

即

命题 $A: a + c = 10$.

命题 $B: b \in \mathbf{Z}$.

命题 $C: c = bd$.

命题 $D: d = a^2$.

命题 $E: e = d$.

命题 $F: \max\{a, b, \dots, f\} - \min\{a, b, \dots, f\} = 7, f > \frac{10}{3}$.

已知 a, b, \dots, f 均为整数、一位小数或两位小数, 满足 $a, b, \dots, f \in (0, 20], a + b + c + d + e + f = 20$, $\max\{a, b, \dots, f\} = c$, 命题 A, B, \dots, F 中恰有一个假命题, 求假命题对应的字母所对应的数.

解:

① 命题 A 为假命题.

设 $a = a, b = n (n \in \mathbf{N}^*, n \in [1, 9])$, 则 $c = na^2, d = a^2, e = a^2$.

若 $a < 1$ 则 $c = na^2 < n = b$, 与 $c = \max\{a, b, \dots, f\}$ 矛盾.

$\therefore a \geq 1. \therefore a \leq a^2 \leq na^2, n \leq na^2$.

又 $f > \frac{10}{3} > \min\{a, b, \dots, f\}$.

i) $\min\{a, b, \dots, f\} = n$.

由命题 F 得 $n + 7 = na^2. \therefore \sqrt{1 + \frac{7}{n}} = a \in \mathbf{Q}$, 无符合条件的解, 舍.

ii) $\min\{a, b, \dots, f\} = a$.

由命题 F 得 $a + 7 = na^2. \therefore na^2 - a - 7 = 0$.

$\sqrt{\Delta} = \sqrt{28n + 1} \in \mathbf{Q}$, 当且仅当 $n = 6$ 或 $n = 8$ 时满足.

$n = 6$ 时, $(6a - 7)(a + 1) = 0, a = \frac{7}{6}$, 舍.

$n = 8$ 时, $(8a + 7)(a - 1) = 0, a = 1$.

此时 $a = d = e = f = 1, c = d = 8$, 与 $f > \frac{10}{3}$ 矛盾, 舍.

② 命题 B 为假命题.

设 $a = a, b = b$, 则 $c = a^2b = 10 - a, d = a^2, e = a^2$.

$\therefore a^2b + a = 10, b = \frac{10 - a}{a^2}$.

若 $a < 1$, 则 $c = 10 - a > 9, c - a > 7$ 与命题 F 矛盾.

$\therefore a \geq 1, \therefore a \leq a^2 \leq c, b \leq c$.

又 $f > \frac{10}{3} > \min\{a, b, \dots, f\}$.

i) $\min\{a, b, \dots, f\} = a$.

由命题 F 得 $a + 7 = 10 - a, a = 1.5, b = \frac{10 - 1.5}{1.5^2} = \frac{34}{9}$, 舍.

$$\text{ii) } \min\{a, b, \dots, f\} = b.$$

$$\text{由命题 } F \text{ 得 } \frac{10-a}{a^2} + 7 = 10 - a,$$

整理得 $a^3 - 3a^2 - a + 10 = 0$, 无正数解, 舍.

③ 命题 C 为假命题.

设 $a = a, b = n$ ($n \in \mathbb{N}^*, n \in [1, 9]$), 则 $c = 10 - a, d = a^2, e = a^2$.

若 $a < 1$, 则 $c = 10 - a > 9, c - a > 7$ 与命题 F 矛盾.

$$\therefore a \geq 1, \therefore a \leq a^2 \leq c, n \leq c.$$

$$\text{又 } f > \frac{10}{3} > \min\{a, b, \dots, f\}.$$

$$\text{i) } \min\{a, b, \dots, f\} = n.$$

$$\text{由命题 } F \text{ 得 } n + 7 = 10 - a. \therefore a + n = 3. \therefore n = 1.$$

$$\text{解得 } a = 2, b = 1, c = 8, d = 4, e = 4, f = 1 \text{ 与 } f > \frac{10}{3} \text{ 矛盾, 舍.}$$

$$\text{ii) } \min\{a, b, \dots, f\} = a.$$

$$\text{由命题 } F \text{ 得 } a + 7 = 10 - a, a = 1.5,$$

$$\therefore a = 1.5, c = 8.5, d = 2.25, e = 2.25, b + f = n + f = 5.5.$$

$n = 2$ 时, $f = 3.5$ 成立.

$$n \geq 3 \text{ 时, } f < \frac{10}{3}, \text{ 舍.}$$

④ 命题 D 为假命题.

