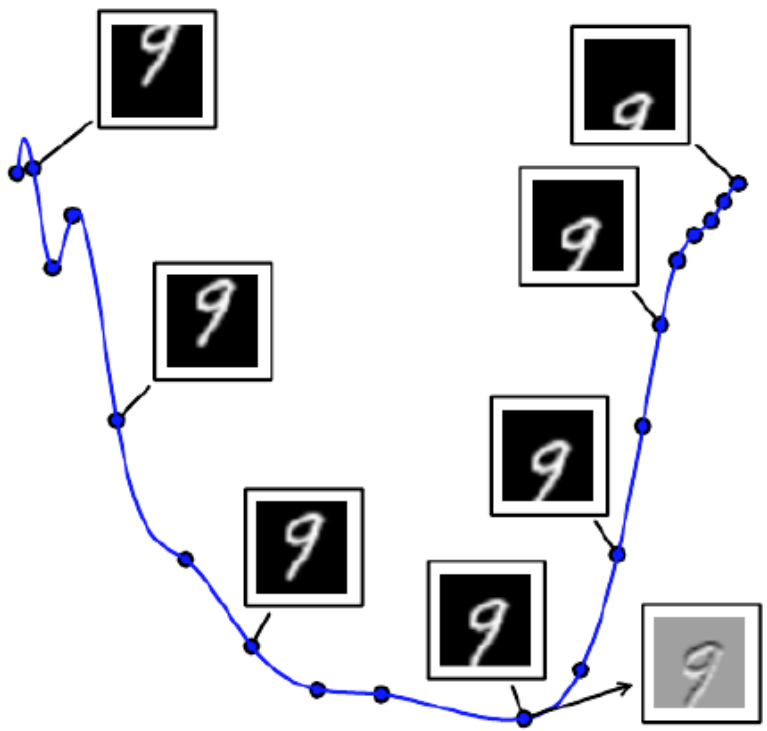
  - $\boldsymbol{J}(\boldsymbol{x})$ summarizes how $\boldsymbol{c}$ changes in terms of $\boldsymbol{x}$



Getting Tangent Vectors II

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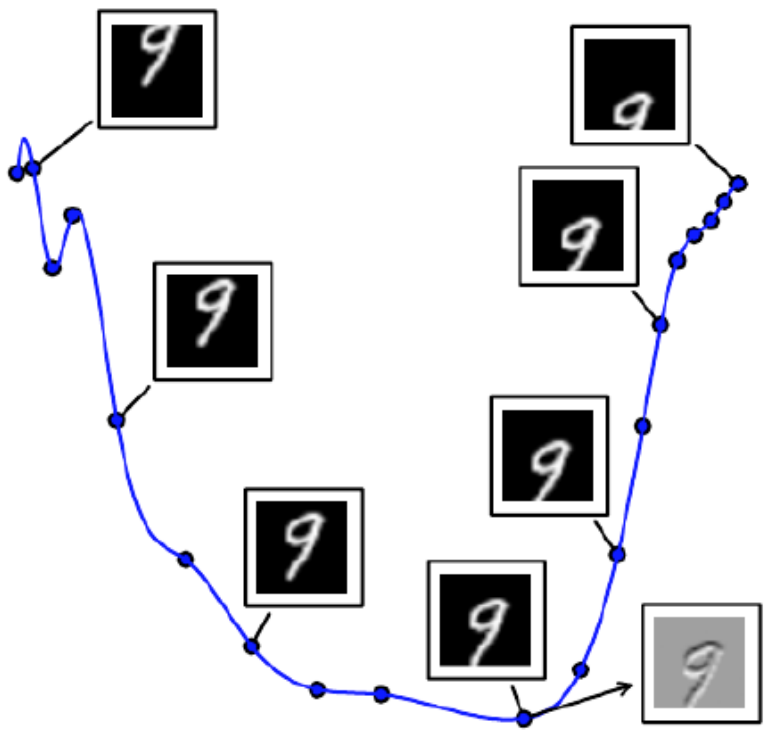
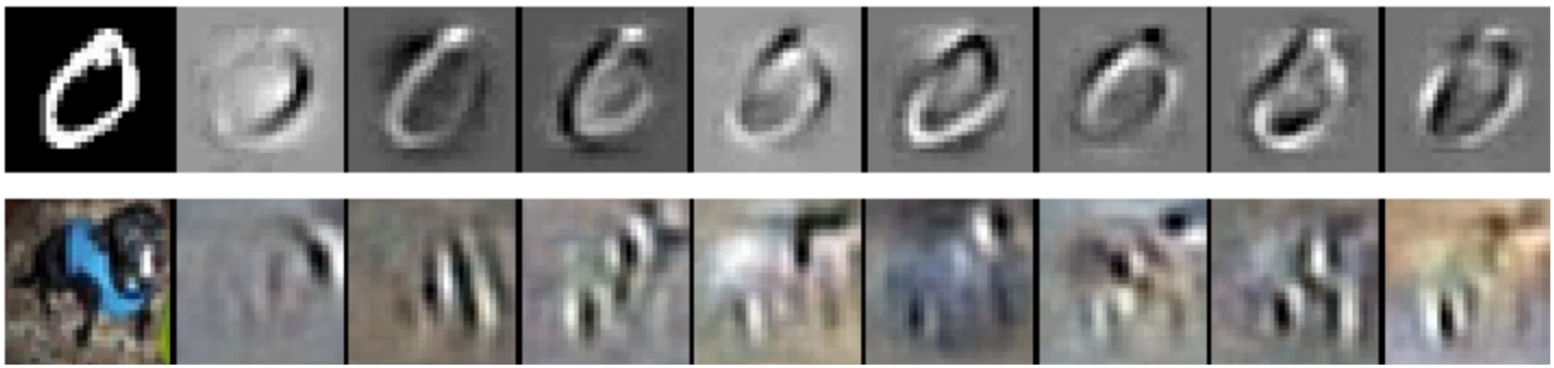
  - $\boldsymbol{J}(\boldsymbol{x})$ summarizes how $\boldsymbol{c}$ changes in terms of $\boldsymbol{x}$

1. Decompose $\boldsymbol{J}(\boldsymbol{x})$ using SVD such that $\boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^\top$
2. Let tangent vectors be ***rows of V corresponding to the largest singular values in D***



Getting Tangent Vectors III

- In practice, $\boldsymbol{J}(\boldsymbol{x})$ usually has few large singular values
- Tangent vectors found by contractive/denoising autoencoders:



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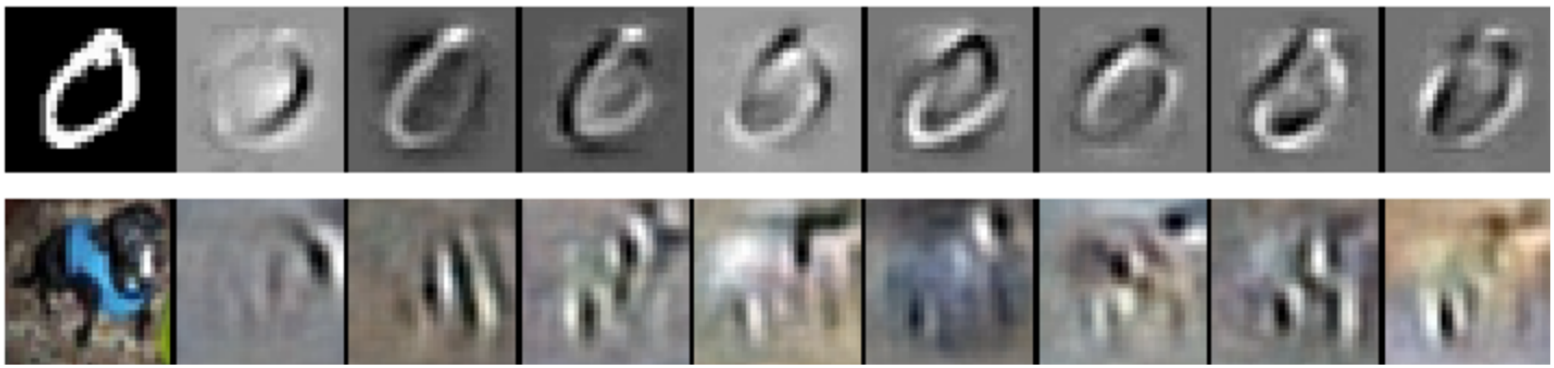
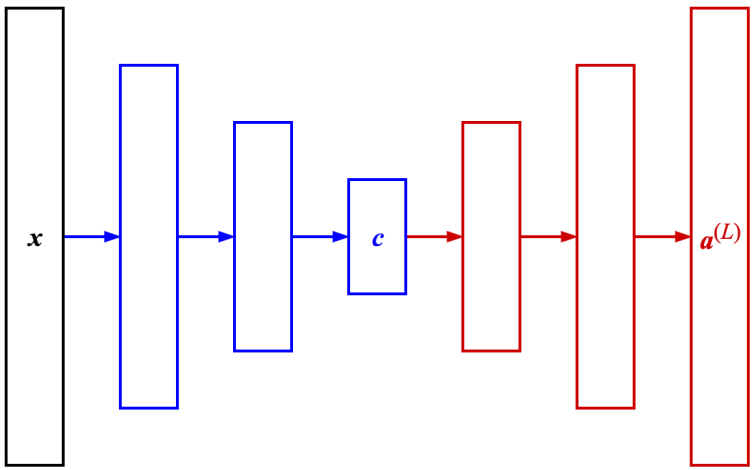


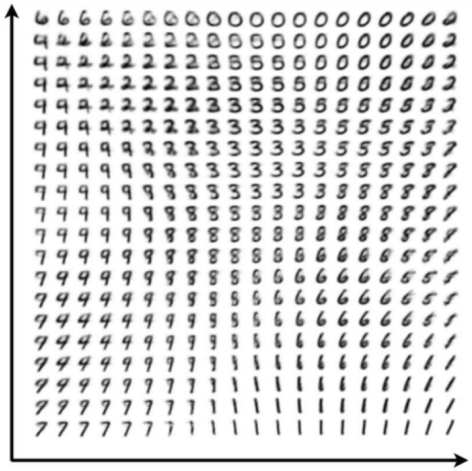
- Can be used by Tangent Prop [22]:
- Let $\{\boldsymbol{v}^{(i,j)}\}_j$ be tangent vectors of each example $\boldsymbol{x}^{(i)}$
- Trains an NN classifier f with cost penalty: $\Omega[f] = \sum_{i,j} \nabla_{\boldsymbol{x}} f(\boldsymbol{x}^{(i)})^\top \boldsymbol{v}^{(i,j)}$
  - Points in the same manifold share the same label

Outline

Decoder as Data Generator

- Decoder of an autoencoder can be used to generate data points even with ***synthetic codes***





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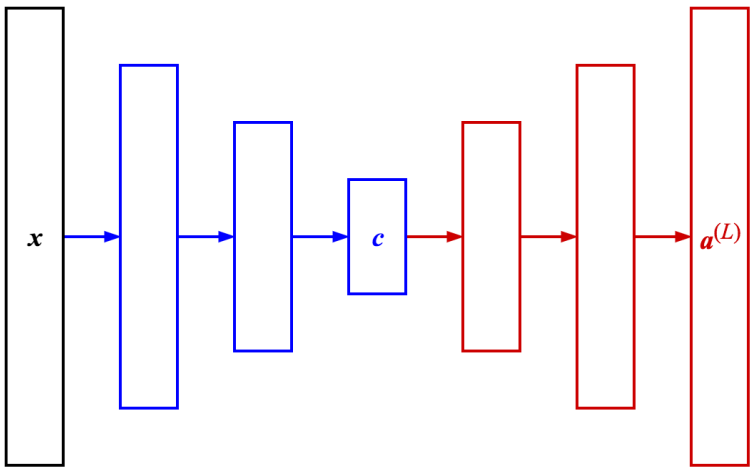

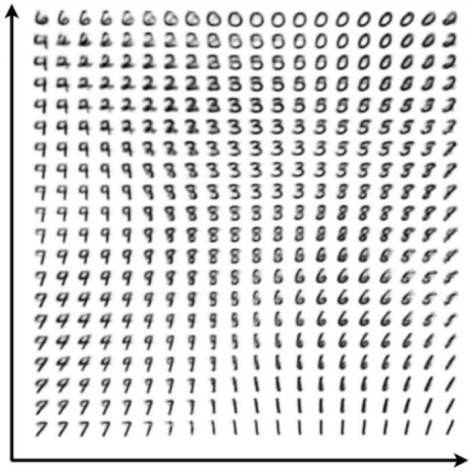

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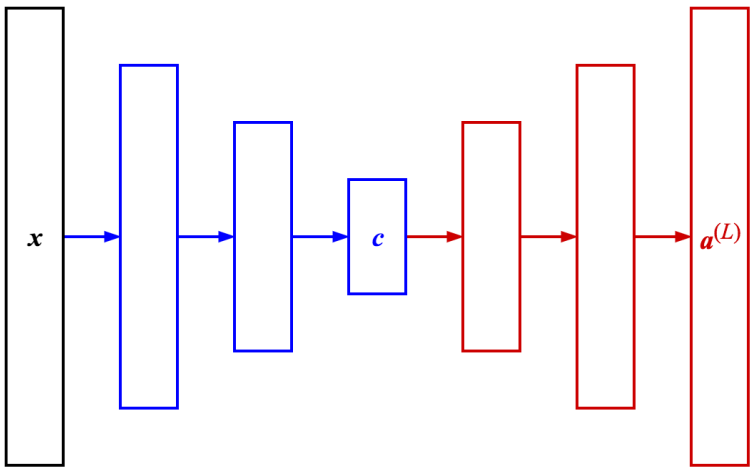

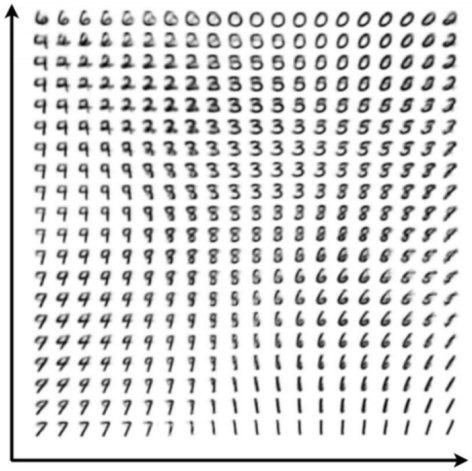

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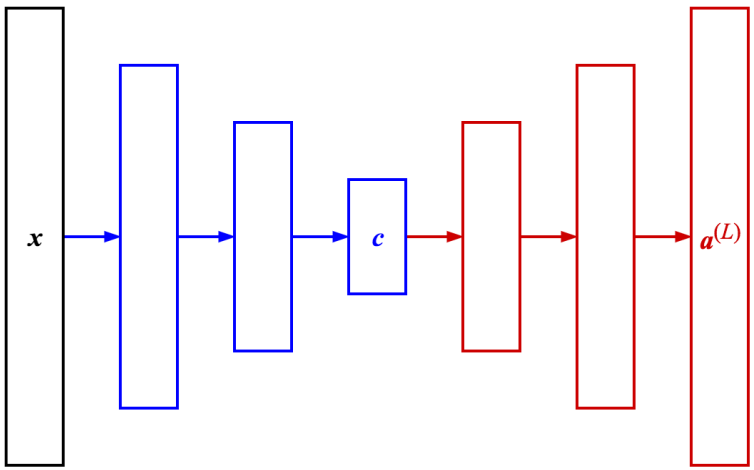

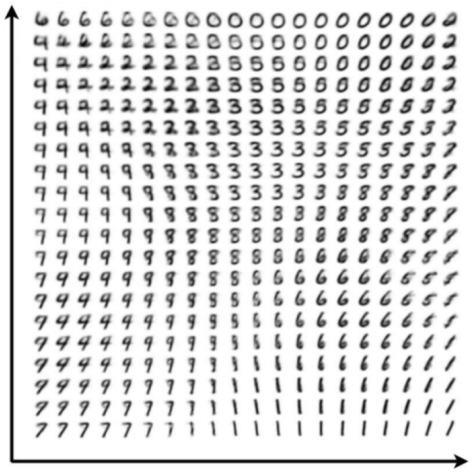

- Problems:
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  - ***Blurry images***

Why Blurry Images?

