# Introduction to Audio Content Analysis

## Module 2.0: Fundamentals — Signals

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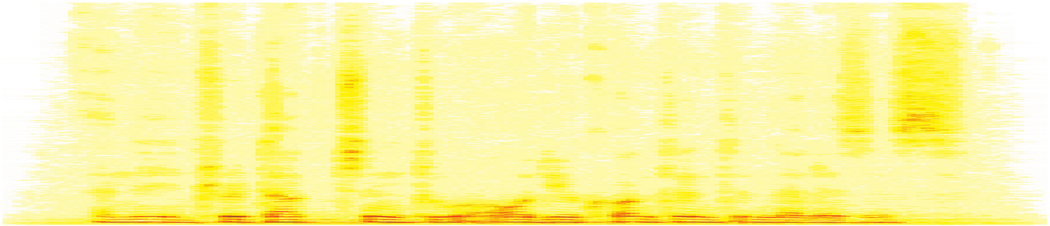

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# introduction
## overview

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**corresponding textbook section**

Chapter 2 — Fundamentals: pp. 7–9
Chapter 2 — Fundamentals: pp. 13–14

- **lecture content**
  - deterministic & periodic signals
  - Fourier Series
  - random signals
- **learning objectives**
  - name basic signal categories
  - discuss the nature of periodic signals with respect to harmonics
  - give a short description of meaning and use of the Fourier Series

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# audio signals

## signal categories

- **deterministic signals**:
  *predictable*: future shape of the signal can be known (example: sinusoidal)
- **random signals**:
  *unpredictable*: no knowledge can help to predict what is coming next (example: white noise)

"real-world" audio signals can be modeled as time-variant combination of

- (quasi-)periodic parts
- (quasi-)random parts

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# audio signals

## periodic signals 1/5



periodic signals: most prominent examples of deterministic signals

$$\begin{aligned} x(t) &= x(t + T_0) \\ f_0 &= \frac{1}{T_0} = \frac{\omega_0}{2\pi} \end{aligned}$$

# audio signals
## periodic signals 1/5



periodic signals: most prominent examples of deterministic signals

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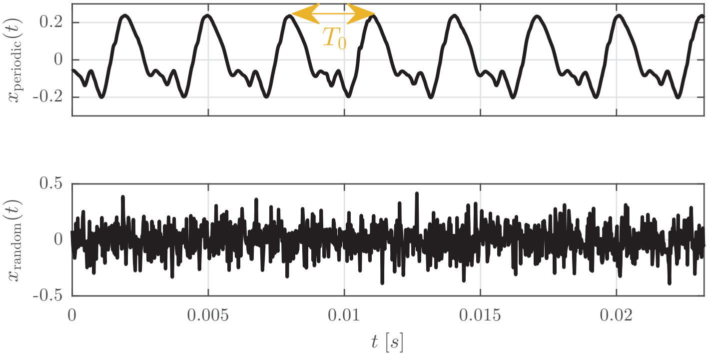


matlab source: matlab/displayPeriodicRandom.m

# audio signals
## periodic signals 2/5

periodic signal ⇒ representation in **Fourier series**[1]

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{\mathrm{j}\omega_0 kt}$$

- $\omega_0 = 2\pi \cdot f_0$
- $k\omega_0$: integer multiples of the lowest frequency
- $e^{\mathrm{j}\omega_0 kt} = \cos(\omega_0 kt) + \mathrm{j}\sin(\omega_0 kt)$
- $a_k$: Fourier coefficients — amplitude of each component

$$a_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-\mathrm{j}\omega_0 kt}\, dt$$

[1] Jean-Baptiste Joseph Fourier, 1768–1830

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# audio signals

## periodic signals 3/5

### Fourier series

- **every** periodic signal can be represented in a Fourier series
- a periodic signal **contains only** frequencies at integer multiples of the fundamental frequency $f_0$

- Fourier series can only be applied to periodic signals
- Fourier series is analytically elegant but only of limited practical use as the fundamental period has to be known

