7th Conference on the Geometric Science of Information

GSI'25

Geometric Structures of Statistical & Quantum Physics, Information Geometry, and Machine Learning

Saint-Malo, 29th to 31st October 2025

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7th International Conference on

GEOMETRIC SCIENCE OF INFORMATION

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Welcome to GSI'25!

We are delighted to organize the 7th edition of the launch of the GSI conferences cycle which were initiated in 2013. As for GSI'13, GSI'15, GSI'17, GSI'19, GSI'21 and GSI'23 (www.franknielsen.github.io/GSI/), the objective of this 7th edition of SEE GSI'25 conference, hosted in Saint-Malo, birthplace of Pierre Louis Moreau de Maupertuis, is to bring together pure and applied mathematicians and engineers, with common interest in geometric tools and their applications for information analysis. GSI emphasizes an active participation of young researchers to discuss emerging areas of collaborative research on the topic of "Geometric Science of Information and their Applications".

In 2025, GSI main theme was on "Geometric Structures of Statistical and Quantum Physics, Information Geometry, and Machine Learning: FROM CLASSICAL TO QUANTUM INFORMATION GEOMETRY", and the conference took place at Palais du Grand Large, in Saint-Malo, France. This is the second time in a row that the GSI conference is held in Saint-Malo.

The GSI conference cycle has been initiated by the Brillouin Seminar Team as soon as 2009 (www.repmus.ircam.fr/brillouin/home). The GSI'21 event has been motivated in the continuity of the first initiative launched in 2013 (www.web2.see.asso.fr/gsi2013) at Mines ParisTech, consolidated in 2015 (www.web2.see.asso.fr/gsi2015) at Ecole Polytechnique, and opened to new communities in 2017 (www.web2.see.asso.fr/gsi2017) at Mines ParisTech, 2019 (www. web2.see.asso.fr/gsi2019) at ENAC Toulouse, 2021 (www.web2.see.asso.fr/gsi2021) at Sorbonne University and 2023 at Saint-Malo.

Let us mention that in 2011, we organized an Indo-French workshop on the topic of "Matrix Information Geometry" (www.lix.polytechnique.fr/~nielsen/MIG/) that yielded an edited book in 2013, and in 2017, collaborate to CIRM seminar in Luminy with the event TGSI'17 "Topological & Geometrical Structures of Information" (www.fconferences.cirm-math.fr/1680.html).

GSI satellites event have been organized in 2019, and 2020 as FGSI'19 "Foundation of Geometric Science of Information" in Montpellier (www.fgsi2019.sciencesconf.org/) and Les Houches Seminar SPIGL'20 "Joint Structures and Common Foundations of Statistical Physics, Information Geometry and Inference for Learning" (www.franknielsen.github.io/SPIG-LesHouches2020/).

The technical program of GSI'25 covers all the main topics and highlights in the domain of the "Geometric Science of Information" including information geometry manifolds of structured data/information and their advanced applications. These Springer LNCS proceedings consist solely of original research papers that have been carefully peer-reviewed by at least two or three experts (sometimes by five experts). Accepted contributions were revised before acceptance.

As for the GSI'13, GSI'15, GSI'17, GSI'19, GSI'21, GSI'23 and GSI'25 addresses inter-relations between different mathematical domains like shape spaces (geometric statistics on manifolds and Lie groups, deformations in shape space, ...), probability/optimization and algorithms on manifolds (structured matrix manifold, structured data/information,...), relational and discrete metric spaces (graph metrics, distance geometry, relational analysis,...), computational and Hessian information geometry, geometric structures in thermodynamics and statistical physics, algebraic/infinite dimensional/Banach information manifolds, divergence geometry, tensor-valued morphology, optimal transport theory, manifold and topology learning, ... and applications like geometries of audio-processing, inverse problems and signal/image processing. GSI'23 topics were enriched with contributions from Lie Group Machine Learning, Harmonic Analysis on Lie Groups, Geometric Deep Learning, Geometry of Hamiltonian Monte Carlo, Geometric & (Poly)Symplectic Integrators, Contact Geometry & Hamiltonian Control, Geometric and structure preserving discretizations, Probability Density Estimation & Sampling in High Dimension, Geometry of Graphs and Networks and Geometry in Neuroscience & Cognitive Sciences.

Topics of interests include but are not limited to: Geometric Learning and Differential Invariants on Homogeneous Spaces, Statistical Manifolds and Hessian information geometry, Renyi Entropy & Information, Geometric Foliation Structures of Dissipation and Machine Learning, Geometric Structures of Quantum Information & Processing, Applied Geometric Learning, Probability, Information and Topology (fundamentals & applications), Divergences in Statistics and Machine Learning, Geometric Statistics, Geometric Methods in Hybrid Classical/Quantum Systems, Computational Information Geometry and Divergences, Geometric Methods in Thermodynamics, The Geometry of Classical & Quantum States, Geometric Mechanics, Geometric Green & Quantum Machine Learning, Stochastic Geometric Dynamics, New trends in Nonholonomic Systems, Learning of Dynamic Processes, Neurogeometry, PINN (Physics-Informed Neural Network) with Geometric Structures, Lie Groups in Machine Learning, Information Geometry, Toric Manifold (Delzant Theory), Symplectic approach to differential equations, Lie Group Based Method in Robotics & Kalman Filters, Geometric and Analytical Aspects of Quantization and Non-Commutative Harmonic Analysis on Lie Groups, Probability and Statistics on manifolds, Deep learning: Methods, Analysis and Applications to Mechanical Systems, Integrable Systems and Information Geometry (From Classical to Quantum), Computing Geometry & Algebraic Statistics.

At the turn of the century, new and fruitful interactions were discovered between several branches of science: Information Sciences (information theory, digital communications, statistical signal processing), Mathematics (group theory, geometry and topology, probability, statistics, sheaves theory, ...) and Physics (geometric mechanics, thermodynamics, statistical physics, quantum mechanics, ...). GSI biannual international conference cycle is a tentative to discover joint mathematical structures to all these disciplines by elaboration of a “General Theory of Information” embracing physics science, information science, and cognitive science in a global scheme.

GSI'25 received 146 submissions among which 124 papers were accepted after single-blind review process where each paper was reviewed by at least two anonymous reviewers.

The GSI'25 conference was structured in 18 sessions of more than 120 papers:

1. Geometric Learning and Differential Invariants on Homogeneous Spaces
2. Statistical Manifolds and Hessian information geometry
3. Applied Geometry-Informed Machine Learning
4. Geometric Green Learning on Groups and Quotient Spaces
5. Divergences in Statistics and Machine Learning
6. Geometric Statistics
7. Computational Information Geometry and Divergences
8. Geometric Methods in Thermodynamics
9. Classical & Quantum Information, Geometry and Topology
10. Geometric Mechanics
11. Stochastic Geometric Dynamics
12. New trends in Nonholonomic Systems
13. Learning of Dynamic Processes
14. Optimization and learning on manifolds
15. Neurogeometry
16. Lie Group in Learning Distributions & in Filters
17. A geometric approach to differential equations
18. Information Geometry, Delzant Toric Manifold & Integrable System

GSI 2025 General Co-Chairs

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Keynote Speakers

Prof. Nina MIOLANE
Assistant Professor, AI, UC Santa Barbara. Co-Director, AI Center, Bowers Women's Brain Health Initiative. Affiliate, Stanford SLAC

TOPOLOGICAL DEEP LEARNING: UNLOCKING THE STRUCTURE OF RELATIONAL SYSTEMSS

Abstract: The natural world is full of complex systems characterized by intricate relations between their components: from social interactions between individuals in a social network to electrostatic interactions between atoms in a protein. Topological Deep Learning (TDL) provides a framework to process and extract knowledge from data associated with these systems, such as predicting the social community to which an individual belongs or predicting whether a protein can be a reasonable target for drug development. By extending beyond traditional graph-based methods, TDL incorporates higher-order relational structures, providing a new lens to tackle challenges in applied sciences and beyond. This talk will introduce the core principles of TDL and provide a comprehensive review of its rapidly growing literature, with a particular focus on neural network architectures and their performance across various domains. I will present open-source implementations that make TDL methods more accessible and practical for real-world applications. All in all, this talk will showcase how TDL models can effectively capture and reason about the complexity of real-world systems, while highlighting the remaining challenges and exciting opportunities for future advancements in the field.

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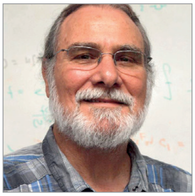

METRIPLECTIC DYNAMICS: A GEOMETRICAL FRAMEWORK FOR THERMODYNAMICALLY CONSISTENT DYNAMICAL SYSTEMS

Abstract: Classical descriptions of matter present many fluid mechanical and kinetic theory dynamical systems. These include, e.g., the Navier-Stokes-Fourier system, the Cahn-Hilliard-Navier-Stokes system for multiphase fluid flow, and various types of collisional kinetic theories for gaseous and plasma modeling. A desirable feature of such modeling is thermodynamic consistency, i.e., conservation of energy and production of entropy, in agreement with the first and second laws of thermodynamics. Metriplectic dynamics is a kind of dynamical system (finite or infinite) that encapsulates in a geometrical formalism such thermodynamic consistency. An algorithmic procedure for building such theories is based on the metriplectic 4-bracket, a bracket akin to the Poisson bracket that maps phase space functions to another. However, the 4-bracket maps 4 such functions and has algebraic curvature symmetries. Metriplectic 4-brackets can be constructed using the Kulkarni-Nomizu product or via a pure Lie algebraic formalism based on the Koszul connection. The formalism algorithmically produces many known and new dynamical systems, and it provides a pathway for constructing structure preserving numerical algorithms.

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EXTENDED VARIATIONAL LEARNING VIA FENCHEL-YOUNG LOSSES

Abstract: Many statistical learning and inference methods, from Bayesian inference to empirical risk minimization, can be unified through a variational perspective that balances empirical risk and prior knowledge. We introduce a new, general class of variational methods based on Fenchel-Young (FY) losses. These losses, derived using Fenchel conjugation (a central tool of convex analysis), generalize the Kullback-Leibler divergence and encompass Bayesian as well as classical variational learning. This FY framework provides generalized notions of free energy, evidence, evidence lower bound, and posterior, while still enabling standard optimization techniques like alternating minimization and gradient backpropagation. This allows learning a broader class of models than previous variational formulations. This talk will review FY losses and then detail this new generalized variational inference approach to machine learning.

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A NONCOMMUTATIVE PERSPECTIVE OF GRAPH NEURAL NETWORKS

Abstract: In this talk we show how the merging of different languages pertaining to K-theory, algebraic geometry, noncommutative geometry and gauge field theory can be fruitfully integrated to give a unified version of discrete differential geometry on graphs.

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THE L^P FISHER-RAO METRICS AND THE ALPHA-CONNECTIONS

Abstract: The information geometry of parametric statistical models is built around two central constructions: the Fisher-Rao metric and the family of dual\ alpha-connections. Both have non-parametric counterparts, which we discuss in this talk. We introduce the L^p-Fisher-Rao metrics, a family of Finsler metrics that generalize the Fisher-Rao metric, and study their links with the \alpha-connections. We show that their geodesics coincide on the space of smooth densities, for p=2/(1-\alpha), while they differ on the space of probability densities. This gives a new variational interpretation of\alpha-geodesics as being energy minimizing curves. Joint work with Martin Bauer, Yuxiu Lu and Cy Maor.

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BICENTENARY OF THERMODYNAMICS AND SADI CARNOT'S SEMINAL WORK: FROM CONSTANTIN CARATHÉODORY'S CONTACT GEOMETRY MODEL TO JEAN-MARIE SOURIAU'S SYMPLECTIC FOLIATION STRUCTURE

Abstract: Sadi Carnot stands as a symbol of the "Ingénieur-Savant" who have profoundly enriched the fertile ground of French innovation. The discipline of thermodynamics, which he founded, today underpins information theory, the development of future quantum computers, and artificial intelligence, as well as climate science, pillars upon which rest both our industrial policies and the future of our societies. Sadi Carnot formulated the second law of thermodynamics in his treatise, "Reflections on the Motive Power of Fire "(1824), that initially received little

attention, but emerged from obscurity years later thanks to the works of Émile Clapeyron, William Thomson (later Lord Kelvin), and Rudolf Clausius. Sadi Carnot drew upon a remarkably wide range of knowledge, nourished by an eclectic and inquisitive mind. Taken by illness only a few years after publishing his Reflections, he remains forever preserved in the brilliance of his genius. The explanation of thermodynamics through geometric models was initiated by seminal figures such as Gibbs, Reeb, Carathéodory and Souriau. We shall trace the narrative of the geometric models, from Carathéodory (1909) to Souriau (1969), that have sought to re-establish Sadi Carnot's thermodynamics upon new foundational variational principles. As observed by Vladimir Arnold, "Every mathematician knows it is impossible to understand an elementary course in thermodynamics. The reason is that thermodynamics is based, as Gibbs has explicitly proclaimed, on a rather complicated mathematical theory, on the contact geometry« . A seminal contribution of Carathéodory lies in the introduction of a differential equation that governs the infinitesimal changes in state functions as the system undergoes infinitesimal transitions between states. This equation is of paramount importance as it embodies the first and second laws of thermodynamics in a single, unified mathematical expression. Carathéodory's axiomatization is intricately linked with the language of differential geometry, later interpretated as contact geometry and has greatly influenced Misha Gromov geometer who introduced the concept of Carnot-Carathéodory spaces. One of the pivotal results of Carathéodory's axiomatization is his theorem, which asserts that, given certain conditions the second law of thermodynamics, as formulated by Clausius, can be derived directly from the fundamental thermodynamic relation. While Carnot thermodynamics was primarily concerned with systems in equilibrium, Carathéodory's axiomatization extends far beyond this, offering a more general framework that accommodates systems in non-equilibrium states. Only recently, however, has the Souriau's Symplectic Foliation Model, introduced within the domain of geometric statistical mechanics, provided a geometric definition of entropy as an invariant Casimir function on symplectic leaves, specifically, the coadjoint orbits of the Lie group acting on the system, where these orbits are interpreted as level sets of entropy. We present a symplectic foliation interpretation of thermodynamics, based on Jean-Marie Souriau's Lie Groups Thermodynamics. This model offers a Lie algebra cohomological characterisation of entropy, viewed as an invariant Casimir function in the coadjoint representation.

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Bicentenary of Seminal Sadi Carnot's Book on Thermodynamics « Réflexions sur la puissance motrice du feu »

Frédéric Barbaresco (THALES)

1. Sadi Carnot Diagrams by Emile Clapeyron

We revisit the context in which Sadi Carnot's work was rediscovered by Émile Clapeyron, who played a pivotal role in enhancing the steam engines power-ing the first locomotives in Paris. During this endeavour, Clapeyron adapted the Southern-Watt diagrams to the thermodynamic theory of Sadi Carnot, subsequently proposing new diagrams for the Carnot cycle.

Following a period of residence in Russia under the tutelage of Gabriel Lamé, Émile Clapeyron (Clapeyron, 1834 & 1958) contributed to the construction of railways in France. In 1832, Clapeyron revisited the ideas of Sadi Carnot (Carnot, 1824, CNRS 1976) concerning the driving force of heat, grounding them in the principle of vis viva (living forces) and providing an analytical translation of these concepts. This work would subsequently influence the formulations of the second law of thermodynamics by William Thomson and Rudolf Clausius (Clausius, 1850 & 1864). In 1834, Clapeyron published a memoir titled «La force motrice de la chaleur» in the Journal de l'École Polytechnique, in which he rescued Carnot's work from obscurity. He provided a graphical representation of Carnot's theorem and derived the thermodynamic equations that would later bear his name. Clapeyron wrote:

[…] Returning to France after the revolution of 1830, my occupations took on a more practical character; However, around this time, I wrote a work on the quantities of heat released or absorbed in the changes in volume or state that bodies experience, and which was published in the Journal de l'Ecole Polytechnique. This work has often been cited by people who have since dealt with the mechanical theory of heat. *(Clapeyron 1958, p.12).*

In the paper of the Journal de l'Ecole Polytechnique, Emile Clapeyron write in the introduction :

[…] Finally, I will cite among the works that have appeared on the theory of heat, a work by M. S. Carnot, published in 1824, under the title "reflections on the motive power of fire". The idea which serves as the basis for his research seems to me fruitful and incontestable, his demonstrations are based on the absurdity that it would be to admit the possibility of creating motive force or heat from scratch. … This new means of demonstration seems to me worthy of attracting the attention of geometers; it seems to me to be safe from any objection, and has acquired a new importance since the verification it finds in the work of Mr. Dulong, who demonstrated through experience the first theorem whose statement I have just recalled. I believe that it is of some interest to take up this theory again; M. S. Carnot, avoiding the use of mathematical analysis, arrives, through a series of delicate and difficult to understand reasoning, at results, which are easily deduced from a more general law, which I will seek to establish. But before entering into the matter, it is useful to return to the fundamental axiom which serves as the basis of Mr. Carnot's research, and which will also be my starting point.*(Clapeyron 1934, p. 16).*

Émile Clapeyron, having undergone training at the École Polytechnique from 1815 to 1818 and subsequently at the École des Mines de Paris from 1818 to 1820, embarked upon a journey to Saint Petersburg alongside his fellow student Gabriel Lamé. There, they were tasked with educating the students of the Institute and the Corps of Communications Engineering, established in 1809 and directed by Augustin Bétancourt. Clapeyron returned to France following the July Revolution of 1830, when diplomatic relations between France and Russia had soured.

During this period, Clapeyron, profoundly impressed by the success of the pioneering railway experiments between Manchester and Liverpool, conceived the idea and subsequently drafted the plans for a railway connecting Paris to Saint-Germain. In 1834, a company was formed to undertake the construction of the Paris-Saint-Germain railway, and Clapeyron, alongside Gabriel Lamé and Stéphane Mony, was entrusted with the management of the project. His enthusiasm for railways grew, and he was appointed professor at the École des Mines de Saint-Étienne (1832–1834), during which time he acquired a copy of Sadi Carnot's memoir at the institution.

Clapeyron's professional focus gradually became more practical in nature; nevertheless, around this time, he authored a treatise on the quantities of heat released or absorbed during the changes

Fig. 1a: Sadi Carnot. Source Wikimedia

Fig. 1b: Emile Clapeyron. Source ENSMP

in volume or state of bodies, a work that was later published in the Journal de l'École Polytechnique. Clapeyron writes:

[…] This work has often been cited by people who have since dealt with the mechanical theory of heat. I can refer on this point to the appreciation that Mr. Reech was kind enough to make in the reading given by him to the Academy in its session of January 11, 1858. *(Clapeyron 1858, p. 22)* .

The reference pertains to Frédéric Reech (1805–1884) (Reech, 1858 & 1869), General Inspector of Maritime Engineering in Lorient, who had known, applied, and cited the work of Sadi Carnot prior to and independently of its rediscovery by later scientists, following the contributions of Thomson.

Indeed, during his time in Russia, Clapeyron was exposed to the British engineers who were engaged in the installation of Watt's engine in the country. It was during this period that he encountered the «indicator diagram» invented by John Southern, which enabled the measurement of a machine's power by recording and graphing the displacement of the piston as a function of pressure. Consequently, Clapeyron did not, in fact, invent the (p, V) «Carnot diagram»; rather, he introduced the Southern-Watt diagrams—previously the domain of mechanical engineers—into the scientific realm. In this regard, he effectively became the pivotal figure bridging the divide between the two communities: that of scholars and that of practitioners, thereby facilitating the integration of Sadi Carnot's ideas into both spheres.

