

***** Review report for the Authors *****

Report on the Manuscript entitled:

Some Universal Insights on Divergences for Statistics, Machine Learning and Artificial Intelligence
by Michel Broniatowski and Wolfgang Stummer

Submitted for publication to the Springer book focusing on *Geometric Structures of Information*

General Comments

The main purpose of this paper concerns with the definition and presentation of general classes of divergences and distances, not only between probability measures but also between general functions. In this framework, divergences between functions are presented and discussed while several subclasses, examples and applications are illuminated. Connections between divergences and geometry are indicated while minimum divergence ideas and methods are presented to obtain parameter estimators which are based on noisy divergence minimization.

I think the authors have given very interesting results on the topic of generalized distances and divergences between functions in a great variety of disciplines and contexts. The paper is well organized and the exposition seems ideal for a so long paper. As far as I have been able to check, the proofs are technically correct, however I have to admit that I didn't check all of them, in a full detail.

The results obtained in the paper may provide a basis for interesting developments and discussions in almost all fields of science and engineering where the concept of the distance or dissimilarity or divergence is omnipresent and plays an important and universal role. I think then that the manuscript may eventually deserve publication on the above mentioned Springer book.

Some minor comments of a stylistic content:

1. Page 1, Abstract: The abbreviation AI should be fixed previously.
2. Page 2, Property (D2): Except of "reflexivity", a suitable terminology for this property is, maybe, "identity of indiscernibles", a terminology which I borrow from the paper by:

David J. Weller-Fahy, Brett J. Borghetti and ASngela A. Sodemann (2015), A Survey of Distance and Similarity Measures Used Within Network Intrusion Anomaly Detection. *IEEE Communication Surveys and Tutorials*, Vol. 17. No.1.

3. Page 4, Line 5 (top): "pricipally" should read as "principally"
4. Page 5, Last paragraph before Subsection 2.3: The following recent paper should be maybe included in the references, appeared in this paragraph:

Soumik Pal and Ting-Kam Leonard Wong (2018), Exponentially concave functions and a new information geometry. *Annals of Probability*, Vol. 46, No 2, 1070-1113.

I believe that, for the sake of completeness, some of such recent bibliography should be mentioned in the References and the relations with the present material should be briefly traced.

5. There are several points which need elaboration, during the formatting process of the manuscript. To mention a few:

- i. Text, formulas and numbers of equations exceed to the margins in almost all the pages of the manuscript.
 - ii. A blank space should be inserted in many places to separate words or punctuations and words. For instance, page 7, Line 7 (top) (mechanism,algorithm) or page 13, Line 13 (bottom), Assertions (1),(2) or page 21, Line 12 (bottom), Basu et al.[10], etc.
 - iii. Numbering (α) at the beginning of Definition 1 of page 30 should be removed.
 - iv. Numbering used inside the same theorem should be of the same type. For instance: Theorem 6 of page 32. Numbering (1), (2) and (3) should be all italic or normal.
6. Given the length of the manuscript, a final ***section of conclusions*** seems to be necessary in order to be summarized the results presented here and to be stated some insights of further development.