- Cost function: $\arg\min_\Theta -\log \mathrm{P}(\mathbb{X}\,|\,\Theta) = \arg\min_\Theta -\sum_n \log \mathrm{P}(\boldsymbol{x}^{(n)}\,|\,\Theta)$
- Image generation: linear output units $\boldsymbol{a}^{(L)} = \boldsymbol{z}^{(L)} = \hat{\mu}$ for $\mathbf{x} \sim \mathcal{N}(\mu, \Sigma)$
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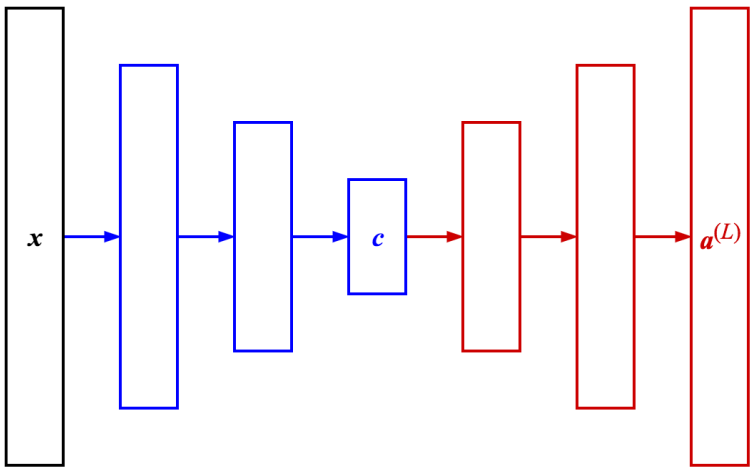

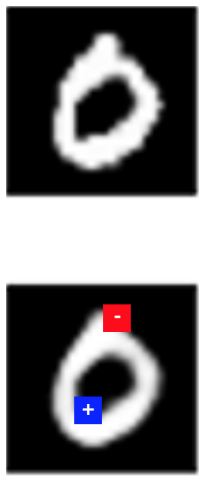

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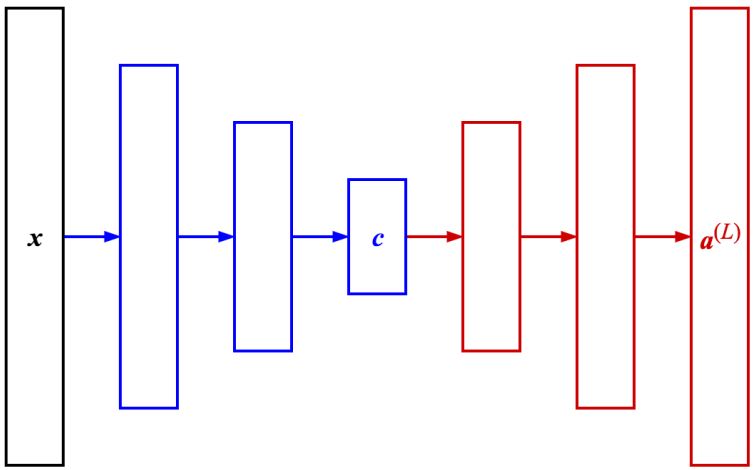

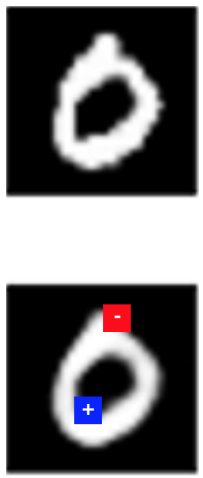

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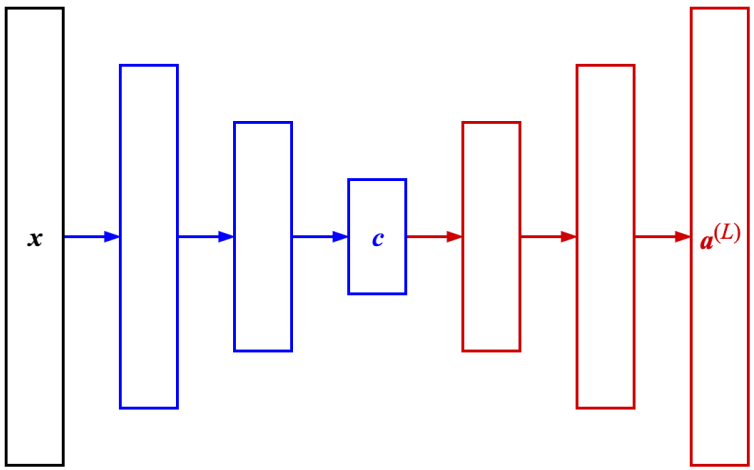

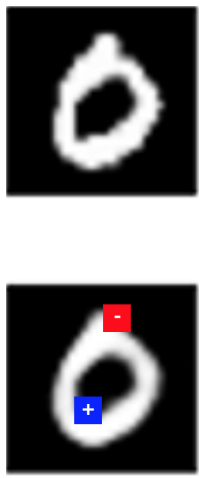

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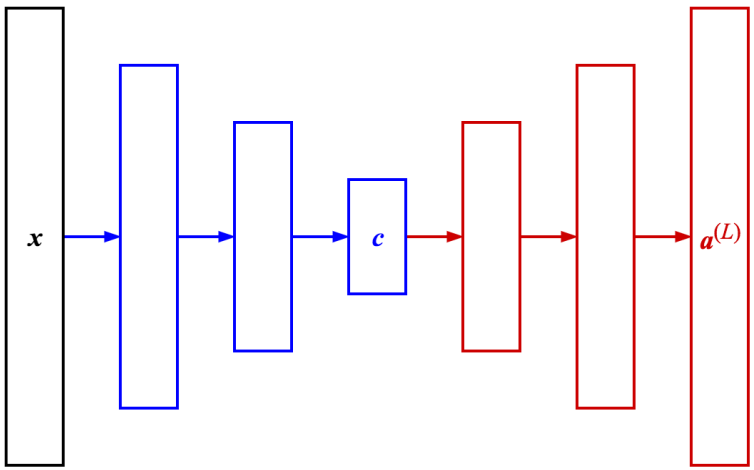
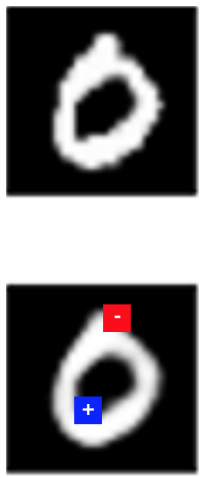


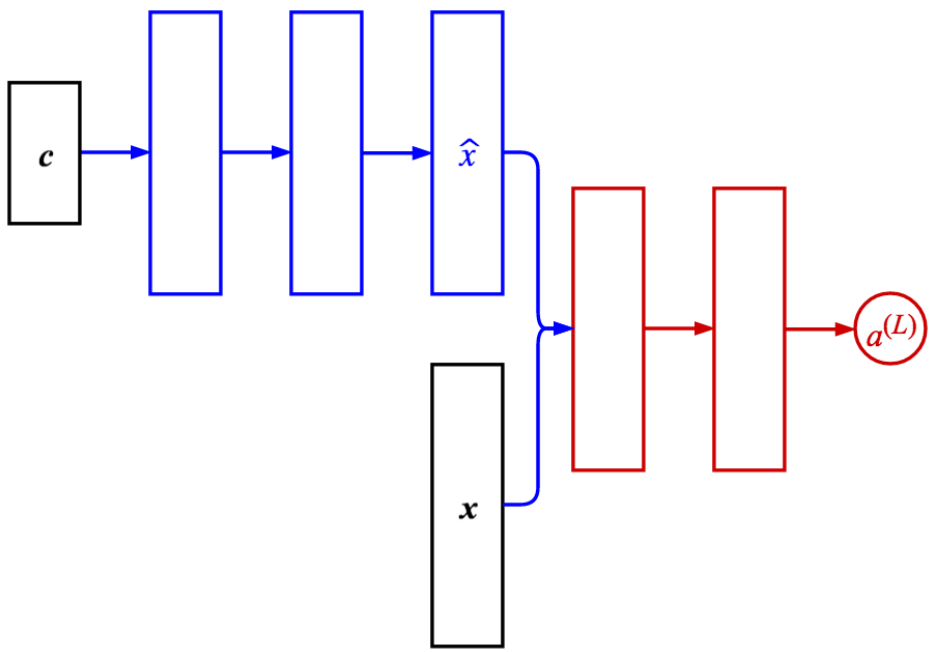


- Better assuming distribution for $\mathbf{x}$? $\mathrm{P}(\mathbf{x})$ may be very complex
- Better "goodness" measure? Why not use an NN to tell if a generated image is of good quality?

Outline

Generative Adversarial Networks (GANs)

- *Generative adversarial network* (*GAN*) [3]:



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- ***Generative adversarial network*** (***GAN***) [3]:
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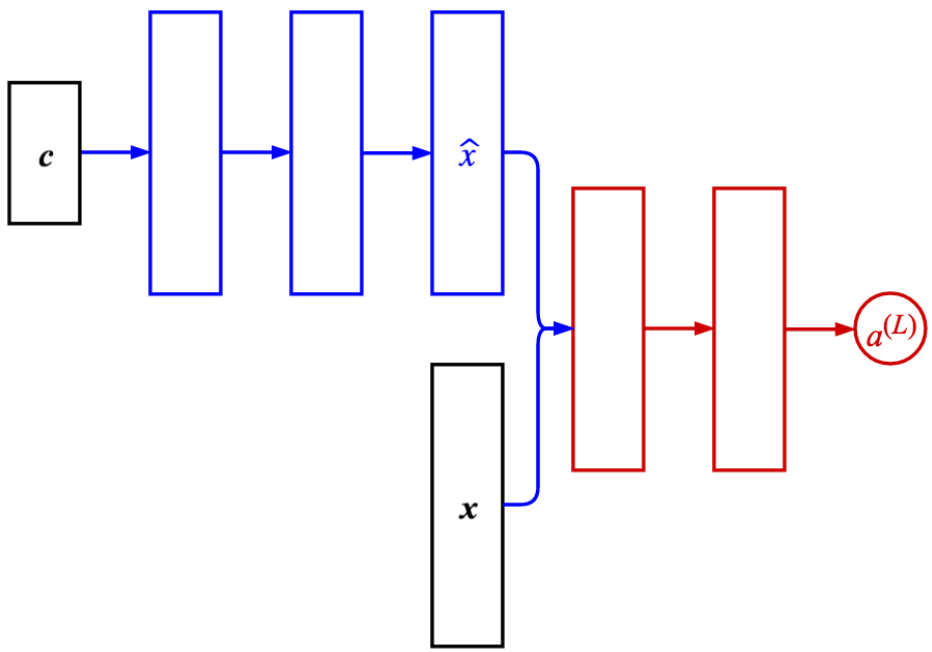
  - No need for "encoder" since the task is data synthesis



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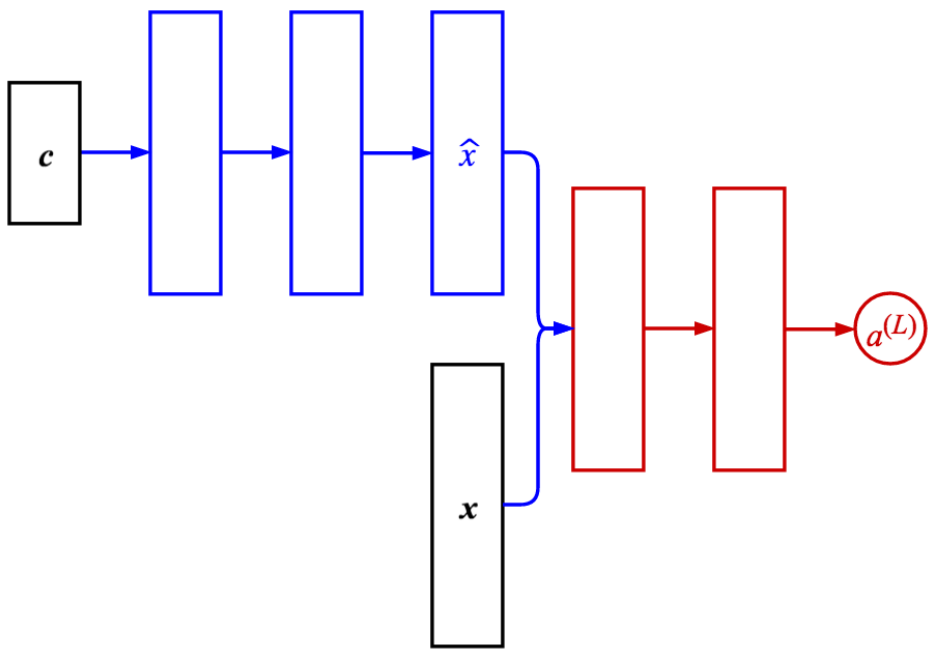
  - A binary classifier with Sigmoid output unit $a^{(L)} = \hat{\rho}$ for $\mathrm{P}(\mathrm{y} = \text{true point} \,|\, \mathbf{x}) \sim \mathrm{Bernoulli}(\rho)$



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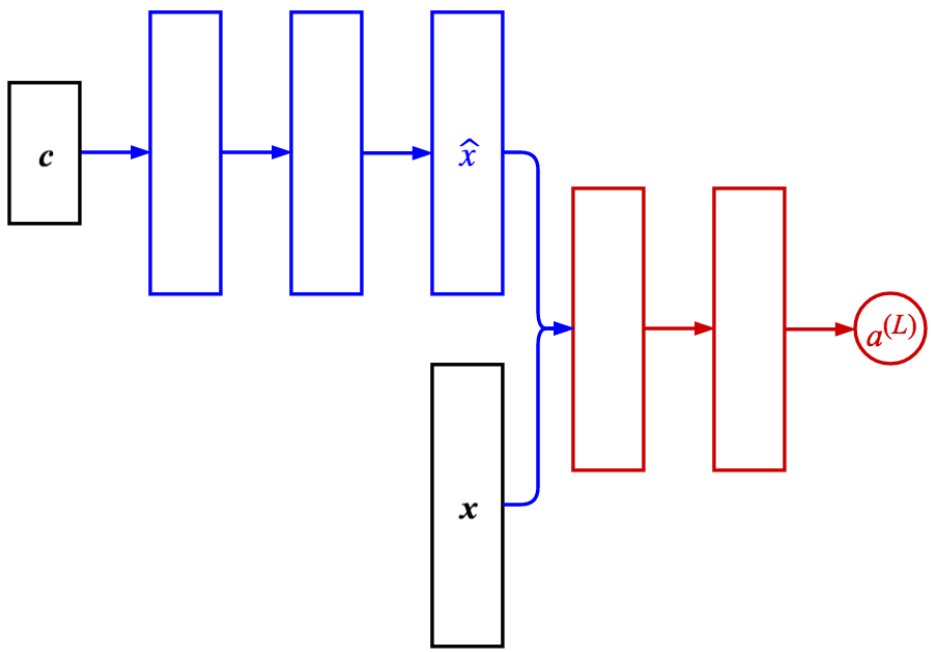
- Goal: to train a g that tricks d into believing $g(\boldsymbol{c})$ is real



Cost Function

- Given N real training points and N generated points:

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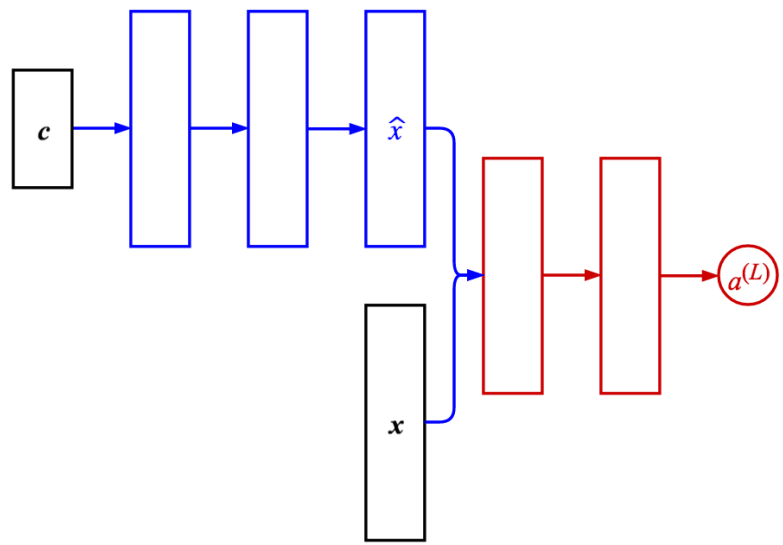

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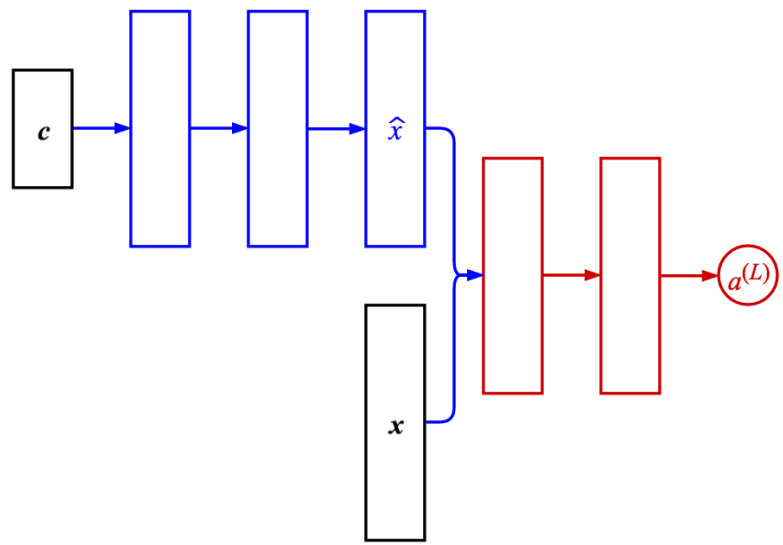