In 1796, Southern devised a rudimentary apparatus to address the problem of power measurement. He affixed a sheet of paper above the indicator, which was rigged to oscillate in unison with the movement of the main piston. A pencil was then attached to the tip of the pressure gauge. As the pressure fluctuated, so too would the pencil's motion, while the paper shifted laterally, in accordance with the engine's cycle. The resulting trace, when the system operated smoothly, formed a closed curve, which Southern termed the «indicator diagram.» From this, the average pressure during the engine's operation could be computed by measuring the mean distance between the upper and lower bounds of the curve.

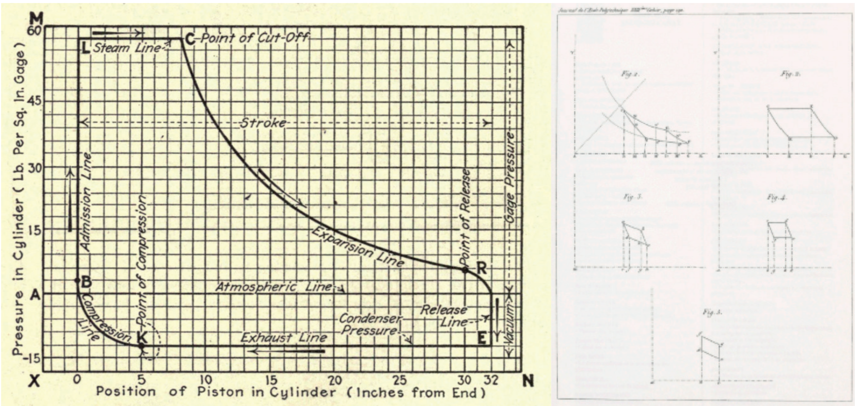

Fig. 2: top) Steam engine designed by Boulton & Watt, England, 1784 and Southern's (Watt employee) "Indicator Diagram", 1796 (Croft 1922 p.41). (bottom) Carnot Cycle figure in Emile Clapeyron paper. Source Galica (Clapeyron 1834, p. 54)

Shortly thereafter, Clapeyron published a note in the Annales des Mines on a mechanical theorem, the formulation of which is as follows: when a solid body moves with uniform motion through a ponderable fluid at rest, the resistance encountered by the solid body in a specified direction is equivalent to the quantity of motion imparted by the solid body to the surrounding fluid in the same direction, per unit of time. Clapeyron observed that from this proposition, one may deduce significant consequences regarding the specific force exerted by winged animals, which appears to be considerably greater than that of quadrupeds, as well as the velocity at which long waves propagate. The operation of the Saint-Germain and Versailles railways drew Emile Clapeyron's attention to the potential improvements in the distribution of steam within locomotive engines. A practice well-known to English engineers, referred to as «drawer advance» (avance du tiroir), served as the starting point for Clapeyron's investigations. In 1842, he presented a memoir to the Academy, detailing the modifications made to most locomotive engines. These modifications resulted in a fixed expansion occurring over a portion of the piston stroke, varying by a quarter to a third, which was achieved through a simple adjustment of the eccentric setting and the width of the drawer flanges. Emile Clapeyron wrote:

Fig. 3: Emile Clapeyron is in charge of the first Parisian railway lines. Meudon station, inauguration of the Paris-Saint-Germain railway line. On August 24, 1837, the first rail link dedicated to passenger transport was inaugurated in France. It connects the Parisian district of Saint-Lazare (la place de l'Europe) to the town of Pecq in less than an hour, shortly before Saint-Germain-en-Laye, 18 km west of the capital. Travelers then reach Saint-Germain-en-Laye, a very popular excursion for Parisians. This line immediately became a huge success. The locomotives stop at Le Pecq because they are not powerful enough to cross the Saint-Germain hill.(Barbaresco C 2023). Source Gallica.

[…] The facts which are related in my Memoir result much more from workshop experiences than from scientific experiments, difficult to combine with the needs of an active service and the absorbing occupations of the practice of engineering art; nevertheless the results were sufficiently salient to convince professional men, and they can be considered definitively acquired from practice.*(Clapeyron 1958, p.43).*

A commission of the Academy of Sciences, led by Poncelet and Lamé, appended an additional note to their report, acknowledging the contributions of various engineers, both in England and France, to this discovery. As a consequence, this method was first implemented in the construction of locomotive engines in England in 1840, and simultaneously in France, where the Le Creusot engine was modified in accordance with Clapeyron's designs. Clapeyron concluded his presentation with the following remarks:

[…] Here I end this rapid presentation of my career, both practical and scientific; I am still at the moment that he is attached with the title of consulting engineer to the Chemins de Fer du Nord et du Midi. I have also been, for many years, in charge of the steam engine course at the École des Ponts et Chaussées. These are the qualifications that I have to present in support of my candidacy and which I have the honor to submit for the consideration of the Academy.*(Clapeyron 1958, p.57).*

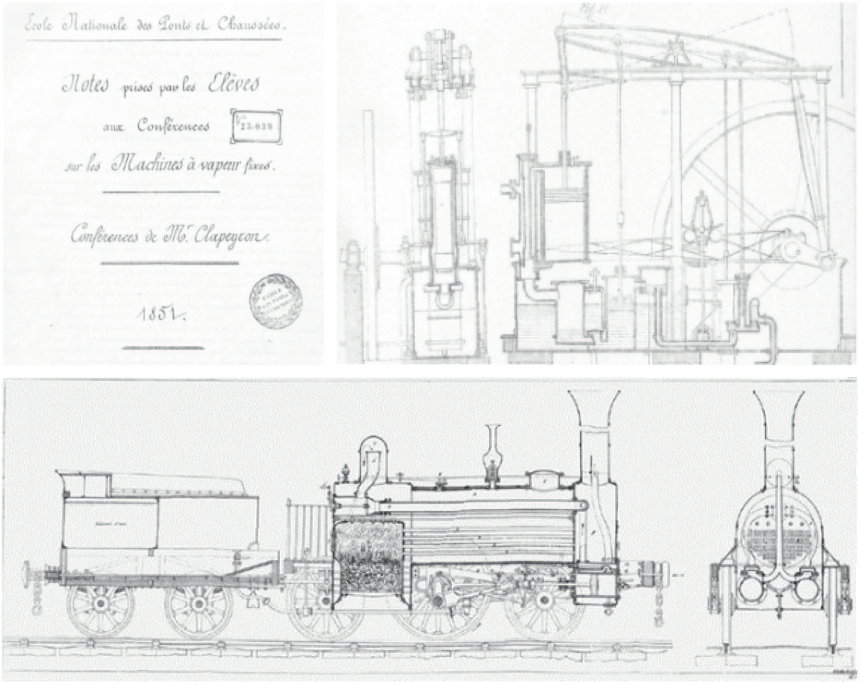

Fig. 4: Lecture Notes of Emile Clapeyron on "Steam Engines" written by Ecole des ponts et chaussées students in 1952.(Clapeyron 1944, p.27). Source Gallica.

2. Constantin Carathéodory's Seminal Idea of foliation for Sadi Carnot's 2nd principle

We present Constantin Carathéodory's seminal 1909 paper, which in-troduced an axiomatic formulation of thermodynamics and provided a geometric interpretation of Sadi Carnot's second law, based on the theory of Pfaffian equations—an area that Carathéodory had explored in a different context. We elucidate the influence of J.W. Gibbs on Carathéodory, particularly with regard to his diagram method. Furthermore, we examine the interpretations of Carathéodory's work by the physicist Wolfgang Pauli, as well as more contemporary scholars who have reinter-preted this model through the lens of foliation theory. We shall elucidate how Constantin Carathéodory introduced his axiomatization of thermodynamics in his seminal 1909 paper, offering the first geometric interpretation of Sadi Carnot's second principle, grounded in the theory of Pfaffian equations *(Goursat, 1922).*

The mathematical foundation of Carathéodory's axiomatization of thermodynamics is predicated upon a carefully constructed set of postulates and mathematical apparatus, principally drawn from the domains of differential geometry and mathematical analysis. Carathéodory's methodology is anchored in the aspiration to derive the laws of thermodynamics from a minimal and self-contained set of axioms, thereby relying upon a purely formal and mathematical framework to describe the dynamics of thermodynamic systems. At the outset, Carathéodory conceptualizes a thermodynamic system as one that can be delineated by a set of state variables. These variables—such as pressure, volume, and temperature—serve to characterise the system's state at any given instant. The state of the system is thus represented as a point within a state space, where each point corresponds to a unique configuration of these variables. The system's behaviour is subsequently described through state functions, which are defined as functions of the state variables. These functions depend exclusively on the system's present state and not on the particular path taken to achieve that state.
Central to Carathéodory's formulation is the concept of entropy, which he identifies as the principal state function in thermodynamics, encapsulating the irreversibility inherent in the processes of the system.

A seminal contribution of Carathéodory lies in the introduction of the fundamental thermodynamic relation, a differential equation that governs the infinitesimal changes in state functions as the system undergoes infinitesimal transitions between states. This equation is of paramount importance as it embodies the first and second laws of thermodynamics in a single, unified mathematical expression. It furnishes a means to calculate variations in energy and entropy within a system and serves as the foundation for deriving numerous other thermodynamic relations. For any given state of the system, there exists an external environment—conceived, for instance, as a heat reservoir or another surrounding system—with which the system may exchange energy. Carathéodory's formulation places particular emphasis on reversible processes, wherein the system can undergo transitions between states without the generation of entropy.

Moreover, the thermodynamic relation must exhibit homogeneity with respect to the state variables, ensuring that the formulation retains its validity across all states of the system. Carathéodory's axiomatization is intricately linked with the language of differential geometry. In this framework, the state of the thermodynamic system is regarded as a point within a manifold, and the transformations of the system are understood as smooth changes between these points. The notion of differential forms is employed to express the manner in which quantities such as energy, entropy, and temperature vary as the system evolves from one state to another. One of the pivotal results of Carathéodory's axiomatization is his theorem, which asserts that, given certain conditions—most notably, the continuity of the system's energy as a function of its state variables—the second law of thermodynamics, as formulated by Clausius, can be derived directly from the fundamental thermodynamic relation. This theorem establishes that entropy must, indeed, be a state function and that it invariably increases in irreversible processes.

While classical thermodynamics was primarily concerned with systems in equilibrium, Carathéodory's axiomatization extends far beyond this, offering a more general framework that accommodates systems in non-equilibrium states. This generalization is of particular significance in the study of irreversible processes and statistical mechanics. It represents one of the major advancements of Carathéodory's approach, as it lays the foundational groundwork for subsequent developments in the study of non-equilibrium thermodynamics.

2.1 Gibbs' influence on Carathéodory

The influence of Josiah Willard Gibbs on Carathéodory's work in thermodynamics is profound, particularly in relation to pivotal conceptual elements, such as the modelling of heterogeneous systems as composed of distinct phases (i.e., as a set of homogeneous systems). It was through this framework that Gibbs conceptualized thermodynamic equilibrium within a geometric representation. It is also with reference to this model (a system as a set of phases) that Carathéodory introduced the quasi-static approximation for the state of equilibrium. Gibbs' geometric model of a heterogeneous system elucidates how Carathéodory's conceptual foundation is intricately intertwined with physical reality. In his 1878 paper, Gibbs introduced the following formulation:

[…] It is an inference naturally suggested by the general increase of entropy which accompanies the changes occurring in any isolated material system that when the entropy of the system has reached a maximum, the system will be in a state of equilibrium. Although this principle has by no means escaped the attention of physicists, its importance does not appear to have been duly appreciated. Little has been done to develop the principle as a foundation for the general theory of thermodynamic equilibrium.

The principle may be formulated as follows, constituting a criterion of equilibrium:

I. For the equilibrium of any isolated system it is necessary and sufficient that in all possible variations of the state of the system which do not alter its energy, the variation of its entropy shall either vanish or be negative.

The following form, which is easily shown to be equivalent to the preceding, is often more convenient in application:

II. For the equilibrium of any isolated system it is necessary and sufficient that in all possible variations of the stale of the system which do not alter its entropy, the variation of its energy shall either vanish or be positive. *(Gibbs 1906, p.354).*

Josiah Willard Gibbs made foundational contributions to the field of thermodynamics, particularly through his employment of geometric models to represent thermodynamic systems. In his works «A Method of Geometrical Representation of the Thermodynamic Properties of Substances by Means of Surfaces» (1873) and «Graphical Methods in the Thermodynamics of Fluids,» Gibbs demonstrated how geometric surfaces could be employed to comprehend thermodynamic properties. His most significant contribution in this domain is his development of the thermodynamic surface—a graphical representation of the fundamental thermodynamic relationships. Gibbs introduced a three-dimensional geometric model to visualize thermodynamic properties in a space representing the various equilibrium states of a substance: entropy (S), volume (V), and energy (U). This surface facilitated the direct visualization of phase transitions and equilibrium conditions. His geometric model proved invaluable in explaining phase stability and the coexistence of phases via surfaces, and how Maxwell's construction could be applied to determine phase equilibria through the use of tangent planes to the thermodynamic surface. Gibbs' graphical approach influenced subsequent developments in thermodynamics and statistical mechanics. Although graphical methods were eventually supplanted by algebraic formulations, his insights laid the foundation for the concepts of Gibbs free energy and the phase rule. His graphical methods, still widely utilized today, were devised to provide a deeper understanding of phase transitions and equilibrium states.

Josiah Willard Gibbs also formulated the theory associated with the thermodynamic state through the development of his graphical method, as expounded in his two seminal publications (Gibbs, 1873a; Gibbs, 1873b). In the first of these works, as indicated by its title, Gibbs introduced a general graphical method that provided profound insights into the relationships between thermodynamic properties, as governed by the first and second laws of thermodynamics. In the opening paragraph of this work, Gibbs explicates:

[…] Although geometrical representations of propositions in the thermodynamics of fluids are in general use, and have done good service in disseminating clear notions in this science, yet they have by no means received the extension in respect to variety and generality of which they are capable. So far as regards a general graphical method, which can exhibit at once all the thermodynamic properties of a fluid concerned in reversible processes, and serve alike for the demonstration of general theorems and the numerical solution of particular problems, it is the general if not the universal practice to use diagrams in which the rectilinear co-ordinates represent volume and pressure. The object of this article is to call attention to certain diagrams of different construction, which afford graphical methods coextensive in their applications with that in ordinary use, and preferable to it in many cases in respect of distinctness or of convenience. (Gibbs 1873, p.3).

The concept of thermodynamic potentials, as employed by Josiah Willard Gibbs, can be traced to the work of François Massieu, a distinguished French physicist. In 1869, Massieu introduced a set of potentials, including the characteristic function and entropy, which he defined as functions capable of describing the equilibrium state of a system in terms of various variables, such as pressure, temperature, and volume. Massieu's significant insight lay

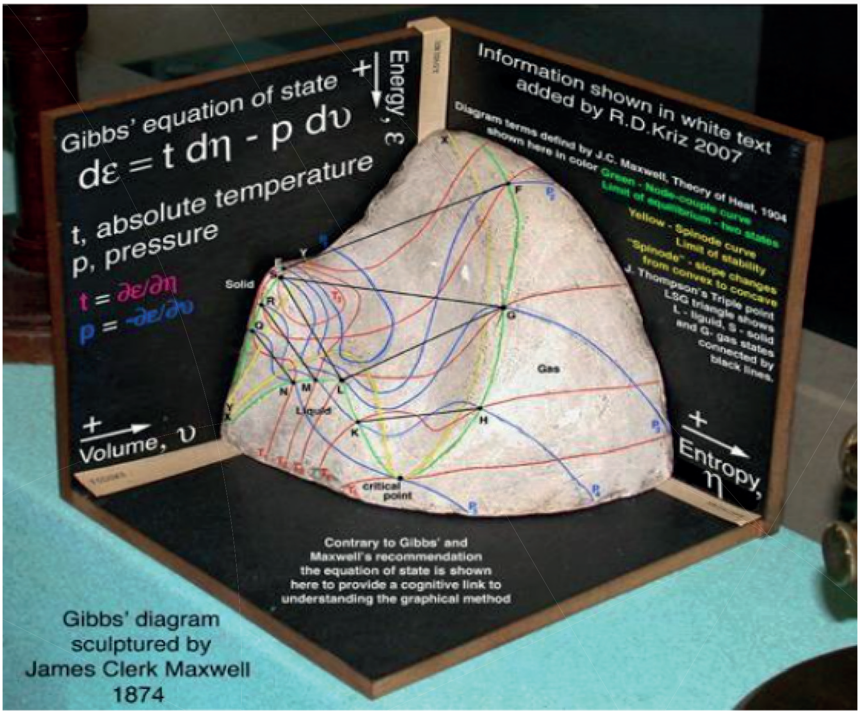

Fig. 5: Gibbs' geometric model of thermodynamics provided a visual and conceptual framework for understanding complex thermodynamic relationships. While modern thermodynamics primarily uses mathematical formulations, his graphical methods were pioneering in the 19th century and influenced the development of statistical mechanics and equilibrium thermodynamics. Source Gallica.

in his recognition that thermodynamic quantities, such as the characteristic function and entropy, could be expressed through potentials, thereby facilitating the study and analysis of thermodynamic systems. His pioneering work laid the essential foundation for subsequent advancements in the field and notably influenced Henri Poincaré, who, upon discovering Massieu's work, introduced the characteristic function in the context of probability theory. Poincaré's lectures at the Sorbonne, during which he encountered Massieu's contributions, further expanded the scope of these ideas.

Josiah Willard Gibbs subsequently developed and popularized the concept of thermodynamic potentials in the 1870s and 1880s, particularly through his seminal contributions to statistical mechanics. In his 1873 paper, «On the Equilibrium of Heterogeneous Substances,» Gibbs introduced the Gibbs free energy and established the fundamental principles for the application of potentials within thermodynamics. He demonstrated how these potentials were connected to the first and second laws of thermodynamics, creating a robust mathematical framework that not only advanced the understanding of equilibrium thermodynamics but also illuminated the conditions necessary for spontaneous processes and chemical reactions. Moreover, Gibbs introduced the concept of thermodynamic equilibrium and made pivotal contributions to the phase rule and chemical potentials, enabling the study of multi-phase, multi-component systems.

It is also essential to acknowledge the contributions of Pierre Duhem, a French physicist and philosopher, whose work, while less focused on the formalism of thermodynamic potentials compared to Gibbs, provided a valuable conceptual framework for their use. Duhem advanced the idea of duality between the Massieu characteristic function and entropy, promoting the understanding that different thermodynamic potentials are merely alternative descriptions of the same thermodynamic behaviour under varying constraints. His work expanded the scope of thermodynamics beyond the confines of purely mechanical systems, paving the way for its application in fields such as chemistry and engineering.

