Conformal Flattening on the Probability Simplex and its applications to Voronoi Partitions and Centroids

May 15, 2018

Information geometry investigates the geometric structures of statistical models. Geometric structures of the statistical models can be deformed using embedding functions. One of the possibilities is using the affine immersions. In the information geometry dually flat manifolds are fundamental objects. Kurose has shown that 1-conformally flat statistical manifolds realized by certain class of affine immersions can be transformed into dually flat ones. In this chapter conformal flattening of the probability simplex and its applications are discussed.

The distributions of discrete random variables over a finite set of n values form a simplex S^n.

$$S^n = \{p = (p_i)/p_i \in R_+, \sum_{i=1}^{n+1} p_i = 1\}$$

An immersion of S^n into R^{n+1} is considered so that (S^n, ∇, h) is a statistical manifold where ∇ is the induced torsion-free connection and h the affine fundamental form. In this chapter immersion f and choice of the transversal vector field ξ on S^n are specified so that (f, ξ) non-degenerate and equiaffine. Conormal vector ν is defined and the fundamental form (Riemannian metric on S^n) h is given in terms of ν. Then geometric divergence ρ is defined which is the contrast function of the geometric structure (S^n, ∇, h). Associated with the geometric divergence the conformal divergence is defined using the conformal factor which is a positive function on S^n. This conformal divergence is a contrast function for 1-conformally transformed geometric structure from (S^n, ∇, h). The problem here is the induced structure from the conformal divergence need not also be dually flat. Then it is demonstrated that by choosing the conformal factor suitably the induced structure $(S^n, \tilde{\nabla}, \tilde{h})$ is dually flat. It is also shown that the conformal divergence is the canonical divergence and the pair of affine coordinates and the pair of potential functions are explicitly given. The dually flat structure is independent of the choice of the transversal vector field. As examples, by taking the immersion as logarithmic function the ± 1-dually flat structure is obtained and by taking the immersion $\frac{t^{1-q}}{1-q}$ the alpha-geometry is obtained and demonstrated the conformal flattening of it.

As an application of these conformal flattening of the geometric structure Voronoi partition (diagram) on S^n is constructed using divergent functions of

Bregman type. But the problem is that the geometric divergence on S^n is not of Bregman type in general. The conformal flattening given in this paper solve this issue since the conformal divergence is of Bregman type and escort probability (dual coordinate) is taken as the coordinate system. Obtaining the Voronoi diagram on S^n using the space of escort distributions is detailed in this chapter. Also the weighted ρ-centroid for given m points on S^n is obtained using escort probability distributions. As an appendix to this chapter basic notions and results in information geometry and affine differential geometry are given.

This chapter is a detailed and lucid account of the conformal flattening of the probability simplex S^n using affine immersions and its applications to construction of Voronoi diagram on S^n and computation of weighted centroids.

Comments to the Editor

1. In Remark 1, in the last two lines replace g by h.

2.In the first paragraph of the introduction one may add the following reference also to 8,6,7 regarding several ways to do the deformation of the statistical manifold structure.

Harsha, K.V; Subrahamanian Moosath, K.S. F-geometry and Amaris α-geometry on a statistical manifold. Entropy, 2014, 16(5), 2472-2487.