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Problem 1

1.a.

$E[(Y-\hat{f}(x))^2][X=x]=E[(y-\hat{f}(x))^2)]$

$=E[y^2]+E[\hat{f}(x)^2]-E[2y\hat{f}(x)]$

$=E[y^2]-E[y]^2+E[y]^2+E[\hat{f}(x)^2]-E[\hat{f}(x)]^2+E[\hat{f}(x)]^2-2f(x)E[\hat{f}(x)]$

$=E[(f(x)+\epsilon-f(x))^2]+E[(\hat{f}(x)-E[\hat{f}(x)])^2]+(E(\hat{f}(x)-f(x))^2$

$=\sigma_\epsilon^2+(E(\hat{f}(x)-f(x))^2+E[(\hat{f}(x)-E[\hat{f}(x)])^2]$

1.b.

σ_ϵ^2 is $Noise$, and here it is gaussian noise of the data

$(E(\hat{f}(x)-f(x))^2$ is $Bias^2$, it is a measure of whether the predictors we choose approximates $f(x)$ well

$E[(\hat{f}(x)-E[\hat{f}(x)])^2]$ is $Variance$, it is a measure of whether a predictor is susceptible to changes

Problem 2

2.a.

Assume Z is the standard normal distribution and the X is a $M\times 1$ random vector with covariance Σ, and we know its mean is μ, then we have,

$X=\mu+VZ$, where V is a $M\times M$ invertible matrix such that $\Sigma=VV^T=V^TV$Then we have,

$f_X(x)=\frac{1}{|det(\Sigma)|}f_Z(V^{-1}(x-\mu))$

$=\frac{1}{|det(V)|}(2\pi)^{-\frac{M}{2}}e^{-\frac{1}{2}(V^{-1}(x-\mu))^T(V^{-1}(x-\mu))}$

$=|det(V)|^{-\frac{1}{2}}|det(V)|^{-\frac{1}{2}}(2\pi)^{-\frac{M}{2}}e^{-\frac{1}{2}(x-\mu)^T(V^{-1})^TV^{-1}(x-\mu)}$

$=(2\pi)^{-\frac{M}{2}}|det(V)det(V)|^{-\frac{1}{2}}e^{-\frac{1}{2}(x-\mu)^T(V^T)^{-1}V^{-1}(x-\mu)}$