In the following figure, we give an Autograph letter from François Massieu (Corps des Mines) to Josiah Willard Gibbs, dated Rennes, October 9, 1878 (Gibbs Archive of Yale University):

[…] You have been kind enough to send me four brochures that you have published on thermodynamics; I would like to thank you as well as for the honor you have done me by citing my work on the characteristic functions of fluids. I fear that you have had only an incomplete idea of this work for the reports of the Institut de France to which you refer and I am sending you a copy of what I have written on this subject. I would be honored if you would be kind enough to bring some interest to it. I am writing to you in French for the reason that if I read English quite easily, I would write it very badly. I would be very happy, Sir, if our relations could continue and if I publish, as is probable, some new work on thermodynamics, which I will hasten to send you a copy. Please accept, Sir, the assurance of my most distinguished sentiments – MASSIEU Chief Engineer of Mines, Professor at the Faculty of Sciences of Rennes to Professor Willard Gibbs, at Yale College, New Haven, Connecticut, United States of America. (Gibbs Archive of Yale University)

As noted by Roger Balian (Balian, 2017) of the French Academy of Sciences, the concept of thermodynamic potential was initially introduced in the papers of François Massieu (Massieu, 1869a, 1869b) and subsequently expanded upon by J.W. Gibbs:

[…] Massieu's pioneering contribution had little impact, maybe because his incentive looked technical (although he wrote in a scientific style and accounted for the

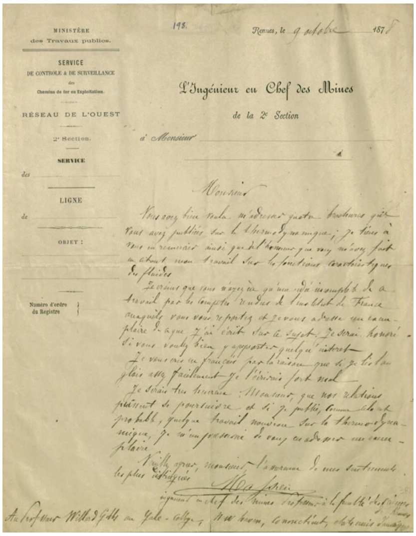
MINISTÈRE des Travaux publics.
SERVICE DE CONTRÔLE & DE SURVEILLANCE des Chemins de fer en Exploitation.
RÉSEAU DE L'OUEST
2e Section.
Rennes, le 9 octobre 1878
L'Ingénieur en Chef des Mines de la 2e Section
à Monsieur

Fig. 6: Autograph letter from François Massieu (Corps des Mines) to Josiah Willard Gibbs, dated Rennes, October 9, 1878 (Gibbs Archive of Yale University).

most recent advances such as Clausius's entropy). Even presently, most textbooks still ignore him, crediting Gibbs for the invention of thermodynamic potentials. In fact, as it is well known, Gibbs introduced in 1876, under the name of "fundamental function" what we now call the "free enthalpy" G(T,p,{Ni})= U – T S + pV for a fluid made of {Ni} molecules of different species. However, Gibbs himself had clearly written in a footnote: "Massieu appears to have been the first to solve the problem of representing all the properties of a body of invariable composition which are concerned in reversible processes by means of a single function." In fact, Gibbs's function can be regarded as an extension of Massieu's second characteristic function to a mixture that may undergo chemical reactions. Likewise, the "free energy" F(T,p,{Ni}) = U – T S introduced in 1882 by Helmholtz appears as an extension to mixtures of Massieu's first characteristic function. Duhem, who proposed to term all these functions "thermodynamic potentials", properly attributes their idea to Massieu, and their introduction in thermochemistry to Gibbs. The same credits are given by Poincaré, who as Duhem presents Massieu's functions in their modified form H and H' of 1876. However, Planck's potential is nothing but the original form ψ' of Massieu's second characteristic function; Planck, as many others, seems to have been unaware of Massieu's work... The fact that Massieu's original functions ψ and ψ' of 1869 should be regarded as the most natural thermodynamic potentials, either as Legendre transforms of the entropy function or as logarithms of partition functions, is slowly getting recognition. Massieu's name, which was not yet mentioned in Gillispie's dictionary, now appears in Wikipedia. (Balian 2017, p.4)

In Chapter IV of his seminal work Elementary Principles in Statistical Mechanics, developed with Especial Reference to the Rational Foundation of Thermodynamics (Gibbs, 1902), Josiah Willard Gibbs explores a generalisation of Gibbs states, constructed using the moment map of the product of the one-dimensional group of translations in time and the three-dimensional group of rotations in space. This generalisation is employed to study systems contained within a rotating vessel, referencing a paper by James Clerk Maxwell published in 1878. Gibbs states:


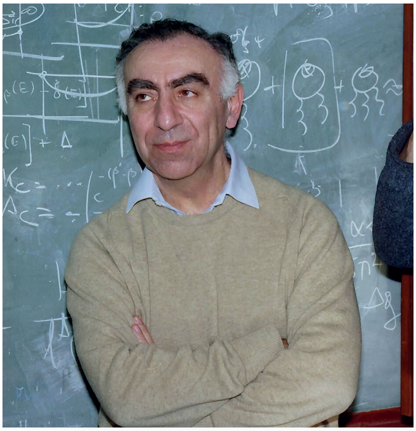

Fig. 7: François Massieu & Roger Balian. Source Gallica and CEA IPhT.

[...] The consideration of the above case of statistical equilibrium may be made the foundation of the theory of the thermodynamic equilibrium of rotating bodies, a subject which has been treated by Maxwell in his memoir On Boltzmann's theorem on the average distribution of energy in a system of material points (Gibbs 1902, p.44)

Jacques Hadamard made a review of this book in 1906 (Hadamard 1906) and wrote:

[...] This book is not one of those that one analyzes hastily; but, on the other hand, the questions it deals with have been greatly agitated in recent times; the ideas defended by Gibbs have been the subject of much controversy; the reasoning with which he supported them has also been criticized. It seems interesting to me to study his work in the light of these controversies and by discussing these criticisms (Hadamard 1906, p. 194)

The generalisation of Gibbs states, which incorporates the moment map of the product of the one-dimensional group of time translations and the three-dimensional group of spatial rotations for the analysis of systems within a rotating vessel, was later developed by Jean-Marie Souriau in 1969. Souriau systematically studied thermodynamics within the context of dynamical systems governed by the action of a Lie group, as part of his application of Symplectic Geometry to Statistical Mechanics.

The physical phenomenon of symmetries, as well as their physical and mathematical conceptualisation, permeates the history of science, a notion extensively elaborated upon by Raffaele Pisano (Pisano, 2024). Herbert Bernard Callen (Callen, 1973, 1974, 1985), a pivotal figure in the development of modern irreversible thermodynamics, is renowned for his influential work Thermodynamics and an Introduction to Thermostatistics (1960). In 1973 and 1974, Callen further advanced the field through the publication of a book chapter, A Symmetry Interpretation of Thermodynamics, and a paper entitled Thermodynamics as a Science of Symmetry, wherein he proposed a novel interpretation of thermodynamics through the symmetry properties of physical laws, mediated by the statistical behaviour of large systems. Callen's work elucidates the inherent symmetries within the fundamental laws of physics, which impose constraints and regularities on the potential properties of matter in thermodynamics. In his discussions, Callen references Noether's theorem.

[...] every continuous symmetry of a system implies a conservation theorem, and vice versa ... The most primitive class of symmetries is the class of continuous spacetime transformations. The (presumed) invariance of physical laws under time translation implies the conservation of energy. Symmetry under spatial translation implies conservation of momentum, and rotational symmetry implies conservation of angular momentum.(Callen, p.425)

Herbert Bernard Callen also referred to dynamical symmetries, particularly in relation to the gauge transformations of the electromagnetic equations, where the scalar and vector potentials undergo transformations while maintaining the invariance of the electric charge. Other symmetries, such as the conservation of baryon and lepton numbers or the invariance of strangeness and isospin in strong interactions, do not arise in conventional thermodynamic systems. The final category of symmetry pertains to the notion of broken symmetry. Concerning the coordinates of thermodynamics, and unaware of Jean-Marie Souriau's work, Herbert Bernard Callen expressed ideas that were indirectly related to the preservation of Souriau's moment map, which can be seen as a geometrisation of Noether's Theorem. He remarked:

[...] The most immediately evident conserved coordinate is, of course, the energy (time-translation symmetry). Its relevance as a thermodynamic coordinate underlies the «first law» of thermodynamics. Time-translation, spatial translation, and spatial rotation symmetries are interrelated in a single class of continuous space-time symmetries. The symmetry interpretation of thermodynamics immediately suggests, then, that energy, linear momentum, and angular momentum should play fully analogous roles in thermodynamics. The equivalence of these roles is rarely evident in conventional treatments, which appear to grant the energy a misleadingly unique status. The momentum and the angular momentum are generally suppressed by restricting the theory to systems at rest, constrained by external «clamps.» Nevertheless, it is evident that in principle the linear momentum does appear in the formalism in a form fully equivalent to the energy, for relativistic considerations imply that the energy in one frame appears partially as linear momentum in another frame. Similarly, the angular momentum is only occasionally introduced explicitly into thermodynamic formalisms (as in astrophysical applications to rotating galaxies); it appears, for instance, in the «Boltzmann factor,» $\exp(-\beta E - \beta V \cdot T)$, additively and symmetrically with the energy. To stress these facts we might well amend the first law to read that «the extended first law of thermodynamics is the symmetry of the laws of physics under space and time translations and under spatial rotation.».(Callen 1974 p.427)

As Callen demonstrated, the symmetry of thermodynamic coordinates led to an extension of the first law of thermodynamics, not merely in terms of energy conservation, but also extending to the preservation of angular momentum. He then examined the second and third laws of thermodynamics more profoundly, considering their relationship to the emerging theory of second-order phase transitions, as well as Onsager's extension to irreversible processes. Callen observed that the equal a priori probability of states constitutes a symmetry principle, wherein entropy depends symmetrically on all permissible states.

Given the unitarity symmetry of quantum mechanics among microstates, it follows that a uniform probability density in phase space remains uniform under the system's intrinsic dynamics. This principle dictates the functional form of entropy. The conservation of phase space volume under an internal unitary transformation corresponds to the Boltzmann H-theorem. The interchangeability of unitarity and time-reversal symmetries further demonstrates that unitarity is regarded as a symmetry condition. Callen concluded that the central foundation of Onsager's theory rests upon the time-reversal symmetry of physical laws.

2.2 Constantin Carathéodory's origin

Constantin Carathéodory was born in Berlin, Germany, in 1873, the son of a Turkish ambassador of Greek descent. From 1875 to 1895, he resided in Brussels, Belgium, where he attended the Belgian École Militaire. Following a period in Greece, London, and Egypt, he returned to Berlin in 1900 to pursue studies in mathematics. In 1904, he defended his PhD at Göttingen on special Euler–Lagrange equations, under the supervision of H. Minkowski. Carathéodory held successive academic appointments in Göttingen, Bonn, and Hanover.

Fig. 9: Constantin Carathéodory at Belgian Military School. Source Wikimedia.

Between 1905 and 1910, while still in Brussels, Carathéodory corresponded with Max Born in 1907, sharing his definition of the concepts of "amount of heat" and "reversibility," derived from the Carnot principle. He returned to Munich in 1924 to focus on variational calculus in the context of optics.

2.3 Constantin Carathéodory axiomatization of Thermodynamics

Carathéodory's work on thermodynamics began during his student years at the Belgian Military School, where he initiated his manuscript "Untersuchungen über die Grundlagen der Thermodynamik". This manuscript was further developed in July 1908 on the Bosphorus in Turkey and completed in Bonn, Germany, in November 1908. It was subsequently published in Mathematische Annalen in 1909. In this seminal paper, Carathéodory introduced a novel definition of the second law of thermodynamics, describing it in terms of the inaccessibility of certain states by adiabatic paths. Additionally, he established the existence of an entropy function.

In 1917, Carathéodory made another significant contribution to thermodynamics, highly regarded by leading physicists, in which he presented a new axiomatic derivation of thermodynamic theory. This approach did not rely on the assumption of heat as a physical quantity, as had been customary in the mechanical formulations of thermodynamics. Carathéodory's paper was written in response to a question raised by Max Born. Born elaborated on his query as follows:

[…] I tried hard to understand the classical foundations of the two theorems, as given by Clausius and Kelvin; they seemed to me wonderful, like a miracle produced by a magician's wand, but I could not find the logical and mathematical root of these marvellous results. A month later, I visited Carathéodory in Brussels where he was staying with his father, the Turkish ambassador, and told him about my worries. I expressed the conviction that a theorem expressible in mathematical terms, namely the existence of a function of state like entropy, with definite properties, must have a proof using mathematical arguments, which for their part are based on physical assumptions or experiences but clearly distinguished from these. Carathéodory saw my point at once and began to study the question. The result was his brilliant paper, published in Mathematische Annalen, which I consider the best and clearest presentation of thermodynamics. *(Born 1921, p.12)*

Carathéodory published another influential paper in 1925. His Foundations of Thermodynamics is underpinned by the following remark:

[…]if the value of entropy has not remained constant in some change of state, then no adiabatic change of state can be found that could transfer the observed system from its final to its initial state. Every change of state in which the entropy value varies is 'irreversible'. *(Carathéodory 1925, p.11).*

In his 1909 paper On the Foundations of Thermodynamics, Carathéodory laid the groundwork for an axiomatic formulation of thermodynamics. He approached the subject from a rigorous and mathematical standpoint, drawing influence from Gibbs' geometric thermodynamics, while also introducing several important new concepts. In the first section of this paper, spanning pages 357 to 362, Carathéodory develops the key ideas related to the axiomatic foundation of thermodynamics. In this section, he elaborates on the principles developed by Gibbs, particularly the geometric concepts used to describe the relationships between equilibrium states, and introduces an axiomatic formalism for thermodynamics. This novel approach extends beyond classical thermodynamics, offering a more rigorous mathematical foundation for the fundamental laws of thermal physics.

Regarding the axiomatic foundation of thermodynamics, Carathéodory sought to establish a solid axiomatic base for the field, beginning with geometric concepts, particularly those derived from Gibbs' ideas on the equilibrium of heterogeneous systems. His methodology intentionally avoids presupposing specific laws regarding thermodynamic equilibrium states. Instead, it relies on broad, fundamental principles that serve as the foundation for the development of thermodynamics.

[…] However, the general principles from which this description can be attained come to light in their full generality when we, in order to make things more concise, specialize the problem and make the same assumptions as – say – Gibbs did in the first part of his ground-breaking treatise "On the equilibrium of heterogeneous substances." *(Carathéodory 1909, p. 357).*

Carathéodory's mathematical model is developed with reference to an ideal physical system undergoing quasi-static change of state. These pivotal notions are developed in Carathéodory's paper chapter 3 entitled "Einfache Systeme" (Carathéodory 1929, p. 364-369). Carathéodory introduces the concept of thermodynamic coordinates, a pivotal element in his formal treatment of thermodynamics. These coordinates serve to describe the state of a thermodynamic system through independent variables (such as temperature, pressure, and volume), thus enabling the formulation of general laws governing the interrelations among these quantities. One of the fundamental propositions in the opening section is the axiom asserting that variations in energy within a system must be reversible. In essence, Carathéodory demonstrates that infinitesimal changes in a state function, such as internal energy, must be expressible through an exact differential relation. This stipulation ensures that thermodynamic transformations can be reversed in the case of a reversible process.

In his discussion of the differential relations governing internal energy, Carathéodory distinguishes between various thermodynamic variables, particularly internal energy and the parameters that influence it. He defines differential relations that link the internal energy of a system to its thermodynamic coordinates. This contributes significantly to the formulation of a more rigorous and mathematical description of thermodynamics and its underlying principles. Furthermore, Carathéodory delves into the continuity of thermodynamic functions—such as internal energy—and their convexity within the space of thermodynamic coordinates. This framework facilitates the coherent modelling of equilibrium transitions while safeguarding the stability of solutions to the thermodynamic equations.

Consistent with his mathematical approach, Carathéodory underscores the importance of ensuring that thermodynamic coordinates are independent of one another within the phase space of a system. Such independence is crucial, as it allows the distinct thermodynamic variables—temperature, volume, and pressure—to evolve autonomously under the influence of external factors.

Carathéodory also notes that the classical thermodynamic perspective entails an over-determinacy, wherein certain assumptions impose redundant constraints on the system's description.

[…] One can derive the entire theory without assuming the existence of a physical quantity that deviates from ordinary mechanical quantities, namely, heat.*(Carathéodory 1925, p.13).*

He then proposes to define an axiomatization of two first thermodynamics laws:

First law:

[…] Then the axiom of the first law can be expressed in such a way that it corresponds exactly to Joule's experimental setup if the calorimeter used for this purpose is considered to be its adiabatically isolated system. *(Carathéodory 1929; p. 356).*

Second law:

[…] For the axiom of the second law, I have chosen a definition that is very similar to Planck's; only the latter had to be modified in a suitable way to take into account the fact that heat and quantity of heat are not yet defined in our way of representation. (Carathéodory 1929; p. 356)

The main Axiom of Constantin Carathéodory of the 2nd law of Thermodynamics is written in 1909 paper:

[…] The formulation of the first law that was given just now includes the assumptions thatwere made at the beginning of this work, that neither distant not capillary forces shall be considered. Namely, had, e.g., capillary forces been considered then one would have to say that the sum over the various volume energies of the phases no longer represents the total energy e of S, and that one must add certain terms to this sum that arise from the separating surface between the phases. Moreover, if distant forces between the phases were noteworthy then new terms would likewise enter in that arise from the interaction between the phases, not from one phase alone, and would be dependent upon several of them.

The second law that now comes into question is of a completely different nature:

Namely, one has found that under all adiabatic changes of state that start *from any given initial state certain final states are not attainable and that such "unattainable" final states can be found in any neighborhood of the initial state.*

However, since physical measurements cannot be absolutely precise this fact of experience includes more than the mathematical content of the aforementioned law, and we must demand that when a point is excluded, the same shall also be true of a small region around this point whose size depends upon the precision of the measurement.

However, in order for us to give this precision no weight it is convenient to give the axiom in question a somewhat more general form, and indeed in the following way:

Axiom II: In any arbitrary neighborhood of an arbitrarily given initial point there is a state that cannot be arbitrarily approximated by adiabatic changes of state. (Carathéodory 1909, p. 363).

Carathéodory's mathematical model is developed with reference to an ideal physical system undergoing quasi-static change of state. These pivotal notions are developed in Carathéodory's paper chapter 3 entitled "Einfache Systeme" *(Carathéodory 1929, p. 364-369).*

3. Modern Epistemology of Sadi Carnot's thermodynamics by Poincaré and Serres

Finally, in this chapter, we engage with the epistemological questions raised by the field of thermodynamics and present elements of Michel Serres' philosophy, which advocates a unified theory of heat and information. First epistemological question on thermodynamics was developed by Henri Poincaré, during his lecture series on Thermodynamics at the Sorbonne, who remarked upon the inherent impossibility of developing a mechanical theory of heat:

[…] All attempts of this nature must therefore be abandoned; the only ones which have any chance of success are those which are based on the intervention of statistical laws such as, for example, the kinetic theory of gases". *(Poincaré 1908, p. 450).*

In a manner reflecting his characteristic «peasant common sense,» Poincaré illustrated the Second Law of Thermodynamics through the allegory of a grain of oats and a pile of wheat, presented in the concluding remarks of his course:

[…] Suppose we want to place a grain of oats in the middle of a pile of wheat; it will be easy; suppose that we then want to find it there and remove it; we will not be able to achieve this. All irreversible phenomena, according to certain physicists, are built on this model [Supposons que nous voulions placer un grain d'avoine au milieu d'un tas de blé ; cela sera facile; supposons que nous voulions ensuite l'y retrouver et l'en retirer; nous ne pourrons y parvenir. Tous les phénomènes irréversibles, d'après certains physiciens, seraient construits sur ce modèle]. *(Poincaré 1908, p. 450).*

Michel Serres, in his exploration of the epistemology of thermodynamics and information theory, delves into the interrelationship between order and disorder, drawing notable parallels between the two disciplines. He

Fig. 14: Henri Poincaré's fable of the grain of oats and the pile of wheat. Source Gallica.

conceived both thermodynamics and information theory as systems concerned with entropy, though in distinct contexts: the former pertains to physical systems, while the latter concerns communication processes. Serres recognized entropy as a measure of disorder in both realms, drawing a connection between physical decay and the degradation of information. He underscored the dynamic interplay between chaos and order, where systems evolve by transforming disorder into order. Furthermore, Serres highlighted the resonance between information theory's focus on signal and noise and the principles of thermodynamics, suggesting that communication processes can be understood as navigating the tension between chaos and clarity. His interdisciplinary approach seamlessly integrates scientific and philosophical concepts to offer a comprehensive understanding of complex systems.

