hw1-S25-pt1

January 17, 2025

1 Homework 1 Part 1

part 1 total 50pt. Partial answers or answers without essential steps receive partial or no points.

This is an individual assignment.

- homework includes two parts: A **graded** part and **ungraded** part. The ungraded part is for your own exercise.

1.1 Description

Provide analytical solutions to the problems below and upload your answers as a PDF. (There is no need to type your answers – just scan your handwritten solutions with an app like CamScanner or Scannable.)

1.2 Problem 1

[TOTAL 16 PT]

Let $\{\Omega, \mathcal{F}, P\}$ be a probability space with $A \in \mathcal{F}$, $B \in \mathcal{F}$ and $C \in \mathcal{F}$ such that $P(A) = 0.4$, $P(B) = 0.3$, $P(C) = 0.2$, and $P(\overline{A \cup B}) = 0.35$. Compute the following probabilities:

1. [4pt] Either A or B occurs.
2. [3pt] Both A and B occur.
3. [3pt] A occurs but B does not occur.
4. [2pt] Are A and B mutually exclusive?
5. [2pt] Is this claim correct? give your reasoning: If C is independent of A, then C^c is independent of A.
6. [2pt] If C is independent of A, compute $P(A \cap C)$ and $P(A \cap C^c)$.

Hint: using the axioms of probability.

1.3 Problem 2

[total 14 pt.]

onsider a card drawn randomly from a standard deck of 52 cards. Let:

- \$ A\$ be the event "the card is a spade,"
- B be the event "the card is a face card (Jack, Queen, King)."

Solve the following two questions:

1 [6pt, 2pt each]. compute $P(A)$, $P(B)$ and $P(A \cap B)$

2 [4pt]. Are A and B independent?

3 [4pt]. Are A and B exclusive?

1.4 Problem 3 (probability calculuation)

[Total 20 pt]

Consider repeatedly rolling a fair 6-sided die.

1 [5pt]. What is the probability that the top face will be 4 at least once on 10 rolls of the die?

2 [5pt]. What is the probability that the top face will be 4 at least once on 20 rolls of the die?

3 [10pt]. How many rolls of the die would you have to do to be 90% confident that you would see at least one 4? In other words, how many rolls of the die would you have to be able to say, with a probability at least 90%, you will see at least one 4 appears.

hint: For 1, consider to solve the probability of the top face be any number except 4 in one roll. And then use two axioms: 1) the probability of the complement of an event; 2) the independence rule.

For 3, use the answer of 1 and 2 to find a function that takes the number of die rolls to the probability of seeing at least one 4. Then, solve an equation when the probability is 90%, what is the number of die rolls.

[ ]:

1.5 The following questions are ungraded exercises:

1. Suppose $P(A) = 0.3$ and $P(B) = 0.5$, where $A \cap B = \emptyset$.

- What is $P(A \cup B)$?
- What is $P(A \cup B^c)$?

2. A fair six-sided die is rolled. Let A be the event "roll is an odd number" and B be the event "roll is less than 4." Compute:

- $P(A)$
- $P(B)$
- $P(A \cup B)$

3. A coin is flipped three times. Define the events:

- A: The first flip is heads.
- B: The second flip is heads.
- C: The third flip is heads.
- Are A, B, and C mutually exclusive? Justify your answer.

- Are A, B, and C independeny? Justify your answer.

4. A single card is drawn from a standard deck of 52 cards. Let:

- A: The card is a heart.
- B: The card is a queen.
- Calculate $P(A^c)$, $P(B^c)$, and $P(A \cap B)$.