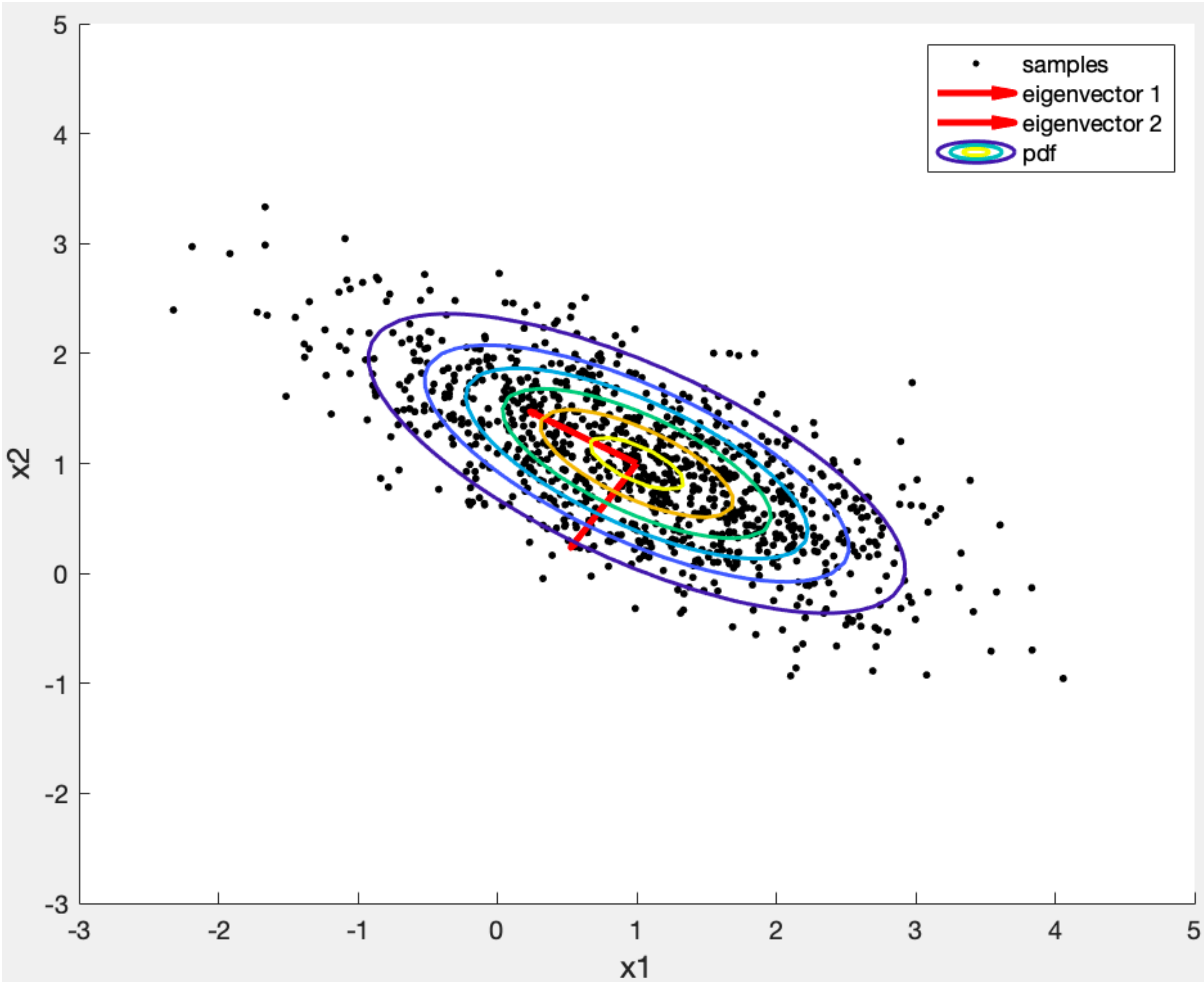
$=(2\pi)^{-\frac{M}{2}}|det(V)det(V^T)|^{-\frac{1}{2}}e^{-\frac{1}{2}(x-\mu)^T(V^T)^{-1}V^{-1}(x-\mu)}$

$=(2\pi)^{-\frac{M}{2}}|det(VV^T)|^{-\frac{1}{2}}e^{-\frac{1}{2}(x-\mu)^T(VV^T)^{-1}(x-\mu)}$