In the 2024 edition of Michel Serres' Cahiers de formation, we find evidence of his intellectual engagement with Léon Brillouin's The Science and Theory of Information (Bertrand, 2023). Serres asserts that "the greatest discovery in history is the connection between entropy and information, a revelation that spans both epistemology and the theory of matter." For Serres, Prometheus—the figure who sought fire from the gods—embodies the patron of fire's arts, first as blacksmiths, and later as the creators of steam engines. The world of Atlas, cold, silent, and pristine, gave way to the world of Prometheus with Sadi Carnot, whose theories of heat and work reshaped our understanding of the physical universe. Serres poignantly contrasts this transformation in his work Turner Translates Carnot (Serres, 1974, p. 212), wherein he writes:

[…] From Garrard to Turner, the path is very simple is the path that runs from Lagrange to Carnot, from simple machines to steam engines, from mechanics to thermodynamics, by the way of industrial Revolution. What is the industrial revolution? A revolution in matter. It happens at the very sources of dynamics. The origins of strength. Force, we take it as it is or we produce it […]. Turner sees matter being transformed by fire. The new material of the world at work, where geometry is short. Everything is reversed, the material, the painting triumphing over the drawing, the geometry, the form. No, Turner is not a pre-impressionist. He is a realist, properly a materialist. He shows the material of 1844 […] and he is the first to see it, the first absolutely. No one had really perceived it, neither scholar nor philosopher, and Carnot was not read. Who knew her? The fire workers and Turner. Turner or the introduction of igneous matter into culture. The first true genius in thermodynamics *(Serres 1974, p.233).*

In his Book "La traduction", Michel Serres opened thermodynamics to biochemistry and physics of life:

[…] Let us reread the preface where Jean Perrin draws his own genealogy. He comes from two lines: that of thermodynamicists, whom he calls analogists, and who arrived, without hypothesis, at the two principles - observe carefully that the first, that of equivalence, is a principle of invariance, that the second, that of the increase in entropy, is a principle of irreversible evolution –; and that of the atomists, whom he calls the intuitives, and who interpret the previous results. Put the closed box down and look inside. Boltzmann forever links the two paths. It is still on the banks of the divine sea, where the song of the bards had been silent for two thousand years, that the drama ends and begins. The great Boltzmann commits suicide there, he who had once again united the story of Prometheus with the story of Pandora. From this torture our time will be born: atomic physics and molecular biochemistry. *(Serres 1974, p.74).*

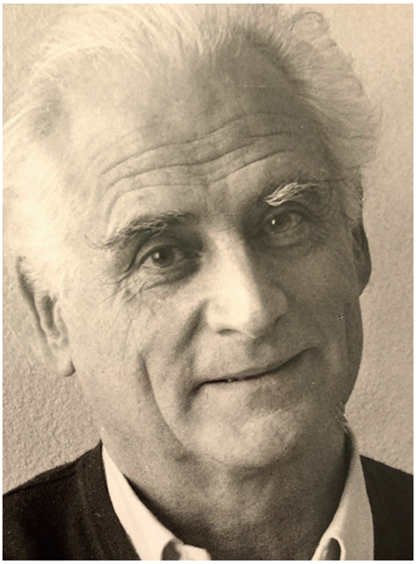

Fig. 15: (top) Michel Serres. Source Wikimedia, (bottom) Painting of "The Fighting Temeraire tugged to her last berth to be broken up, 1838" by Joseph Mallord William Turner, commented by M. Serres in Hermès III. Source Wikimedia

4. From Contantin Carathéodory contact Geometry Model to Jean-Marie Souriau Symplectic Foliation Structures of Thermodynamics and Information Geometry

While Carnot thermodynamics was largely concerned with systems in equilibrium, Carathéodory's axiomatization is far more general. It allows for the description of systems in non-equilibrium states, which is particularly important in the study of irreversible processes and statistical mechanics. This generalization is one of the major advancements of Carathéodory's approach, as it lays the groundwork for later developments in the field of non-equilibrium thermodynamics.

In 1909, Carathéodory advanced a geometric framework for thermodynamics, introducing the concept of a thermodynamic state space. Within this framework, he described the system through a set of thermodynamic variables (such as temperature, pressure, volume, and others), and established a set of axioms that governed the evolution of these variables. A key result of Carathéodory's work lies in his contributions to the theory of partial differential equations and Hamiltonian systems, establishing a direct connection with contact geometry. Specifically, he articulated a fundamental principle concerning differential forms, expressed as follows: a differential form θ satisfies a certain condition $\theta \wedge d\theta \neq 0$ if and only if it cannot be completely integrated. This principle underpins the concept of contact structures, with such a form θ being referred to as a contact form. A manifold equipped with such a form is termed a contact manifold, which can be considered as a kind of «boundary» of a symplectic manifold. In classical mechanics, symplectic geometry is employed to describe the phase space—comprising positions and momenta—of conservative systems. Contact geometry emerges when an additional dimension, associated with time or energy, is introduced, capturing dissipative effects or energy constraints. Starting from a symplectic manifold, one may restrict the dynamics to a submanifold of codimension one, thereby obtaining a contact structure. Conversely, a contact manifold can be viewed as a structure derived from a higher-dimensional symplectic manifold.

In 1969, Jean-Marie Souriau introduced a symplectic model of thermodynamics, known as «Lie Groups Thermodynamics.» In both Carathéodory's and Souriau's frameworks, thermodynamic potentials assume a central role. Carathéodory introduced these potentials within the context of differential forms, while Souriau connected them to the symmetries of the system, as encoded in Lie groups. In Souriau's framework, thermodynamic potentials are generated by the action of a symmetry group on the system: entropy is viewed as an invariant Casimir function on the leaves of the symplectic foliation, which is generated by the coadjoint orbit of the symmetry group. Souriau formalized the notion that the dynamics of a physical system can be described geometrically by a symplectic manifold, interpreting the trajectories of Hamiltonian systems as curves within a symplectic manifold. He precisely formulated the manner in which symmetry groups act on phase spaces, which are themselves symplectic manifolds, and elucidated the connection between these symmetries and the invariants of the system. Souriau further explored the relationship between symplectic geometry and the coadjoint orbits of a Lie group acting on a manifold, particularly in mechanical systems where symmetries are often described by Lie groups (e.g., the Galileo group). He also recognized that a contact structure, which generalizes the odddimensional symplectic structure, is capable of describing dissipative or non-reversible dynamics. This insight linked contact geometry to broader classes of dynamical systems, thereby enriching the understanding of symplectic geometry, which traditionally applies to reversible systems.

In contemporary research, Souriau's symplectic model of statistical mechanics has largely supplanted Carathéodory's contact geometry model for applications in the emerging field of «Statistical Learning on Lie Groups.» This field extends statistics and machine learning to Lie groups, drawing upon the theory of representations and cohomology of Lie algebras. Building upon Jean-Marie Souriau's work in «Lie Groups Thermodynamics,» new geometric statistical tools have been developed to define probability densities (such as the Gibbs density of maximum entropy) on Lie groups or homogeneous manifolds for supervised methods. Additionally, the extension of the Fisher metric from information geometry to unsupervised methods in metric spaces has gained prominence. The symplectic model of thermodynamics is also applied in the context of Thermodynamics-Informed Neural Networks (TINNs), which augment AI-driven engineering applications.
The geometric structures of TINNs are studied through their metriplectic flow (also known as GENERIC flow), which models both non-dissipative dynamics (governed by the first thermodynamic law of energy conservation) and dissipative dynamics (characterized by the second thermodynamic law of entropy production). The thermodynamics of Lie groups, as conceptualized by Souriau, facilitates the geometric characterization of the metriplectic flow by employing a structure of «webs» composed of symplectic foliations and transversely Riemannian foliations. From the symmetries of the system, the coadjoint orbits of the Lie group, through the moment map (a geometrization of Noether's theorem), generate the symplectic foliation, defined as the level sets of entropy, where entropy functions as an invariant Casimir function on these symplectic leaves. The metric on these symplectic leaves is provided by the Fisher metric, while the dynamics along the leaves, governed by the Poisson bracket, characterizes the non-dissipative component of the system's evolution. The dissipative dynamics are described by the transverse Poisson structure and a metric flow bracket, with the evolution from one leaf to another constrained by entropy production. The transverse foliation, which is a Riemannian foliation, is endowed with a metric defined by the dual of the Fisher metric (the Hessian of entropy).

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About the signage image of GSI: «Woman teaching geometry»

Detail of a scene within the letter "P" of an illumination showing a woman holding a set square and a compass. She is being observed by a group of students. In the Middle Ages, it was unusual to see women depicted as teachers, especially when the students appear to be monks. She is more likely the personification of Geometry, based on the famous book by Martianus Capella, De Nuptiis Philologiae et Mercurii (5th century), a standard source for allegorical images of the seven liberal arts. Illustration at the beginning of Euclid's Elements, in the translation attributed to Adelard of Bath.

Organization of the Sessions

SESSIONS OCTOBER 29TH

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
09:00	«Conference Opening Session Frédéric Barbaresco & Frank Nielsen (GSI'25 General Chairs)»		
09:30	«Topological Deep Learning: Unlocking the Structure of Relational Systems Nina Miolane (UC Santa Barbara, USA)»		
10:30	Coffee Break + GSI'25 Posters Session		
11:00–12:20	**«Geometric Statistics (Session 1) Chair: Xavier Pennec, Stefan Sommer, Benjamin Eltzner»**	**«A geometric approach to differential equations (Session 1) Chair: Javier de Lucas Araujo»**	**«Lie Group in Learning Distributions & in Filters (Session 1) Chair: Eren M. Kiral, Koichi Tojo, Ha Q. Minh»**
11:00	«Ridge Regression for Manifold-valued Time-Series with Application to Hurricane Forecasting (7) Esfandiar Nava-Yazdani»	«Reduction of hybrid Hamiltonian systems with non-equivariant momentum maps (18) Asier López-Gordón»	«F-t Joint Distribution on a Real Siegel Domain and Simultaneous Hypothesis Test (61) Hiroto Inoue»
11:20	«On the approximation of the Riemannian barycenter (63) Simon Matagne»	«New Lie systems from Goursat distributions: reductions and reconstructions (121) Oscar Carballal»	«Note on harmonic exponential families on homogeneous spaces (77) Koichi Tojo»
11:40	«Accelerated Stein Variational Gradient Flow (33) Viktor Stein»	«Symplectic approach to global stability (124) Jordi Gaset Rifà»	«The Fisher metric and the Amari–Chentsov tensor of the family of Poincaré distributions (76) Koichi Tojo»
12:00	«Geodesic Non-completeness of the Truncated Normal Family (68) Baalu Ketema»	«Reduction of exact symplectic manifolds and energy hypersurfaces (145) Bartosz Zawora»	«Fast equivariant k-means on SPD matrices (90) Gabriel Trindade»
12:20–12:40	Lunch break + GSI'25 Posters Session		
14:00	«The Lp Fisher-Rao metrics and the alpha-connections Alice Le Brigant (Université Paris 1 Panthéon-Sorbonne, France)»		
15:00–16:00	**«Geometric Statistics (Session 2) Chair: Xavier Pennec, Stefan Sommer, Benjamin Eltzner»**	**«A geometric approach to differential equations (Session 2) Chair: Bartosz Zawora»**	**«Lie Group in Learning Distributions & in Filters (Session 2) Chair: Eren M. Kiral, Koichi Tojo, Ha Q. Minh»**

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
15:00	«Intrinsic LDA for 3D Shape Classification via Parallel Transport (94) Maria Victoria Ibáñez-Gual»	«Novel pathways in SkS-contact geometry (142) Tomasz Sobczak, Tymon Frelik»	«A new geometric regression with inputs-outputs on matrix Lie groups (88) Serigne Daouda Pene»
15:20	«Eigengap Sparsity for Covariance Parsimony (99) Tom Zwagjer»	«A relation between SkS-symplectic and SkS-contact Hamiltonian systems (129) Silvia Vilariño»	«Equivariant Filter: navigation on the rotating round-earth model using a left-error state (89) Alexandre Cellier-Devaux»
15:40	«RNA Structure Correction – the Importance of Small Clusters (138) Benjamin Eltzner»	«Applications of standard and Hamiltonian stochastic Lie systems (139) Javier de Lucas Araujo»	«Sequential parallel Metropolis-Adjusted Langevin Algorithm on Matrix Lie Groups (131) Enzo Lopez»
16:00–16:30	**Coffee Break + GSI'25 Posters Session**		
16:30–17:50	**«Neurogeometry Chair: Alessandro Sarti, Giovanna Citti, Giovanni Petri»**	**«New trends in Nonholonomic Systems Chair: Manuel de Leon, Leonardo Colombo»**	**«Learning of Dynamic Processes Chair: Stéphane Chretien»**
16:30	«Geometric neural fields for cortical activity (36) Emre Baspin»	«Virtual nonlinear nonholonomic constraints from a symplectic point of view (43) Alexandre Simoes»	«Memory capacity of nonlinear recurrent networks: is it informative? (128) Giovanni Ballarin»
16:50	«Log-Euclidean Frameworks for Smooth Brain Connectivity Trajectories (48) Olivier Bisson»	«Geometric Stabilization of Virtual Nonlinear Nonholonomic Constraints (108) Efstathios Stratoglou»	«Lie-Adaptive Inversion of Signature via Pfeffer–Siegul–Sturmfels Algorithm (140) Vailati Giovanni»
17:10	«A heterogeneous model of boundary and figure completion in V1 (91) Mattia Baspinar»	«Trajectory generation for nonholonomic control systems using reconstruction techniques on SE(2) (130) Nicola Sansonetto»	«Hypergraphs on high dimensional time series set using signature transform (141) Rémi Vaucher»
17:30	«Geometry of Cells Sensible to Curvature and Their Receptive Profiles (97) Vasiliki Liontou»	«Homogeneous bi-Hamiltonian structures and integrable contact systems (4) Asier López-Gordón»	«Using Signatures and Koopman operators to learn non-linear dynamics (146) Stéphane Chretien»
17:50	«Orientation Scores should be a Piece of Cake (60) Finn Sherry»	«Deep Dirac Neural Networks for Holonomic Mechanical Systems (80) Kenshin Okuyaki»	«A kernel-based global method for the learning of elastic potentials on Lie groups (152) Jianyu Hu»
18:45	**Conference Group Photo**		
19:00	**Cocktail**		

SESSIONS OCTOBER 30TH

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
09:00	**Metriplectic Dynamics: A Geometrical Framework for Thermodynamically Consistent Dynamical Systems - Philip J. Morrison (The University of Texas at Austin, USA)**		
10:00 - 11:00	**"Geometric Methods in Thermodynamics (Session 1) Chair: François GAY-BALMAZ, Hiroaki YOSHIMURA"**	**"Stochastic Geometric Dynamics Chair: Ana Bela CRUZEIRO, Jean-Claude ZAMBRINI, Stefania UGOLINI"**	**"Optimization and learning on manifolds (Session 1) Chair: Cyrus MOSTAJERAN, Salem SAID"**
10:00	Thermodynamic Functionality of Non-Detailed Balance Finite-Tape Information Ratchet (26) - Lock Yue Chew	Stochastic perturbation of geodesics on the manifold of Riemannian metrics (81) - Ali Suri	Geometric design of the tangent term in landing algorithms for orthogonality constraints (101) - Florentin Goyens
10:20	Entropy functionals and equilibrium states in mixed quantum-classical dynamics (32) - Cesare Tronci	"10:20 - Stochastic Maupertuis's principles and Jacobi's integration theorem (79) Qiao Huang"	Role of Riemannian geometry in double-bracket quantum imaginary-time evolution (50) - Marek Gluza
10:40	Variational approach to the stochastic thermodynamics of Langevin systems (53) - Héctor Vaquero del Pino	Lagrangian averaging of singular stochastic actions for fluid dynamics (49) - Ruiao Hu	Numerical techniques for geodesic approximation in Riemannian shape optimization (55) - Kathrin Welker
11:00	Variational Principle for Stochastic Nonholonomic Systems Part I: Continuous-Time Formulation (57) - Tianzhi Li	Continuous-time filtering in Lie groups: estimation via the Fréchet mean of solutions to stochastic differential equations (95) - Magalie Bénéfice	Geometric Gaussian Approximations of Probability Distributions (93) - Nathaël Da Costa
11:20	**Coffee Break + GS1'25 Posters Session (Chair: Rita FIORESI)**		
	"Geometric Methods in Thermodynamics (Session 2) Chair: François GAY-BALMAZ, Hiroaki YOSHIMURA"	**"Classical & Quantum Information, Geometry and Topology Chair: Florio M. CIAGLIA, FABIO DI COSMO, Pierre BAUDOT and Grégoire SERGEANT-PERTHUIS"**	**"Optimization and learning on manifolds (Session 2) Chair: Cyrus MOSTAJERAN, Salem SAID"**
11:50	Variational Principle for Stochastic Nonholonomic Systems Part II: Stochastic Nonholonomic Integrator (83) - Tianzhi Li	The category of non-commutative probabilities in Information Geometry (105) - Laura Gonzalez Bravo	A probabilistic view on Riemannian machine learning models for SPD matrices (9) - Thibault de Surrel
12:10	Interconnection and variational principles for fluid-bubble dynamics (78) - François Gay-Balmaz	Independent States are Orthogonal: a Categorical Framework to Treat Probability Geometrically (118) - Paolo Perrone	Efficiency of the Generalized Method of Moments from the Viewpoint of Differential Geometry (14) - Hitoshi Tanaka
12:30	Hamilton-Dirac formulation for thermodynamic systems with reaction and diffusion (84) - Hiroaki Yoshimura	Two-typed Tangent Vectors in Quantum Statistical Mechanics (31) - Jan Naudts	p-Laplacians for Manifold-valued Hypergraphs (110) - Jo Stokke

Thursday, October 30th

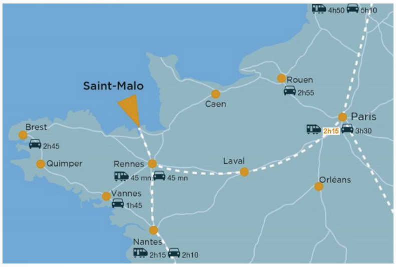
Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
12:50	On a generalisation of metriplectic systems (92) - Jonas Kirchhoff	A Historical Perspective on the Schützenberger-van-Trees Inequality: A Posterior Uncertainty Principle (109) - Olivier Rioul	Universal kernels via harmonic analysis on Riemannian symmetric spaces (59) - Cyrus Mostajeran
13:10 - 13:10		Tree inference with varifold distances (136) Elodie Maignant	
13:30	**Lunch Break + GS1'25 Posters Session (Chair: Rita FIORESI)**		
15:00	**Bicentenary of Thermodynamics and Sadi Carnot's Seminal Work: From Constantin Carathéodory's Contact Geometry Model to Jean-Marie Souriau's Symplectic Foliation Structure - Frédéric BARBARESCO (THALES, France)**		
16:00	**Coffee Break + GS1'25 Posters Session (Chair: Rita FIORESI)**		
16:30 - 18:30	**"Information Geometry, Delzant Toric Manifold & Integrable System Chair: Mathieu MOLITOR, Hajime FUJITA, Daisuke TARAMA and Frédéric BARBARESCO"**	**"Geometric Green Learning on Groups and Quotient Spaces Chair: Alice Barbara TUMPACH, Diarra FALL and Levin MAIER"**	**"Applied Geometry-Informed Machine Learning (Session 1) Chair: Pierre-Yves LAGRAVE, Santiago VALASCO-FORERO and Teodora PETRISOR"**
16:30	Adler-Kostant-Symes Theorem and Algebraic Complete Integrability of Information Geometry and Souriau Lie Groups Thermodynamics (19) - Frédéric Barbaresco	Enhancing CNNs robustness to occlusions with bioinspired filters for border completion (56) - Rita Fioresi	Riemannian Integrated Gradients: A Geometric View of Explainable AI (6) - Lachlan Simpson
16:50	Statistical transformation models of multivariate normal distributions and their -geodesic flows (75) - Daisuke Tarama	A new model for natural groupings in high-dimensional data (71) - Mireille Boutin	A Geometric Deep Learning Approach to Forecast the Time Series of Covariance Matrices (10) - Michele Palma
17:10	Geodesic flow of a statistical manifold associated to Souriau's thermodynamics (86) - Jérémie Pierard de Maujoy	Information Geometry on the T^2-Simplex via the q-Root Transform (85) - Levin Maier	Learning Riemannian Metrics for Interpolating Animations (25) - Sarah Kushner
17:30	Moment polytopes of toric exponential families (40) - Mathieu Molitor	K-P Quantum Neural Networks (87) - Elija Perrier	Conditioning Surface Shape Processes with Neural Operators (143) - Jingchao Zhou
17:50	A compactification of the orthogonal foliation via toric geometry (58) - Hajime Fujita	Infinite-dimensional Siegel disc as symplectic and Kähler quotient (106) - Alice Barbara Tumpach	GNN-Enhanced TCN Algorithms for ECG Signal Quality Recognition (66) - Angelica Simonetti
18:10	Torsion of -connections on the density manifold (35) - Lorenz Schwachhöfer	The hyperkähler marriage between the sphere and the hyperbolic space (107) - Alice Barbara Tumpach	I-WAN2DNS-PM: Weak adversarial networks for solving 2D incompressible Navier-Stokes equations in porous media (51) - Frederic Cadet
19:30	**Conference Group Photo at Maison du Corsaire**		
20:00	**Gala Dinner at Maison du Corsaire**		

