- Franck[1] Nielsen, "Output-sensitive peeling of convex and maximal layers," Information processing letters, 59.5 (1996): 255-259.

  Let $\mathcal{P}$ be a finite set of n points in the plane, in general position. Define by $\mathcal{H}_1$ the subset of $h_1 = |\mathcal{H}_1|$ extreme points $\mathcal{E}(\mathcal{P})$ calculated in $O(n \log h_1)$ [2]. Similarly, the i-th convex extreme point layer is defined as $\mathcal{H}_i = \mathcal{E}\left(\mathcal{P} \backslash \cup_{j=1}^{i-1} \mathcal{H}_j\right)$. The *convex depth* [1] of $\mathcal{P}$ is $\max_j \{|\mathcal{H}_j| \neq 0\}$ calculated by iteratively onion peeling $\mathcal{P}$ (Fig. 1). In [3], we describe a $O(n \log \sum_{j=1}^{i} h_j)$ algorithm to compute the first i convex layers of $\mathcal{P}$. This improves over the basic technique which consists in repeatedly applying k times the output-sensitive algorithm of [2] to get a $O(\sum_{i=1}^{k} n \log h_i) = O(nk \log \frac{H_k}{k})$ time algorithm. As a byproduct, we obtain a $O(n \log n)$ algorithm to compute the convex depth of $\mathcal{P}$. This output-sensitive algorithm also generalizes similarly to maxima layers (also called Pareto fronts), and more generally to compute the first $\leq i$-level sets of univariate function graph envelopes. The paradigm used is "grouping and querying" [4], a general paradigm to get output-sensitive algorithms.

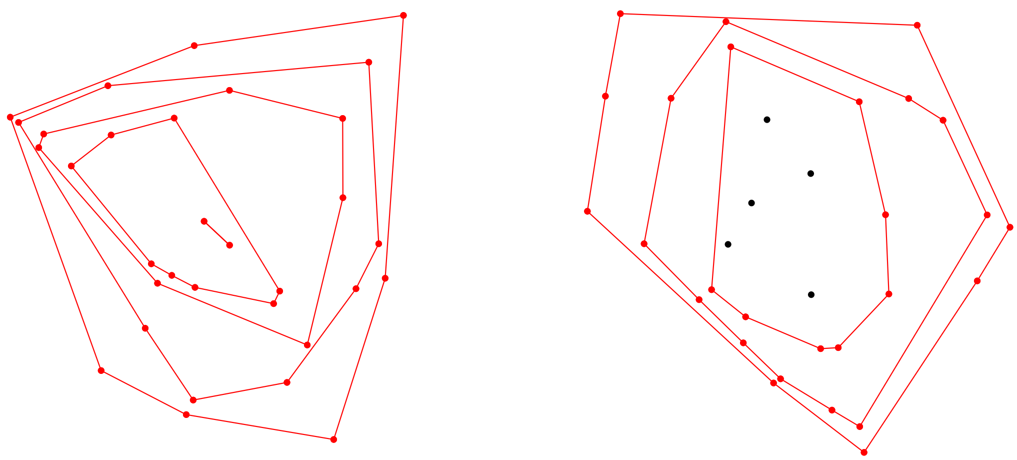

Figure 1: Full onion peeling with last layer indicating the convex depth (left) and partial onion peeling of the first $k = 3$ layers.

References

[1] Bernard Chazelle. On the convex layers of a planar set. *IEEE Transactions on Information Theory*, 31(4):509–517, 1985.

[2] David G Kirkpatrick and Raimund Seidel. The ultimate planar convex hull algorithm? *SIAM journal on computing*, 15(1):287–299, 1986.

[3] Franck Nielsen. Output-sensitive peeling of convex and maximal layers. *Information processing letters*, 59(5):255–259, 1996.

[4] Frank Nielsen. Grouping and querying: A paradigm to get output-sensitive algorithms. In *Japanese Conference on Discrete and Computational Geometry*, pages 250–257. Springer, 1998.

[1] Frank is usually spelled with a 'c' in French.