Review of book chapter 'Warped Riemannian metrics for location-scale models'

This contribution illuminates the interplay between information geometry and warped Riemannian structures, which are particular Riemannian metrics that have long been appreciated in differential geometry and mathematical physics for their relevance and practical utility. The computation of many key quantities in Riemannian geometry often becomes significantly more efficient if the metric has a warped structure. Therefore, the prominent appearance of such structures in information geometry in the presence of invariance of the model under the action of a Lie group as shown in this work may be of significant theoretical and practical importance. The paper further illustrates the main ideas with examples consisting of two important location-scale models: (1) the von Mises-Fisher model of directional data and (2) the Riemannian Gaussian model of data in spaces of covariance matrices. Overall, the paper is technically sound, the mathematics is thorough, and the topic is interesting and appears novel. The division of the content of the paper into the main body of text and the appendices has also been carried out in a judicious manner. As such, I recommend the acceptance of the contribution as a book chapter. However, there are some comments that the authors should consider with some care and incorporate into a revised version of the contribution. The authors and the editor should consider most of the comments below as suggestive, rather than demands.

- I feel that the paper would see a considerable improvement if its length was reduced. Overall, the paper seemed longer than it should be and there were places where it seemed somewhat repetitive in both language and concepts. I would recommend that the authors review the paper with the aim of making the writing, language, and mathematics as concise as possible without eliminating any section, main example, or key result. As a rough suggestion, a 20% reduction in length seems achievable without affecting the content too much.

  As very basic examples of possible cuts in language, I would suggest the elimination of the first sentence on page 6 'It was felt ... ', which seems unnecessary, as well as the last two sentences in Remark 8 on page 15 'Clearly, ... Riemannian metrics', which seems to be a statement of an immediately obvious mathematical fact. These are only two examples, but there could be many others little cuts like this.

- The presentation of Section 9 could be improved. For instance, the part of the observations that are not proved should be stated more clearly, perhaps in the form of a clear conjecture. In any case, the significance of Hadamard manifolds in this context as well as the observations deserves a clearer discussion and interpretation. For instance the interpretation of the results in Table 1 within a geometric information context would be of value.

  Some of the language on page 32 is too strong. For instance, in Remark 16 the authors speak of 'the fact that the sectional curvature is negative', which has yet to be established as a *fact*. *Strongly suggested result* may be a more reasonable substitute for the word 'fact'.

- The presentation of Figures 1 to 3 should change significantly. The existing presentation takes far more space than is necessary. The plots should be combined into one figure, showing the 6 curves converging to the various limits. The 6 curves could be distinguished by a choice of 3 colors and 2 textures, say. A combined figure allows a more immediate comparison of the results as well.

- The chapter should include a Conclusion as a final section, which would offer a summary of the chapter and convey the main essence of the results. There are many different technical sections, and a concluding discussion should be presented to tie everything together.

Overall, I have no doubt that the authors will be able to address the above comments with some time and care without too much difficulty.