Financial Quantitative Risk Management and Decision Making March 19, 2021

Problem Set 1

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1. Given any continuous r.v. X of our choice (Whose CDF $F_X(x)$ is strictly increasing), **prove:**
   (a) $\tilde{u} := F_X(X) \sim \mathcal{U}[0,1]$
   (b) $X \sim Q_X(\tilde{u})$
   (c) Using two examples to simulate $Q_X(U)$ and check the correctness of question (b). For example you can choose $X \sim \mathrm{N}(\mu, \sigma)$ and $X \sim \mathrm{E}(\lambda)$.

2. There have ten stocks in the attachment, please calculating the location indices values of each stock, respectively. And then, answering one of the bellow two questions.
   (a) Suppose you are a financial product manager, which stock do you like best? In order to selling this stock to consumers successfully, which location index should you show to customer? which location index should you hide to customer? and why?
   (b) Suppose you are a consumer, which stock would you like to buy? When buying this stock, which location index you most want to know? which location index is usually useless? and why?

3. Check formula (1.36) and explain why it is right.

$$Z^2_{a+bX} = Z^2_X, \tag{1.36}$$

   where

$$Z_X \equiv \frac{X - \mathrm{Loc}\{X\}}{\mathrm{Dis}\{X\}}. \tag{1.35}$$

4. (a) Choose and download three stocks from China stock market, from 2018 to 2021 about three years daily data.
   (b) Using the data you download in question (a) and formula (2.87)

$$\mathcal{E}^q_{\mathrm{E,Cov}} \equiv \left\{ \boldsymbol{x} | (\boldsymbol{x} - \mathrm{E})^\top \mathrm{Cov}^{-1} (\boldsymbol{x} - \mathrm{E}) \leq q^2 \right\} \tag{2.87}$$

   draw a location-dispersion ellipsoid ball, where we assume that the q is the mean value of *Mahalanobis distance* of all the samples.

(c) Changing q into $(1 \pm 5\%)q$ and $(1 \pm 10\%)q$, and draw location-dispersion ellipsoid ball with different q (namely, there have five different q values which are q, $(1 \pm 5\%)q$ and $(1 \pm 10\%)q$) in one figure.

(d) Calculating the probability of coverage by each location-dispersion ellipsoid ball, namely, for each location-dispersion ellipsoid ball, you need to calculate the percentage of the points covered by the ball.

(e) With the same stocks in question (a), please download the daily data from Jan. 1, 2021 to Mar. 19, 2021. Assume the q value is the values calculated in question (b) and (c), and then, please calculate the probability of coverage by using the daily date download in question (e). What do you find and discuss it.