HW5

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Problem 1

To prove that exact 4-SAT is NP complete, we need to show:

1) Exact 4-SAT is NP
   Proof: for each clause C, in O(1) time can check at least one literals in the clause is satisfied. Assuming m clauses in the problem, runtime is O(m). So in linear time can verify solution. 4-SAT is NP.

2) 3SAT -> Exact 4-SAT
   In formular f of 3SAT, there's n variable and m clauses. To create an input formular f' to Exact 4SAT, we pick any clause in f. The length of clause in f can be 1 or 2 or 3.

   Say c1 in f has length 3, and assume c1 = (a or b or c), we replace c1 with below clauses c1': (a or b or c or x1) and (a or b or c or (not x1)). c1' is satisfiable iff c1 is satisfiable as at least a or b or c needs to be true.

   Say c2 in f has length 2, and assume c2 = (a or b), then we replace c1 with below clauses c2': (a or b or x2 or x3) and (a or b or (not x2) or x3) and (a or b or x2 or not(x3)) and (a or b or(not x2) or(not x4)). So c2' is satisfiable iff c2 is satisfiable

   Say c3 in f has length 1. Assume c3 = (a). we use c3' to replace c3:
   (a or x5 or x6 or x7) and (a or (not x5) or x6 or x7) and (a or x5 or (not x6) or x7) and (a or x5 or x6 or (not x7)) and (a or (not x5) or (not x6) or x7) and (a or (not x5) or x6 or (not x7)) and (a or x5 or (not x6) or (not x7)) and (a or (not x5) or (not x6) or (not x7)). Again c3' is satisfiable iff c3' is satisfiable.

   So we replace f by f', and f is satisfiable iff f' is satisfiable. On the other hand, if we have a solution to f', we only need ignore the added variables to get the solution to f. So we proved 3SAT-> exact 4 SAT

   Thus, exact 4-SAT is NP complete problem.

Problem 2

To prove the given problem is NP complete, we need to show:

1) The given problem is NP
   Proof: for solution S including 2 set of vertices (s1, s2), and assume input graph G = (V,E) we can prove in O(|V| ^2) time we can iterate all pairs of vertices(x,y) in s1 are connected, and all pair of vertices(x,y) in s2 are independent
2) Clique -> given problem
   In Clique problem, we need to output S where s is a clique of size |S| >= k. In this problem we need a clique with size |S| = k. So if we satisfy the Clique problem, we can say there is a clique with size |S| = k. Consider the input of Clique problem G = (V,E), we can form a new graph G'(V+k, E) with k added vertices, but we do not add any edges to the graph. In this case, we know this k vertices are an independent set with size k because they are not connected with any edges. Since they are not connected with any edges, we do not modify the cliques in the original graph G. So we replaced G by G', and G has a clique with size k iff G' has a clique with size k and a independent size with size k. if we have the solution to the given problem in G', we only need to include the clique to get the solution to Clique problem in G.

With the above proofs we can say that the given problem is NP-complete.