SESSIONS OCTOBER 31ST

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
08:30	A Noncommutative perspective of Graph Neural Networks - Rita FIORESI (University of Bologna, Italy)		
09:30 - 11:10	**"Geometric Mechanics (Session 1) Chair: Gery de SAXCE, Zdravko TERZE, François DUBOIS"**	**"Computational Information Geometry and Divergences (session 1) Chair: Frank NIELSEN and Olivier RIOUL"**	**"Statistical Manifolds and Hessian information geometry (Session 1) Chair: Michel NGUIFFO BOYOM, Stéphane PUECHMOREL"**
09:30	Symplectic bipotentials for the dynamics of dissipative systems with non associated constitutive laws (8) - Géry de Saxcé	Geometric Jensen-Shannon divergence between Gaussian measures on Hilbert space (17) - Minh Ha Quang	On Invariant Conjugate Symmetric Statistical Structures on the Space of Zero-Mean Multivariate Normal Distributions (64) - Hikoro Kobayashi
09:50	Debreu's 3-webs and Affinely Flat Bi-Lagrangian Manifolds links with Transverse Symplectic Foliation of Souriau's Dissipative Lie Groups Thermodynamics (117) - Frédéric Barbaresco	Confidence Bands for Multiparameter Persistence Landscapes (67) - Anthea Monod	Bi-forms Approach to Potential Functions in Information Geometry (102) - Marco Pacelli
10:10	Applied Conformal Carroll Geometry (23) - Eric Bergshoeff	Wasserstein KL-divergence for Gaussian distributions (37) - Adwait Datar	A Foliation by Escort Distributions of Exponential Families and Extended Divergences (123) - Keiko Uohashi
10:30	A variational symplectic scheme based on Lobatto's quadrature (24) - François Dubois	f-Divergence Approximation for Gaussian Mixtures (52) - Amit Vishwakarma	Flat f-manifolds on Statistical Manifolds of Hyperboloid type (135) - Guilherme Feitosa de Almeida
10:50	Lifting of some dynamics on the set of bilagrangian structures (29) - Bertuel Tangue Ndaiva	A dimensionality reduction technique based on the Gromov-Wasserstein distance (62) - Rafael Eufrazio	Maximum likelihood estimation for the -exponential family (34) - Ting-Kam Leonard Wong
11:10 - 11:40	Coffee Break + GS1'25 Posters Session		
11:40 - 13:00	**"Geometric Mechanics (Session 2) Chair: Gery de SAXCE, Zdravko TERZE, François DUBOIS"**	**"Computational Information Geometry and Divergences (session 2) Chair: Frank NIELSEN and Olivier RIOUL"**	**"Applied Geometry-Informed Machine Learning (Session 2) Chair: Pierre-Yves LAGRAVE, Santiago VALASCO-FORERO and Teodora PETRISOR"**
11:40	The contact Eden bracket and the evolution of observables (72) - Victor Jiménez Morales	KDS^{2}SM: An unifying framework for feature knowledge distillation (74) - Eduardo Montesuma	Space filling positionality and the Spiroformer (125) - Pablo Suarez-Serato
12:00	A new symmetry group for Physics to revisit the Kaluza-Klein theory (114) - Géry de Saxcé	Curved representational Bregman divergences and their applications (112) - Frank Nielsen	Image Recognition via Vaisman-Neifeld's Geometry (45) - Noémie Combe

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
12:20	Defects in unidimensional structures (122) - Mewen Crespo	Tangent Groupoid and Information Geometry (111) - Jun Zhang	The Stick Model for Distance Geometry (120) - Antonio Mucherino
12:40	A gradient structure for isotropic non-linear morphoelastic bodies (151) Adam Ouzeri	Two types of matching priors for non-regular statistical models (82) - Masaki Yoshioka	Generating random hyperfractad cities (150) - Geoffrey Deperle
13:00	Towards Full 'Galilei General Relativity': Gravitational Kinematics in Bargmann Spacetimes (155) - Christian Cardall	Hyperbolic decomposition of Dirichlet distance for ARMA models (98) - Jaehyung Choi	Shape Theory via the Atiyah–Molino Reconstruction and Deformations 2 (46) - Noémie Combe
13:20 - 14:50	Lunch Break + GS1'25 Posters Session		
14:50	**Extended Variational Learning via Fenchel–Young Losses - Mário A.T. FIGUEIREDO (Universidade de Lisboa, Portugal)**		
15:50	**"Divergences in Statistics and Machine Learning Chair: Michel BRONIATOWSKI and Wolfgang STUMMER"**	**"Statistical Manifolds and Hessian information geometry (Session 2) Chair: Michel NGUIFFO BOYOM, Stéphane PUECHMOREL"**	**"Geometric Learning and Differential Invariants on Homogeneous Spaces Chair: Remco DUITS, Erik BEKKERS"**
15:50 - 17:50	Minimum of Divergences with Relaxation: a Hilbertian Alternative to Duality Approach (20) - Valérie Girardn	Rényi partial orders for BISO channels (133) - Christoph Hirche	Global Positioning on Earth (21) - Mireille Boutin
16:10	Relationship between Hölder Divergence and Functional Density Power Divergence: Intersection and Generalization (47) - Masahiro Kobayashi	The Fisher-Rao distance between finite energy signals (3) - Franck Florin	Analysis and Computation of Geodesic Distances on Homogeneous Spaces (44) - Remco Duits
16:30	Bayesian-like estimation with unnormalized model (126) - Takashi Takenouchi	Statistical models built on sub-exponential random variables (103) - Barbara Trivellato	Universal Collection of Euclidean Invariants between Pairs of Position-Orientations (69) - Gijs Beillaard
16:50	Some smooth divergences for α||β-approximations (42) - Wolfgang Stummer	Production of labelled foliations (154) - Michel Boyom	Roto-Translation Invariant Metrics on Position-Orientation Space (70) - Bart Smets
17:10	A Connection Between Learning to Reject and Bhattacharyya Divergences (39) - Alexander Soen	Coherent States on the Statistical Manifold (100) - Carlos Alcalde	Group Morphology Fixed Points on Homogeneous Spaces for Deep Learning Equivariant Networks (153) - Jesus Angulo
17:30			Flow Matching on Lie Groups (41) - Finn Sherry
17:50	Closing Session (Paper Awards)		

Travel Information to GSI'25



Arriving by plane:

› Landing at Roissy Charles De Gaulle (CDG) airport, you may either:

- Take a TGV train to Rennes and then a connection to Saint-Malo (around 3,5-hour trip – 3 direct trains per day - 8:48 - 12:16 - 18:49)

- Take an Air France bus shuttle to Montparnasse railway station and then take a TGV to Saint-Malo • Rent a car in the airport for a 4.5-hour trip by motorways A11/A13 (through Rennes) or A13/A84(through Caen)

› Landing at Orly airport, you may either:

- Take an Air France bus shuttle to Montparnasse railway station and then take a TGV to Saint-Malo (either direct or via Rennes)

- Rent a car at the airport for a 4.5-hour trip by motorways A11/A13 (via Rennes) or A13/A84 (via Caen)

› Landing at Rennes –Saint Jacques Airport, you may either:

- Take a bus or a taxi to Rennes railway station. Then take a direct train to Saint-Malo.

- Rent a car or take a taxi (*) Saint-Malo is 75 km away, less than 1-hour trip.

(*) Direct shuttles from Rennes Airport may be organized when there are at least 2 passengers

Arriving by train:

› From Paris Montparnasse railway station (town center):

- Direct TGV train to Saint Malo in 2h15 or via Rennes with a connection to Saint Malo (around 2h30 trip)

- To book you ticket on line: https://www.oui.sncf/

› Direct TGV trains to Rennes with connection to St Malo (45 à 60 min trip) from:
- Lille (3h30)
- Roissy (2h30)
- Massy Palaiseau (1h35)
- Strasbourg (4h50)
- Lyon (4h)
- Marseille (6h).

Arriving by boat:

› From Portsmouth (UK):
Direct and daily cruises with Brittany Ferries

Arriving by car:

› From Paris, 3.5 hours by highways:
- A11/A13 (through Rennes)
- A13/A84 (through Caen)

› From Rennes: 1 hour by highways

› From Nantes: 2 hours by highways

Wednesday, October 29th
GSI'25 Opening Day

DAY 1 OF AUTHOR SESSIONS

9:00 - 10:30

OPENING SESSION, **Topological Deep Learning: Unlocking the Structure of Relational Systems - Nina MIOLANE**

GEOMETRIC STATISTICS - XAVIER PENNEC, STEFAN SOMMER, BENJAMIN ELTZNER

› Ridge Regression for Manifold-valued Time-Series with Application to Hurricane Forecasting - Esfandiar Nava-Yazdani

Abstract: We propose a natural intrinsic extension of the ridge regression from Euclidean spaces to general manifolds, which relies on Riemannian least-squares fitting, empirical covariance, and Mahalanobis distance. We utilize it for time-series prediction and apply the approach to forecast hurricane tracks and their intensities (maximum wind speeds).

› On the approximation of the Riemannian barycenter - Simon Mataigne, P.-A. Absil, Nina Miolane

Abstract: We present a method to compute an approximate Riemannian barycenter of a collection of points lying on a Riemannian manifold. Our approach relies on the use of theoretically proven under- and overapproximations of the Riemannian distance function. We compare it to the exact computation of the Riemannian barycenter and to an approach that approximates the Riemannian logarithm using lifting maps. Experiments are conducted on the Stiefel manifold.

› Accelerated Stein Variational Gradient Flow - Viktor Stein, Wuchen Li

Abstract: Stein variational gradient descent (SVGD) is a kernel-based particle method for sampling from a target distribution, e.g., in generative modeling and Bayesian inference. SVGD does not require estimating the gradient of the log-density, which is called score estimation. In practice, SVGD can be slow compared to score-estimation based sampling algorithms. To design fast and efficient high-dimensional sampling algorithms, we introduce ASVGD, an accelerated SVGD, based on an accelerated gradient flow in a metric space of probability densities following Nesterov's method. We then derive a momentum-based discrete-time sampling algorithm, which evolves a set of particles deterministically. To stabilize the particles' momentum update, we also study a Wasserstein metric regularization. For the generalized bilinear kernel and the Gaussian kernel, toy numerical examples with varied target distributions demonstrate the effectiveness of ASVGD compared to SVGD and other popular sampling methods.

› Geodesic Non-completeness of the Truncated Normal Family – Baalu, Nicolas Bousquet, Francesco Costantino

Abstract: Motivated by robustness studies under uncertainty of computer codes that simulate the behavior of a physical system, we are brought to inspect geodesic completeness of parametric families of truncated probability distributions. Specifically, we focus on the parametric family of truncated normal distributions with fixed truncation interval. Endowed with the Fisher information metric, this family can be seen as a Riemannian manifold. We prove that it is not geodesically complete and conjecture a potential candidate for the completion.

› Intrinsic LDA for 3D Shape Classification via Parallel Transport – Maria Victoria Ibáñez-Gual, Jorge Valero-Zorraquino, Amelia Simó

Abstract: In this paper we propose a novel methodology that extends Linear Discriminant Analysis (LDA) to Kendall's shape space to classify 3D shapes and analyze which features most influence class differentiation. Our approach adapts LDA to the non-Euclidean geometry of the shape space, generalizing assumptions about the probability distribution of data in Euclidean spaces and incorporating parallel transport to improve the estimation of shape variability between clusters. A simulation study is performed to show the effectiveness of the proposed methodology.

› Eigengap Sparsity for Covariance Parsimony – Tom Szwagier, Guillaume Olikier, Xavier Pennec

Abstract: Covariance estimation is a central problem in statistics. An important issue is that there are rarely enough samples n to accurately estimate the p (p+1) / 2 coefficients in dimension p. Parsimonious covariance models are therefore preferred, but the discrete nature of model selection makes inference computationally challenging. In this paper, we propose a relaxation of covariance parsimony termed ``eigengap sparsity" and motivated by the good accuracy–parsimony tradeoffs of eigenvalue-equalization in covariance matrices. This penalty can be included in a penalized-likelihood framework that we propose to solve with a projected gradient descent on a monotone cone. The algorithm turns out to resemble an isotonic regression of mutually-attracted sample eigenvalues, drawing an interesting link between covariance parsimony and shrinkage.

› RNA Structure Correction – the Importance of Small Clusters – Stephan Huckemann, Benjamin Eltzner, Kanti Mardia

Abstract: RNA residues come in a plethora of geometric conformational shapes, some of which are very common and some of which are rather rare. In this contribution we extend the previously developed clustering algorithm MINT-AGE in order to find within a large database rare conformational clusters of very small sizes. To this end in the MINT step we replace the previous nonparametric circular mode hunting by a parametric version. In validation, this allows to identify conformational classes of sizes ≥ 2 via statistical unsupervised learning on the gold standard database, hand curated by the Richardson Laboratory.

A GEOMETRIC APPROACH TO DIFFERENTIAL EQUATIONS - JAVIER DE LUCAS ARAUJO, BARTOSZ ZAWORA

› Reduction of hybrid Hamiltonian systems with non-equivariant momentum maps - Leonardo Colombo, María Emma Eyrea Irazú, María Eugenia García, Asier López-Gordón, Marcela Zuccalli

Abstract: We develop a reduction scheme à la Marsden-Weinstein-Meyer for hybrid

Hamiltonian systems. Our method does not require the momentum map to be equivariant, neither to be preserved by the impact map. We illustrate the applicability of our theory with an example.

› New Lie systems from Goursat distributions: reductions and reconstructions - Oscar Carballal

Abstract: We show that types of bracket-generating distributions lead to new classes of Lie systems with compatible geometric structures. Specifically, the n-trailer system is analysed, showing that its associated distribution is related to a Lie system if n = 0 or n = 1. These systems allow symmetry reductions and the reconstruction of solutions of the original system from those of the reduced one. The reconstruction procedure is discussed and indicates potential extensions for studying broader classes of differential equations through Lie systems and new types of superposition rules.

› Symplectic approach to global stability - Verónica Errasti Díez, Jordi Gaset Rifà, Manuel Lainz Valcázar

Abstract: We present a new approach to the problem of proving global stability, based on symplectic geometry and with a focus on systems with several conserved quantities. We also provide a proof of instability for integrable systems whose momentum map is everywhere regular. Our results take root in the recently proposed notion of a confining function and are motivated by ghost-ridden systems, for whom we put forward the first geometric definition.

› Reduction of exact symplectic manifolds and energy hypersurfaces - Julia Lange, Bartosz Zawora

Abstract: This article introduces two reduction schemes for Hamiltonian systems on an exact symplectic manifold admitting Lie group symmetries. It is demonstrated that these reduction procedures are equivalent, by employing a modified Marsden–Meyer–Weinstein reduction theorem for exact symplectic manifolds and contact manifolds given by energy hypersurfaces. Each approach is illustrated through an example.

› Novel pathways in k-contact geometry - Tomasz Sobczak, Tymon Frelik

Abstract: Our study of Goursat distributions originates new types of k-contact distributions and Lie systems with applications. In particular, families of generators for Goursat distributions on \mathbb{R}^4, \mathbb{R}^5 and \mathbb{R}^6 give rise to Lie systems, and we characterise Goursat structures that are k-contact distributions. Our results are used to study zero-trailer and other systems via Lie systems and k-contact manifolds. New ideas for the development of superposition rules via geometric structures and the characterisation of k-contact distributions are given and applied. Some relations of k-contact geometry with Cartan theory are inspected.

› A relation between k-symplectic and k-contact Hamiltonian system - Silvia Vilariño

Abstract: Systems of partial differential equations which appear in classical field theories can be studied geometrically using different geometrical structures, for example, k-symplectic geometry, k-cosymplectic geometry, multisymplectic geometry, etc.

In recent years, there has been a notable increase in the study of k-contact Hamiltonian systems. These are based on the description of the dynamics of field theories using the

so-called k-contact manifolds. Such structures are generalizations of contact structures and k-symplectic structures.
The relation between k-symplectic manifolds and k-contact manifolds was established in \cite{LRS24}. In light of the above relation, this work seeks to explore the relationship between k-symplectic Hamiltonian systems and k-contact Hamiltonian systems. .

› Applications of standard and Hamiltonian stochastic Lie systems - Javier de Lucas Araujo, Marcin Zając

Abstract: A stochastic Lie system on a manifold M is a stochastic differential equation whose dynamics is described by a linear combination with functions depending on \mathbb{R}^\ell-valued semi-martigales of vector fields on M spanning a finite-dimensional Lie algebra. We analyse new examples of stochastic Lie systems and Hamiltonian stochastic Lie systems, and review the coalgebra method for Hamiltonian stochastic Lie systems. We apply the theory to biological and epidemiological models, stochastic oscillators, stochastic Riccati equations, coronavirus models, etc.

LIE GROUP IN LEARNING DISTRIBUTIONS & IN FILTERS – EREN M. KIRAL, KOICHI TOJO, HA Q. MINH

› F-t Joint Distribution on a Real Siegel Domain and Simultaneous Hypothesis Test - Hiroto Inoue

Abstract: This article introduces the F-t distributions on the real Siegel domain associated to the cone of positive definite symmetric matrices.
These distributions arise from a random variable defined in a group theoretical way using the quadratic map to the cone.
Its density function is also provided based on analysis on the real Siegel domains.
As an application, we present a numerical experiment for an invariant simultaneous test for two-sample problem.

› Note on harmonic exponential families on homogeneous spaces - Koichi Tojo, Taro Yoshino

Abstract: In [5], we proposed a method to construct a G-invariant exponential family on a homogeneous space G/H by using a representation of G. In this paper, we prove that for any G-invariant exponential family P on G/H, we can construct a family P_0 by the method such that P ⊂ P_0.

› The Fisher metric and the Amari–Chentsov tensor of the family of Poincaré distributions - Koichi Tojo, Taro Yoshino

Abstract: We give a simple description of the Fisher metric and the Amari–Chentsov tensor of the family of Poincaré distributions on the upper half plane by using a coordinate compatible with SL(2, R)-action.

