Referee Report for "Information geometry in portfolio theory" by Ting-Kam Leonard Wong

As the paper's title states, the author discusses the role of information geometry in the field of stochastic portfolio theory. He builds on former impressive work by himself and his coauthor Pal. The paper introduces stochastic portfolio theory and information geometry. The first three chapters provide an overview of previous results and help making the paper very understandable. In particular, functionally generated portfolios are introduced. These objects are of major importance in stochastic portfolio theory and very worthwile to be studied.

The paper's main contribution is then discussed in Section 4. Using an information-geometric characterization of portfolio-generating functions, the author is able to embed two known generations into one framework. At the same time, this approach provides new intuition and generalizes the existing literature. Section 5 provides further related curiosities connected to the divergence terms used in the characterization of portfolio-generating functions. Two open problems are also formulated.

———-

I do not understand (31) in the paper's main definition. This does not seem to be a definition. The process η appears on both sides of the equality. Somehow this needs to be fixed. The same problem appears in Section 4.5. There, it is not clear to me at all how this can be made rigorous (in the discrete case, I have no doubts that this problem can be fixed). More generally, Section 4.5 suffers from a lack of rigor. Fortunately, this section is not relevant for the rest of the paper. Hence, I'd suggest to either remove this section or make it rigorous. For example, η needs to be integrable with respect to μ – something not even mentioned.

In the following, some minor typos.

- P. 4, line 7: maybe write "MIGHT" have different impacts on the portfolio.
- Definition 1, 2nd sentence: correct the grammar.
- First display on page 7: I am not sure whether I undestand the appearance of $\sum X_i(0)$. In any case, I'd suggest to remove the whole sentence, starting with "We may interpret [...]".
- P. 9, line -8: add an "a" to "Note that a change in ..."
- Proposition 2: typo $\alpha := ...$
- P. 14, line 8: add "the" to "characterizes the multiplicatively"
- Theorem 3: What is the meaning of "self-financed"?
- Two lines after (29): Correct the grammar of the sentence starting "If condition ..."
- P. 26, line 6: change "is" to "are" in the sentence "The main properties ..."

- Theorem 8: Check the formatting of the enumeration.
- Reference [22]: check the journal.