Introduction to Audio Content Analysis

Module 2.2: Fundamentals — Quantization

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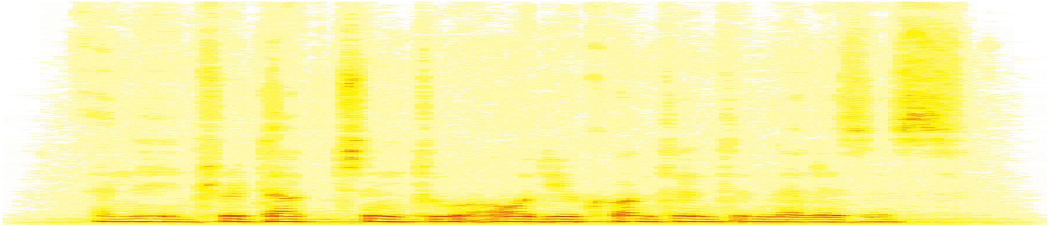

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introduction
overview

corresponding textbook section

Chapter 2 — Fundamentals: pp. 11–13

- **lecture content**
  - discretization of signals in amplitude
  - properties of the quantization error
  - number representation in Matlab

- **learning objectives**
  - understanding the principle and impact of quantization
  - representing audio signals in Matlab

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digital signals
introduction

digital signals are represented with a limited number of values

$\Rightarrow$

1. **sampling**: time discretization
   continuous time $\mapsto$ discrete equidistant points in time
2. **quantization**: amplitude discretization
   continuous amplitude $\mapsto$ discrete, pre-defined, set of values

digital signals
quantization

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- continuous amplitude values are mapped to pre-defined, equidistant set of values
- signal stored in binary $\Rightarrow$ # quantization steps equals **power of 2**
- **example: 4-bit quantization**
  - *word length*: $w = \log_2(\mathcal{M}) = 4\,\mathrm{bit}$
  - *number of quantization steps*: $\mathcal{M} = 2^w = 16$

matlab source: matlab/displayQuantization.m

digital signals

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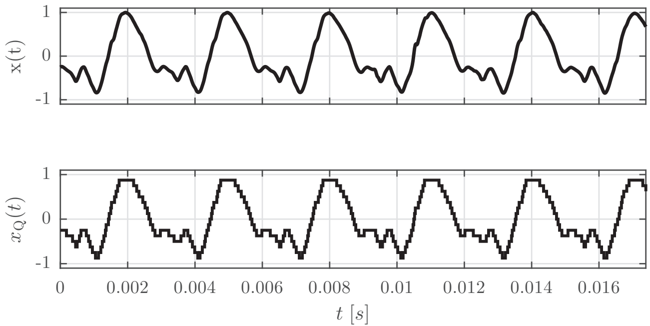

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digital signals

quantization wordlength

What are typical wordlengths?

digital signals
quantization wordlength

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- 8 bit: speech
- 12–14 bit: low quality audio/music
- 16 bit: (consumer) audio/music
- >16 bit: production audio

digital signals
quantization wordlength

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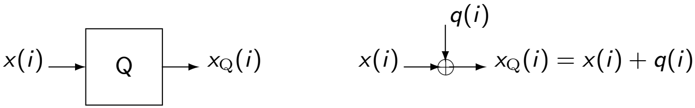
w	16 bit	12 bit	8 bit	4 bit	2 bit

digital signals

quantization error



model for quantization:
quantization noise q is added to input signal x

$$\begin{aligned} x_{\mathrm{Q}}(i) &= x(i) + q(i) \\ q(i) &= x(i) - x_{\mathrm{Q}}(i) \end{aligned}$$

digital signals

quantization error

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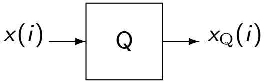

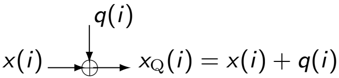

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digital signals

quantization error magnitude

What is the maximum amplitude of the quantization error?

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digital signals

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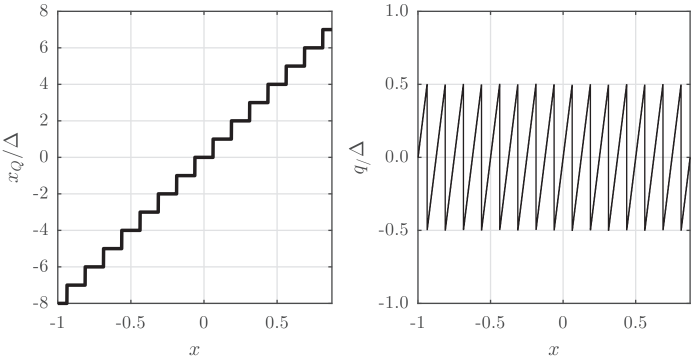

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digital signals

quantization error properties

Under the assumption that the signal has a variance much higher than the quantization step size (no derivation), we find that the quantization error

- is white noise and uncorrelated to signal,
- is uniformly distributed, and
- its power W_Q is directly related to the wordlength.

The quantizer quality is usually given by its *Signal-to-Noise Ratio (SNR)*

$$SNR = 10 \cdot \log_{10}\left(\frac{W_\mathrm{S}}{W_\mathrm{Q}}\right) \; [dB]$$

digital signals

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digital signals
quantization: SNR

signal-to-noise ratio (quantizer)

$$SNR = 6.02 \cdot w + c_S \quad [dB]$$

- every additional bit adds app. 6 dB SNR
- constant c_S depends on *signal* (scaling and PDF)

- square wave (full scale): $c_S = 10.80\,\mathrm{dB}$
- sinusoidal wave (full scale): $c_S = 1.76\,\mathrm{dB}$
- rectangular PDF (full scale): $c_S = 0\,\mathrm{dB}$
- Gaussian PDF (full scale $= 4\sigma_g$): $c_S = -7.27\,\mathrm{dB}$

digital signals
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digital signals
amplitude in DSP

- when represented as integer, different wordlengths lead to different maximum amplitude ranges
- most common: normalize to the absolute maximum integer value and represent the signal in **floating point format**

⇒ signal amplitude:

$$-1 \leq x_\mathrm{Q} < 1$$

⇒ level:

$$\text{max. amplitude} \mapsto 0dBFS$$

- floating point representation

$$x_\mathrm{Q} = M_\mathrm{G} \cdot 2^{E_\mathrm{G}}$$

- internal float point representation usually treated as signal being **not quantized**

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summary
lecture content

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- **quantization process**
  - modeled as addition of noise
- **quantization error properties**
  - maximum amplitude is half the step size
  - number of steps depends on wordlength
- **SNR**
  - SNR depends on input signal characteristic and wordlength
  - SNR increases linearly (6 dB/bit) with wordlength
- **signals in Matlab**
  - maximum amplitude independent of wordlength (-1...1)