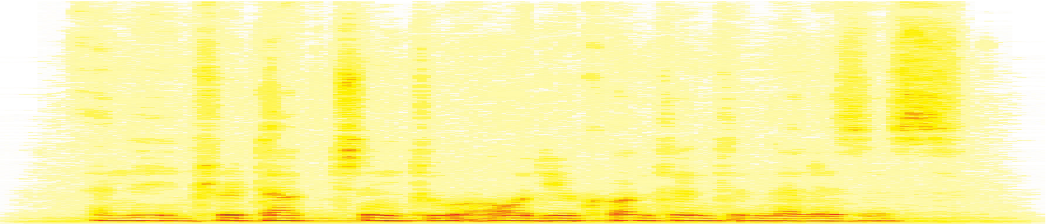
Introduction to Audio Content Analysis

Module 2.6: Fundamentals — Non-Fourier Time-Frequency Transforms

alexander lerch



Georgia Tech Center for Music Technology
College of Design

introduction

overview

Georgia Tech Center for Music Technology College of Design

corresponding textbook section

Chapter 2 — Fundamentals: pp. 9–11

- **lecture content**
  - constant-Q transform (CQT)
  - Gammatone filterbank
- **learning objectives**
  - discussing the advantages and disadvantages of different time-frequency transforms
  - explaining the principles of the CQT and auditory filterbanks

introduction

overview

Georgia Tech Center for Music Technology College of Design

corresponding textbook section

Chapter 2 — Fundamentals: pp. 9–11

- **lecture content**
  - constant-Q transform (CQT)
  - Gammatone filterbank

- **learning objectives**
  - discussing the advantages and disadvantages of different time-frequency transforms
  - explaining the principles of the CQT and auditory filterbanks

other time frequency transforms
introduction

Georgia Tech Center for Music Technology College of Design

- Fourier transform continues to be much-used tool in audio signal processing and MIR
- but there are disadvantages, e.g.
  - frequency axis does not directly map to (perceptual) pitch axis
  - frequency and time resolution inversely related

$\Rightarrow$ **alternative transforms** can be used

other time frequency transforms

introduction

Georgia Tech Center for Music Technology College of Design

- Fourier transform continues to be much-used tool in audio signal processing and MIR
- but there are disadvantages, e.g.
  - frequency axis does not directly map to (perceptual) pitch axis
  - frequency and time resolution inversely related

  $\Rightarrow$ **alternative transforms** can be used

constant-Q transform

introduction

- DFT has a *linear* frequency axis:
  - not perceptually meaningful: *logarithmic* is better match
  - low frequency resolution at low frequencies

⇒ compute DFT-like transform at specific frequencies
  - space frequencies logarithmically (constant Q)
  - resulting abscissa resolution is pitch-related

constant-Q transform

introduction

- DFT has a *linear* frequency axis:
  - not perceptually meaningful: *logarithmic* is better match
  - low frequency resolution at low frequencies

⇒ compute DFT-like transform at specific frequencies

- space frequencies logarithmically (constant $\mathcal{Q}$)
- resulting abscissa resolution is pitch-related

$$\mathcal{Q} = \frac{f}{\Delta f} = \frac{1}{2^{1/c} - 1}$$

constant Q transform

implementation 1/2

Georgia Tech Center for Music Technology College of Design

$$X_{\mathrm{CQ}}(k,n) = \frac{1}{\mathcal{K}(k)} \sum_{i=i_{\mathrm{s}}(n)}^{i_{\mathrm{e}}(n)} w_k(i-i_{\mathrm{s}}) \cdot x(i) e^{\mathrm{j}2\pi \frac{\mathcal{Q}\cdot(i-i_{\mathrm{s}})}{\mathcal{K}(k)}}$$

$$\mathcal{K}(k) = \frac{f_{\mathrm{S}}}{f(k)}\mathcal{Q}$$

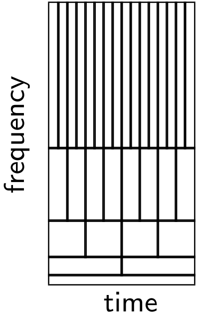
- $f(k)$: frequency of bin index k
- $\mathcal{K}(k)$: blocklength for bin index k
- $\mathcal{Q}$: measure of pitch res.
- w_k: window function
- $i_{\mathrm{s}}, i_{\mathrm{e}}$: start and stop time indices of block
- f_{S}: sample rate