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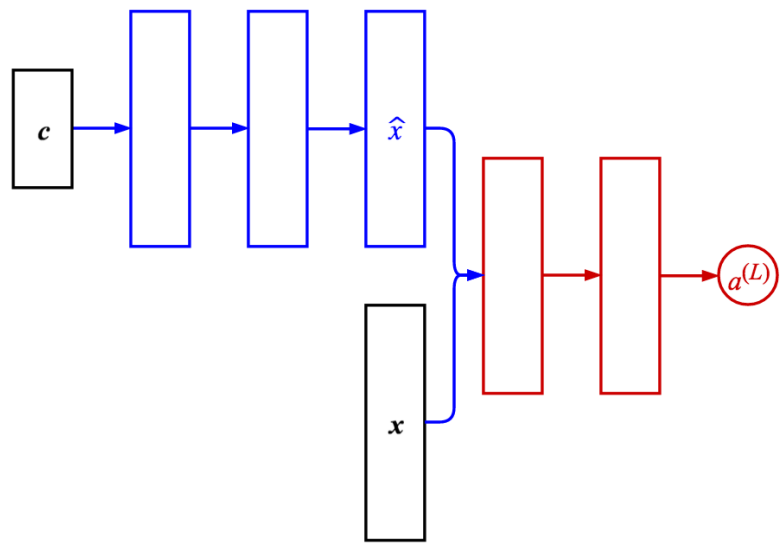

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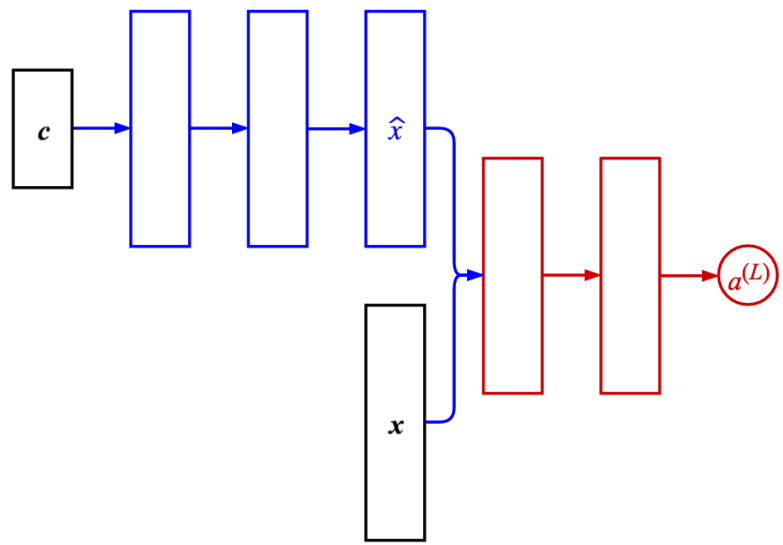
- $\hat{\rho}^{(n)}$ depends on Θ_f only
- $\hat{\rho}^{(m)}$ depends on both Θ_f and Θ_g



Training: Alternative SGD

$$\arg \min_{\Theta_g} \max_{\Theta_f} \sum_n \log f(\boldsymbol{x}^{(n)}) + \sum_m \log(1 - f(g(\boldsymbol{c}^{(m)})))$$

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- Why limiting the steps (K) when updating Θ_f?

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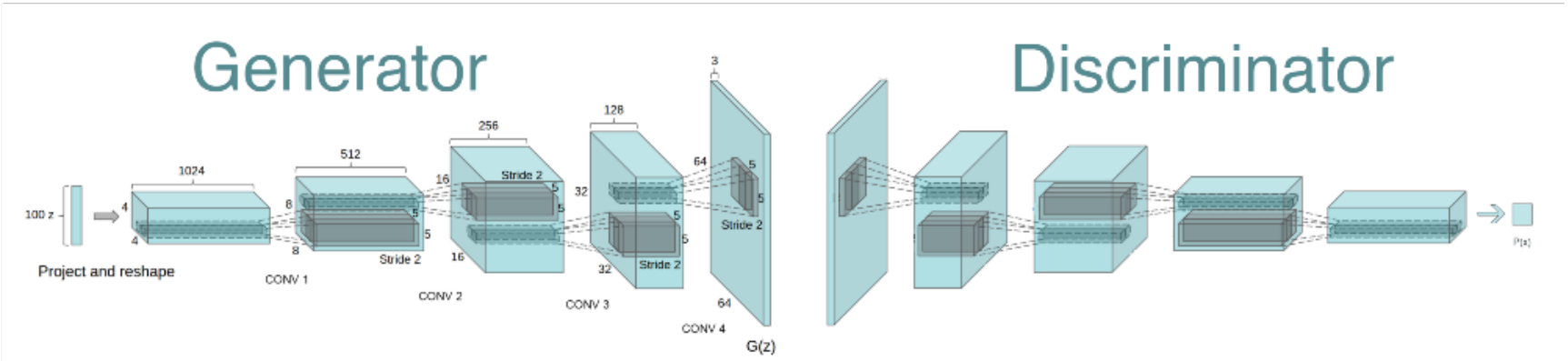
- Why limiting the steps (K) when updating Θ_f?
  - f may overfit data and give very different values once g is updated
  - Limiting K so to prevent g from being updated for "wrong" target

Results

- Domain-specific architecture, e.g., DC-GAN [17]



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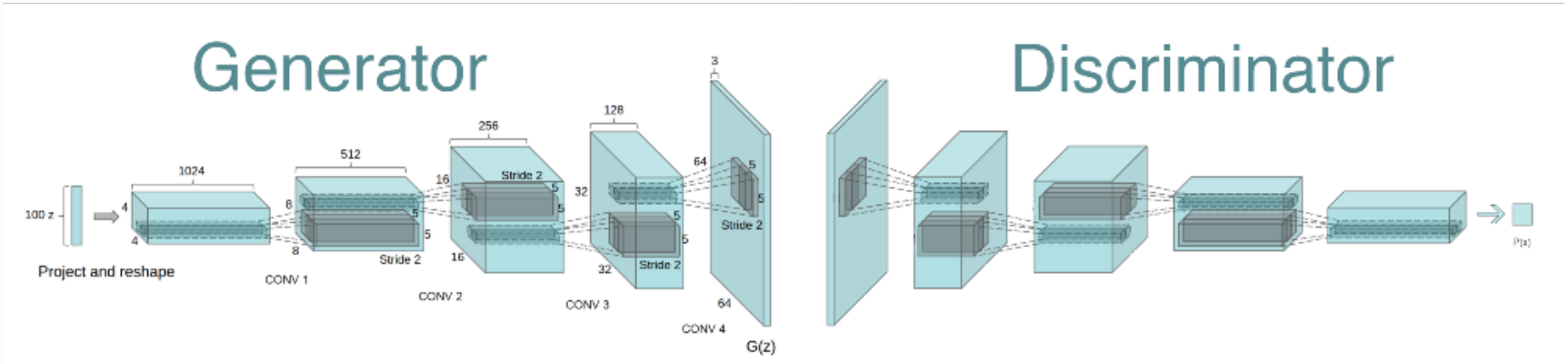

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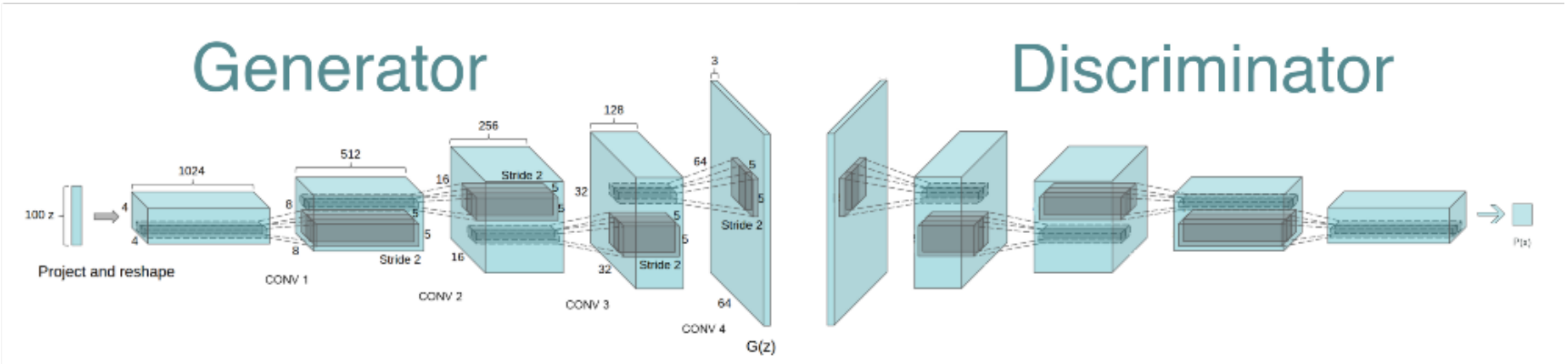

- GANs are well-known to be hard to train
  - GAN hacks on GitHub

Outline

Challenge: Non-Convergence

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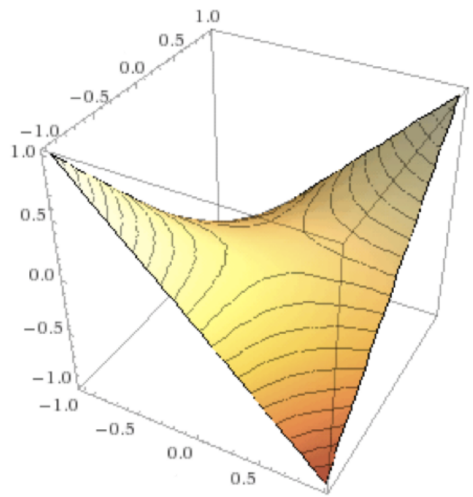

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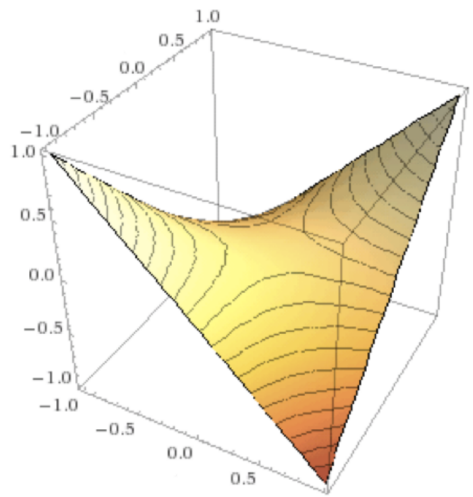

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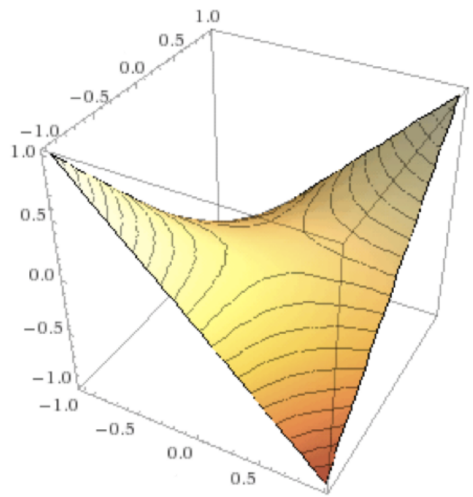
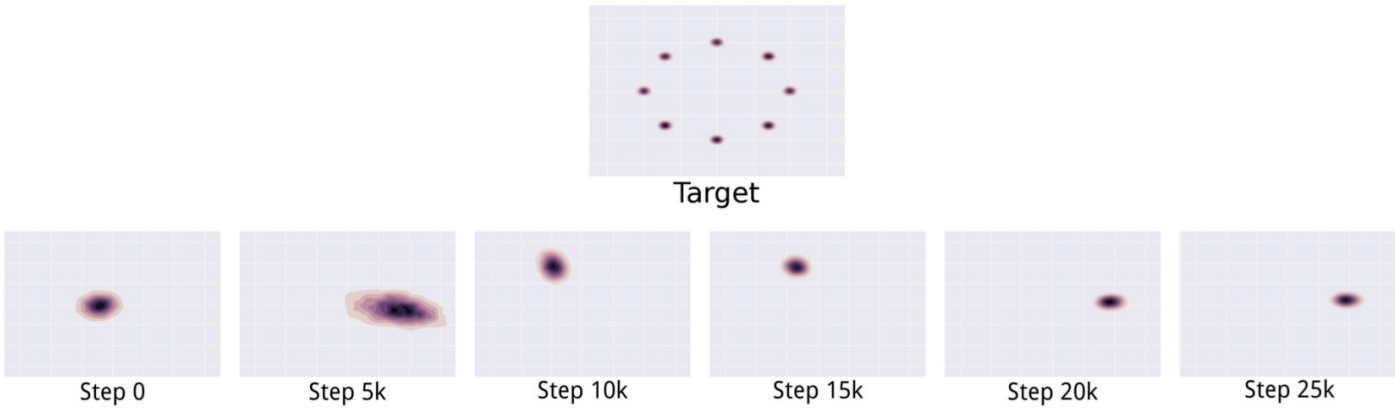


- The updated Θ_f and Θ_g may cancel each other's progress
- Requires human monitoring and termination

Mode Collapsing

- Even worse: ***mode collapsing***
  - g may oscillate from generating one kind of points to generating another kind of points



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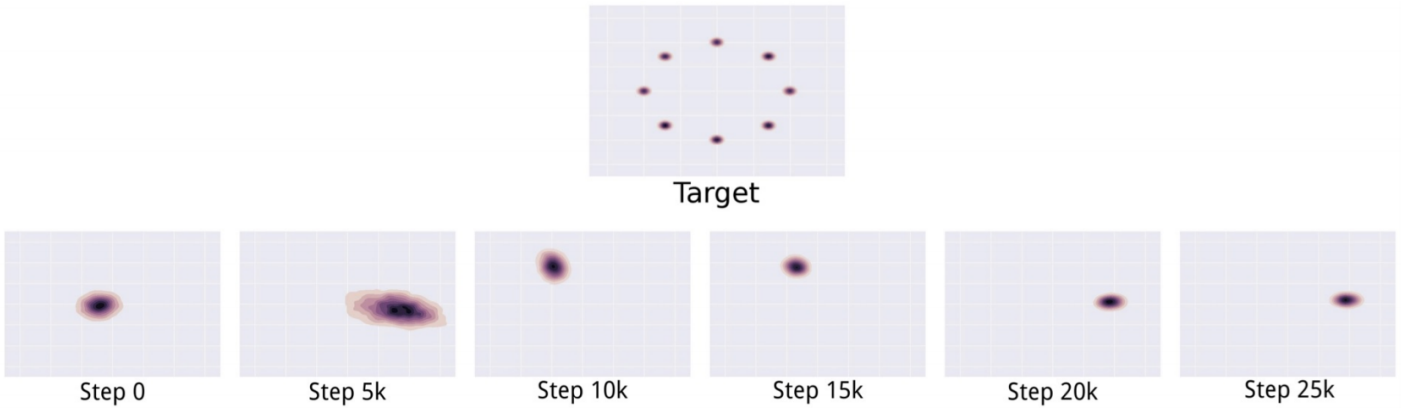

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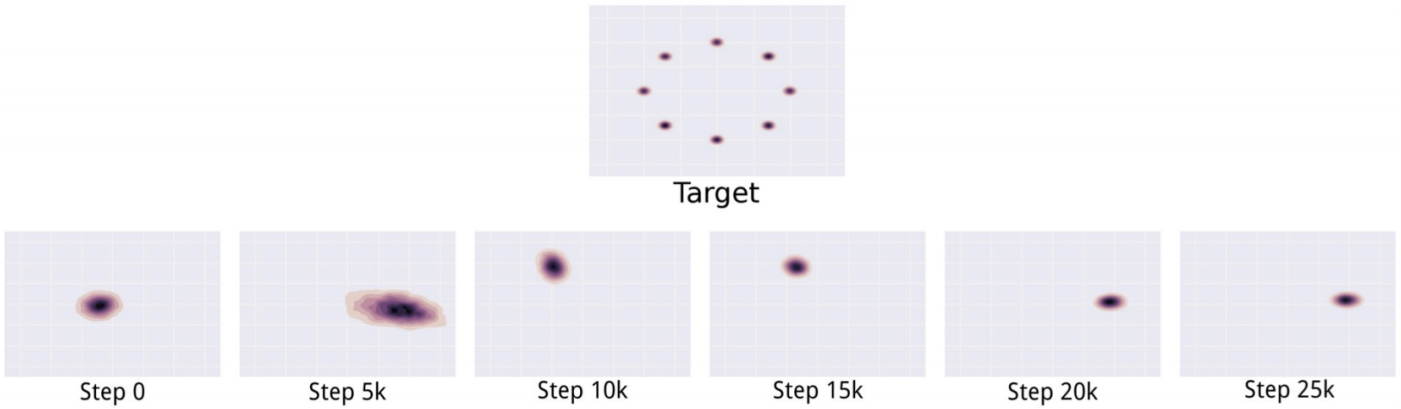

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- $\max_{\Theta_f}\min_{\Theta_g}$? g is encouraged to map every code to the "mode" that f believes is most likely to be real

Solutions

- Minibatch discrimination [21]
  - In $\max_{\Theta_f} \min_{\Theta_g}$ case, g collapses because $\nabla_{\Theta_f} C$ are computed independently for each point
  - Why not augment each $\boldsymbol{x}^{(n)}/\hat{\boldsymbol{x}}^{(n)}$ with ***batch features***?