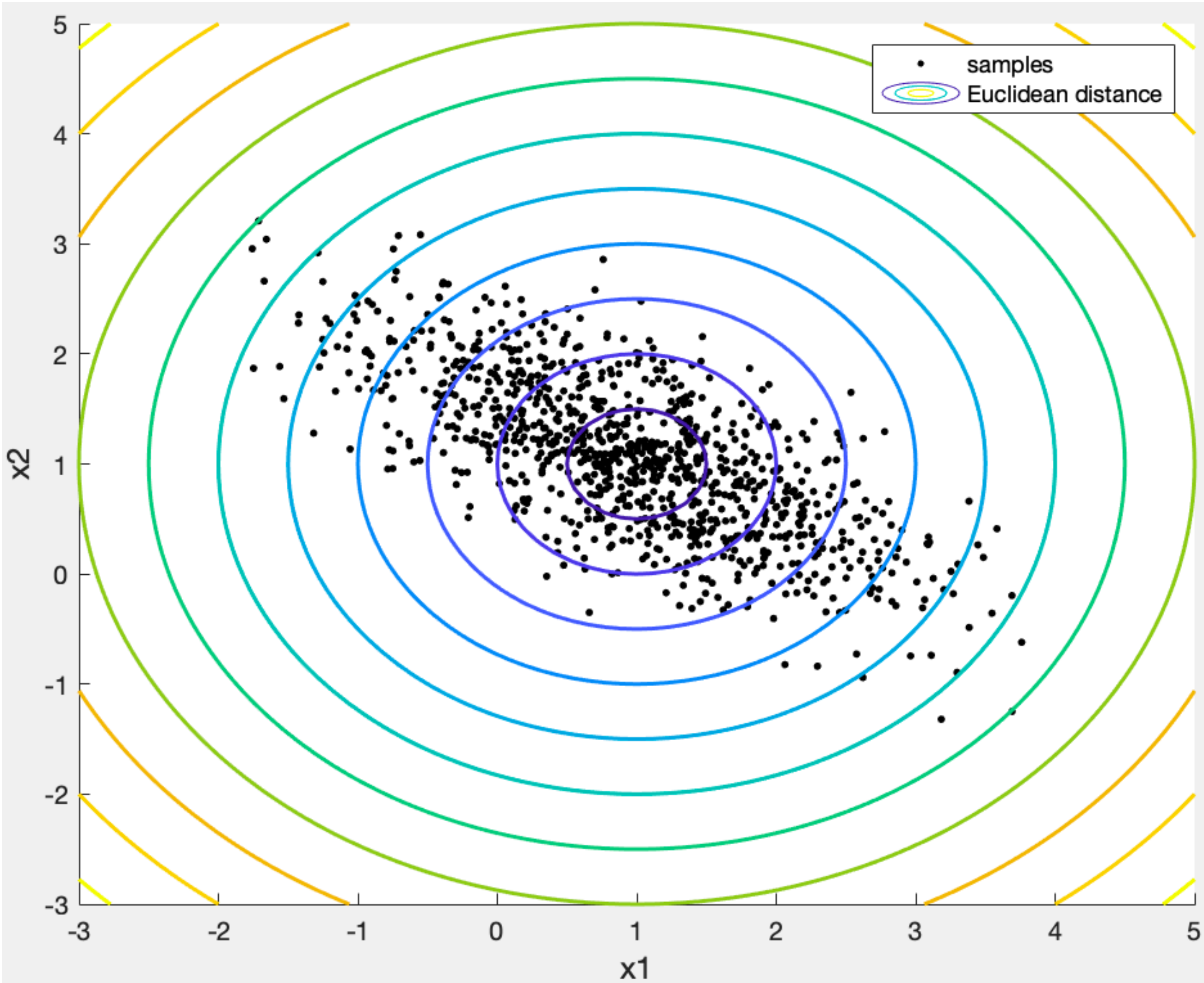
$=(2\pi)^{-\frac{M}{2}}|det(\Sigma)|^{-\frac{1}{2}}e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$

The existence of a matrix V satisfying $\Sigma=VV^T=V^TV$ is guaranteed by the fact that Σ is symmetric and positive definite.