› Fast equivariant k-means on SPD matrices – Gabriel Trindade, Chevallier Emmanuel, André Nicolet, Frank Nielsen

Abstract: In this paper, we propose an efficient alternative to the affine-invariant Riemannian k-means algorithm on symmetric positive definite matrices. Recently introduced log-extrinsic means are coupled with the Jensen-Bregman log-det divergence, as a replacement for the Riemannian Fréchet mean and the Riemannian distance. Performances and computation times are compared for several frameworks on point clouds sampled from

Riemannian Gaussians. Results show that our algorithm matches the clustering accuracy of the affine-invariant Riemannian k-means, while achieving runtimes comparable to those of log-Euclidean k-means.

› A new geometric regression with inputs-outputs on matrix Lie groups - Serigne Daouda Pene, Samy Labsir, Julien Lesouple, Jean-Yves Tourneret

Abstract: This paper investigates a new Lie group regression model for input-output data belonging to Lie groups. The originality of the model lies in the fact that the unknown weights also lie in Lie groups and are learned using an intrinsic optimization algorithm based on maximum likelihood estimation. The model is validated through numerical simulations conducted using synthetic data belonging to the Lie group SO(3), which is commonly used in robotics to represent rotational observations.

› Equivariant Filter: navigation on the rotating round-earth model using a left-error state - Alexandre Cellier-Devaux, Grimaud Salmon

Abstract: For navigation problems based on Flat Earth equations with inertial sensor biases, the system's equivariance principle on the Semi-Direct Group enables the design of filters that improves estimation error and covariance compared to legacy Extended or Invariant Kalman Filtering. We derived the equivariance principle for comprehensive navigation on a rotating, round Earth by choosing appropriate reference frames and filter architecture to preserve natural symmetries in the system equations. For the filter design, we chose a right-equivariance structure with a left error (in the local reference frame) of the estimation state and compared it to the usual academic choice of right error (in the global reference frame). This approach aims to take advantage of inertial sensor outputs during filter propagation and avoid making the observation matrix dependent on attitude error. In the end, we compare the performances of bias estimation, covariance dynamics, and estimated error accuracy of our L-EqF filter against both EKF ($SO(3)xR^{12}$) and IEKF ($SE_2(3)$ xR^6) in a simulation representative of a GNSS-denied scenario and fast alignment.

› Sequential parallel Metropolis-Adjusted Langevin Algorithm on Matrix Lie Groups - Enzo Lopez (ONERA), Karim Dahia, Nicolas Merlinge, Benedicte Winter-Bonnet, Alain Maschiella, Christian Musso

Abstract: Langevin-based Monte Carlo Markov Chain methods provide a powerful framework for nonlinear state estimation. Using Langevin dynamics for efficient state transitions, these methods offer a robust alternative to traditional nonlinear filtering techniques. However, standard approaches suffer from high computational costs, the curse of dimensionality, and sensitivity to local maxima. To address these challenges, we extend sequential Metropolis Adjusted Langevin Algorithm (MALA) techniques to parallel chains on Lie Groups, leading to the Lie Group parallel Metropolis-Adjusted Langevin Algorithm (LG-pMALA) filter.

NEUROGEOMETRY - ALESSANDRO SARTI, GIOVANNA CITTI, GIOVANNI PETRI

› Geometric neural fields for cortical activity - Emre Baspinar

Abstract: Neural fields refer to integro-differential equations which model the average neural activity of a neural population in a coarse-grained limit. In classical neural fields, which follow Wilson-Cowan-Amari formalism,

the neural interactions are modeled based on a distance-based connectivity, without taking into account the modulatory effects of functional properties of neurons on the connectivity. Such effects are observed in particular in the primary visual cortex (V1). In this work, we consider a neural field which takes into account these effects in the connectivity by focusing on the functional architecture of V1. This model was applied to a specific family of visual illusions, to reproduce the cortical activity generating the illusions. We will explain this model, and discuss its potential to an extension towards pathological cortical activity.

Log-Euclidean Frameworks for Smooth Brain Connectivity Trajectories - Olivier Bisson, Xavier Pennec, Yanis Aeschlimann, Samuel Deslauriers-Gauthier

Abstract: The brain is often studied from a network perspective, where functional activity is assessed using functional Magnetic Resonance Imaging (fMRI) to estimate connectivity between predefined neuronal regions. Functional connectivity can be represented by correlation matrices computed over time, where each matrix captures the Pearson correlation between the mean fMRI signals of different regions within a sliding window. We introduce several Riemannian Log-Euclidean framework for constructing smooth approximations of functional brain connectivity trajectories. Representing dynamic functional connectivity as time series of full-rank correlation matrices, we leverage recent theoretical Log-Euclidean diffeomorphisms to map these trajectories in practice into Euclidean spaces where polynomial interpolation becomes feasible. Pulling back the interpolated curve ensures that each estimated point remains a valid correlation matrix, enabling a smooth, interpretable, and geometrically consistent approximation of the original brain connectivity dynamics. Experiments on fMRI-derived connectivity trajectories demonstrate the geometric consistency and computational efficiency of our approach.

A heterogeneous model of boundary and figure completion in V1 - Mattia Galeotti, Giovanna Citti, Alessandro SARTI I

Abstract: We propose a neurally based model of joint boundary and figure completion, which takes into account the arrangements of simple cells in orientation maps. This map is modeled as a regular surface in a sub-Riemannian structure and the propagation process is studied on the surface. Application to Kanizsa triangle is considered, and results compared with high-resolution fMRI measurements of completion phenomena in V1.

Geometry of Cells Sensible to Curvature and Their Receptive Profiles - Vasiliki Liontou

Abstract: We propose a model of the functional architecture of curvature sensible cells in the visual cortex that associates curvature with scale. The feature space of orientation and position is naturally enhanced via its oriented prolongation, yielding a 4-dimensional manifold endowed with a canonical Engel structure. This structure encodes position, orientation, signed curvature, and scale. We associate an open submanifold of the prolongation with the quasi-regular representation of the similitude group SIM(2), and find left-invariant generators for the Engel structure. Finally, we use the generators of the Engel structure to characterize curvature-sensitive receptive profiles.

Orientation Scores should be a Piece of Cake - Finn Sherry, Chase van de Geijn, Erik Bekkers, Remco Duits

Abstract: We axiomatically derive a family of wavelets for an orientation score, lifting from position space R^2 to position and orientation space R^2 x S^1, with fast reconstruction property, that minimise position-orientation uncertainty.
We subsequently show that these minimum uncertainty states are well-approximated by cake wavelets: for standard parameters, the uncertainty gap of cake wavelets is less than 1.1, and in the limit, we prove the uncertainty gap tends to the minimum of 1. Next, we complete a previous theoretical argument that one does not have to train the lifting layer in (PDE-)G-CNNs, but can instead use cake wavelets.
Finally, we show experimentally that in this way we can reduce the network complexity and improve the neurogeometric interpretability of (PDE-)G-CNNs, with only a slight impact on the model's performance.

NEW TRENDS IN NONHOLONOMIC SYSTEMS – MANUEL DE LEON, LEONARDO COLOMBO

› Virtual nonlinear nonholonomic constraints from a symplectic point of view - Efstratios Stratoglou, Alexandre Simoes, Anthony Bloch, Leonardo Colombo

Abstract: In this paper, we provide a geometric characterization of virtual nonlinear nonholonomic constraints from a symplectic perspective. Under a transversality assumption, there is a unique control law making the trajectories of the associated closed-loop system satisfy the virtual nonlinear nonholonomic constraints. We characterize them in terms of the almost-tangent and a symplectic structure on TQ. In particular, we show that the closed-loop vector field satisfies a geometric equation of Chetaev type. Moreover, the closed-loop dynamics is obtained as the projection of the uncontrolled dynamics to the tangent bundle of the constraint submanifold defined by the virtual constraints.

› Geometric Stabilization of Virtual Nonlinear Nonholonomic Constraints - Efstratios Stratoglou, Alexandre Simoes, Anthony Bloch, Leonardo Colombo

Abstract: In this paper, we address the problem of stabilizing a system around a desired manifold determined by virtual nonlinear nonholonomic constraints. Virtual constraints are relationships imposed on a control system that are rendered invariant through feedback control. Virtual nonholonomic constraints represent a specific class of virtual constraints that depend on the system's velocities in addition to its configurations. We derive a control law under which a mechanical control system achieves exponential convergence to the virtual constraint submanifold, and rendering it control-invariant. The proposed controller's performance is validated through simulation results in an application to the control of an unmanned surface vehicle (USV) navigating a stream.

› Trajectory generation for nonholonomic control systems using reconstruction techniques on SE(2) - Nicola Sansonetto, Marta Zoppello

Abstract: In this note we investigate the trajectory generation problem for nonholonomic mechanical shape- control systems, focusing in the case in which the symmetry group is SE(2), by using techniques from reconstruction theory.

› Homogeneous bi-Hamiltonian structures and integrable contact systems - Leonardo Colombo, Manuel de León, María Emma Eyrea Irazú, Asier López-Gordón

Abstract: Bi-Hamiltonian structures can be utilised to compute a maximal set of functions in involution for certain integrable systems, given by the eigenvalues of the recursion operator relating both Poisson structures. We show that the recursion operator relating two compatible Jacobi structures cannot produce a maximal set of functions in involution. However, as we illustrate with an example, bi-Hamiltonian structures can still be used to obtain a maximal set of functions in involution on a contact manifold, at the cost of symplectisation.

› Deep Dirac Neural Networks for Holonomic Mechanical Systems - Kenshin Okuwaki, Hiroaki Yoshimura

Abstract: We propose a physics-informed machine learning method for mechanical systems using the framework of Dirac dynamical systems. Specifically, we focus on mechanical systems with holonomic constraints. Our approach enables the learning of the generalized energy, which is theoretically derived from a Lagrangian, using both training and target data. Notably, it does not require prior knowledge of constraint forces to separately learn the generalized energy and constraint forces. This is achieved by enforcing a nonenergic condition through a loss function that ensures the constraint forces perform no work. The proposed approach outperforms existing methods by eliminating the need for predefined holonomic constraints as prerequisites for learning. We demonstrate the effectiveness of our method using a double pendulum as an example and conduct a comparative analysis of both approaches.In this paper, we propose a physics-informed machine learning method for mechanical systems using the framework of Dirac dynamical systems. Specifically, we focus on mechanical systems with holonomic constraints. Our approach enables the learning of the generalized energy, which is theoretically derived from a Lagrangian, using both training and target data. Notably, it does not require prior knowledge of constraint forces to separately learn the generalized energy and constraint forces. This is achieved by enforcing a nonenergic condition through a loss function that ensures the constraint forces perform no work. The proposed approach outperforms existing methods by eliminating the need for predefined holonomic constraints as prerequisites for learning. We demonstrate the effectiveness of our method using a double pendulum as an example and conduct a comparative analysis of both approaches.

LEARNING OF DYNAMIC PROCESSES – STÉPHANE CHRETIEN

› Memory capacity of nonlinear recurrent networks: Is it informative? - Giovanni Ballarin, Lyudmila Grigoryeva, Juan-Pablo Ortega

Abstract: The total memory capacity (MC) of linear recurrent neural networks (RNNs) has been proven to be equal to the rank of the corresponding Kalman controllability matrix, and it is almost surely maximal for connectivity and input weight matrices drawn from regular distributions. This fact questions the usefulness of this metric in distinguishing the performance of linear RNNs in the processing of stochastic signals. This note shows that the MC of random nonlinear RNNs yields arbitrary values within established upper and lower bounds depending just on the input process scale. This confirms that the existing definition of MC in linear and nonlinear cases has no practical value.

› Lie-Adaptive Inversion of Signature via Pfeffer-Seigal-Sturmfels Algorithm - Remi Vaucher

Abstract: Since rough path signatures were introduced into machine learning by Terry Lyons, the practical inversion of the Signature transform remains an open problem. Several approaches have been proposed, ranging from insertion methods to optimal transport techniques. Each of these methods is only an approximation of the inversion, based on optimization problems. Our work extends the framework of Pfeffer, Seigal, and Sturmfels to incorporate the Lie group structure of G^N(\mathbb{R}^d), the signature space. The original framework use an expression of the i-th level of signature S^{(i)} as a sequence of k-mode tensor product between a given functional base and the decomposition matrix of the aimed path in this base. In this paper, we aim to go beyond a mean square loss by constructing a sequence of tensors that adhere to the Lie group structure. Additionally, we propose a method to recover the exact-length path rather than the shortest one. Finally, we evaluate our approach on multi-dimensional Brownian paths.

› Hypergraphs on high dimensional time series sets using signature transform - Remi Vaucher, Paul Minchella

Abstract: Over the past decades, hypergraphs and their study with topological data analysis (TDA) have become first-rate tools. Accordingly to this phenomena, a significant amount of tools appeared to build hypergraphs (named simplicial complexes in TDA) on top of data. Such structures allow us to create edges between more than two vertices.

In this paper, we adress the problem of constructing an hypergraph on top of multiple multivariate time series. The case of an hypergraph over a single multivariate time series has been addressed multiple times these past years. We succed to this task by generalizing a pre-existing algorithm for multivariate time series in the case of multiple multivariate time series. In addition, we exploit the properties of the signature transform to propose the introduction of some randomness in the algorithm in order to robustify the construction. Finally, our method is tested on synthetic data, and gives promising results.

› Using Signatures and Koopman Operator to learn Non-linear Dynamics - Stephane Chretien, Ben Gao, Jordan Patracone, Olivier Atala

Abstract: We propose a novel framework for predicting the evolution of dynamical systems by learning the Koopman operator in the space of linear functionals on the Signature transform of trajectory data. The Signature, a central object in rough path theory, provides a universal and compact representation of paths through iterated integrals, enabling linear models to approximate a wide class of nonlinear functionals. By restricting observables to lie in the span of truncated Signatures, we construct a finite-dimensional approximation of the Koopman operator, which we estimate directly from data using regularized linear regression. This approach merges the expressiveness of operator-theoretic methods with the structural richness of Signature features.

› A kernel-based global method for the learning of elastic potentials on Lie groups - Jianyu Hu, Domenico Campolo, Juan-pablo Ortega, Daiying Yin

Abstract: We propose a structure-preserving kernel ridge regression method for learning elastic potentials on Lie groups from noisy observations of force and torque fields. The approach is demonstrated on the special Euclidean group SE(3), where the elastic potential acts as an external control. A key advantage of our method is that the potential function estimator admits a globally defined closed-form solution, with provable convergence analysis. Numerical experiments confirm the effectiveness of the proposed scheme.

| Thursday, October 30th |

DAY 2 OF AUTHOR SESSIONS

9:15 - 10:00

OPENING SESSION,
Metriplectic Dynamics: A Geometrical Framework for Thermodynamically Consistent Dynamical Systems - Philip J. MORRISON

GEOMETRIC METHODS IN THERMODYNAMIC - FRANÇOIS GAY-BALMAZ, HIROAKI YOSHIMURA

› Thermodynamic Functionality of Non-Detailed Balance Finite-Tape Information Ratchet - Lock Yue Chew, Jian Wei Cheong, Andri Pradana

Abstract: In this paper, we investigate into the functionality of the finite-tape information ratchet when its thermal transition is non-detailed balance. First, we construct an analytical framework of the information ratchet from stochastic thermodynamics by generalizing over that of [1] with detailed balance broken. This leads to special cases of the information processing first and second law stipulated by Semaan et. al. [2] with the appearance of housekeeping heat. Through the application of Kullback-Leibler divergence as a statistical distance, we observe theoretically the mathematical condition for the finite-tape information ratchet to serve as an heat engine: its cumulative change in entropy should exceed that of the reduction in statistical distance of its initial to stationary state. While this is true for both the equilibrium and nonequilibrium stationary state, the heat extraction from the latter is exacerbated by the flow of housekeeping heat. We demonstrate the validity of our results by a Markov transition model which displays statistical dynamics that is non-detailed balance.

› Entropy functionals and equilibrium states in mixed quantum-classical dynamics - Cesare Tronci

Abstract: The computational challenges posed by many-particle quantum systems are often overcome by mixed quantum-classical (MQC) models in which certain degrees of freedom are treated as classical while others are retained as quantum. One of the fundamental questions raised by this hybrid picture involves the characterization of the information associated to MQC systems. Based on the theory of dynamical invariants in Hamiltonian systems, here we propose a family of hybrid entropy functionals that consistently specialize to the usual Rényi and Shannon entropies. Upon considering the MQC Ehrenfest model for the dynamics of quantum and classical probabilities, we apply the hybrid Shannon entropy to characterize equilibrium configurations for simple Hamiltonians. The present construction also applies beyond Ehrenfest dynamics.

› Variational principle in stochastic thermodynamics: Discrete systems - Héctor Vaquero del Pino, François Gay-Balmaz, Hiroaki Yoshimura, Lock Yue Chew

Abstract: In this paper, the first insights into a variational formulation of stochastic thermodynamics is presented for the finite-dimensional case of discrete systems. Following the variational approach in (Gay-Balmaz & Yoshimura 2019), the fundamental variational principle of classical mechanics is systematically extended to include irreversible and stochastic forces. By including thermodynamic entropy as an independent state variable in phase space, the conditions for thermodynamic consistency are derived extending the tools of stochastic thermodynamics, providing the fluctuation-dissipation relation. The method is illustrated with mechanical dissipative forces.

› Variational Principle for Stochastic Nonholonomic Systems Part I: Continuous-Time Formulation - Tianzhi Li, Francois Gay-Balmaz, Donghua Shi, Jinzhi Wang

Abstract: Nonholonomic mechanics has received considerable attention in dynamics and control area. However, due to a wide range of fluctuations in the physical world, the ideal mathematical models of mechanical systems with nonholonomic constraints suffer from issues of ignoring the real-world perturbations and physically difficult to realize. Motivated by recent developments in stochastic and constrained mechanics, here we present a stochastic variational formulation for mechanical systems with or without stochastic nonholonomic constraints. We give stochastic variational principles for both stochastically unconstrained and nonholonomic cases under the same framework by deriving the stochastic implicit Hamel equations. Moreover, an interesting example of the stochastic rolling disk is provided to illustrate the proposed method.

› Variational Principle for Stochastic Nonholonomic Systems Part II: Stochastic Nonholonomic Integrator - Tianzhi Li, Francois Gay-Balmaz, Donghua Shi, Jinzhi Wang

Abstract: Nonholonomic integrators are a class of geometric numerical integration schemes that are designed to simulate mechanical systems with nonholonomic constraints. To the best of our knowledge, so far there have been no variational integrators designed for stochastic systems with noisy nonholonomic constraints, which are extensively studied in robotics and control area. Based on the stochastic nonholonomic variational formulation introduced in Part I, we present a stochastic integrator for both stochastically unconstrained and stochastically nonholonomic systems under the same framework. The numerical integration scheme is obtained by deriving a discrete counterpart of the stochastic variational principle discussed in Part I.

› Interconnection and variational principles for fluid-bubble dynamics - Francois Gay-Balmaz, Hiroaki Yoshimura

Abstract: We consider the dynamics of a barotropic fluid interacting with a bubble filled with uniform gas from the perspective of system interconnection in Lagrangian mechanics. Extending the existing geometric framework to an infinite-dimensional setting requires careful consideration of the appropriate duality pairing underlying the relationship between interaction forces and distribution constraints. We address both inviscid and viscous cases, including surface tension, and consider both free-slip and no-slip interface conditions. This work represents a first step toward

building the geometric foundations for Rayleigh-Plesset equation and its related models.