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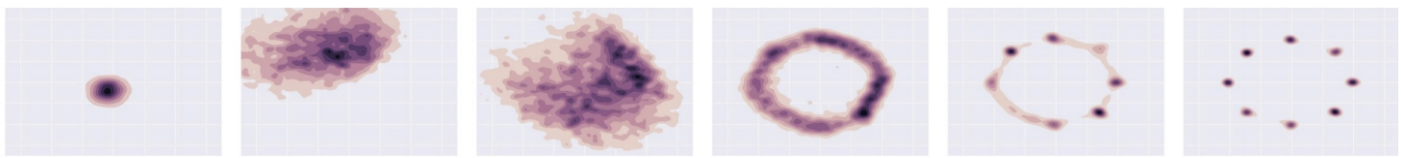
- Unrolled GANs [14]: to back-propagate through ***several*** $\max$ ***steps*** when computing $\nabla_{\Theta_g} C$



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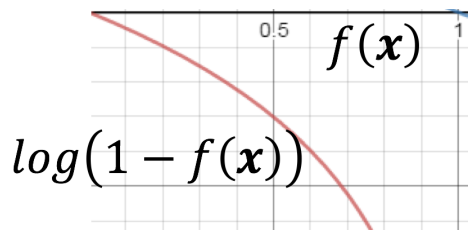

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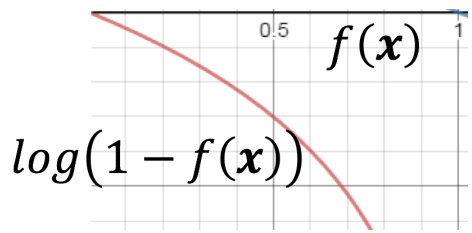

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Solution: Wasserstein GAN [1]

- Let f be a regressor ***without*** the sigmoid output layer
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$$\arg\min_{\Theta_g}\max_{\Theta_f}\sum_n f(\boldsymbol{x}^{(n)}) - \sum_m f(g(\boldsymbol{c}^{(m)}))$$

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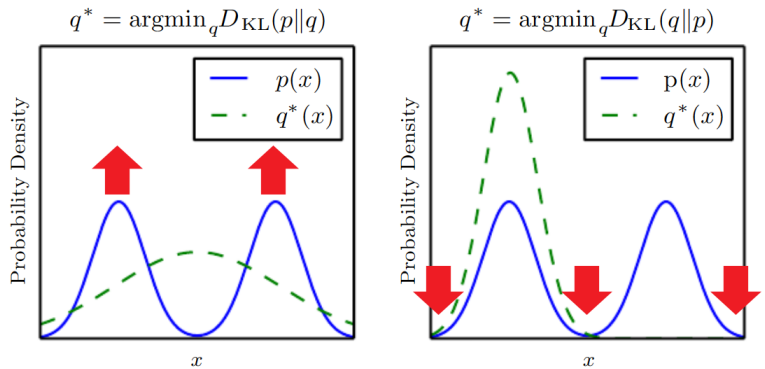

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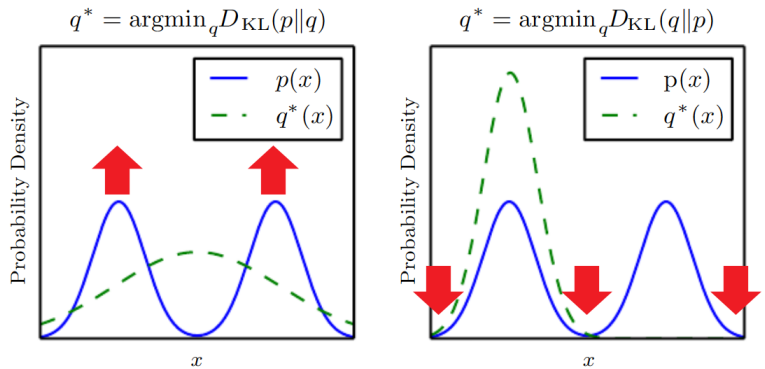

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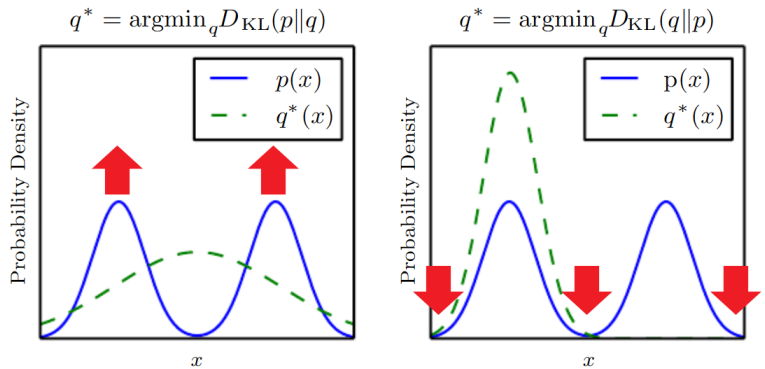

- GAN: $\arg\min_{\Theta_g} \max_{\Theta_f} \sum_n \log f(\boldsymbol{x}^{(n)}) + \sum_m \log(1 - f(g(\boldsymbol{c}^{(m)})))$
- Actually, the max term measures Jensen-Shannon divergence (a.k.a. symmetric KL divergence):

$$\mathrm{D}_{\mathrm{JS}}(\mathrm{P}_{\text{data}} \| \mathrm{P}_g) = \frac{1}{2}\mathrm{D}_{\mathrm{KL}}(\mathrm{P}_{\text{data}} \| \mathrm{Q}) + \frac{1}{2}\mathrm{D}_{\mathrm{KL}}(\mathrm{P}_g \| \mathrm{Q}), \text{ where } \mathrm{Q} = \frac{1}{2}(\mathrm{P}_g + \mathrm{P}_{\text{data}})$$

Why Jensen-Shannon Divergence? I

- Given a fixed g, we have

$$C^* = \max_{\Theta_f} \sum_n \log f(\boldsymbol{x}^{(n)}) + \sum_m \log(1 - f(g(\boldsymbol{c}^{(m)})))$$

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- To have C^*, we can find f maximizing

$$\mathrm{P}_{\mathsf{data}}(\boldsymbol{x}) \log f(\boldsymbol{x}) + \mathrm{P}_g(\boldsymbol{x}) \log(1 - f(\boldsymbol{x}))$$

  for each $\boldsymbol{x}$
  - Assuming that f has infinite capacity

Why Jensen-Shannon Divergence? II

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- Given P_{data}, P_g, and $\boldsymbol{x}$, what is the $f(\boldsymbol{x})$ that maximizes

$$\mathrm{P}_{\text{data}}(\boldsymbol{x}) \log f(\boldsymbol{x}) + \mathrm{P}_g(\boldsymbol{x}) \log(1 - f(\boldsymbol{x}))?$$

- $f^*(\boldsymbol{x}) = \frac{\mathrm{P}_{\text{data}}(\boldsymbol{x})}{\mathrm{P}_{\text{data}}(\boldsymbol{x}) + \mathrm{P}_g(\boldsymbol{x})} \in [0, 1]$ [Proof]
- That is,

$$C^* = \max_{\Theta_f} \int_{\boldsymbol{x}} [\mathrm{P}_{\text{data}}(\boldsymbol{x}) \log f(\boldsymbol{x}) + \mathrm{P}_g(\boldsymbol{x}) \log(1 - f(\boldsymbol{x}))] d\boldsymbol{x}$$

Why Jensen-Shannon Divergence? II

- Given P_{data}, P_g, and $\boldsymbol{x}$, what is the $f(\boldsymbol{x})$ that maximizes

$$\mathrm{P}_{\text{data}}(\boldsymbol{x}) \log f(\boldsymbol{x}) + \mathrm{P}_g(\boldsymbol{x}) \log(1 - f(\boldsymbol{x}))?$$

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&= -2\log 2 + 2\mathrm{D}_{\mathrm{JS}}(\mathrm{P}_{\text{data}} \| \mathrm{P}_g)
\end{aligned}$$

Balance between g and f, Revisited I

- Cost function of GAN:

$$\begin{aligned}&\arg\min_{\Theta_g} \max_{\Theta_f} \sum_n \log f(\boldsymbol{x}^{(n)}) + \sum_m \log(1 - f(g(\boldsymbol{c}^{(m)}))) \\ &= \arg\min_{\Theta_g} -2\log 2 + 2\mathrm{D}_{\mathsf{JS}}(\mathrm{P}_{\mathsf{data}} \| \mathrm{P}_g)\end{aligned}$$

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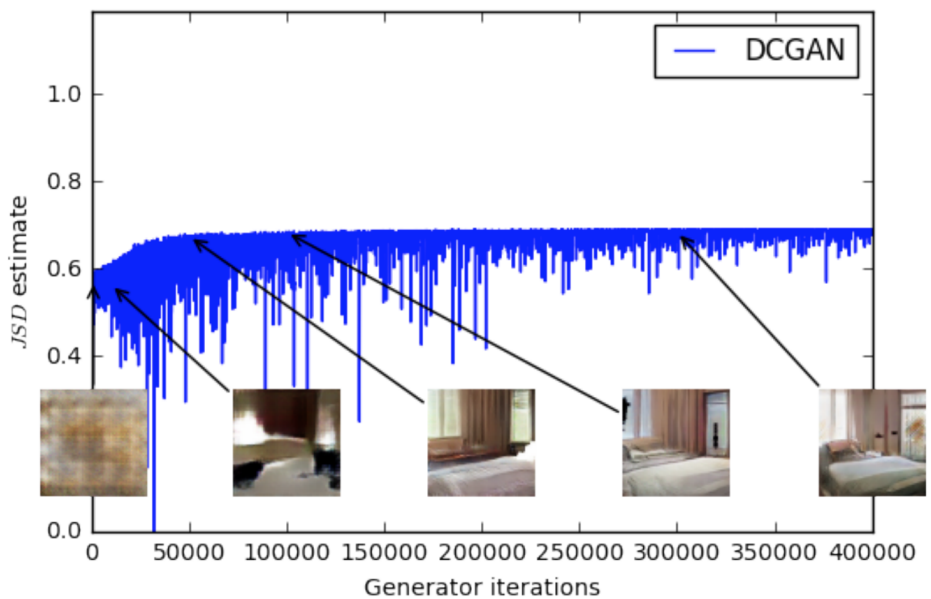
- However, no mater how g changes, $\mathrm{D}_{\mathrm{JS}}(\mathrm{P}_{\mathrm{data}} \| \mathrm{P}_g)$ remains high during the GAN training process



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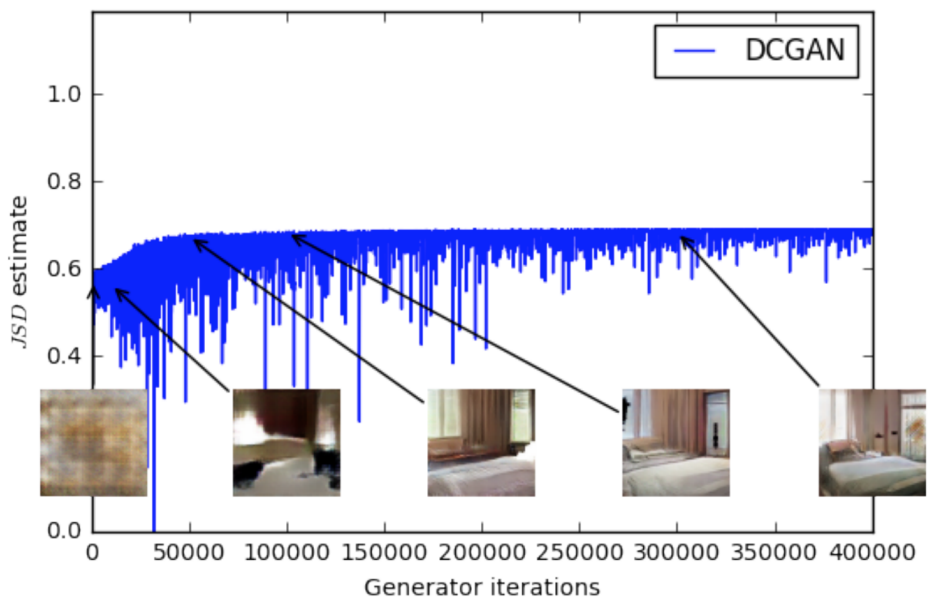
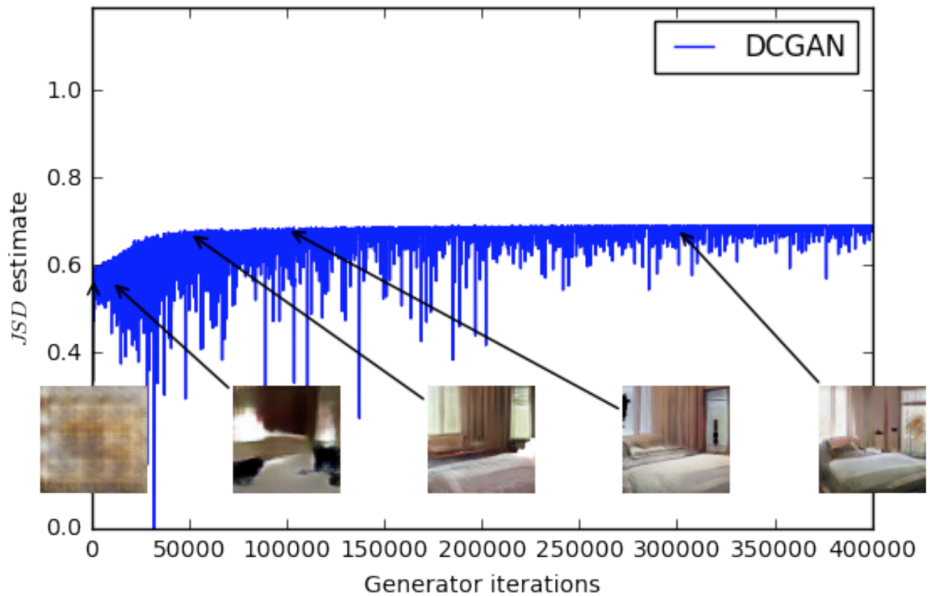


- There's something wrong with the design of the inner max problem!

Balance between g and f, Revisited II

- GAN: $\arg\min_{\Theta_g} D_{JS}(P_{data} \| P_g)$



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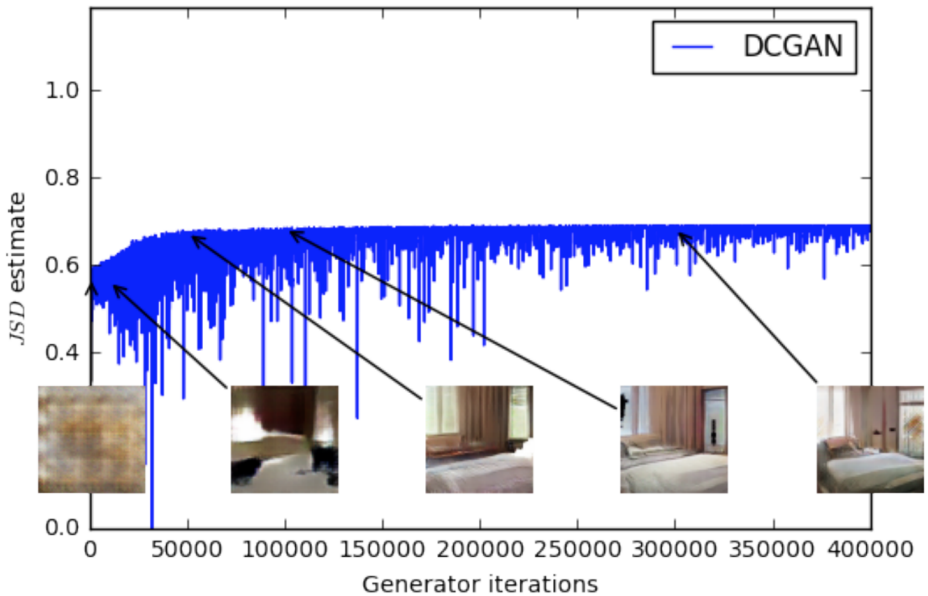

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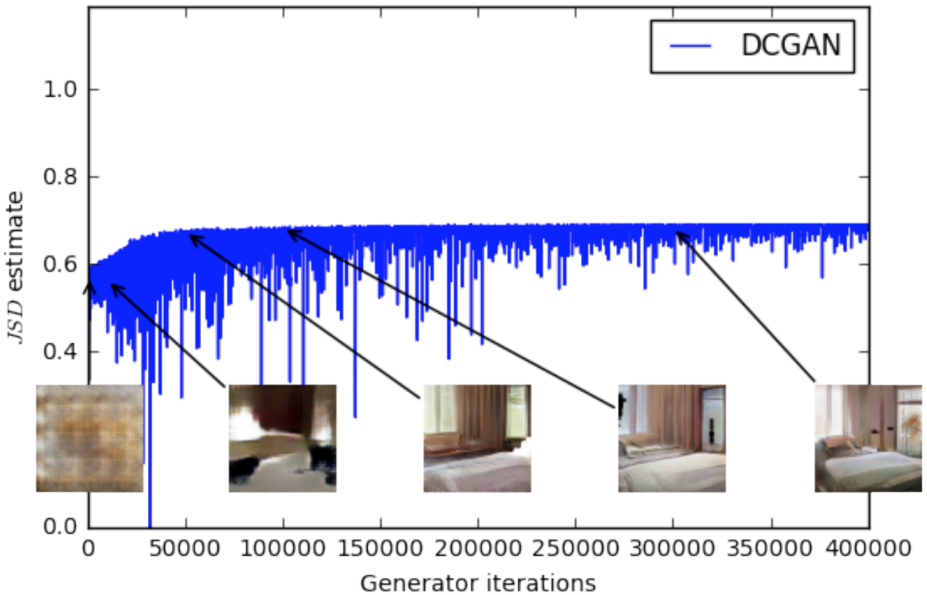

