HW5

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Problem 1

(c)

From the original max flow construction, we only need to include a variable for lower bound flow in each edge.

Input: directed graph G = (V, E). Let c_e > 0 be the positive capacity for each edge e, let l_e >= 0 be none negative lower bound flow for each edge e,

Create decision variables f_e be flow of each edge e

Objective Function: $\max \sum_{sv \in e} f_{sv}$
S.T:
For every edge e in E, $l_e \leq f_e \leq c_e$
For every vertex v in V - {s, t}, $\sum_{wv \in E} f_{wv} = \sum_{vz \in E} f_{vz}$

(d)

We need to make sure the out flow of each node captures the loss of inbound flow.

Input: directed graph G = (V, E). Let C_e > 0 be the positive capacity for each edge e, let ε_v be the loss factor for each vertex v

Create decision variables f_e be flow of each edge e,

Objective Function: $\max \sum_{sv \in e} f_{sv}$
S.T:
For every edge e in E, $0 \leq f_e \leq c_e$
For every vertex v in V - {s, t}, $\sum_{wv \in E} f_{wv} = \sum_{vz \in E} f_{vz} / (1 - \varepsilon_v)$

Problem 2

To prove the given problem is NP complete, we need to show:

1) The given problem is NP
   Proof: for solution set S, we need to check size (S) < b. Assuming number of elements in S = m, it will take O(m) time, and we also need to check if $s \cap S_I \neq \emptyset$. This wil take O(n*m*m) time. Overall Polynomial time can we verify the solution.
2) Vertex cover -> Hitting Set
   Proof: consider a vertex cover problem, we can create an instance for the Hitting Set problem. In the vertex cover problem, we have the input G = (V,E), for each edge e in the graph, we create a set Si, and the budge for Hitting Set b = budget b in vertex cover. Then if the vertex cover problem has a solution vertex cover S <=b iff there is a hitting set of size b for the new constructed hitting set problem. Thus we are able to reduce vertex cover problem to hitting set problem

With above statements, we can say Hitting Set is NP-complete.