

CS532100 Numerical Optimization Homework 3

Student ID name

Due Jan 12, 2022

1. (25%) Consider the problem

$$\begin{array}{ll} \min_{x_1, x_2} & 0.1 \times (x_1 - 3)^2 + x_2^2 \\ \text{s.t.} & x_1^2 + x_2^2 - 1 \leq 0 \end{array} \quad (1)$$

- (a) Write down the KKT conditions for (1).
- (b) Solve the KKT conditions and find the optimal solutions, including the Lagrangian parameters.
- (c) Compute the reduced Hessian and check the second order conditions for the solution.

[Answers are put here.](#)

2. (20%) Consider the problem

$$\begin{array}{ll} \min_{x_1, x_2, x_3} & (x_1 - 1)^2 + (x_2 - 1)^2 + (x_3 - 1)^2 \\ \text{s.t.} & 0 \leq x_1, x_2, x_3 \leq 2. \end{array} \quad (2)$$

Find the optimization solution x^* for (2) with gradient projection method, with initial guess at $\vec{x}_0 = (x_1, x_2, x_3) = (0, 0, 2)^T$.
(Find out all segments and the minimizers of all segments, and determine whether the solution you got from this method is optimal solution. Justify your answer.)

[Answers are put here.](#)

3. (15%) Consider the problem

$$\begin{array}{ll} \min_{x_1, x_2} & z = 8x_1 + 5x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 100 \\ & 3x_1 + 4x_2 \leq 240 \\ & x_1, x_2 \geq 0 \end{array} \quad (3)$$

Formulate this problem to the equation of the interior point method, and derive the gradient of the Lagrangian and the Jacobian of the function F. (The "gradient" means F and the Jacobian is the derivative of F.)

[Answers are put here.](#)

4. (20%) Consider the following constrained optimization problem

$$\begin{array}{ll} \min_{x_1, x_2} & x_1 + x_2 \\ \text{s.t.} & x_1^2 + x_2^2 = 2 \end{array} \quad (4)$$

- (a) Write the augmented Lagrangian penalty function L and Hessian of L of this problem.
- (b) To make the augmented Lagrangian function L exact, what is the penalty parameter μ should be ?

[Answers are put here.](#)

5. (5%) Find the condition number $\kappa(A)$ of matrix A. Describe how ill- conditioned and good-conditioned matrices behave in matrix computation.

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & 0 \end{bmatrix}$$

[Answers are put here.](#)

6. (15%) The problem 15.4 in textbook shows an example of Maratos effect.

$$\begin{array}{ll} \min_{x_1, x_2} & f(x_1, x_2) = 2(x_1^2 + x_2^2 - 1) - x_1 \\ \text{s.t.} & x_1^2 + x_2^2 - 1 = 0 \end{array}$$

The optimal solution is $\vec{x}^* = (1, 0)$. Suppose $\vec{x}_k = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$, $\vec{p}_k = \begin{pmatrix} \sin^2 \theta \\ -\sin \theta \cos \theta \end{pmatrix}$. The Maratos effect says although \vec{p}_k is a good step, but the filter method will reject it. In the textbook and slides, it says the remedy to this problem is using the second order correction. Read the textbook or the slides to understand the reason why the second order correction can help avoiding this problem. And explain it in your own words.

[Answers are put here.](#)