Smooth Manifolds Conjugate Connection Manifolds  $^{LC}\nabla = \frac{\nabla + \nabla^*}{2}$ Riemannian Manifolds  $(M, g, \nabla, \nabla^*)$  $(M, g) = (M, g, {}^{LC}\nabla)$  $(M, g, C = \Gamma^* - \overline{\Gamma})^- \qquad Se^{\frac{1}{2}} = \frac{1+\alpha}{2} \nabla + \frac{1-\alpha}{2} \nabla^* + \frac{1-\alpha}{2} \nabla^*$   $(X, G, C) = \Gamma^* - \overline{\Gamma}$   $(X, G, C) = \Gamma^* - \overline{\Gamma}$  (X, G,Self-dual Manifold  $(M, g, \alpha C)$ Divergence Manifold  $g = ^{\text{Fisher}} g$ Fisher-Riemannian  $(M, D^g, {}^D\nabla, {}^D\nabla^* = {}^{D^*}\nabla)$ Fisher  $g_{ij} = E[\partial_i l \partial_j l]$ Manifold  $^{D}\nabla$  - flat  $\Leftrightarrow$   $^{D}\nabla^{*}$  - flat Multinomial  $I[p_{\theta}:p_{\theta'}] = D(\theta:\theta')$ Location-scale Parametric family on exponential families family families KL on mixture families Conformal divergences on deformed families f-divergences Etc. Bregman divergence Location canonical Expected Manifold family Hyperbolic Manifold divergence  $(M, ^{\mathrm{Fisher}}g, \nabla^{-\alpha}, \nabla^{\alpha})$ Spherical Manifold  $\alpha$ -geometry ▲ Dually flat Manifolds

Euclidean Manifold

 $(M, F, F^*)$ 

(Hessian Manifolds)

Dual Legendre potentials

Bregman Pythagorean theorem

 $C_{ijk} = E[\partial_i l \partial_j l \partial_k l]$  Bregman Py  $\alpha C = \nabla^{\alpha \text{Fisher}} g$ Distance = Non-metric divergence

Cubic skewness tensor

Distance = Metric geodesic length