ELEC4631 Lab 2

1 General information

1. This lab is conducted as a both a face-to-face and remote labs.
2. This lab does not require you to take control of the remote computer. It is recommended that tasks be run on a copy of Matlab on your own computer.
3. The lab tasks must be carried out and completed during the lab time to be marked by demonstrators.
4. Lab tasks are divided into well defined time checkpoints: tasks that are scheduled to be completed in first **50 minutes**, in the first **1 hour 40 minutes**, and in **2 hours 30 mins.** Students must pace themselves to complete the tasks and have them assessed at the defined time checkpoints.
5. Students must not arrive late at the lab by more than 10 minutes. Demonstrators can refuse to let in late students.

2 Preparatory tasks

To prepare for this lab, you will need to:

1. Learn the function of, and how to use, the following Matlab commands:
   (a) `mesh`, `meshgrid`
   (b) `quiver`
   (c) `plot`, `plot3`, `hold`
2. Learn to use the Matlab Simulink graphical environment for system simulation.

3 Laboratory

This lab will mainly involve plotting various interesting figures and is intended to help you to further visualise concepts that were taught in Lecture 3. This lecture should be reviewed before attending the lab.

Consider the damped pendulum in the lecture notes with the equation of motion given by the ODE:

$$\begin{bmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} \omega(t) \\ -\frac{g}{r}\sin\theta(t) - D\omega(t) \end{bmatrix}.$$

So, the vector field f here is

$$f(\theta,\omega) = \begin{bmatrix} \omega \\ -\frac{g}{r}\sin\theta - D\omega \end{bmatrix}.$$

Take the values $g = 10$ m/s^2, $r = 1$ m, and $D = 1$ s^{-1}. Complete all of the following sub-tasks:

Assessment Exercise 1 (3 points, assessed at 50 minutes):

1. Using the Matlab functions **meshgrid** and **quiver**, plot the vector field f of the ODE using the variable θ as the horizontal axis and the variable ω as the vertical axis. The range of values of θ and ω should be [-4,4] and they should be discretised with a spacing of 0.2.

2. In the lecture notes we initially used the system's total energy as a Lyapunov function (take the mass of the pendulum to be $m = 1$ kg):

$$V(\theta,\omega) = \frac{1}{2}mr^2\omega^2 + mgr(1-\cos\theta). \tag{1}$$

   On the same plot as for the vector field in sub-task 1 (you can use the Matlab command **hold on** to do this), use the **quiver** command to plot the gradient vectors $\nabla V(x)$ of the Lyapunov function given above but using a red colour to distinguish them from the vector field f (you have an explicit expression for the gradient, so do **not** use the Matlab gradient command). What happens along the horizontal axis where $\omega = 0$, what does it mean?

3. On a separate figure, make a 3D plot of V given in (1) in the area $(\theta,\omega) \in [-4,4] \times [-4,4]$ using the Matlab functions **meshgrid** and **mesh**. You can use a grid which is finer than in sub-task 1 (i.e., a spacing finer than 0.2). Examine the plot that you obtain (in Matlab you can grab 3D plots and rotate them with a mouse).

Assessment Exercise 2 (3 points, assessed at 1 hour 40 minutes):

1. On a separate figure, repeat sub-tasks 1 and 2 from Assessment Exercise 1 for the alternative Lyapunov function:

$$V(\theta, \omega) = \frac{1}{2}\omega^2 + \frac{1}{2}(\frac{1}{D}\omega + \theta)^2 + \frac{g}{r}(1 + \frac{1}{D^2})(1 - \cos\theta). \tag{2}$$

   Compare the plot that you obtain with the one in Assessment Exercise 1 sub-task 2, how do the plots compare along the horizontal axis where $\omega = 0$? What important difference between (1) and (2) can you see?

2. On a separate figure, repeat Assessment Exercise 1 sub-task 3 for V given by (2). Use the same gridding as in that sub-task. Examine your plot and compare with the one that you obtained in Assessment Exercise 1 sub-task 3.

Assessment Exercise 3 (4 points, assessed at 2 hours 30 minutes):

1. Use Matlab **Simulink** to simulate a trajectory of the ODE from time $t = 0$ to $t = 20$ starting at the state $(\theta(0), \omega(0)) = (3, -2)$. Save the time vector and the values of $\theta(t)$ and $\omega(t)$ for each simulation time point t onto the Matlab workspace via the Simulink **Scope** block. Using the data that you saved, plot the evolution of the system state $(\theta(t), \omega(t))$ in $\mathbb{R}^2$ for the given starting state on a separate figure. That is, for each point t in the time vector plot the points $(\theta(t), \omega(t))$ on $\mathbb{R}^2$.

2. Using the Simulink simulation data for the trajectory in sub-task 1 of this assessment exercise, on a separate figure use the Matlab function **plot3** to plot the coordinates $(\theta(t), \omega(t), V(\theta(t), \omega(t)))$ for the function V given in (1) for all your simulation time points t. What do you see?

3. On a separate figure, repeat sub-task 2 of this assessment exercise for the function V given in (2). Compare this with the plot from sub-task 7. Do both trajectories converge to the origin?