Bregman spheres: Thales' theorem

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[1]

space of Bregman spheres [3]

Lemma 1 (Corollary 3.11 [2], Problem 11.6 [4]) *Consider a sphere* $S = \{q : D(c : q) = r\}$ *of center* c *and radius* r. *Then the* ∇*-geodesic passing through* c *intersects the sphere orthogonally. The tangent autoparallel manifolds are* ∇^**-hyperplane.*

Proof: ... □

Recall Thales' theorem in Euclidean geometry

Theorem 1 (Thales' theorem) *The triangle circumscribed to a circle with one side being a diameter is right-angle.*

Proof: ... □

This theorem is not to be confused with Thales' intercept theorem (basic proportionality theorem). There are many proofs of Thales's (circle) theorem (e.g., using the sum of angles in a triangle to be π, using Pythagoras' theorem).

In a dually flat space, there are $2^3 = 8$ geodesic triangles[1] (6 pseudo-triangles and 2 dually flat triangles) passing through three vertices p, q and r, depending on whether we choose the primal or dual geodesic for linking ay two of those points. In general a dually flat space is not conformal but flat: The angles of (pseudo)-triangle (which triangle type?, purely primal or dual) sum up to π and there exists a Pythagoras' theorem.

Let (ab) denote the primal geodesic segment passing through a and b, and $(ab)^*$ the dual geodesic segment.

Theorem 2 (Thales' Bregman theorem) *The triangle* $(pq)^*(qr)^*$ *circumscribed to a* ∇*-circle with the side* (pr) *being a diameter is right-angle at* q.

Note that (pr) intersects the ∇-circle orthogonally.

References

[1] Shun-ichi Amari. *Information geometry and its applications.* Springer, 2016.

[2] Shun-ichi Amari and Hiroshi Nagaoka. *Methods of information geometry*, volume 191. American Mathematical Soc., 2007.

[3] Jean-Daniel Boissonnat, Frank Nielsen, and Richard Nock. Bregman Voronoi diagrams. *Discrete & Computational Geometry*, 44(2):281–307, 2010.

[4] Ovidiu Calin and Constantin Udrişte. *Geometric modeling in probability and statistics.* Springer.

[1] And 2^n types of geodesic n-gons.

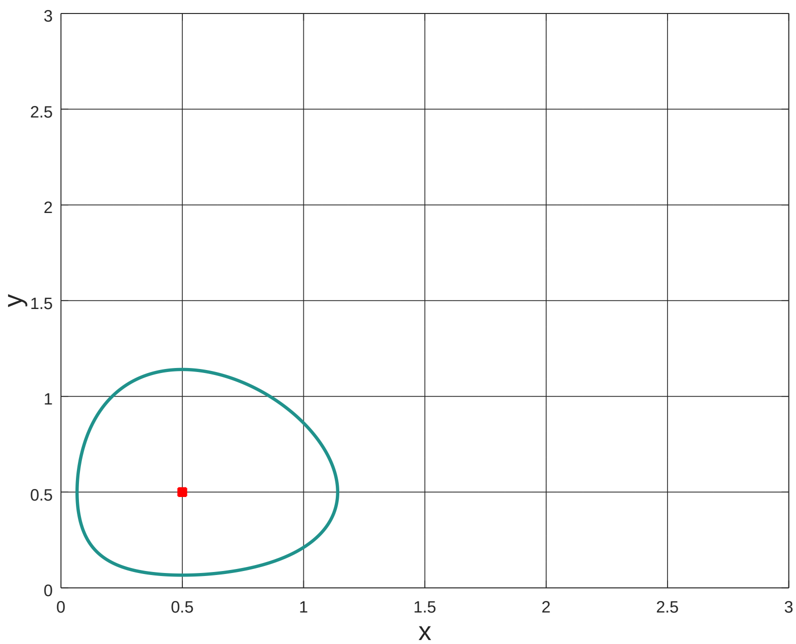

Figure 1: An extended Kullback-Leibler ball of center $(\frac{1}{2}, \frac{1}{2})$ and radius $\frac{3}{10}$ (defined on positive measures).