RESEARCH PAPER

Local equivalence problem in hidden Markov model

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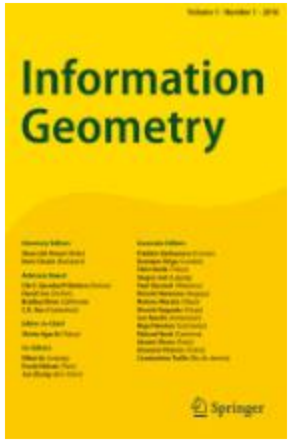
Abstract

In the hidden Markov process, there is a possibility that two different transition matrices for hidden and observed variables yield the same stochastic behavior for the observed variables. Since such two transition matrices cannot be distinguished, we need to identify them and consider that they are equivalent, in practice. We address the equivalence problem of hidden Markov process in a local neighborhood by using the geometrical structure of hidden Markov process. For this aim, we introduce a mathematical concept to express Markov process, and formulate its exponential family by using generators. Then, the above equivalence problem is formulated as the equivalence problem of generators. Taking this equivalence problem into account, we derive several concrete parametrizations in several natural cases.

Keywords Hidden Markov · Equivalence problem · Information geometry · Exponential family



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RESEARCH PAPER

Information geometry of modal linear regression

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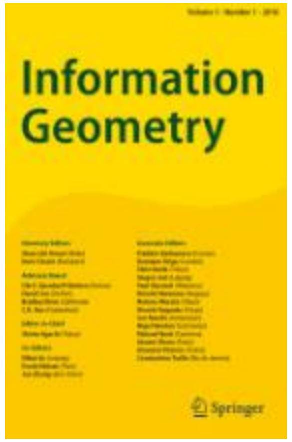
Abstract

Modal linear regression (MLR) is used for modeling the conditional mode of a response as a linear predictor of explanatory variables. It is an effective approach to dealing with response variables having a multimodal distribution or those contaminated by outliers. Because of the semiparametric nature of MLR, constructing a statistical model manifold is difficult with the conventional approach. To overcome this difficulty, we first consider the information geometric perspective of the modal expectation–maximization (EM) algorithm. Based on this perspective, model manifolds for MLR are constructed according to observations, and a data manifold is constructed based on the empirical distribution. In this paper, the *em* algorithm, which is a geometric formulation of the EM algorithm, of MLR is shown to be equivalent to the conventional EM algorithm of MLR. The robustness of the MLR model is also discussed in terms of the influence function and information geometry.

Keywords Modal linear regression · Information geometry · Kernel density estimation



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From Hessian to Weitzenböck: manifolds with torsion-carrying connections

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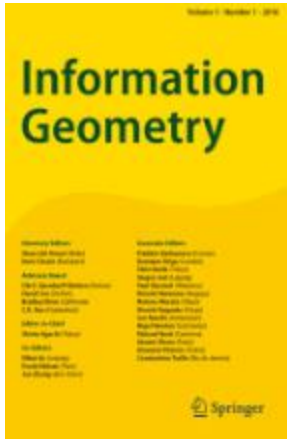
Abstract
We investigate affine connections that have zero curvature but not necessarily zero torsion. Slightly generalizing from what is known as Weitzenböck connections, such non-flat connections (we call "pseudo-Weitzenböck connections") can be constructed from any frame (a set of linearly independent vector fields), not just the orthonormal frames, with their torsions vanishing if and only if the frame is a coordinate frame. In such situations, the notion of biorthogonal frames generalizes the notion of biorthogonal coordinates of a Hessian manifold, with respect to any given Riemannian metric g (not necessarily Hessian). Our main theorem shows that the pair of pseudo-Weitzenböck connections, each adapted to one of the pair of g-biorthogonal frames, are g-conjugate to each other. As a result, the pseudo-Weitzenböck connection pair generalize dually flat connections characteristic of Hessian manifolds, by being both curvature-free yet admitting (generally unequal) torsions. These results allow us to construct a pseudo-Weitzenböck connection for the manifold of parametric statistical models and treat it as a "statistical manifold admitting torsion".

Keywords Codazzi coupling · Conjugate connection · Biorthogonal · Curvature · Torsion · Dually flat · Partially flat



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