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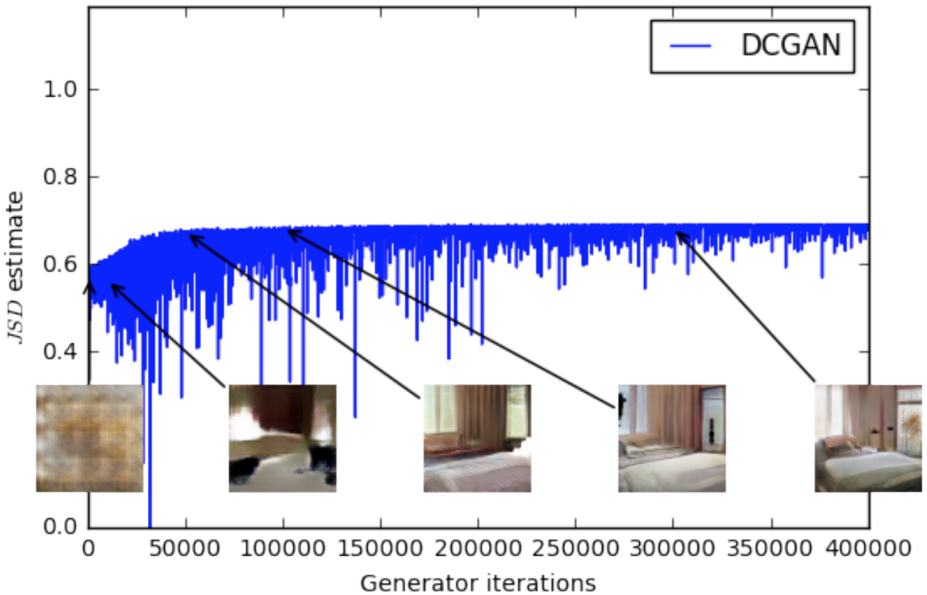
- Let's see the case when P_g and P_{data} are "disjointed"



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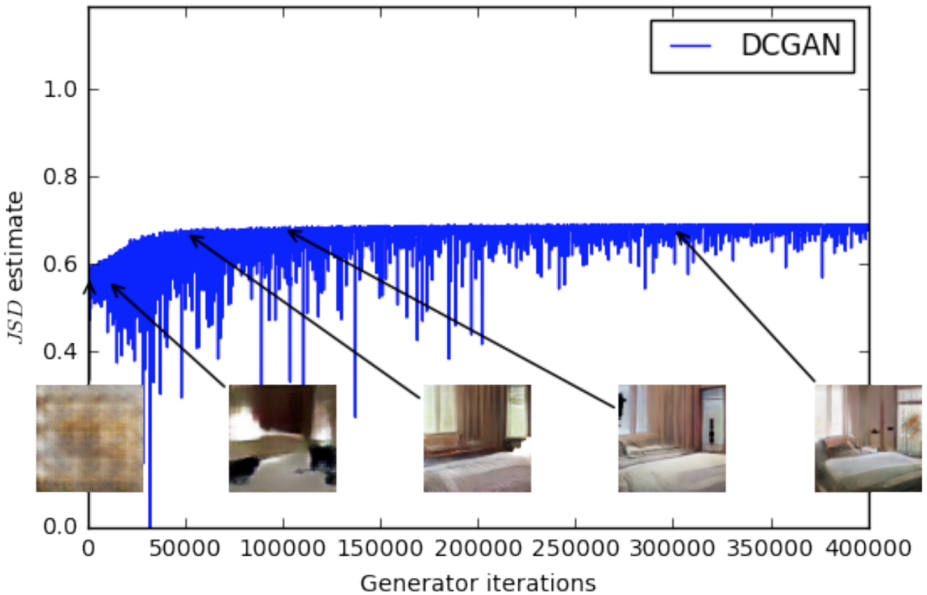

- Suppose $P_g(\boldsymbol{x}) \neq 0 \Leftrightarrow P_{data}(\boldsymbol{x}) = 0$ and $P_g(\boldsymbol{x}) = 0 \Leftrightarrow P_{data}(\boldsymbol{x}) \neq 0$, we have

$$D_{JS}(P_g \| P_{data}) = \frac{1}{2} D_{KL}(P_g \| \frac{P_g + P_{data}}{2}) + \frac{1}{2} D_{KL}(P_{data} \| \frac{P_g + P_{data}}{2})$$

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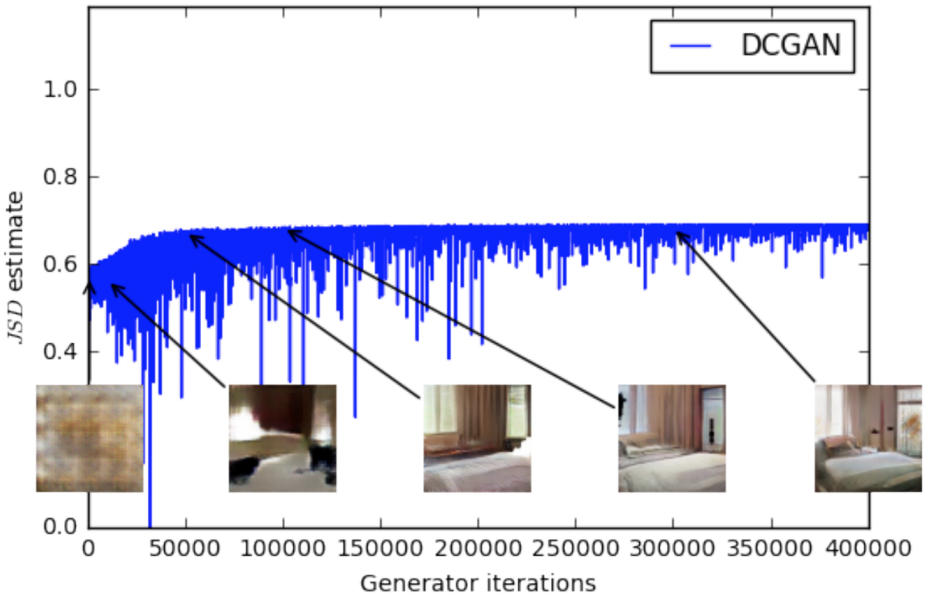

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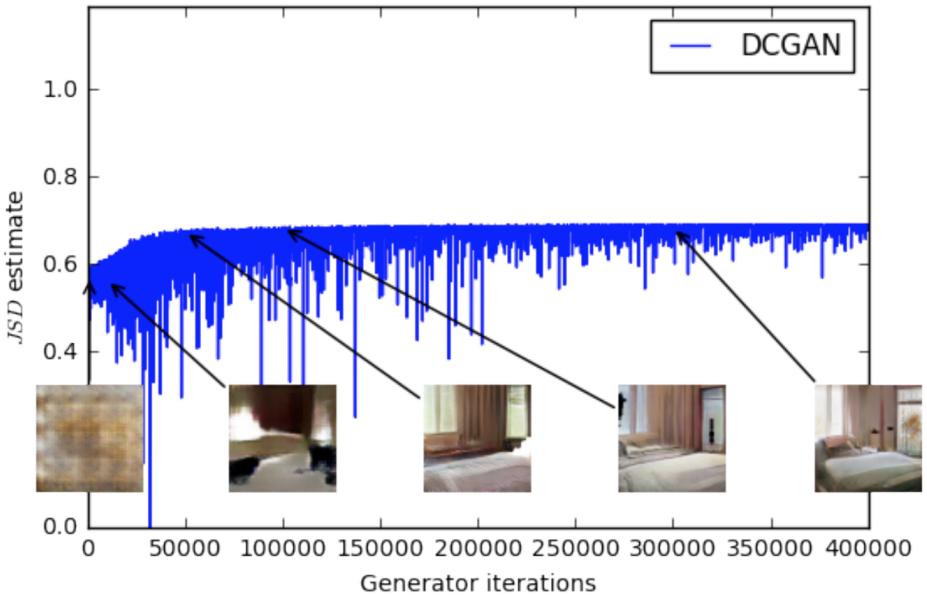

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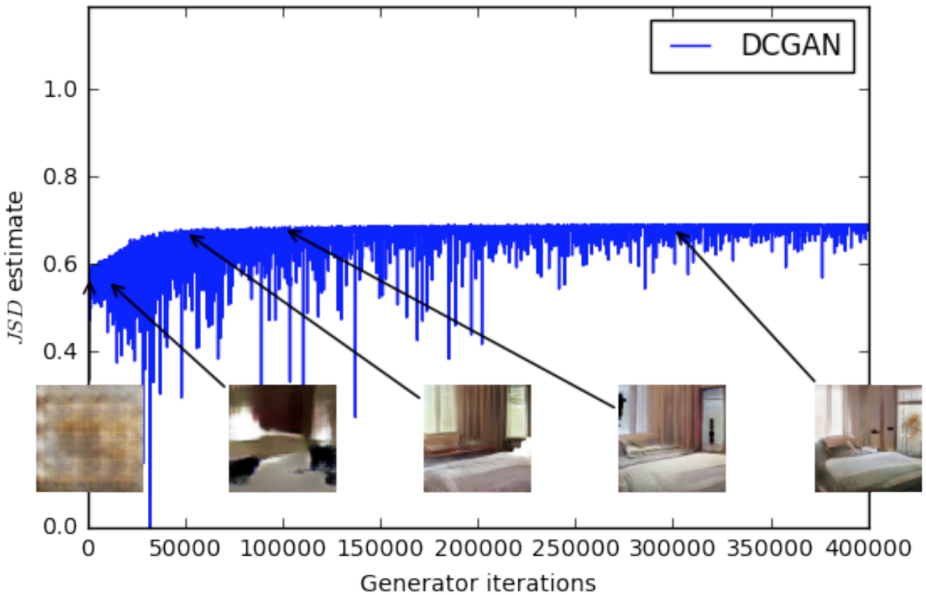

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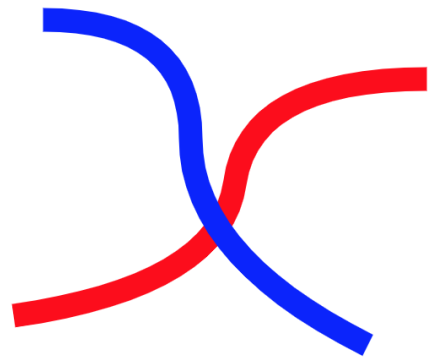
- Are P_g and $\mathrm{P_{data}}$ really disjointed during the GAN training?

Disjoining $\mathbf{P}_g$ and $\mathbf{P}_{\text{data}}$

- In a high dimensional space, $\mathbf{x}$ and $g(\mathbf{z})$ may resides in low dimensional manifolds
  - P_g and P_{data} may have values only on the manifolds



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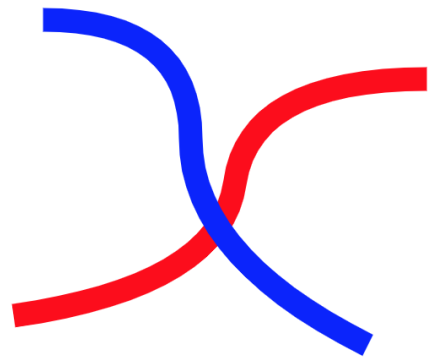

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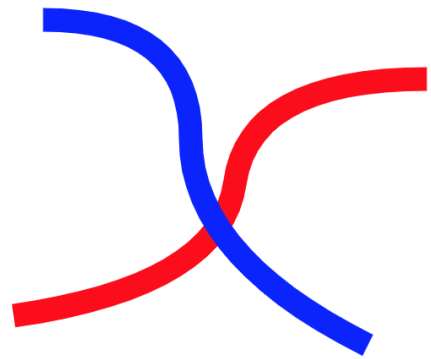

- P_g and P_{data} can be very different initially during GAN training
- The intersections where $\mathrm{P}_g(\boldsymbol{x}) \neq 0$ and $\mathrm{P}_{\text{data}}(\boldsymbol{x}) \neq 0$ can be neglected
  - $\mathrm{P}_g(\boldsymbol{x}) \neq 0 \Leftrightarrow \mathrm{P}_{\text{data}}(\boldsymbol{x}) = 0$ and $\mathrm{P}_g(\boldsymbol{x}) = 0 \Leftrightarrow \mathrm{P}_{\text{data}}(\boldsymbol{x}) \neq 0$ almost surely
  - Maximum JS divergence at all time

Better Divergence Measure?

- GAN: $\arg\min_{\Theta_g} \mathrm{D_{JS}}(\mathrm{P_{data}} \| \mathrm{P}_g)$
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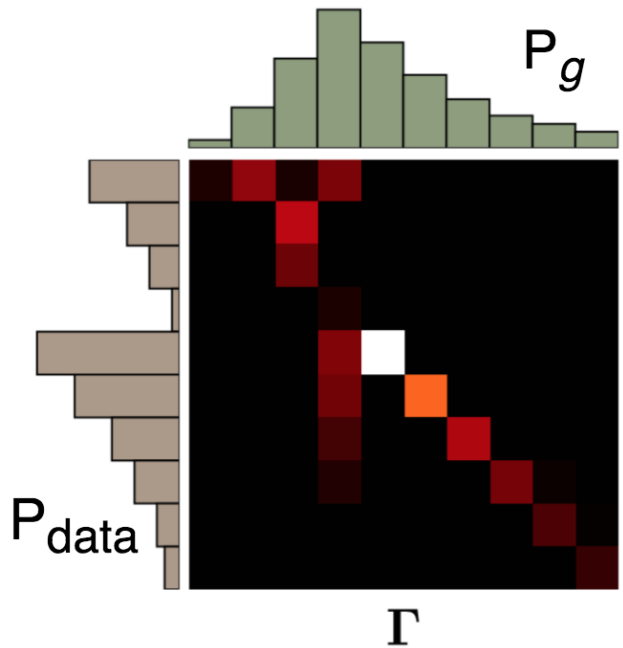

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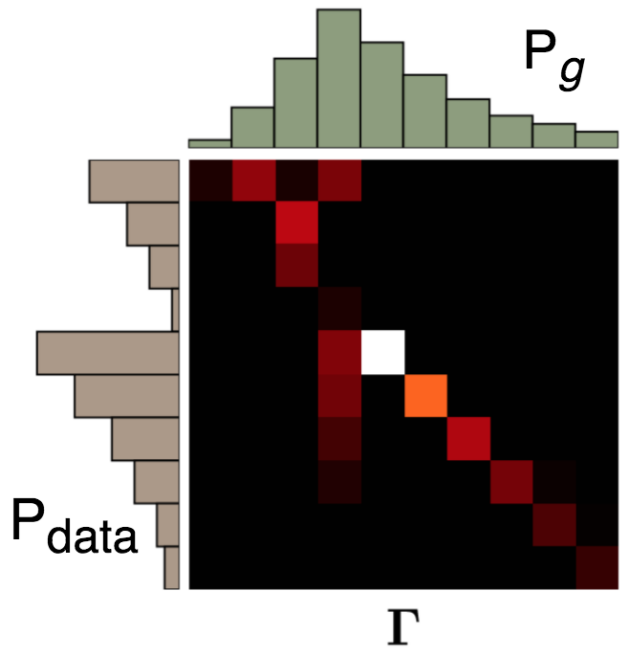

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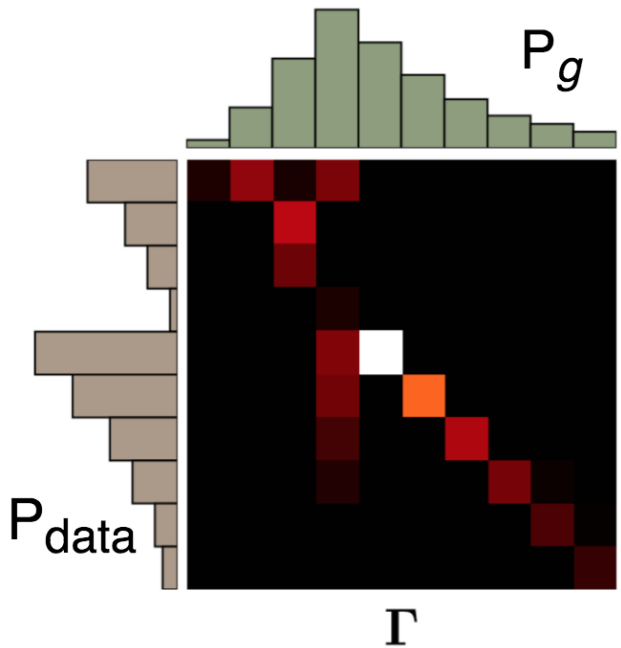
- Intuitively, the minimal "cost" to change $\mathrm{P_{data}}$ into P_g
- $\mathrm{W}(\mathrm{P_{data}}, \mathrm{P}_g)$ measures the "divergence" between P_g and $\mathrm{P_{data}}$ ***even when they are disjointed***



Wasserstein GAN I

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- Unfortunately, $\mathrm{W}(\mathrm{P}_{\mathsf{data}}, \mathrm{P}_g)$ is hard to solve directly

