Referee report of “Jean-Louis Koszul and the elementary structures of Information Geometry” by F. Barbaresco, submitted as a chapter for the Springer volume "Geometric Structures of Information."

This detailed exposition of the life and work of the distinguished French mathematician Jean-Louis Koszul is very well done, and certainly belongs in a volume titled "Geometric Structures of Information." The manuscript consists of two components. The first is an academic biography of Koszul, and the second is a rederivation of key elements of Koszul’s work, work which laid much of the foundation of Hessian structures in information geometry. This manuscript makes a great contribution as it gatherers together much of Koszul’s work in an English rendition of original papers written mostly in French. The manuscript is well documented, with 114 references.

The manuscript centers on the power of the French pure mathematical agenda, dating back to the Bourbaki group. This agenda includes Koszul, who was a second-generation member of Bourbaki. However, information geometry extends beyond the French border, and the author certainly emphasizes the international connections Koszul forged over his life, particularly the Japanese connection. I do think, however, that the efforts of the Roman Ingarden group of mathematical physicists in Torun maybe deserve to be referenced in this manuscript. Perhaps reference Ingarden’s book “Information Dynamics and Open Systems,” written with Kossakowski and Ohya. Unless I overlooked it, none of the authors of this book were named in the manuscript. Perhaps the reason for this is that they did not influence Koszul’s thinking that much. In terms of mathematical structures associated with the Polish group, let me mention as well the contact structures that Mrugala and Janyszek have worked on. How do these fit into the picture of Koszul? These themes all relates to the section on fluctuation theory in section 4 of the manuscript. Also, unless I am in error, Eq. (47) is the thermodynamic metric of Weinhold, Ruppeiner, and Andresen, etc. I know that the thermodynamic metric is related to the earlier Fisher information metric, but, nevertheless, the “thermodynamic metric” generates lots of cites in its own right, and the author might just briefly make a connection.

On page 9, the author refers to Koszul’s “important book”: “Homology and Cohomology of Lie Algebras.” However, this book was not cited in the manuscript, and I could not find it in Google Scholar. Could the author please cite this book, or give the reader a better idea of its status.

I found a few typos in the manuscript:

Page 1 line 1: “we go tribute”->”we pay tribute.”

Page 1 line 1, and elsewhere: The author might write in the first person with “we”->”I”?

Page 1 line 7: “Strong of the French mathematical tradition, .. real avant-garde” seems a bit unclear. Reword as “A strong disciple of the French mathematical tradition… real avant-garde”? But this might not be so clear either. Some might think that a strong mathematical tradition could be somewhat rigid, a rigidity that might work against the new and the experimental. I guess basically, I am not so sure what the author means here, and perhaps he might clarify.

Page 3, bullet point below Fig. 2, check some of the spaces. There are too many.

Page 8, line 1, comma after Koszul, and “son a”->”son in a.”

Page 9, first line in paragraph starting “In 1950”: “62-pages” -> 62-page.

Page 10, fourth line from the bottom: “sharp is there is no”->”sharp as there are no.”

Page 11, third line from the bottom: “2 formes”->”two forms.”

Page 15, first line: “winter 39”->”winter of 1939.”

Page 18, first line in section 4: “Structures is”->”Structures are.”

Page 21: the equations look a bit cramped.

Page 21: I am not sure that Figure 6 is well communicated. It is not clear where this figure begins. In a Table (which this appears to be) there is a distinct tabular form, with a caption above or below the table, and column and row labels. Could the author clarify?

Page 21, following the last word: is a period missing?

Page 23: the equations look a bit cramped.

Page 24, and elsewhere: in the equation following Eq. (74), there are spurious horizontal lines.

Page 33, sixth line of conclusion: “XXth century”->XX’th century.

Generally, there should be no space before a comma. A document search for <space><comma> should lead to the elimination of several typos.