

REPORT ON THE PAPER:

“A class of non-parametric deformed exponential statistical models”

by L. Montrucchio and G. Pistone.

N. J. Newton proposed a general way to define a *non parametric* manifold M of positive probability densities modeled on an Hilbert space. The paper under review investigates a generalization of this approach by considering a class on non-parametric deformed statistical models where the deformed exponential \exp_A has linear growth at infinity and is sub-exponential at zero.

For such a class of models, as usual for nonparametric information geometry, several technical objects are of peculiar interest and the paper under review derives the properties of some of them. More specifically, the convexity and regularity of the normalization operator K_p are investigated in detail (recall here that, roughly speaking, the normalization operator is acting on general random variables u and ensures that deformed exponential curves of the form $t \mapsto \exp_A(tu + \log_A p)\mu$ are probability densities). The Gateaux derivative of K_p is related to the notion of escort density. The authors also investigate the properties of the deformed statistical divergences and their convex duality. The affine manifold structure of the statistical Hilbert bundle is then discussed in details.

The paper is interesting and well-written and provides a nice account of the use of deformed exponential to non parametric information geometry. As far as I can see, mathematical details are correct. I strongly suggest to approve its publication as a Chapter of the volume “Geometric Structures of Information”.

I suggest the following slight changes to improve the presentation.

- (1) In the Introduction, the authors should provide a definition for $L_0^2(\mu)$.
- (2) In Section 2.1, I suggest to unify the presentation: Newton A -logarithm is presented with first the introduction of the corresponding function A whereas Kaniadakis exponential is first introduced and the definition of the corresponding A function is derived from it. I suggest in all cases to start with the definition of the A function.
- (3) In the middle of page 5, replace “bounded converge” with “dominated convergence”.
- (4) Page 7, the notation $\dot{\mu}_0$ is misleading. Somehow, both statements are correct since the second one has to be interpreted as the mapping $t \mapsto \mu_t$ in L^1 . Some changes of notations would be appreciated here.
- (5) P 10, the first point of the proof is not clear. The displayed formula seems a definition of the function $F(t, \kappa)$ and not an equation which should be $F(t, \kappa) = 0$.
- (6) In several places, I suggest to unify the reference to previously stated Propositions: both Prop. and Propositions are used and I suggest to stick to one choice.