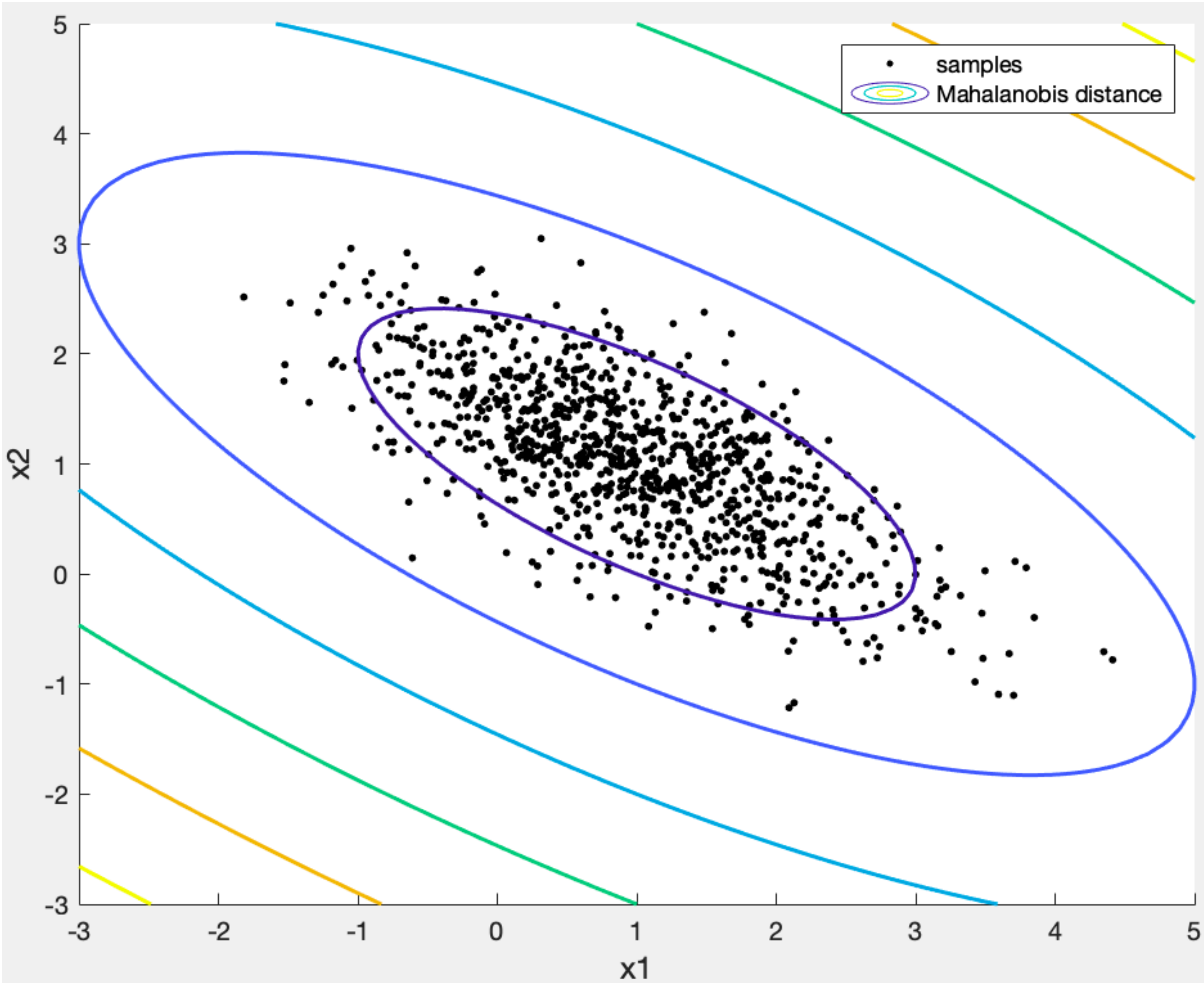
2.b.



