Geometric Sciences of Information: Random musings

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Last updated, November 11, 2021

Aitchison distance vs Hilbert simplex distance

- Aitchison distance:

$$\begin{aligned}\rho_{\mathrm{Aitchison}}(p,q) &= \sqrt{\sum_{i=1}^{d}\left(\log\frac{p^i}{G(p)}-\log\frac{q^i}{G(q)}\right)^2}\\ &= \left\|\log\frac{p}{G(p)}-\log\frac{q}{G(p)}\right\|_2\end{aligned}$$

where $G(p)=\left(\prod_{i=1}^{d}p^i\right)^{\frac{1}{d}}=\exp\left(\frac{1}{d}\sum_{i=1}^{d}\log p^i\right)$ is the geometric mean of coordinates

- Hilbert simplex distance:

$$\begin{aligned}\rho_{\mathrm{Hilbert}}(p,q) &= \log\frac{\max_{i\in\{1,\ldots,d\}}\frac{p_i}{q_i}}{\min_{j\in\{1,\ldots,d\}}\frac{p_j}{q_j}}\\ &= \left\|\log\frac{p}{G(p)}-\log\frac{q}{G(q)}\right\|_{\mathrm{var}}\end{aligned}$$

where $\|\cdot\|_{\mathrm{var}}$ denotes the variation norm:

Geometry of Hamming spaces

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