Answers to Reviewers for paper :

Jean-Louis Koszul and the elementary structures of Information Geometry

<u>**Reviewer R1:**</u>

- the author might just briefly make a connection with works of Ingarden, Mrugala and Janyszek, and Weinhold & Ruppeiner.
  - o I have added a section to contextualize these references that are posterior to Koszul work (during 50's). These authors didn't know Koszul paper.
- On page 9, the author refers to Koszul's "important book": "Homology and Cohomology of Lie Algebras." However, this book was not cited in the manuscript, and I could not find it in Google Scholar. Could the author please cite this book, or give the reader a better idea of its status.
  - o I have added the following reference:

    [128] Koszul, J.M., Homologie et cohomologie des algèbres de Lie, Bulletin de la Société Mathématique de France, Tome 78, p. 65-127, 1950
- Corrections of few typos in the manuscript
  - o All typos have been corrected.

<u>**Reviewer R3:**</u>

- Corrections of few typos in the manuscript
  - o All typos have been corrected.

<u>**General Comments:**</u>

I have added:

- a chapter 6 about Koszul Lecture on Geometric and Analytics Mechanics, related to Geometric Theory of Heat (Souriau's Lie Group Thermodynamics) and Information (Information Geometry). SPRINGER will published soon a translation of Koszul's lecture "Introduction to symplectic geometry" that makes links between Koszul's works and Souriau's works.
- A chapter 8, an appendix about Clairaut(-Legendre) Equation of Maurice Fréchet associated to "distinguished functions" as fundamental equation of Information geometry
- References :

  - o Barbaresco F., Jean-Louis Koszul et les structures élémentaires de la géométrie de l'information, revue MATAPLI 2018, SMAI, 2018
  - o Malgrange B., Quelques souvenirs de Jean-Louis KOSZUL, Gazette des Mathématiciens - n°156, pp. 63-64, Avril 2018
  - o Cartier P., In memoriam Jean-Louis KOSZUL, Gazette des Mathématiciens - n°156, pp. 64-66, Avril 2018
  - o Kosmanek E.-E., Hommage à mon directeur de thèse, Gazette des Mathématiciens - n°156, pp. 67-67, Avril 2018
  - o Amari S.I., Differential Geometry of Statistical Models, SPRINGER Series Lecture Notes in Statistics, vol. 28, 1985

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- Cartan, H., Les travaux de Koszul, II, Séminaire Bourbaki, Tome 1, Exposé no. 8 , p. 45-52, 1948-1951
- Cartan, H., Les travaux de Koszul, III, Séminaire Bourbaki, Tome 1, Exposé no. 12 , p. 71-74, 1948-1951
- Haefliger, A., Des espaces homogènes à la résolution de Koszul Annales de l'institut Fourier, Tome 37, no. 4 , p. 5-13, 1987