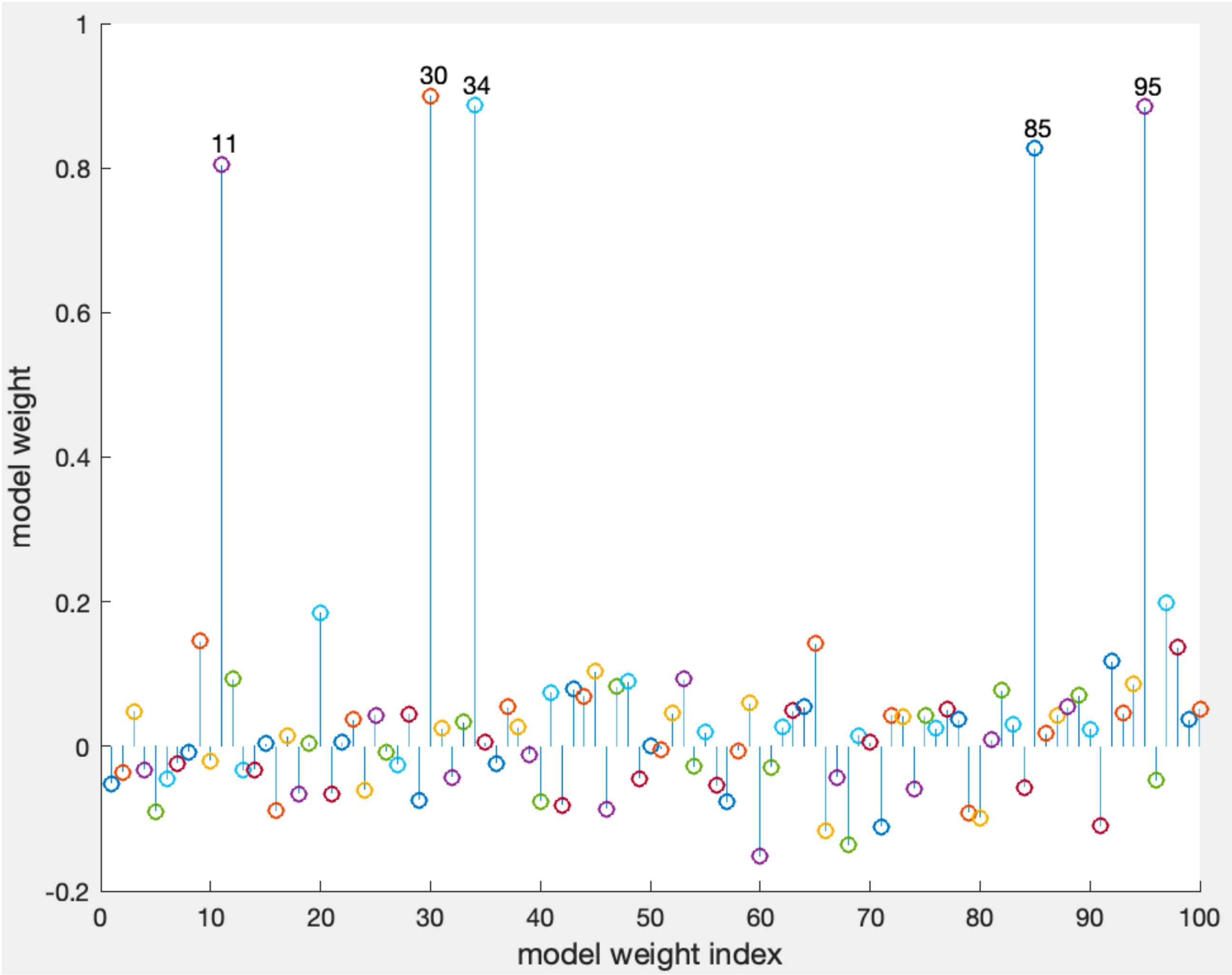
2.c.



2.d.