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## periodic signals 3/5

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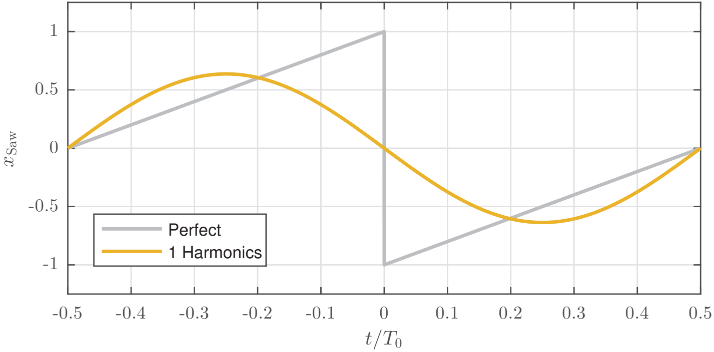
## periodic signals 4/5



reconstruction of periodic signals with limited number of sinusoidals:

$$\hat{x}(t) = \sum_{k=-\mathcal{K}}^{\mathcal{K}} a_k e^{\mathrm{j}\omega_0 k t}$$




matlab source: matlab/displayAdditiveSynthesis.m

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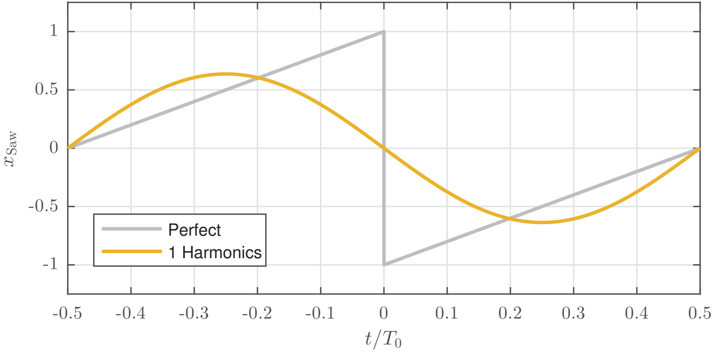


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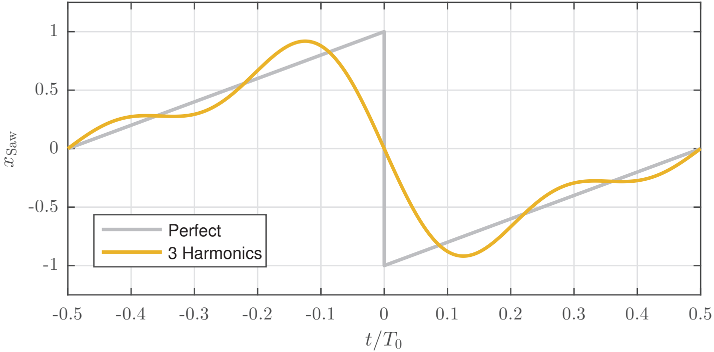


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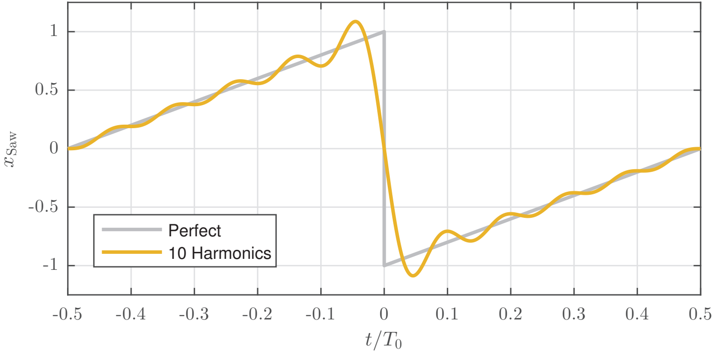


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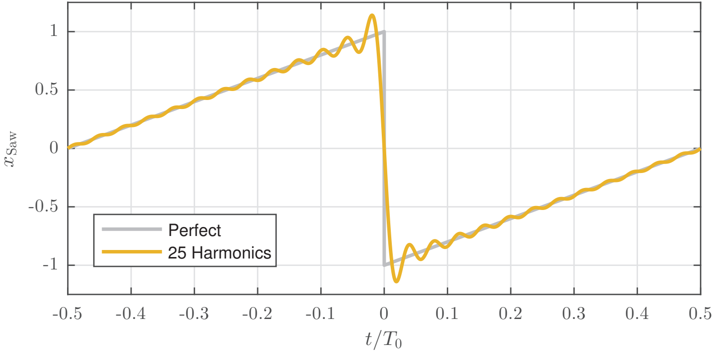


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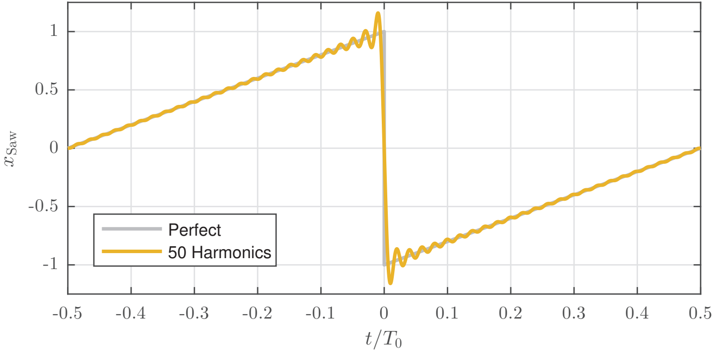