› Hamilton-Dirac formulation for thermodynamic systems with reaction and diffusion - Hiroaki Yoshimura, Francois Gay-Balmaz

Abstract: In this paper, we propose a Hamilton-Dirac formulation for non-simple nonequilibrium thermodynamic systems with chemical reactions and diffusion. A key feature of these systems is the degeneracy of their associated Lagrangian function. To address this, we build upon Dirac's theory of constraints for degenerate Lagrangians and develop a Hamiltonian variational formulation for nonholonomic systems with nonlinear thermodynamic constraints, as well as primary constraints arising from the degeneracy. We introduce the constrained Hamiltonian on the primary constraint and clarify the underlying geometric structure using Dirac structures. Finally, we illustrate our Hamilton-Dirac formulation with an example of a membrane undergoing matter reaction and diffusion.

› On a generalisation of metriplectic systems - Jonas Kirchhoff, Bernhard Maschke

Abstract: This note presents a notion of generalised metriplectic systems which includes not necessarily holonomic constraints using methods from port-Hamiltonian systems theory. Metriplectic systems can be written as particular dissipative Hamiltonian systems with the exergy as Hamiltonian. Within the generalised dissipative Hamiltonian systems, a generalisation of metriplectic systems is identified.

STOCHASTIC GEOMETRIC DYNAMICS - ANA BELA CRUZEIRO, JEAN-CLAUDE ZAMBRINI, STEFANIA UGOLINI

› Stochastic perturbation of geodesics on the manifold of Riemannian metrics - Ana Bela Cruzeiro, Ali Suri

Abstract: In this paper, using diffusion processes with values in the manifold of Riemannian metrics, we compute the evolution equation for the Lagrangian induced by the L^2 stochastic kinetic energy functional.

› Stochastic Maupertuis's principles and Jacobi's integration theorem - Qiao Huang

Abstract: Building on stochastic geometric mechanics on Riemannian manifolds, we shall focus on extensions of the classical Maupertuis's variational principle to a class of diffusion processes as extremals of a stochastic action functional preserving the expectation of energy. We shall also mention a recent and related stochastic Jacobi integration theorem, whose consequence will be analyzed elsewhere.

› Lagrangian averaging of singular stochastic actions for fluid dynamics - Theo Diamantakis, Ruiao Hu

Abstract: We construct sub-grid scale models of incompressible fluids by considering expectations of semi-martingale Lagrangian particle trajectories. Our construction is based on the Lagrangian decomposition of flow maps into mean and fluctuation parts,

and it is separated into the following steps. First, through Magnus expansion, the fluid velocity field is expressed in terms of fluctuation vector fields whose dynamics are assumed to be stochastic. Second, we use Malliavin calculus to give a regularised interpretation of the product of white noise when inserting the stochastic velocity field into the Lagrangian for Euler's fluid. Lastly, we consider closures of the mean velocity by making stochastic analogues of Taylor's frozen-in turbulence hypothesis to derive a version of the anisotropic Lagrangian averaged Euler equation.

› Continuous-time filtering in Lie groups: estimation via the Fréchet mean of solutions to stochastic differential equations - Magalie Bénéfice, Marc Arnaudon, Audrey Giremus

Abstract: We compute the Fréchet mean \SE_t of the solution X_t to a continuous-time stochastic differential equation in a Lie group. It provides an estimator with minimal variance of X_t. We use it in the context of Kalman filtering and more precisely to infer rotation matrices. In this paper, we focus on the prediction step between two consecutive observations. Compared to state-of-the-art approaches, our assumptions on the model are minimal.

OPTIMIZATION AND LEARNING ON MANIFOLDS – CYRUS MOSTAJERAN, SALEM SAID

› Geometric design of the tangent term in landing algorithms for orthogonality constraints - Florentin Goyens

Abstract: We propose a family a metrics over the set of full-rank n\times p real matrices, and apply them to the landing framework for optimization under orthogonality constraints. The family of metrics we propose is a natural extension of the \beta-metric, defined on the Stiefel manifold.

› Role of Riemannian geometry in double-bracket quantum imaginary-time evolution - Marek Gluza, René Zander, Raphael Seidel, Li Xiaoyue

Abstract: Double-bracket quantum imaginary-time evolution (DB-QITE) is a quantum algorithm which coherently implements steps in the Riemannian steepest-descent direction for the energy cost function. DB-QITE is derived from Brockett's double-bracket flow which exhibits saddle points where gradients vanish. In this work, we perform numerical simulations of DB-QITE and describe signatures of transitioning through the vicinity of such saddle points. We provide an explicit gate count analysis using quantum compilation programmed in Qrisp.

› Numerical techniques for geodesic approximation in Riemannian shape optimization - Kathrin Welker, Estefania Loayza-Romero

Abstract: Shape optimization is commonly applied in engineering to optimize shapes with respect to an objective functional relying on PDE solutions. In this paper, we view shape optimization as optimization on Riemannian shape manifolds. We consider so-called outer metrics on the diffeomorphism group to solve PDE-constrained shape optimization problems efficiently. Commonly, the numerical solution of such problems relies on the Riemannian version of the steepest

descent method. One key difference between this version and the standard method is that iterates are updated via geodesics or retractions. Due to the lack of explicit expressions for geodesics, for most of the previously proposed metrics, very limited progress has been made in this direction. Leveraging the existence of explicit expressions for the geodesic equations associated to the outer metrics on the diffeomorphism group, we aim to study the viability of using such equations in the context of PDE-constrained shape optimization. However, solving geodesic equations is computationally challenging and often restrictive. Therefore, this paper discusses potential numerical approaches to simplify the numerical burden of using geodesics, making the proposed method computationally competitive with previously established methods.

› Geometric Gaussian Approximations of Probability Distributions - Nathaël Da Costa, Bálint Mucsányi, Philipp Hennig

Abstract: Approximating complex probability distributions, such as Bayesian posterior distributions, is of central interest in many applications. We study the expressivity of geometric Gaussian approximations. These consist of approximations by Gaussian pushforwards through diffeomorphisms or Riemannian exponential maps. We first review these two different kinds of geometric Gaussian approximations. Then we explore their relationship to one another. We further provide a constructive proof that such geometric Gaussian approximations are universal, in that they can capture any probability distribution. Finally, we discuss whether, given a family of probability distributions, a common diffeomorphism can be found to obtain uniformly high-quality geometric Gaussian approximations for that family.

› A probabilistic view on Riemannian machine learning models for SPD matrices - Thibault de Surrel, Florian Yger, Fabien Lotte, Sylvain Chevallier

Abstract: The goal of this paper is to show how different machine learning tools on the Riemannian manifold \P_d of Symmetric Positive Definite (SPD) matrices can be united under a probabilistic framework. For this, we will need several Gaussian distributions that have been defined in \P_d. We will show how popular classifiers on \P_d can be reinterpreted as Bayes Classifiers using these Gaussian distributions. These distributions will also be used for outlier detection and dimension reduction. By showing that those distributions are pervasive in the tools used on \P_d, we allow for other machine learning tools to be extended to \P_d.

› Efficiency of the Generalized Method of Moments from the Viewpoint of Differential Geometry - Hisatoshi Tanaka

Abstract: The generalized method of moments (GMM) attains the semiparametric efficiency bound when the optimal weight matrix is chosen. In this study, we characterize the efficient choice of the weight matrix from the viewpoint of differential geometry. We induce a metric for the GMM manifold from a linear space in which the model set is embedded. Simultaneously, using the asymptotic normality of the GMM estimators, we define another metric of the manifold. In conclusion, we prove that the two metrics coincide when an optimal weight matrix is employed.

› p-Laplacians for Manifold-valued Hypergraphs - Jo Stokke, Ronny Bergmann, Martin Hanik, Christoph von Tycowicz

Abstract: Hypergraphs extend traditional graphs by enabling the representation of N-ary relationships through higher-order edges. Akin to a common approach of deriving graph Laplacians, we define function spaces and corresponding symmetric products on the nodes and edges to derive hypergraph Laplacians. While this has been done before for Euclidean features, this work generalizes previous hypergraph Laplacian approaches to accommodate manifold-valued hypergraphs for many commonly encountered manifolds.

› Universal kernels via harmonic analysis on Riemannian symmetric spaces - Cyrus Mostajeran, Franziskus Steinert, Salem Said

Abstract: The universality properties of kernels characterize the class of functions that can be approximated in the associated reproducing kernel Hilbert space and are of fundamental importance in the theoretical underpinning of kernel methods in machine learning. In this work, we establish fundamental tools for investigating universality properties of kernels in Riemannian symmetric spaces, thereby extending the study of this important topic to kernels in non-Euclidean domains. Moreover, we use the developed tools to prove the universality of several recent examples from the literature on positive definite kernels defined on Riemannian symmetric spaces, thus providing theoretical justification for their use in applications involving manifold-valued data.

CLASSICAL & QUANTUM INFORMATION, GEOMETRY AND TOPOLOGY – FLORIO M. CIAGLIA, FABIO DI COSMO, PIERRE BAUDOT AND GRÉGOIRE SERGEANT-PERTHUIS

› The category of non-commutative probabilities in Information Geometry - Laura Gonzalez Bravo, Florio Ciaglia, Fabio Di Cosmo

Abstract: The category of non-commutative probabilities (NCP) is introduced to provide a categorical framework for classical and quantum information geometry. This framework enables the classification of field of covariances, as functors from NCP to Hilb, the category of Hilbert spaces and contractions, a description which is suited to both finite and infinite dimensional settings. Additionally, NCP sets an environment to provide a detailed description of statistical models.

› Independent States are Orthogonal: a Categorical Framework to Treat Probability Geometrically - Paolo Perrone, Matthew Di Meglio, Chris Heunen, Dario Stein

Abstract: . Dagger categories (a.k.a. *-categories) can be seen as categories with a notion of « transpose », generalizing the transposition of matrices in linear algebra. This allows us to extend the ideas of orthogonality and orthogonal projector from Euclidean geometry and Hilbert space theory to a much more general and abstract context.
By means of a dagger category of probability spaces and transport plans, we show that this abstract notion of orthogonality can model exactly independence and conditional independence of random variables.
Moreover, orthogonal projectors correspond

exactly to conditioning, giving a unified description of « observations » for both quantum and classical experiments.

› Two-typed Tangent Vectors in Quantum Statistical Mechanics - Jan Naudts

Abstract: The tangent space at a KMS state (Kubo Martin Schwinger state) can be decomposed into two subspaces in such a way that the time evolution, which is described by the modular automorphism group, satisfies the KMS condition w.r.t. each of these two subspaces. In particular, this implies that these subspaces are invariant for the time evolution and that they determine a discrete conserved quantity. The tangent space is said to be two-typed because each tangent vector is the sum of two vectors, one in each subspace. The pair of subspaces is non-unique. It is determined by the choice of an orthonormal basis diagonalizing the modular operator. The paper is restricted to the finite-dimensional case. In this way the technicalities of handling unbounded operators are not exposed.

› A Historical Perspective on the Schützenberger-van~Trees Inequality: A Posterior Uncertainty Principle - Olivier Rioul

Abstract: The Bayesian Cramér-Rao Bound (BCRB) is generally attributed to Van~Trees who published it in 1968. According to Stigler's law of eponymy, no scientific discovery is named after its first discoverer. This is the case not only for the Cramér-Rao bound itself—due in particular to the French mathematicians Fréchet and Darmois—but also for the van Trees inequality: The French physician, geneticist, epidemiologist and mathematician Marcel-Paul (Marco) Schützenberger, in a paper of just fifteen lines written in 1956—more than a decade before van Trees—had not only demonstrated the BCRB but, as a close examination of his proof shows, used a very original approach based on the Weyl-Heisenberg uncertainty principle on the posterior distribution. This work reviews and extends Schützenberger's approach to Fisher information matrices, which opens up new perspectives.

› Tree inference with varifold distances - Elodie Maignant, Tim Conrad, Christoph von Tycowicz c

Abstract: In this paper, we consider a tree inference problem motivated by the critical problem in single-cell genomics of reconstructing dynamic cellular processes from sequencing data. In particular, given a population of cells sampled from such a process, we are interested in the problem of ordering the cells according to their progression in the process. This is known as trajectory inference. If the process is differentiation, this amounts to reconstructing the corresponding differentiation tree. One way of doing this in practice is to estimate the shortest-path distance between nodes based on cell similarities observed in sequencing data. Recent sequencing techniques make it possible to measure two types of data: gene expression levels, and RNA velocity, a vector that predicts changes in gene expression. The data then consist of a discrete vector field on a Euclidean space of dimension equal to the number of genes under consideration. By integrating this velocity field, we recover for each single cell its trajectory from some initial stage to its current stage. Eventually, we assume that we have a faithful embedding of the tree in a Euclidean space, but which we only observe through the curves connecting the root to the nodes. Using varifold distances between such curves, we define a similarity measure

between nodes which we prove approximates the shortest-path distance in a tree that is isomorphic to the target tree.

INFORMATION GEOMETRY, DELZANT TORIC MANIFOLD & INTEGRABLE SYSTEM – MATHIEU MOLITOR, HAJIME FUJITA, DAISUKE TARAMA, FRÉDÉRIC BARBARESCO

› Adler-Kostant-Symes Theorem and Algebraic Complete Integrability of Information Geometry and Souriau Lie Groups Thermodynamics - Frederic Barbaresco

Abstract: Y. Nakamura established that gradient systems defined on specific statistical manifolds, such as those associated with Gaussian and multinomial distributions, satisfy the conditions of Liouville complete integrability. Furthermore, he demonstrated that gradient flows on statistical manifolds may be linearised through the application of dual coordinates from information geometry, in a manner analogous to the action-angle coordinates employed in Hamiltonian mechanics to characterise integrable systems. We extend this line of inquiry to Souriau's symplectic model of Information Geometry for Lie groups. Subsequently, we examine algebraic complete integrability in the sense of Adler and van Moerbeke, as well as the symplectic structure underlying Lax pairs, which can be formulated in terms of algebraic-geometric structures. This approach underlines the interplay between the analytical and group-theoretical methodologies in the study of integrable systems. Within this framework, we study the Adler-Kostant-Symes theorem as a principal tool for the construction of integrable systems, leveraging its capacity to establish algebraic integrability.

› Statistical transformation models of multivariate normal distributions and their \alpha-geodesic flows - Daisuke Tarama

Abstract: This paper deals with the geodesic flows of the \alpha-connections arising from the statistical transformation model for the multivariate normal distributions on \mathbb{R}^d.
The probability density functions are parameterized by the semi-direct product Lie group GL_+\left(d,\mathbb{R}\right)\ltimes \mathbb{R}^d.
The Fisher-Rao semi-definite metric and the Amari-Chentsov cubic tensor are left-invariant tensors on the Lie group.
One can then describe the geodesic flows of the \alpha-connections as left-invariant system.
It is interesting that the geodesic flow of the Fisher-Rao semi-definite metric can be formulated in terms of the subriemannian geometry.
In fact, the Fisher-Rao geodesic flow is a subriemannian geodesic flow for a step-two left-invariant subriemannian structure on the semi-direct product Lie group.
In this paper, an explicit formula of the Amari-Chentsov cubic tensor is obtained and consequently the equation for the \alpha-geodesic flows is found concretely.
As preliminaries of the current paper, the general framework of information geometry is briefly reviewed in relation to the Fisher-Rao metric and the Amari-Chentsov cubic tensor, particularly in the case of statistical transformation models.

› Geodesic flow of a statistical manifold associated to Souriau's thermodynamics - Jérémie Pierard de Maujouy, Daisuke Tarama

Abstract: We investigate the relation between Souriau's Lie group thermodynamics and statistical transformation models.

Souriau associated with a statistical mechanical system a Gibbs set that described the states of thermodynamical equilibrium. These Gibbs sets can be understood as statistical transformation models constructed through a general procedure that involves a representation of the symmetry group. As such, they have a Fisher-Rao metric which is consistent with the usual construction for group-parametrized statistical transformation models.

As an example, we consider the case of the Fisher distributions on the 2-sphere, viewed as a Hamiltonian SO(3)-manifold. We compute the Fisher-Rao metric, which is invariant under rotations. The geodesic flow on so(3) associated with the Fisher distributions on the 2-sphere is integrable and provides a first example of dynamical systems on Gibbs sets.

› **Moment polytopes of toric exponential families - Mathieu Molitor**

Abstract: We show that the moment polytope of a Kähler toric mani-fold, constructed as the torification (in the sense of M. Molitor, « Kähler toric manifolds from dually flat spaces », arXiv:2109.04839) of an exponential family defined on a finite sample space, is the projection of a higher-dimensional simplex.

› **A compactification of the orthogonal foliation via toric geometry - Hajime Fujita**

Abstract: We provide a compactification of the orthogonal foliation for the dually flat structure on the probability simplex. In particular we examine the orthogonality of the e-foliation and m-foliation on the boundary. We use a toric geometric aspect of the probability simplex.

› **Torsion of \alpha-connections on the density manifold - Lorenz Schwachhöfer**

Abstract: We study the torsion of the \alpha-connections defined on the density manifold in terms of a regular Riemannian metric.

In the case of the Fisher-Rao metric our results confirm the fact that all \alpha-connections are torsion free. For the \alpha-connections obtained by the

Otto metric, we show that, except for \alpha = -1, they are not torsion free.

GEOMETRIC GREEN LEARNING ON GROUPS AND QUOTIENT SPACES – ALICE BARBARA TUMPACH, DIARRA FALL AND LEVIN MAIER

› **Enhancing CNNs robustness to occlusions with bioinspired filters for border completion - Rita Fioresi, Catarina Coutinho, Aneeqa Merhab, Janko Petkovic, Ferdinando Zanchetta**

Abstract: We exploit the mathematical modeling of the visual cortex mechanism for border completion to define custom filters for CNNs. We see a consistent improvement in performance, particularly in accuracy, when our modified LeNet 5 is tested with occluded MNIST images.

› A new model for natural groupings in high-dimensional data - Mireille Boutin, Evzenie Coupkova

Abstract: Clustering aims to divide a set of points into groups. The current paradigm assumes that the grouping is well-defined (unique) given the probability model from which the data is drawn.
Yet, recent experiments have uncovered several high-dimensional datasets that form different binary groupings after projecting the data to randomly chosen one-dimensional subspaces. This paper describes a probability model for the data that could explain this phenomenon. It is a simple model to serve as a proof of concept for understanding the geometry of high-dimensional data. Our construction makes it clear that one needs to make a distinction between « groupings » and « clusters » in the original space. It also highlights the need to interpret any clustering found in projected data as merely one among potentially many other groupings in a dataset.

› Information Geometry on the \ell^2-Simplex via the q-Root Transform - Levin Maier

Abstract: In this paper, we introduce \emph{\ell^p-information geometry}, an infinite dimensional framework that shares key features with the geometry of the space of probability densities \(\mathrm{Dens}(M) \) on a closed manifold, while also incorporating aspects of measure-valued information geometry. We define the \emph{\ell^2-probability simplex} with a noncanonical differentiable structure induced via the \emph{q-root transform} from an open subset of the \ell^p-sphere. This structure renders the q-root map an \emph{isometry}, enabling the definition of \emph{Amari–Čencov \alpha-connections} in this setting. We further construct \emph{gradient flows} with respect to the \ell^2 Fisher–Rao metric, which solve an infinite-dimensional linear optimization problem. These flows are intimately linked to an \emph{integrable Hamiltonian system} via a \emph{momentum map} arising from a Hamiltonian group action on the infinite-dimensional complex projective space. \keywords{infinite-dimensional information geometry \and \ell^p-information geometry \and Amari–Čencov \alpha-connections \and integrable Hamiltonian systems \and infinite-dimensional linear programming}

› K-P Quantum Neural Networks - Elija Perrier

Abstract: We present an extension of K–P time-optimal quantum control

solutions using global Cartan KAK decompositions for geodesic-based solutions. Extending recent time-optimal \emph{constant–\theta} control results, we integrate Cartan methods into equivariant quantum neural network (EQNN) for quantum control tasks. We show that a finite-depth limited EQNN ansatz equipped with Cartan layers can replicate the constant–\theta sub-Riemannian geodesics for K–P problems. We demonstrate how for certain classes of control problem on Riemannian symmetric spaces, gradient-based training using an appropriate cost function converges to certain global time-optimal solutions when satisfying simple regularity conditions. This generalises prior geometric control theory methods and clarifies how optimal geodesic estimation can be performed in quantum machine learning contexts. \keywords{Quantum control \and K–P problem \and Equivariant QNN \and Cartan decomposition \and Optimal geodesics \and Sub-Riemannian geometry \and Machine learning.