Wasserstein GAN II

Theorem

Consider f's that are Lipschitz continuous with constant 1, i.e.,

$$|f(\boldsymbol{x}) - f(\hat{\boldsymbol{x}})| \leq 1 \cdot \|\boldsymbol{x} - \hat{\boldsymbol{x}}\|, \forall \boldsymbol{x}, \hat{\boldsymbol{x}},$$

we have [a]

$$W(P_{\text{data}}, P_g) \quad = \sup_f E_{\mathbf{x} \sim P_{\text{data}}}[f(\mathbf{x})] - E_{\mathbf{x} \sim P_g}[f(\mathbf{x})]$$

Wasserstein GAN II

Theorem

Consider f's that are Lipschitz continuous with constant 1, i.e.,

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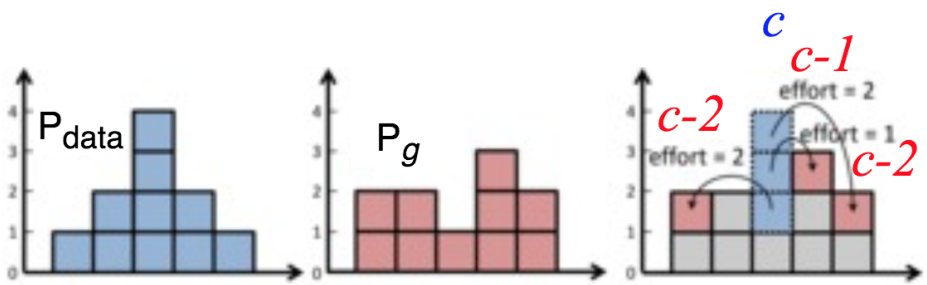
[a] https://vincentherrmann.github.io/blog/wasserstein/



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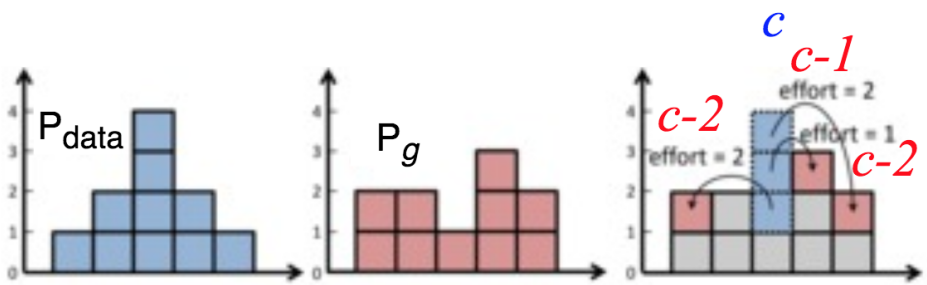

- W-GAN [1]: $\arg\min_{\Theta_g} \max_{\Theta_f} \mathrm{E}_{\mathbf{x} \sim \mathrm{P}_{\text{data}}}[f(\mathbf{x})] - \mathrm{E}_{\mathbf{x} \sim \mathrm{P}_g}[f(\mathbf{x})]$

Alternate SGD for W-GAN

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- f a regressor ***without*** the sigmoid output layer
- Initialize Θ_g for g and Θ_f for f
- At each SGD step/iteration:

1. Repeat K times (with fixed Θ_g):
   1. Sample N real points $\{\boldsymbol{x}^{(n)}\}_n$ from $\mathbb{X}$ and N codes from $\mathbf{c} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I})$
   2. $\Theta_f \leftarrow \Theta_f + \eta\, clip(\nabla_{\Theta_f}[\sum_n f(\boldsymbol{x}^{(n)}) - \sum_m f(g(\boldsymbol{c}^{(m)}))])$
2. Execute once (with fixed Θ_f):
   1. Sample N codes from $\mathbf{c} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I})$
   2. $\Theta_g \leftarrow \Theta_g - \eta \nabla_{\Theta_g}[-\sum_m f(g(\boldsymbol{c}^{(m)}))]$

Why Gradient Clipping?

- Update rule for Θ_f:

$$\Theta_f \leftarrow \Theta_f + \eta \, clip(\nabla_{\Theta_f}[\sum_n f(\boldsymbol{x}^{(n)}) - \sum_m f(g(\boldsymbol{c}^{(m)}))])$$

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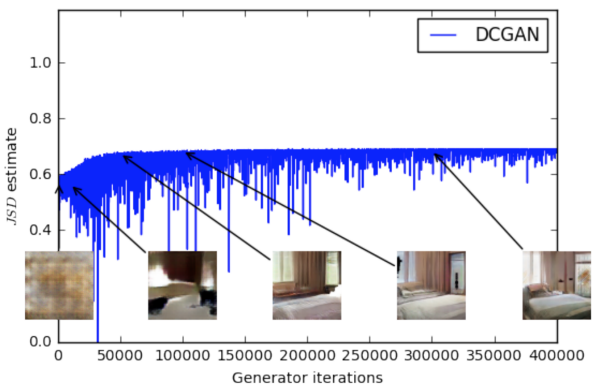
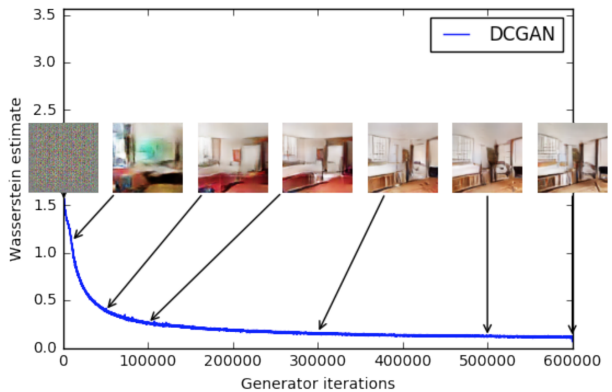
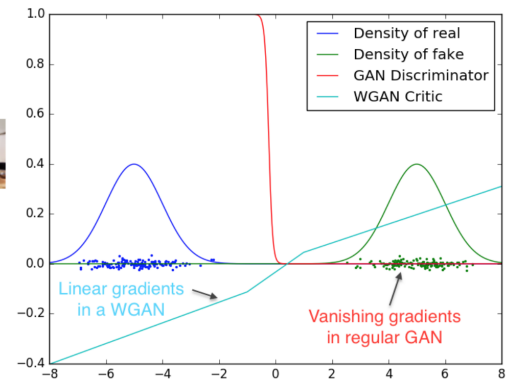
- Heuristics for making f 1-Lipchitz

Advantages of W-GAN

- "Corrects" the inner $\max$ problem
  - Wasserstein distance guides g even when P_g and P_{data} "disjointed"
  - Training less sensitive to K (balance between g and f)
- The $\max$ value can be used as a "stop" indicator
- f a regressor, avoids vanishing gradients for g







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- However, too small a τ severely limits the capacity of f such it cannot actually maximize

$$\max_{\Theta_f} \mathrm{E}_{\mathbf{x} \sim \mathrm{P}_{\mathsf{data}}} [f(\mathbf{x})] - \mathrm{E}_{\mathbf{x} \sim \mathrm{P}_g} [f(\mathbf{x})]$$

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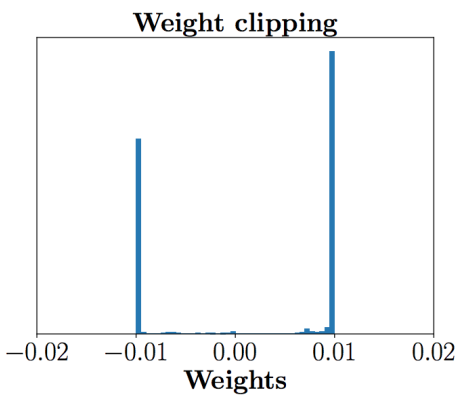
- Distribution of weight values of f ($\tau = 0.01$):



- Exploding and vanishing gradients

Improved W-GAN II

- If f is 1-Lipschitz, then $\|\nabla f(\boldsymbol{x})\| \leq 1$ for all $\boldsymbol{x}$
- Why not just panelize $\|\nabla f(\boldsymbol{x})\| > 1$ for all $\boldsymbol{x}$?
- Cost function:

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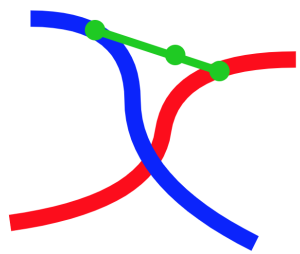

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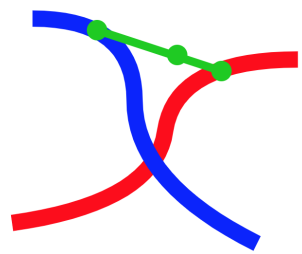
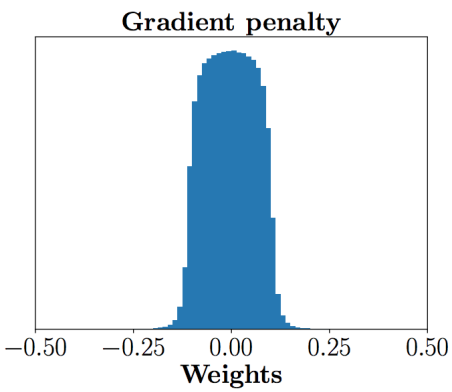


- W-GAN-GP [4]:

$$\arg\min_{\Theta_g} \max_{\Theta_f} \sum_n f(\boldsymbol{x}^{(n)}) - \sum_m f(g(\boldsymbol{c}^{(m)})) - \lambda \sum_p (\|\nabla f(\boldsymbol{x}^{(p)})\| - 1)^2$$

  - The larger $f(\boldsymbol{x}^{(p)})$ the better (subject to $\|\nabla f(\boldsymbol{x}^{(p)})\| \leq 1$)
  - Faster convergence

Results



DCGAN	LSGAN	WGAN (clipping)	WGAN-GP (ours)
Baseline (G: DCGAN, D: DCGAN)			
G: No BN and a constant number of filters, D: DCGAN			
G: 4-layer 512-dim ReLU MLP, D: DCGAN			
No normalization in either G or D			
Gated multiplicative nonlinearities everywhere in G and D			
tanh nonlinearities everywhere in G and D			
101-layer ResNet G and D			

Challenge: Global Coherence

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- Large images generated by GANs usually lack global coherency

Counting

Perspective

Shape

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- A CNN, when used as f, detects ***existence*** of patterns more than their ***relative positions***
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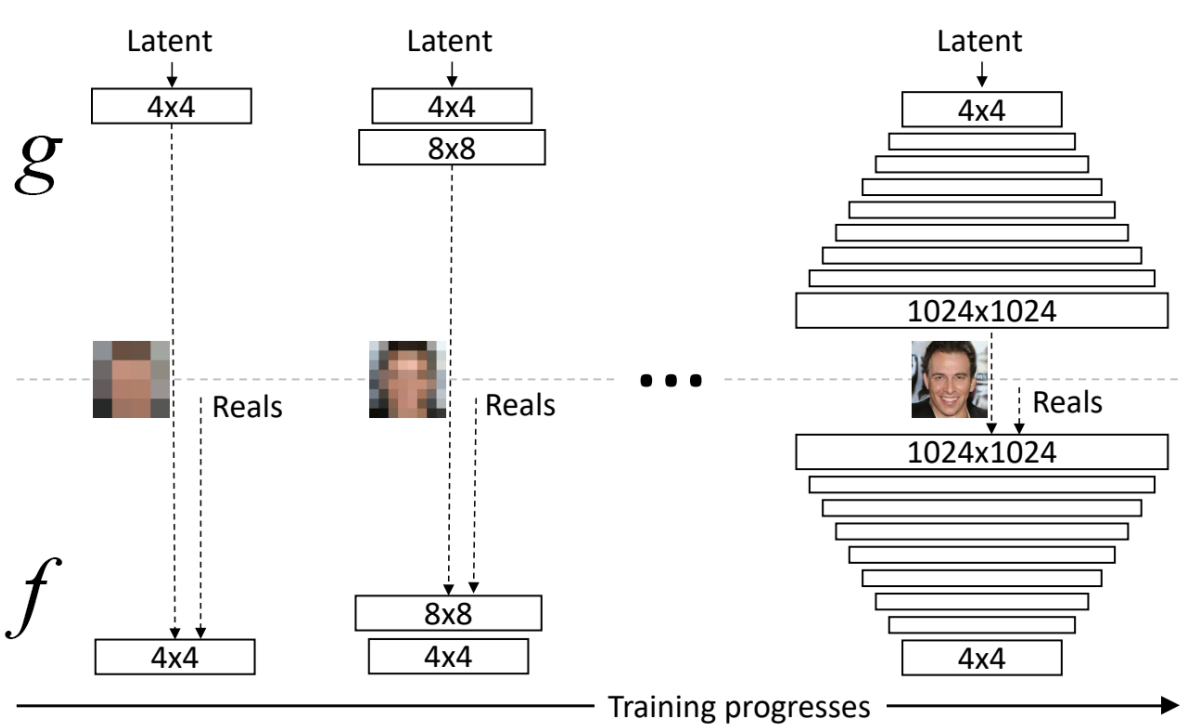
  - f loses track of the position of a pattern after several pooling layers
  - Relative position of patterns in $\mathbb{X}$ may change due to different view angels
- Solutions? A better f, such as the CapsuleNet [20]?

Progressive Growing of GANs [7]



- Incrementally adds new layers in sequential GAN trainings
  - Convolution + upsampling for g each time
  - Convolution + downpooling for f each time
- Real images are downscaled to match the current resolution of g

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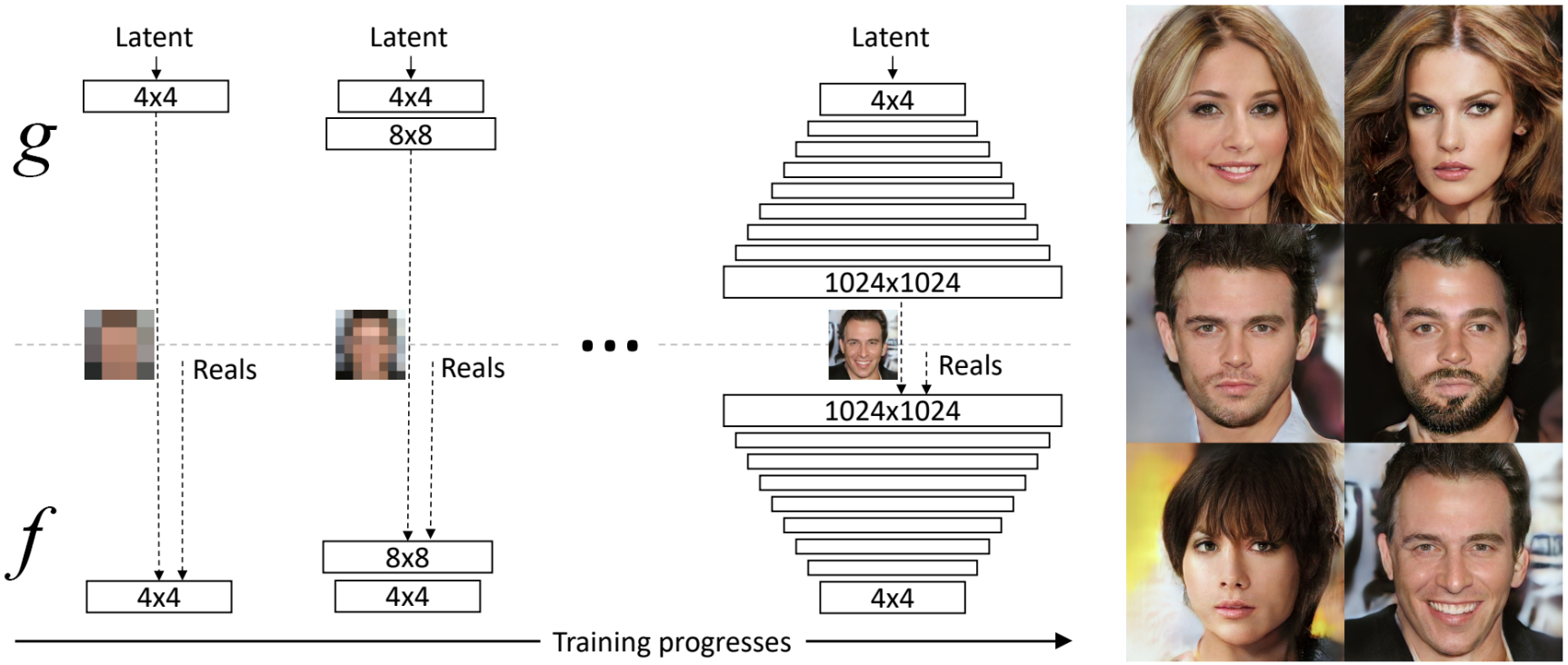
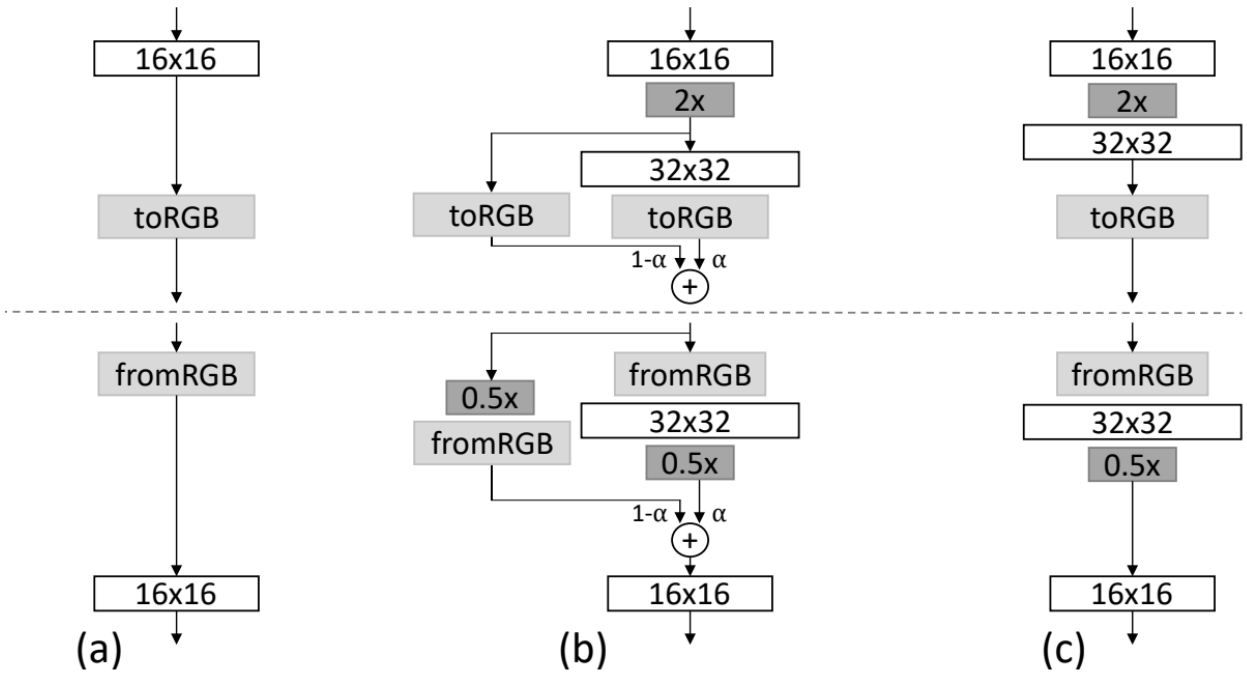