- long window for low frequencies (high freq res, low time res)
- short window for high frequencies (low freq res, high time res)

constant Q transform

implementation 2/2

non-overlapping



overlapping

- define transformation matrix with maximum window length
- zeropad higher frequencies (left & right)

⇒ independent definition of block and hop length

constant Q transform
CQT vs. DFT

CQT:

+ perceptually/musically adapted frequency resolution
– time resolution depends on frequency
– not invertible
– no optimized implementation (compare FFT)

constant Q transform
CQT vs. DFT

Georgia Tech Center for Music Technology College of Design

CQT:

+ perceptually/musically adapted frequency resolution
– time resolution depends on frequency
– not invertible
– no optimized implementation (compare FFT)

constant Q transform

CQT vs. DFT

Georgia Tech Center for Music Technology College of Design

CQT:

+ perceptually/musically adapted frequency resolution
– time resolution depends on frequency
– not invertible
– no optimized implementation (compare FFT)

constant Q transform
CQT vs. DFT

CQT:

+ perceptually/musically adapted frequency resolution
− time resolution depends on frequency
− not invertible
− no optimized implementation (compare FFT)

auditory filterbanks
introduction

Georgia Tech Center for Music Technology
College of Design

FT and related transforms bad models of physiological properties of the human ear:

- frequency resolution (critical bands)
- frequency scale (pitch resolution)
- loudness & masking
- event perception & time integration

$\Rightarrow$ **auditory filterbanks**

not as widely used as one might think because

- computationally inefficient
- analysis only: no invertibility (mostly)
- not proven to be superior

auditory filterbanks

introduction

Georgia Tech Center for Music Technology College of Design

FT and related transforms bad models of physiological properties of the human ear:

- frequency resolution (critical bands)
- frequency scale (pitch resolution)
- loudness & masking
- event perception & time integration

$\Rightarrow$ **auditory filterbanks**

not as widely used as one might think because

1. computationally inefficient
2. analysis only: no invertibility (mostly)
3. not proven to be superior

auditory filterbanks

introduction

FT and related transforms bad models of physiological properties of the human ear:

- frequency resolution (critical bands)
- frequency scale (pitch resolution)
- loudness & masking
- event perception & time integration

$\Rightarrow$ **auditory filterbanks**

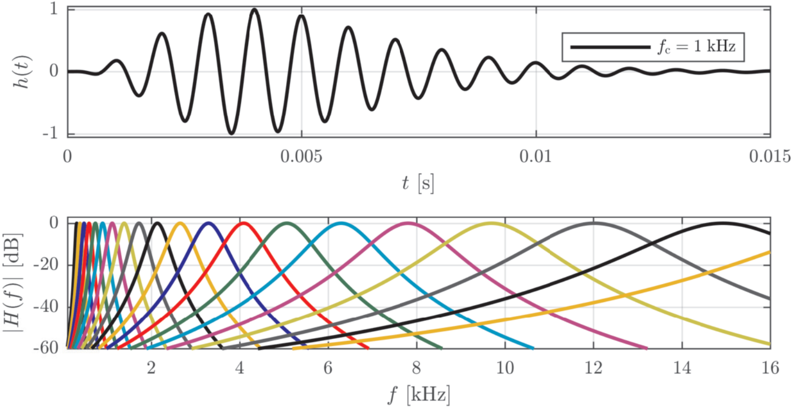
not as widely used as one might think because

1. computationally inefficient
2. analysis only: no invertibility (mostly)
3. not proven to be superior

auditory filterbanks

gammatone filterbank

$$h(i) = \frac{a \cdot \left(i/f_\mathrm{S}\right)^{\mathcal{O}-1} \cdot \cos\left(2\pi \cdot f_\mathrm{c} \frac{i}{f_\mathrm{S}}\right)}{e^{2\pi i \Delta f / f_\mathrm{S}}}$$



matlab source: matlab/displayGammatone.m

summary
lecture content

Georgia Tech Center for Music Technology College of Design

- **DFT has disadvantages**
  - low frequency resolution for low pitches
  - non-logarithmic/perceptually relevant pitch resolution
- **CQT**
  - similar to Fourier Transform but logarithmically spaced frequency bins
  - not invertible and inefficient
- **Filterbanks**
  - good model of human physiology
  - not invertible and inefficient
  - not proven to be superior