

Problem 1

1. $T(i, j)$ is the length of the longest string $Z(i, j)$ that appears to be both a substring of X that ends in $X[i]$ and a subsequence in $Y[1], \dots, Y[j]$
2. Base Case: $T(i, 0) = 0, T(0, j) = 0. 0 \leq i \leq n, 0 \leq j \leq m$ Recurrence: $T(i, j) = T(i - 1, j - 1) + 1$, if $X(i) = Y(j) = T(i, j - 1)$, if $X(i) \neq Y(j)$ Where $0 < i \leq n, 0 < j \leq m$
3. for $i = 0$ to $n, T(i, 0) = 0$ for $j = 0$ to $m, T(0, j) = 0$ for $i = 1$ to n for $j = 1$ to m if $X(i) = Y(j)$, then $T(i, j) = T(i - 1, j - 1) + 1$, else $T(i, j) = T(i, j - 1)$ Return $\max(T(1, m), T(2, m) \text{ to } T(n, m))$
4. for $i = 0$ to $n, T(i, 0) = 0 : O(n)$ for $j = 0$ to $m, T(0, j) = 0 : O(m)$ for $i = 1$ to n for $j = 1$ to $m : O(nm)$ Overall running time: $O(nm)$

Problem 2

1. $T(i) = 1$, if value i can make change by denominations $x(1), \dots, x(n), T(l) = 0$, else
2. Base cases: $T(0) = 1$ Recurrence: In all $x(1), \dots, x(n)$, if exist $x(j)$, that $T(i - x(j)) = 1$, and $x(j) \leq i$, then $T(i) = 1$ else $T(i) = 0$ where $0 < i < n$
3. $T(0) = 1$ for $i = 1$ to $v: T(i) = 0$ for $i = 1$ to v for $j = 1$ to n if $x(j) < i$, then $T(i) = T(i - x(j))$ if $T(i) == 1$: break return($T(v)$)
4. for $i = 1$ to $v: T(i) = 0: O(v)$ for $i = 1$ to v for $j = 1$ to $n : O(nv)$ Overall running time: $O(nv)$