- Incrementally adds new layers in sequential GAN trainings
  - Convolution + upsampling for g each time
  - Convolution + downpooling for f each time
- Real images are downscaled to match the current resolution of g
- Goal: to let new layers ***add details without ruining the context***

Transition when Adding a New Layer



- Gradually increases α
  - New convolution layer in g/f learns to generate/detect details first
  - Then learns the "context"

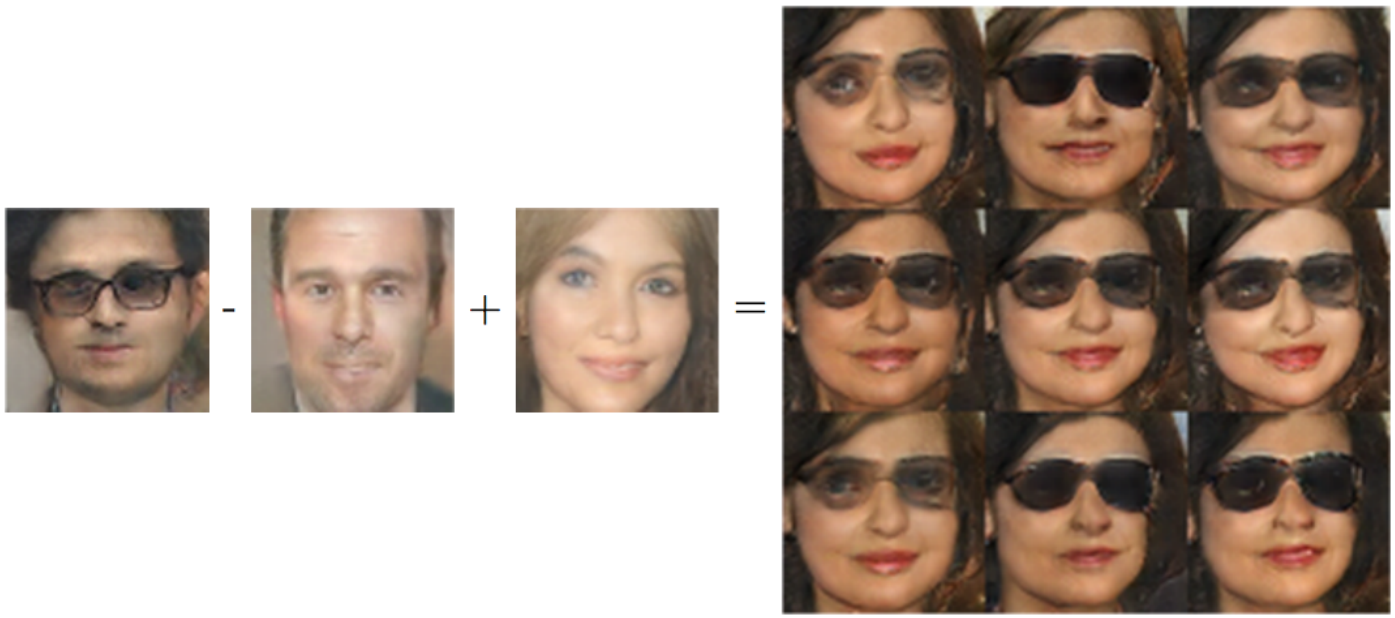
Results

- Minibatch discrimination [21] + W-GAN-GP [4] + progressive growing [7] + other tricks
- $2 \sim 6$ times faster

Outline

Code Space Arithmetics

- DC-GAN [17] can learn to use codes in meaningful ways:



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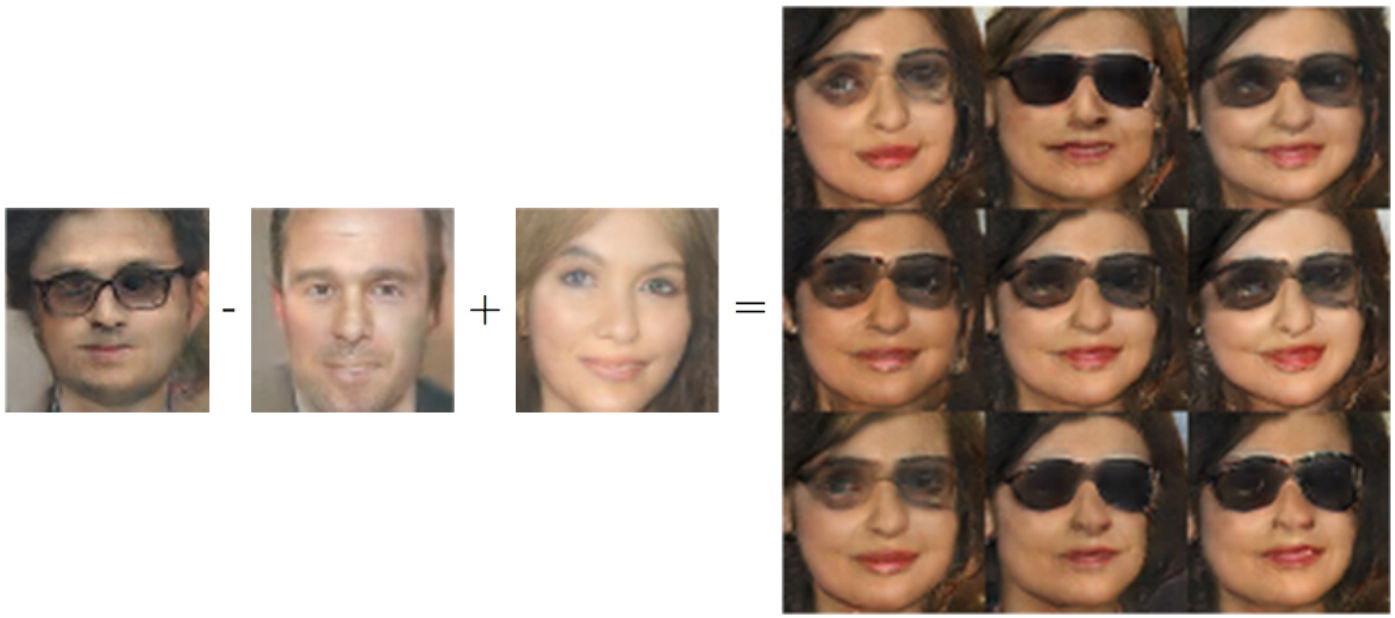


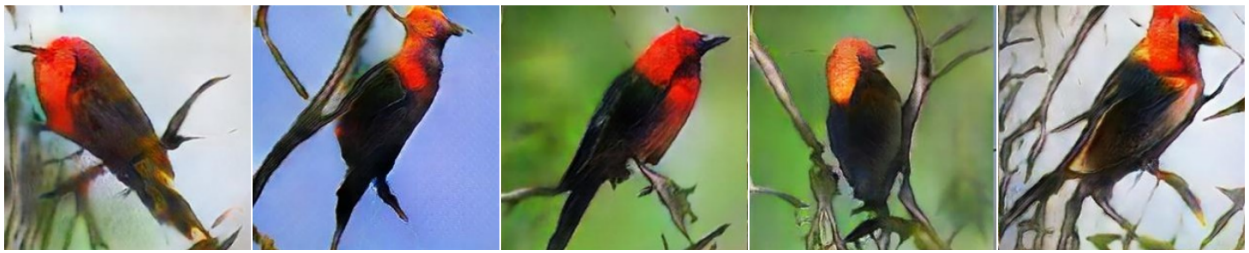
- Finding codes for images with constraints [28, 2] ▸ Demo 1 ▸ Demo 2

$$\arg\min_{\boldsymbol{c}} \|mask(g(\boldsymbol{c})) - \text{constraint}\|_F$$

Conditional GAN I

- Text to image synthesis [18]: $\mathbb{X} = \{(\boldsymbol{x}^{(n)}, \boldsymbol{\phi}^{(n)})\}_n$

  "*This bird is completely red with black wings and pointy beak.*"



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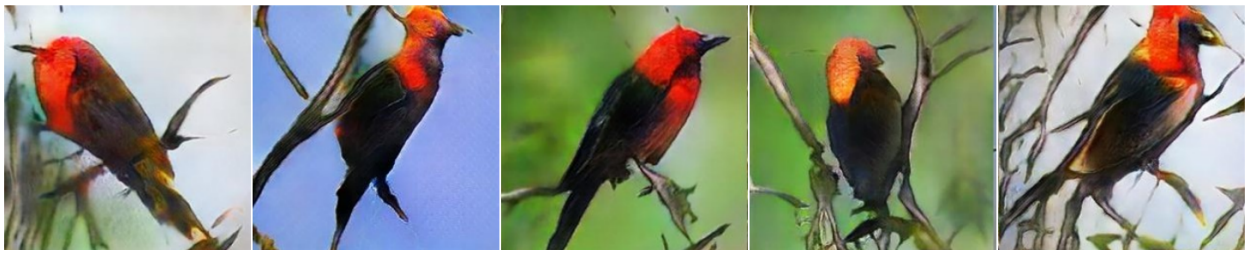

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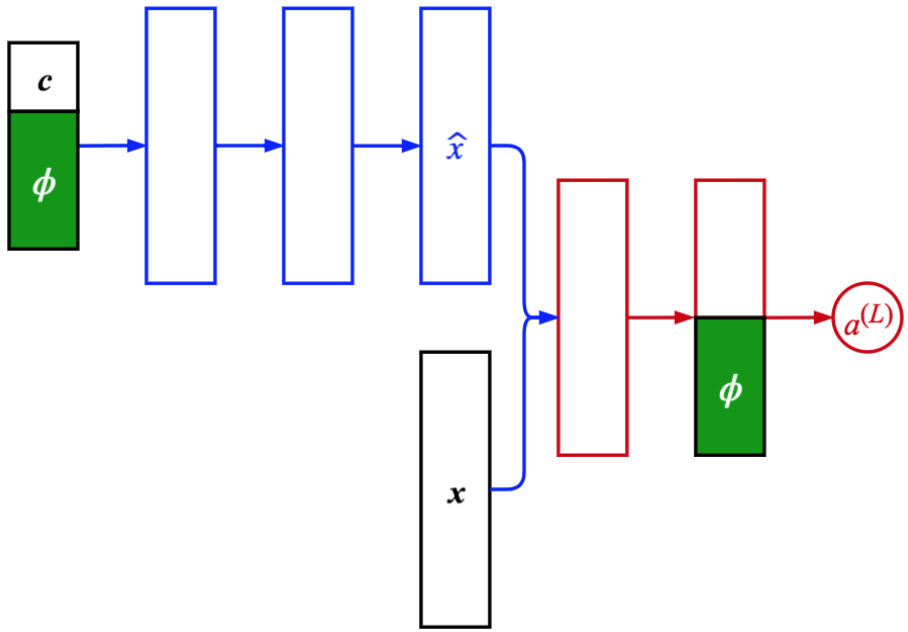
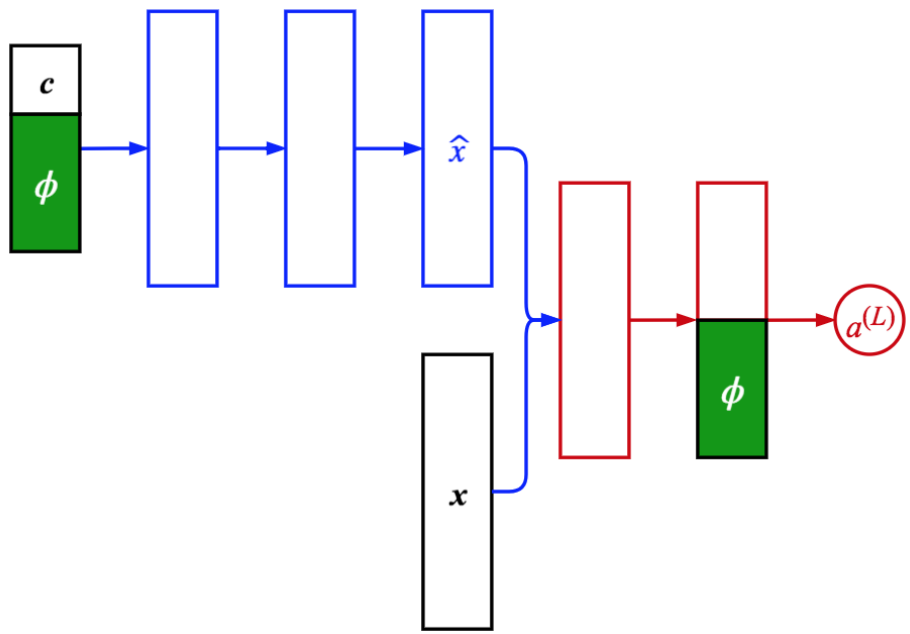


Conditional GAN II

- Pitfall: g and f can choose to ignore the condition ϕ altogether
- Solution?