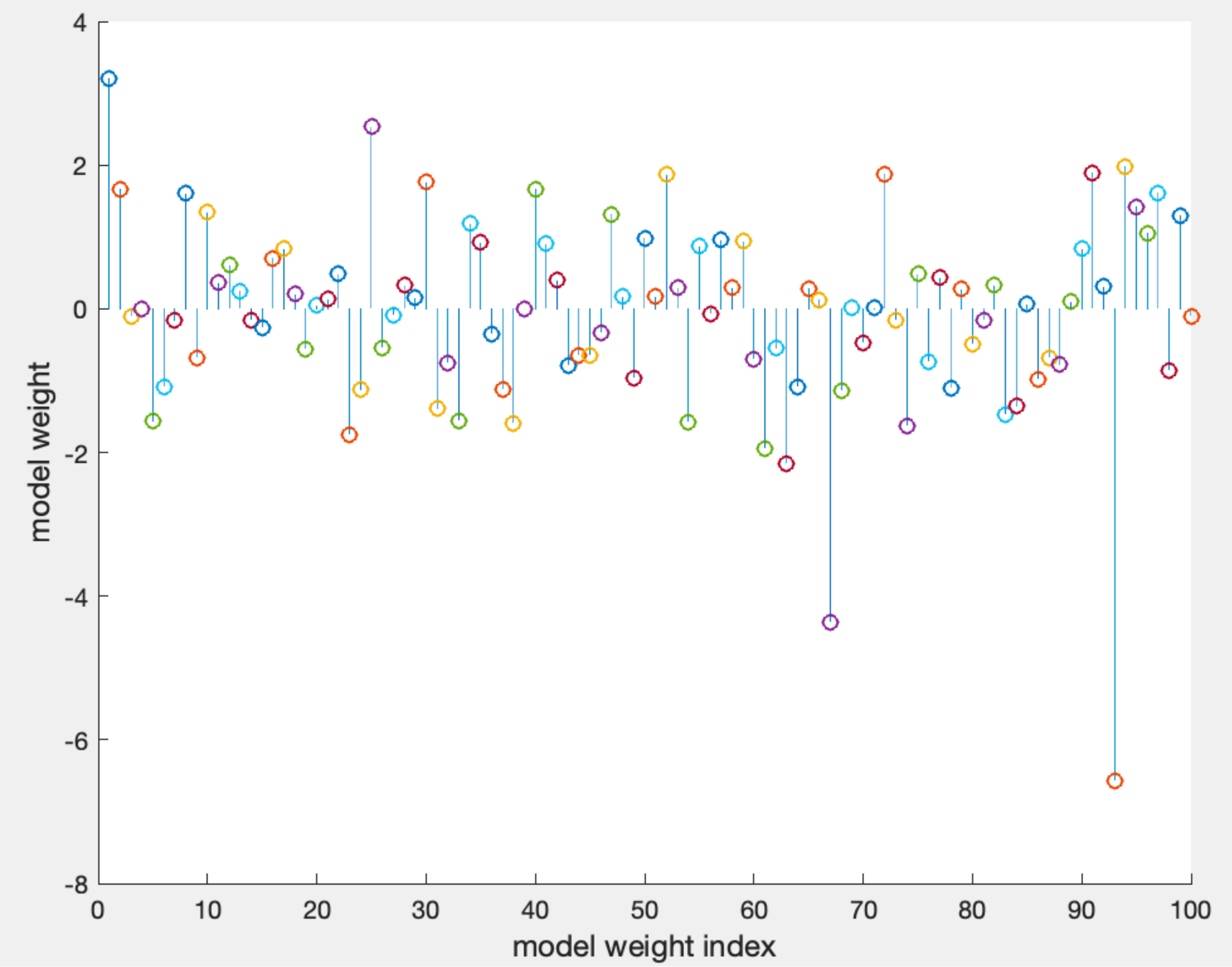


3.a.



From the graph we can see the indices of the nonzero model weights are 11,30,34,85,95

3.b.



3.c.

For ordinary least square model:

mean squre error:46.69630 |bias|:6.257846 variance:8.046464

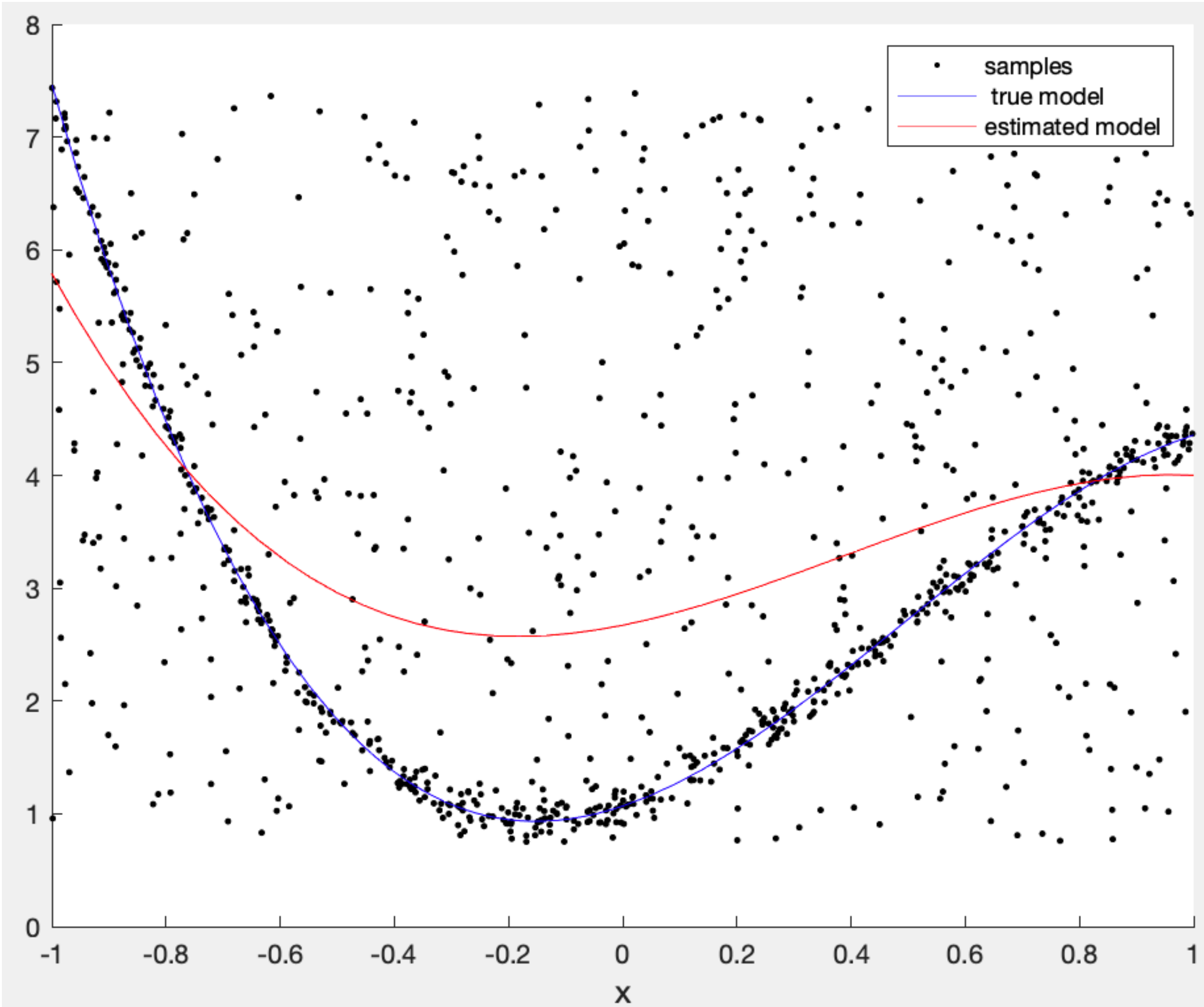
For LASSO:

mean squre error:1.864484e-04

|bias|:-8.174617e-05 variance:4.044406e-01

The above result is from the 3.a and 3.b program. Apparently the performance of LASSO is far better than oridinary least square model.