设 $a = a, b = n$ ($n \in \mathbb{N}^*, n \in [1, 9]$), $d = d$, 则 $c = 10 - a = nd, e = d$.

$$\text{又 } f > \frac{10}{3} > \min\{a, b, \dots, f\}.$$

$$\text{i) } \min\{a, b, \dots, f\} = n.$$

$$\text{由命题 } F \text{ 得 } n + 7 = 10 - a. \therefore a + n = 3. \therefore n = 1.$$

$$\text{解得 } a = 2, b = 1, c = 8, d = 8, e = 8, f < 0 \text{ 舍.}$$

$$\text{ii) } \min\{a, b, \dots, f\} = a.$$

$$\therefore 10 - a = c = nd. \therefore a = 10 - nd.$$

$$\text{由命题 } F \text{ 得 } 10 - nd + 7 = nd.$$

$$c = nd = 8.5, a = 1.5.$$

$$d = \frac{8.5}{n}, \text{ 当且仅当 } n = 1 \text{ 或 } n = 2 \text{ 或 } n = 5 \text{ 时满足.}$$

$$n = 1 \text{ 时, } c = d = e = 8.5, \text{ 舍.}$$

$$n = 2 \text{ 时, } d = e = 4.25, f < 0, \text{ 舍.}$$

$$n = 5 \text{ 时, } d = e = 1.7, f = 1.6 \text{ 与 } f > \frac{10}{3} \text{ 矛盾, 舍.}$$

$$\text{iii) } \min\{a, b, \dots, f\} = d.$$

$$\text{由命题 } F \text{ 得 } d + 7 = nd.$$

$$\therefore d = \frac{7}{n-1}.$$

$n=2$ 时, $d=7>n$, 舍.

$n=3$ 时, $d=4.5>n$, 舍.

$n=4$ 时, $d=\frac{7}{3}$, 舍.

$n=5$ 时, $d=1.75, a=10-nd=1.25<d$, 舍.

$n=6$ 时, $d=1.4$, 解得 $a=1.6, b=6, c=8.4, d=e=1.4, f=1.2<d$, 舍.

$n=7$ 时, $d=\frac{7}{6}$, 舍.

$n=8$ 时, $d=1$, 解得 $a=2, b=8, c=8, d=e=1, f=0$, 舍.

$n=9$ 时, $d=0.875$, 舍.

⑤ 命题 E 为假命题.

设 $a=a, b=n (n \in \mathbf{N}^*, n \in [1,9])$, 则 $c=10-a=na^2, d=a^2$.

$\because 10-a=na^2. \therefore na^2+a-10=0$.

$\sqrt{\Delta} = \sqrt{40n+1} \in \mathbf{Q}$, 当且仅当 $n=2$ 或 $n=3$ 或 $n=9$ 时满足.

$n=2$ 时, $(a-2)(2a+5)=0, a=2$, 解得 $a=2, b=2, c=8, d=4, e+f=4. \because f > \frac{10}{3}, \therefore e < \frac{2}{3}$,

$\therefore c-e > 7$, 与命题 F 矛盾, 舍.

$n=3$ 时, $(a+2)(3a-5)=0, a=\frac{5}{3}$, 舍.

$n=9$ 时, $(a-1)(9a+10)=0, a=1$, 解得 $a=1, b=9, c=9, d=1, e+f=0$, 舍.

⑥ 命题 F 为假命题.

设 $a=a, b=n (n \in \mathbf{N}^*, n \in [1,9])$, 则 $c=10-a=na^2, d=a^2, e=a^2$.

$\because 10-a=na^2. \therefore na^2+a-10=0$.

$\sqrt{\Delta} = \sqrt{40n+1} \in \mathbf{Q}$, 当且仅当 $n=2$ 或 $n=3$ 或 $n=9$ 时满足.

$n=2$ 时, $(a-2)(2a+5)=0, a=2$, 解得 $a=2, b=2, c=8, d=4, e=4, f=0$, 舍.

$n=3$ 时, $(a+2)(3a-5)=0, a=\frac{5}{3}$, 舍.

$n=9$ 时, $(a-1)(9a+10)=0, a=1$, 解得 $a=1, b=9, c=9, d=1, e+f=0$, 舍.

综上, 假命题为命题 $C, c=8.5$, 当且仅当 $a=1.5, b=2, c=8.5, d=2.25, e=2.25, f=3.5$ 时成立.

by Frank