Conditional GAN II

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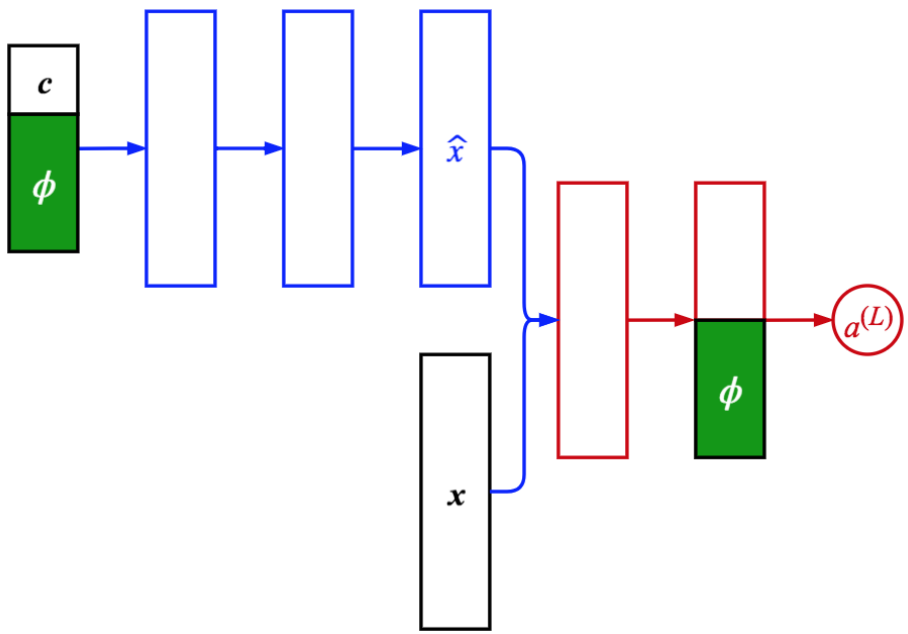
- Solution? Conditioned labeling

$$(\boldsymbol{x}^{(n)}, \boldsymbol{\phi}^{(n)}) \Rightarrow \text{true}$$
$$(\boldsymbol{x}^{(n)}, \boldsymbol{\phi}') \Rightarrow \text{false}, \forall \boldsymbol{\phi}' \neq \boldsymbol{\phi}^{(n)}$$
$$(\hat{\boldsymbol{x}}^{(m)}, \boldsymbol{\phi}^{(m)}) \Rightarrow \text{false}$$



Super Resolution [10]

bicubic
(21.59dB/0.6423)

SRResNet
(23.53dB/0.7832)

SRGAN
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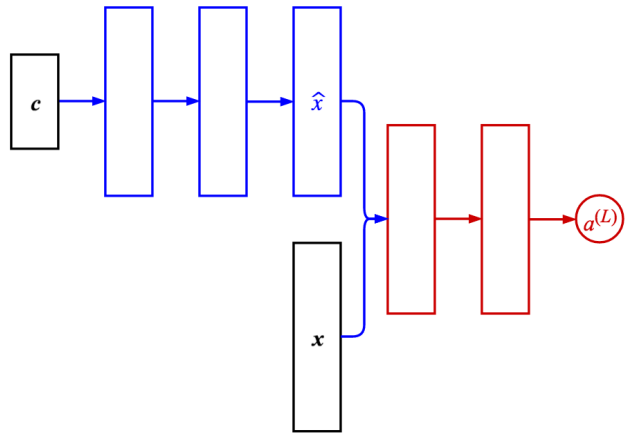
original

Super Resolution [10]

- g : low res img $\rightarrow$ high res img
  - Training: $\boldsymbol{c}$'s are downscaled images



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- No "creativity," f acts as a better lose metric

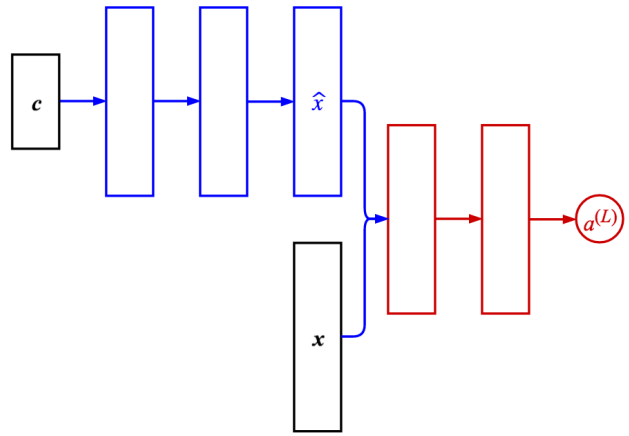

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Image-to-Image Translation [5] I

- Given an image $\boldsymbol{x}_{\text{src}}$ in source domain, generate image(s) $\boldsymbol{x}_{\text{target}}$ in target domain
  - $\boldsymbol{x}_{\text{src}}$ and $\boldsymbol{x}_{\text{target}}$ are semantically aligned

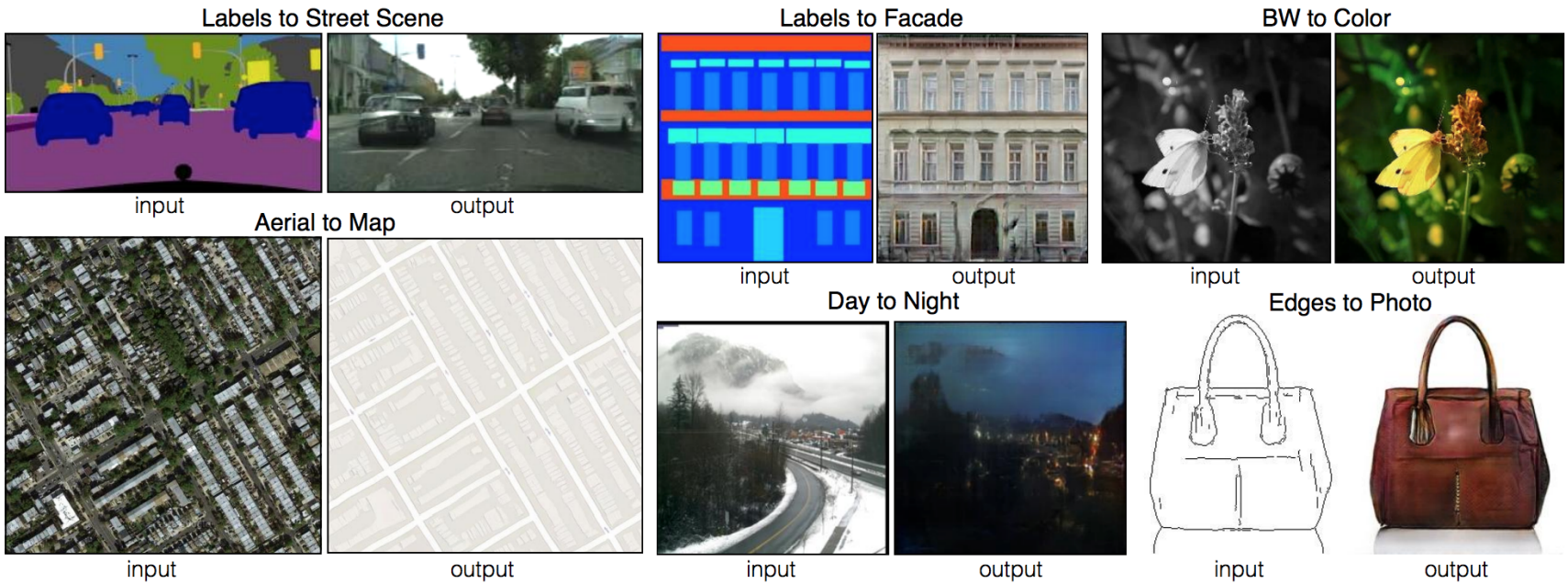
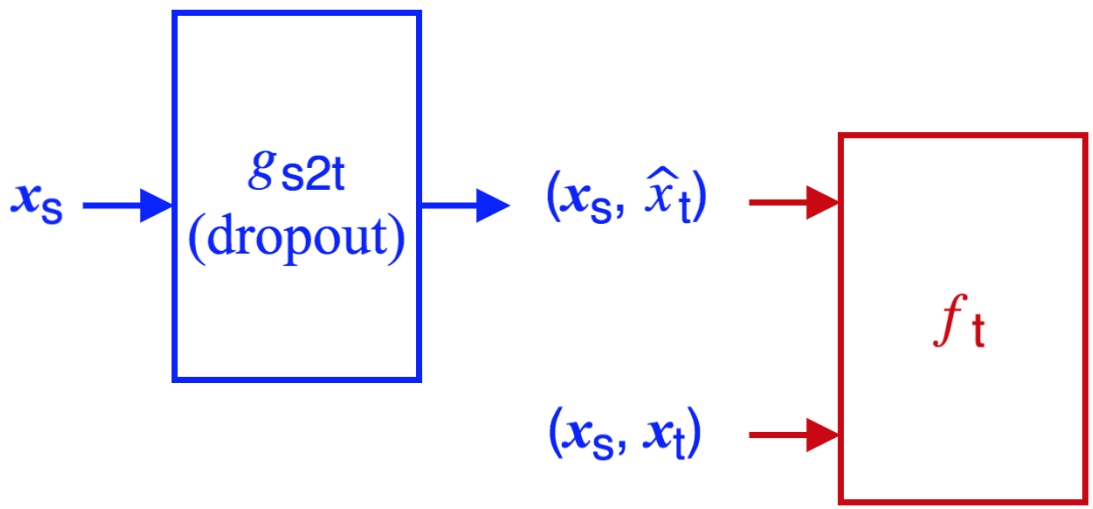


Image-to-Image Translation [5] II

- Based on conditional GAN:
  - $\boldsymbol{c} = \boldsymbol{x}_{\text{src}}$; $g(\boldsymbol{c}) = \boldsymbol{x}_{\text{target}}$
  - Conditioned labels
  - Uses dropout layers to create diversity (if needed)



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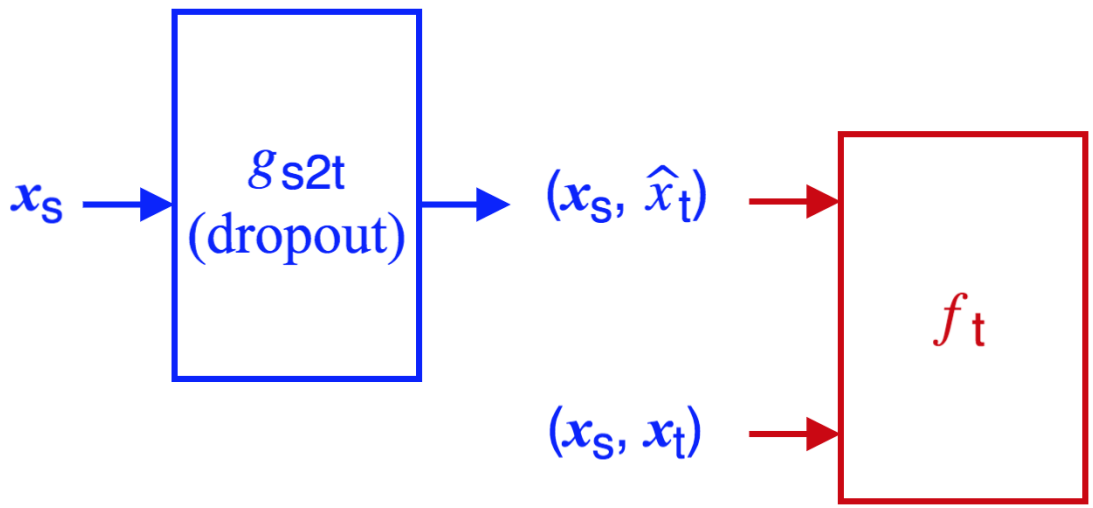
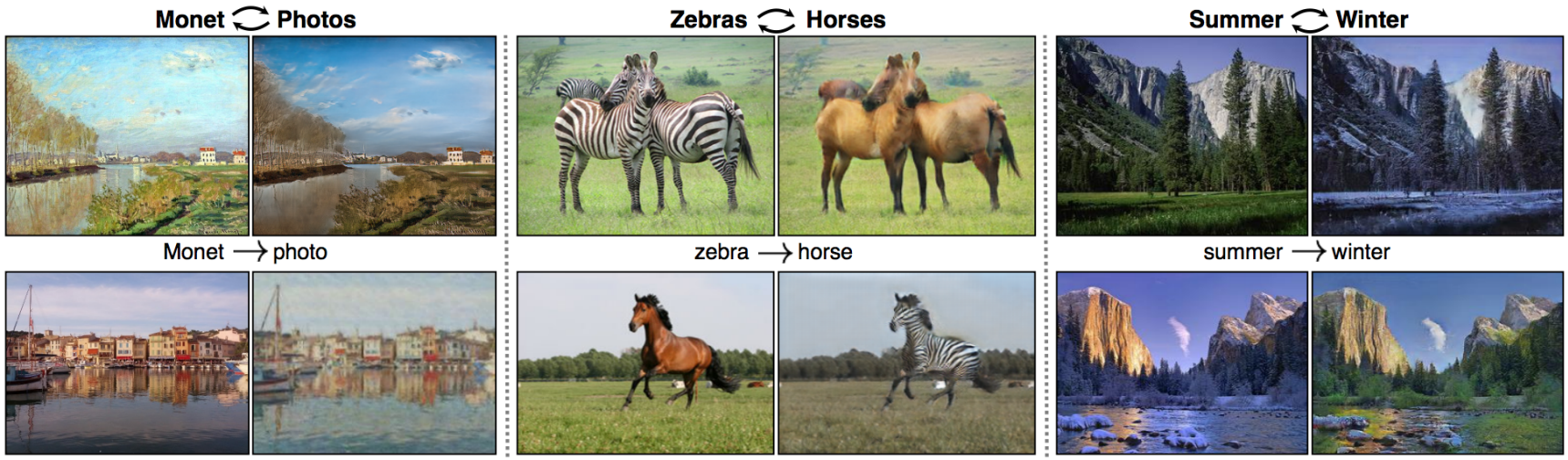
  - Uses dropout layers to create diversity (if needed)
- Requires ***paired*** examples $\mathbb{X} = \{(\boldsymbol{x}_{\text{src}}^{(n)}, \boldsymbol{x}_{\text{target}}^{(n)})\}_n$



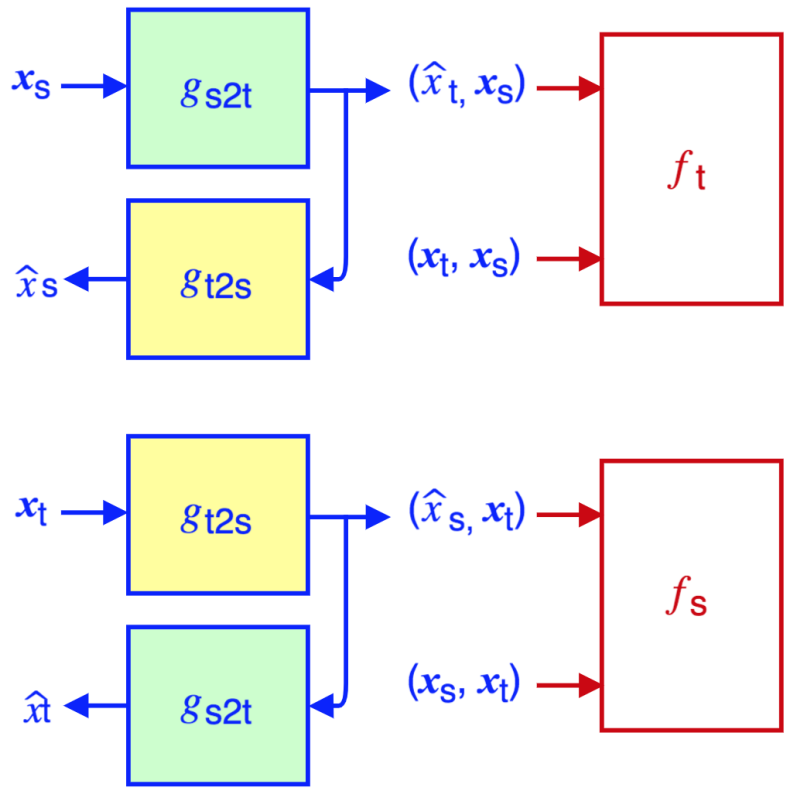
Unpaired Image-to-Image Translation I

- What if the images in different domains are ***unpaired***?
  - $\mathbb{X} = \{\boldsymbol{x}_{\text{src}}^{(n)}\}_n \cup \{\boldsymbol{x}_{\text{target}}^{(n)}\}_n$



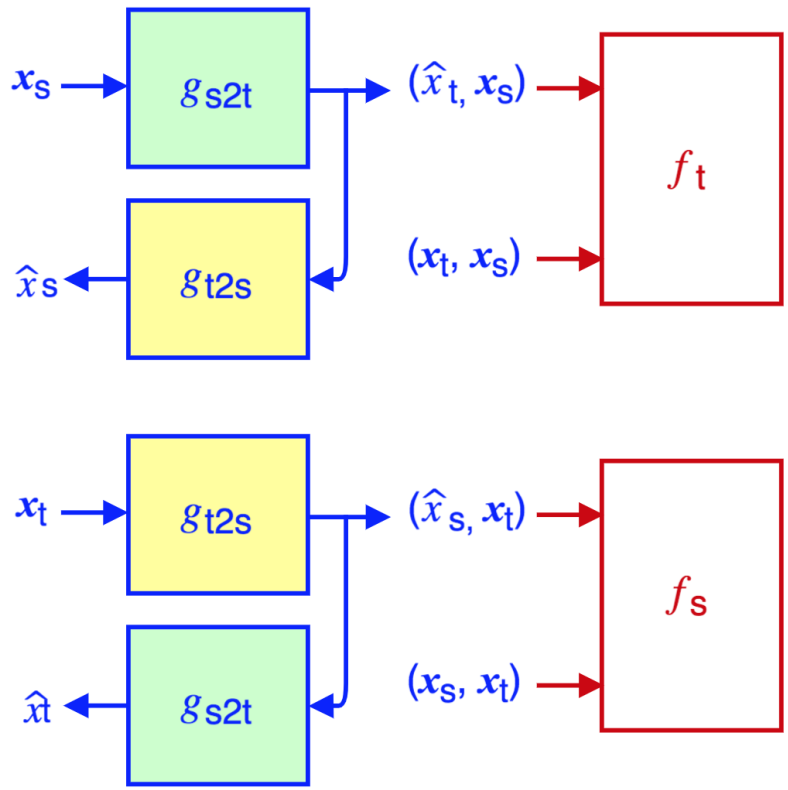
Unpaired Image-to-Image Translation II

- Cycle GAN [29]: to train two generators $g_{\text{src2target}}$ and $g_{\text{target2src}}$ simultaneously in two GANs



Unpaired Image-to-Image Translation II

- Cycle GAN [29]: to train two generators $g_{\text{src2target}}$ and $g_{\text{target2src}}$ simultaneously in two GANs
- Add a loss term $\sum_n \|\boldsymbol{x}_{\text{src}}^{(n)} - g_{\text{target2src}}(g_{\text{src2target}}(\boldsymbol{x}_{\text{src}}^{(n)}))\|_F$ and $\sum_n \|\boldsymbol{x}_{\text{target}}^{(n)} - g_{\text{src2target}}(g_{\text{target2src}}(\boldsymbol{x}_{\text{target}}^{(n)}))\|_F$ for $g_{\text{src2target}}$ and $g_{\text{target2src}}$



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