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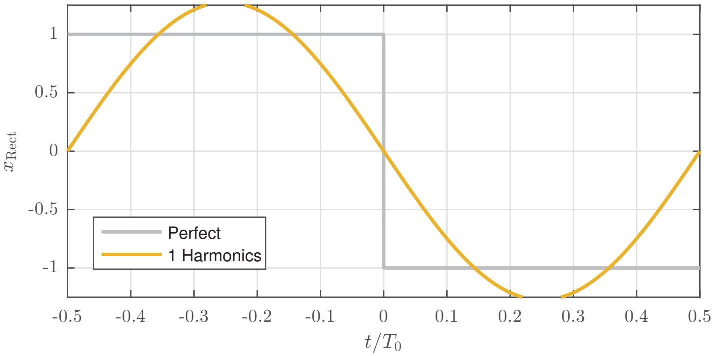


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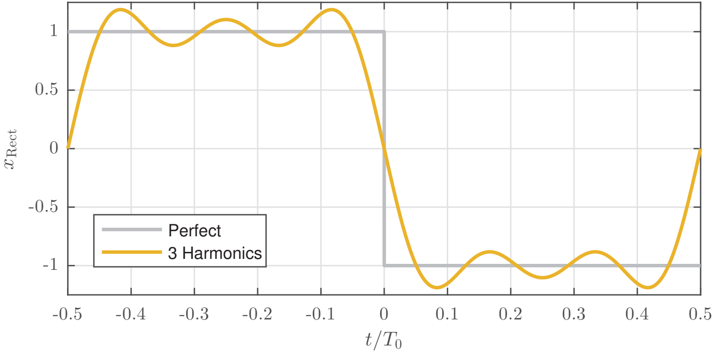


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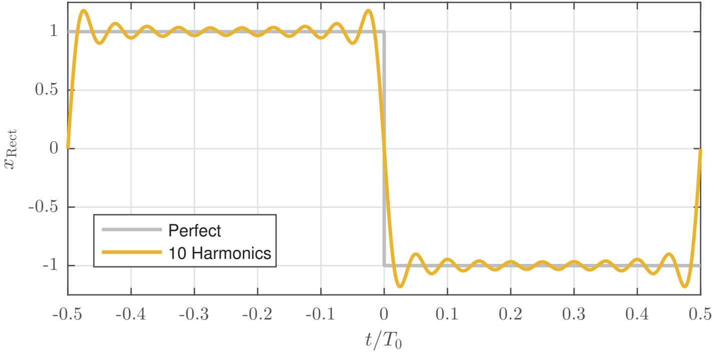


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## periodic signals 4/5



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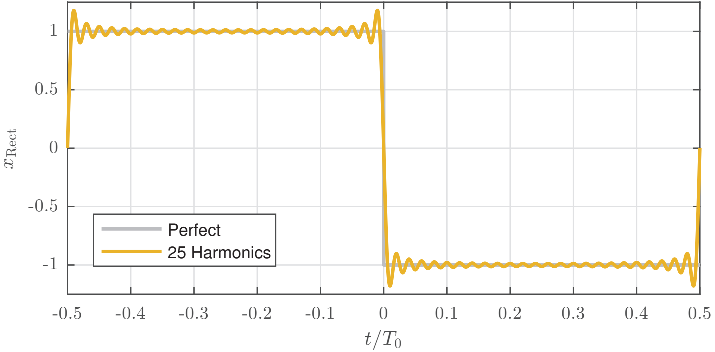


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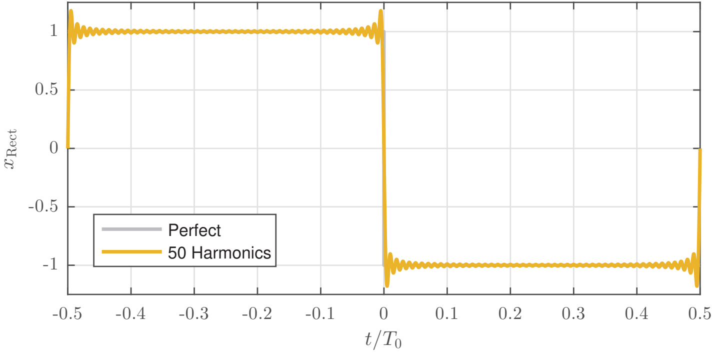


matlab source: matlab/displayAdditiveSynthesis.m

# audio signals
## periodic signals 5/5

youtube example — mechanical additive synthesis:

youtu.be/8KmVDxkia_w

# audio signals

## random process 1/2

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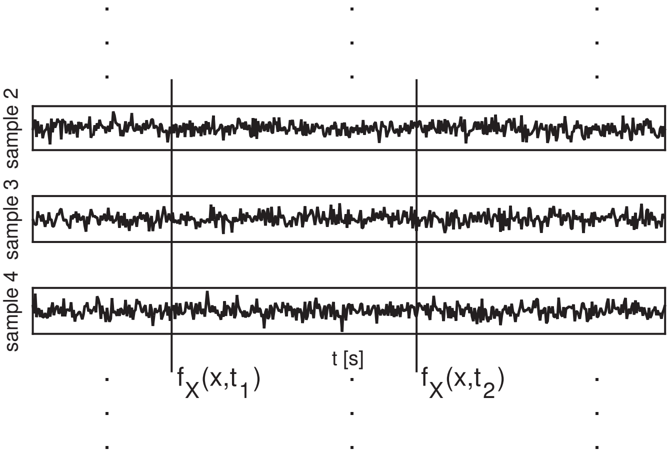
**random process**: ensemble of random series




matlab source: matlab/displayRandomProcess.m

# audio signals

## random process 2/2



### random process

- ensemble of random series
- each series represents a *sample* of the process
- the following value is *indetermined*, regardless of any amount of knowledge

- special case: **stationarity**
  statistical properties such as the mean are time invariant
- example: white noise

# statistical signal description

## probability density function

PDF $p_x(x)$

- abscissa: possible (amplitude) values
- ordinate: probability

$$
\begin{aligned}
p_x(x) &\geq 0, \mathit{and} \\
\int_{-\infty}^{\infty} p_x(x)\,dx &= 1
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$$

RFD—Relative Frequency Distribution (sample of PDF)

histogram of (amplitude) values

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# statistical signal description

## PDF examples

**What is the PDF of the following prototype signals:**

# statistical signal description

## PDF examples



**What is the PDF of the following prototype signals:**

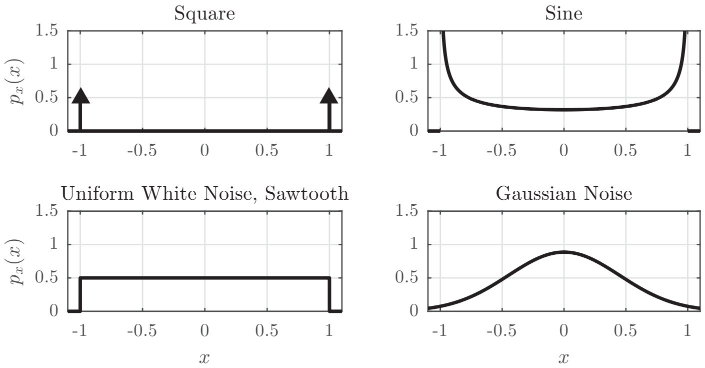
- square wave
- sawtooth wave
- sine wave
- white noise (uniform, gaussian)
- DC

# statistical signal description

## PDF examples



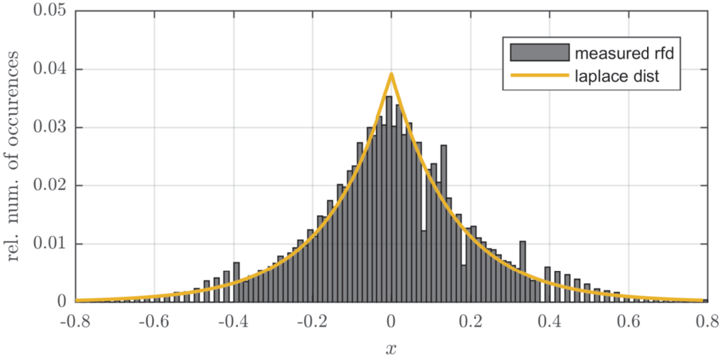
**What is the PDF of the following prototype signals:**




matlab source: matlab/displayPdfExamples.m

# statistical signal description

## RFD: real world signals




matlab source: matlab/displayRfd.m

# summary
## lecture content

- signals can be categorized into **deterministic and random signals**
  - deterministic signal can be described in a mathematical function
  - random processes can only be described by their general properties
- **periodic signals**
  - periodic signals are probably the most music-related deterministic signal
  - any periodic (pitched) signal is a sum of weighted sinusoidals
  - frequencies *only* at the fundamental frequency and integer multiples
- **random** signals
  - noise, unpredictable
- **real-world** signals
  - can be seen as a time-varying mixture of these two signal categories