4.a.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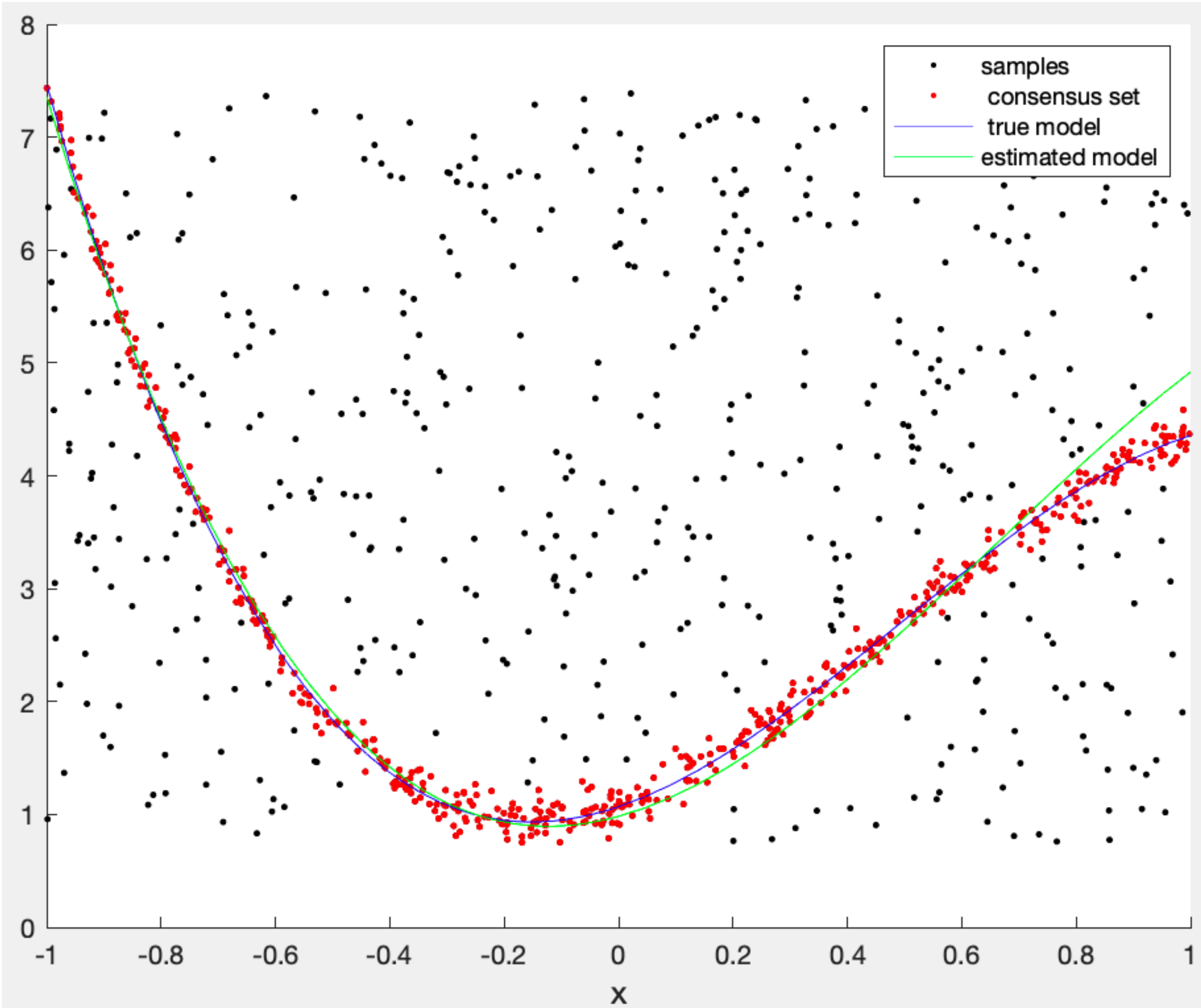
true model
estimated model
8
7
6
5
4
3
2
1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x

4.b.


samples
consensus set
true model
estimated model
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7
6
5
4
3
2
1
0
-1
-0.8
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-0.2
0
0.2
0.4
0.6
0.8
1
x