› Infinite-dimensional Siegel disc as symplectic and Kähler quotient - Alice Barbara Tumpach

Abstract: In this paper, we construct the restricted infinite-dimensional Siegel disc as a Marsden-Weinstein symplectic reduced space and as Kähler quotient of a weak Kähler manifold. The obtained symplectic form is invariant with respect to the left action of the infinite-dimensional restricted symplectic group and coincides with the Kirillov-Kostant-Souriau symplectic form of the restricted Siegel disc obtained via the identification with an affine coadjoint orbit of the restricted symplectic group, or equivalently with a coadjoint orbit of the universal central extension of the restricted symplectic group.

› The hyperkähler marriage between the sphere and the hyperbolic space - Alice Barbara Tumpach

Abstract: We review the construction of the hyperkähler metric on the complexification of the projective space which extends the Kähler metric of the 2-sphere. The hyperbolic space sits in this complexification. In this paper, we are interested in the complex structure inherited on the hyperbolic space by the hyperkähler extension of the 2-sphere. Contrary to what is generally believed, we show that it differs from the natural complex structure of the hyperbolic disc inherited from its embedding in C.

APPLIED GEOMETRY-INFORMED MACHINE LEARNING (SESSION 1) - PIERRE-YVES LAGRAVE, SANTIAGO VALASCO-FORERO AND TEODORA PETRISOR

› Riemannian Integrated Gradients: A Geometric View of Explainable AI - Lachlan Simpson, Federico Costanza

Abstract: We introduce Riemannian Integrated Gradients (RIG); an extension of Integrated Gradients (IG) to Riemannian manifolds. We demonstrate that RIG restricts to IG when the Riemannian manifold is Euclidean space. We show that feature attribution can be phrased as an eigenvalue problem where attributions correspond to eigenvalues of a symmetric endomorphism.

› A Geometric Deep Learning Approach to Forecast the Time Series of Covariance Matrices - Michele Palma, Andrea Bucci

Abstract: The forecasting approaches of time-varying covariance matrices often overlook the geometric properties of symmetric positive definite matrices, ignoring the fact that these are points on a Riemannian manifold. This may lead to suboptimal forecast accuracy and might result in overparameterized model, making it infeasible to work with high-dimensional matrices. This paper introduces an innovative approach to forecasting time series of covariance matrices using a deep learning method grounded in Riemannian optimization. In an application with simulated data, we show that when geometric properties of the predicted object are taken into account, the prediction accuracy significantly improves.

› Learning Riemannian Metrics for Interpolating Animations - Sarah Kushner, Vismay Modi, Nina Miolane

Abstract: We leverage a family of Riemannian metrics to upsample low frame rate animations for creative design and compression applications in computer graphics. Our method interpolates animated characters' bone orientations along various geodesics from a family of invariant Riemannian metrics on a product of SO(3) manifolds. For compression, an optimization step selects the best-fitting metric. We show that our approach outperforms existing techniques.

› Conditioning Surface Shape Processes with Neural Operators - Jingchao Zhou, Gefan Yang, Stefan Sommer

Abstract: We present a novel method for simulating infinite-dimensional conditional stochastic processes governing surface shape evolution. Given boundary conditions represented as spherical functions, we consider a function-valued diffusion process X with initial state X_0, conditioned on X_T. To address the simulation challenge, we develop a neural operator architecture leveraging spherical harmonic transforms to approximate the intractable drift term arising from Doob's h-transform. The proposed operator demonstrates discretization equivariance, enabling direct application to spherical meshes at arbitrary resolutions without architectural modifications or retraining. We validate our method on several synthetic shape evoluation scenarios.

› GNN-Enhanced TCN Algorithms for ECG Signal Quality Recognition - Ferdinando Zanchetta, Angelica Simonetti

Abstract: Temporal Convolutional Networks (TCNs) are among the most effective algorithms to deal with time series dataTo a time series can be also given the structure of directed graph, opening the doors to the usage of Graph Neural Networks (GNNs) in this context. In this paper we develop two distinct Geometric Deep Learning models that merge the capabilities of ordinary TCNs with the ones of GNNs, a supervised classifier and an autoencoder-like model that we apply to solve a quality detection problem on electrocardiogram signals.

› WAN2DNS-PM: Weak adversarial networks for solving 2D incompressible Navier-Stokes equations in porous media - Wenran LI, Miloud Bessafi, Cedric Damour, Yu LI, Alain Miranville, Rong Yang, Xinguang Yang, Frederic CADET

Abstract: The use of neural networks has shown significant potential to reduce the computational costs associated with the dynamics of industrial computational fluids. Weak adversarial networks (WAN) leverage weak solution theory to transform the problem of solving PDEs into a Min-Max optimization problem, which is then solved by training a generative adversarial network. Although this method has been successfully applied to two-dimensional (2D) Navier-Stokes (NS) equations, previous work says nothing about the NS equation in porous media. In this study, we first leverage stream function to introduce the biharmonic formulation of NS in porous media. Then, we extend the WAN framework to solve NS equations in porous media (WAN2DNS-PM) and provide free surface flow as a numerical experiment. Our results demonstrate the stability and accuracy of the proposed method, highlighting its advantages over the traditional Physic-Informed Neural Networks (PINNs) algorithm, particularly for problems lacking strong solutions. This work contributes to the growing research on AI-driven numerical methods for complex fluid dynamics problems, offering a promising approach for industrial applications.

Friday, October 1st

DAY 3 OF AUTHOR SESSIONS

8:30-9:30

OPENING SESSION,
A Noncommutative perspective of Graph Neural Networks - Rita FIORESI

GEOMETRIC MECHANICS - GERY DE SAXCE, ZDRAVKO TERZE, FRANÇOIS DUBOIS

› Symplectic bipotentials for the dynamics of dissipative systems with non associated constitutive laws - Géry de Saxcé

Abstract: In a previous paper, we proposed a symplectic version of the Brezis-Ekeland-Nayroles principle. We applied it to the standard plasticity. The object of this work is to extend the previous formalism to non associated laws. For this aim, we introduce the concept of symplectic bipotential which extends that of bipotential to dynamical systems. We present a method to build it from a bipotential. Next, we generalize the symplectic Brezis-Ekeland-Nayroles principle to the non associated dissipative laws. As example, we apply it to the unilateral contact law with Coulomb's dry friction.

› Debreu's 3-webs and Affinely Flat Bi-Lagrangian Manifolds links with Transverse Symplectic Foliation of Souriau's Dissipative Lie Groups Thermodynamics - Frederic Barbaresco

Abstract: We shall elucidate the foliation structures, namely, the 3-web and the bi-Lagrangian structure, that were jointly employed by the physicist and mathematician Jean-Marie Souriau in his Lie Groups Thermodynamics, extended to include dissipation models, and by Gérard Debreu, the Nobel Laureate in Economics, within the context of preferences and utility theory. Debreu examined the conditions under which a web is considered trivial, that is, whether there exists a change of coordinates rendering the web equivalent to a standard orthogonal grid, integrable into a potential function. He employed the Frobenius theorem and Pfaffian forms to investigate whether certain distributions, in the sense of foliations, satisfy the necessary conditions for integrability. Souriau developed a symplectic model of thermodynamics based on symplectic foliation, wherein dissipation dynamics are defined on a transverse Riemannian foliation, thereby inducing a web structure linked to a bi-Lagrangian manifold. A bi-Lagrangian manifold may be endowed with a metric 3-web structure via a Riemannian metric. For every 3-web, one can associate a canonical Chern connection, whose flatness guarantees additive separability. This connection, originally introduced by Hess for Lagrangian 2-webs, also known as bipolarised symplectic manifolds, is particularly employed in the study of bi-Lagrangian manifolds.

› Applied Conformal Carroll Geometry - Eric Bergshoeff, Patrick Concha, Octavio Fierro, Evelyn Rodríguez, Jan Rosseel

Abstract: We construct conformal Carroll geometry by gauging the conformal Carroll algebra. In doing so, we pay special attention to the way the so-called intrinsic torsion tensor components enter into the transformation rules of the geometric fields.

As an application of our results, we couple a single electric/magnetic massless scalar to conformal Carroll gravity and show how, upon gauge-fixing the dilatations, we obtain a non-conformal version of electric/magnetic Carroll gravity.

› A variational symplectic scheme based on Lobatto's quadrature - François Dubois

Abstract: We present a variational integrator based on the Lobatto quadrature for the time integration of dynamical systems issued from the least action principle. This numerical method uses a cubic interpolation of the states and the action is approximated at each time step by Lobatto's formula. Numerical analysis is performed on a harmonic oscillator. The scheme is conditionally stable, sixth-order accurate, and symplectic. It preserves an approximate energy quantity. Simulation results illustrate the performance of the proposed method.

› Lifting of some dynamics on the set of bilagrangian structures – Bertuel TANGUE NDAWA

Abstract: A triplet (\omega, \mathcal{F}_{1},\mathcal{F}_{2}) is a bilagrangian structure on a manifold M, if \omega is a 2-form, closed and non-degenerate (called symplectic form) on M, and (\mathcal{F}_{1},\mathcal{F}_{2}) is a pair of transversal Lagrangian foliations on the symplectic manifold (M,\omega). The quadruplet (M, \omega, \mathcal{F}_{1},\mathcal{F}_{2}) is called a bilagrangian manifold.
 We prolong a bi-Lagrangian structure on M on its tangent bundle TM, and its cotangent bundle T^{*}M in different ways. As a consequence, some dynamics on the set of bi-Lagrangian structures of M can be prolonged as dynamics on the set of the bi-Lagrangian structures of TM and T^{*}M.

› The contact Eden bracket and the evolution of observables - Víctor Jiménez Morales, Manuel De León

Abstract: In this paper we discuss nonholonomic contact Lagrangian and Hamiltonian systems, that is, systems with a kind of dissipation that are also subject to nonholonomic constraints. We introduce the so-called contact Eden bracket that allows us to simplify the calculation of the evolution of any observable. Finally, we present a particular vector subspace of observables where the dynamics remain unconstrained.

› A new symmetry group for Physics to revisit the Kaluza-Klein theory - Géry de Saxcé

Abstract: In this work, we revisit the Kaluza-Klein theory from the perspective of the classification of elementary particles based on the coadjoint orbit method. We propose a symmetry group for which the electric charge is invariant and, on this basis, a cosmological scenario in which the three former spatial dimensions inflate quickly while the fifth one shrinks, leading to a 4D era where the particles correspond to the coadjoint orbits of this group. By this mechanism, the elementary particles can acquire electric charge as a by-product of the 4 + 1 symmetry breaking of the Universe. By pullback over the space-time, we construct the non-Riemannian connection corresponding to this symmetry group, allowing to recover conservation of the charge and the equation of motion with the Lorentz force. On this ground, we develop a five dimensional extension of the variational relativity allowing to deduce in the classical limit Maxwell's equation.

› Defects in unidimensional structures - Mewen Crespo, Guy Casale, Loïc Le Marrec, Patrizio Neff

Abstract: In a previous work of the first authors, a non-holonomic model, generalising the micromorphic models and allowing for curvature (disclinations) to arise from the kinematic values, was presented. In the present paper, a generalisation of the classical models of Euler-Bernoulli and Timoshenko bending beams based on the mentioned work is proposed. The former is still composed of only one unidimensional scalar field, while the latter introduces a third unidimensional scalar field, correcting the second order terms. The generalised Euler-Bernoulli beam is then shown to exhibit curvature (i.e. disclinations) linked to a third order derivative of the displacement, but no torsion (dislocations). Parallelly, the generalised Timoshenko beam is shown to exhibit both curvature and torsion, where the former is linked to the non-holonomy introduced in the generalisation. Lastly, using variational calculus, asymptotic values for the value taken by the curvature in static equilibrium are obtained when the second order contribution becomes negligible; along with an equation for the torsion in the generalised Timoshenko beam.

› A gradient structure for isotropic non-linear morphoelastic bodies - Adam Ouzeri

Abstract: Morphoelastic bodies are elastic materials that undergo complex shape changes due to intrinsic growth, remodelling, or active internal processes. In a theoretical context, these materials are typically represented by non-Euclidean material manifolds characterized by an evolving metric structure. In this work, we formulate remodelling on such manifolds through a gradient system, where the dynamics of the system are driven by the steepest descent of an energy functional within an appropriate metric space. We obtain a gradient flow equation by combining isotropic non-linear elasticity with growth-induced dissipative mechanisms and illustrate the formalism through numerical simulations of stress relaxation at fixed strain.

› Towards Full `Galilei General Relativity': Gravitational Kinematics in Bargmann Spacetimes - Christian Cardall

Abstract: Because of the strict separation of mass and energy in Galilei physics, a Galilei-invariant tensor formalism is most at home in a 5-dimensional extended spacetime associated with the Bargmann-Galilei (traditionally `Bargmann') group, a central extension of the Galilei group that explicitly exhibits the transformation properties of kinetic energy. While not necessary for a tensor formalism fully embodying Poincar\'e physics, a similar central extension of the Poincar\'e group to the Bargmann-Poincar\'e group may illuminate a path towards a strong-field `Galilei general relativity'. Here the Bargmann metric is generalized to curved spacetime by extending the usual 1+3 (traditionally `3+1) formalism of general relativity on 4-dimensional spacetime to a 1+3+1 formalism, whose spacetime kinematics is shown to be consistent with that of the usual 1+3 formalism. On Bargmann spacetime, tensor laws governing the motion of an elementary classical material particle and the dynamics of a simple fluid reference the foliation of spacetime in a manner that partially reverts the Einstein perspective (accelerated fiducial observers, and geodesic material particles and fluid elements) to a Newton-like perspective (geodesic fiducial observers, and accelerated material particles and fluid elements subject to a gravitational force).

COMPUTATIONAL INFORMATION GEOMETRY AND DIVERGENCES – FRANK NIELSEN AND OLIVIER RIOUL

› Geometric Jensen-Shannon divergence between Gaussian measures on Hilbert space - Minh Ha Quang, Frank Nielsen

Abstract: This work studies the Geometric Jensen-Shannon divergence, based on the notion of geometric mean of probability measures, in the setting of Gaussian measures on an infinite-dimensional Hilbert space. On the set of all Gaussian measures equivalent to a fixed one, we present a closed form expression for this divergence that directly generalizes the finite-dimensional version. By utilizing the notion of Log-Determinant divergences between positive definite unitized trace class operators, we then define a Regularized Geometric Jensen-Shannon divergence that is valid for any pair of Gaussian measures and that recovers the exact Geometric Jensen-Shannon divergence between two equivalent Gaussian measures when the regularization parameter approaches zero.

› Confidence Bands for Multiparameter Persistence Landscapes - Ines Garcia-Redondo, Anthea Monod, Qiquan Wang

Abstract: Multiparameter persistent homology is a generalization of classical persistent homology, a central and widely-used methodology from topological data analysis, which takes into account density estimation and is an effective tool for data analysis in the presence of noise. Similar to its classical single-parameter counterpart, however, it is challenging to compute and use in practice due to its complex algebraic construction. In this paper, we study a popular and tractable invariant for multiparameter persistent homology in a statistical setting: the multiparameter persistence landscape. We derive a functional central limit theorem for multiparameter persistence landscapes, from which we compute confidence bands, giving rise to one of the first statistical inference methodologies for multiparameter persistence landscapes. We provide an implementation of confidence bands and demonstrate their application in a machine learning task on synthetic data.

› Wasserstein KL-divergence for Gaussian distributions - Adwait Datar, Nihat Ay

Abstract: We introduce a new version of the KL-divergence for Gaussian distributions which is based on Wasserstein geometry and referred to as WKL-divergence. We show that this version is consistent with the geometry of the sample space {\Bbb R}^n. In particular, we can evaluate the WKL-divergence of the Dirac measures concentrated in two points which turns out to be proportional to the squared distance between these points.

› f-Divergence Approximation for Gaussian Mixtures - Amit Vishwakarma, K.S. Subrahamanian Moosath

Abstract: Gaussian Mixture Models (GMMs) are important tool for modeling complex data in many tasks such as image recognition and retrieval, pattern recognition, speaker recognition and varification etc. Various GMM similarity measures are in place but most of them consume large computing resources and high computation time. This is mainly due to the lack of a closed form expression for divergence on GMM. We address this by using the embedding of the manifold of K-component GMMs into the manifold of symmetric positive definite (SPD) matrices. The manifold of SPD matrices is identified with the manifold of centered multivariate normal distribution which provides a computationally efficient formula for the divergence. First, we prove that the f -divergence between any two GMMs is greater than or equal to the f -divergence computed between their corresponding centered multivariate normal representations. A local second-order analysis via Taylor series expansion shows that, under small perturbations of the GMM parameters, the difference between the f -divergences is quadratic. This enables to have a closed form formula for divergence on GMMs.

To demonstrate the computational efficiency of this divergence we conducted an audio classification experiment, where Mel Frequency Cepstral Coefficient (MFCC) features extracted from audio signals are modeled by GMMs and classify them using closed-form Symmetric KL divergence. The analysis indicate that the proposed method shows competative accuracy and significantly reduced the computational time compared to the existing methods.

› A dimensionality reduction technique based on the Gromov-Wasserstein distance. - RAFAEL EUFRAZIO, Eduardo Fernandes Montesuma, Charles Casimiro Cavalcante

Abstract: Analyzing relationships between objects is a pivotal problem within data science. In this context, dimensionality reduction (DR) techniques are employed to generate smaller and more manageable data representations. This paper proposes a new method for dimensionality reduction, based on optimal transportation theory and the Gromov Wasserstein (GW) distance. We offer a new probabilistic view of the classical multidimensional scaling (MDS) algorithm and the nonlinear dimensionality reduction algorithm, Isomap (Isometric mapping or Isometric feature mapping) that extends the classical MDS, in which we use the GW distance between the probability measure of high-dimensional data, and its low-dimensional representation. Through gradient descent, our method embeds high-dimensional data into a lower-dimensional space, providing a robust and efficient solution for analyzing complex highdimensional datasets.

› KD^{2}M: An unifying framework for feature knowledge distillation - Eduardo Montesuma

Abstract: Knowledge Distillation (KD) seeks to transfer the knowledge of a teacher, towards a student neural net. This process is often done by matching the networks' predictions (i.e., their output), but, recently several works have proposed to match the distributions of neural nets' activations (i.e., their features), a process known as \emph{distribution matching}. In this paper, we propose an unifying framework, Knowledge Distillation through Distribution Matching (KD^{2}M), which formalizes this strategy. Our contributions are threefold. We i) provide an overview of distribution metrics used in distribution matching, ii) benchmark on computer vision datasets, and iii) derive new theoretical results for KD.

› Curved representational Bregman divergences and their applications - Frank Nielsen

Abstract: By analogy to curved exponential families, we define curved Bregman divergences as restrictions of Bregman divergences to sub-dimensional parameter subspaces, and prove that the barycenter of a finite weighted parameter set with respect to a curved Bregman divergence amounts to the Bregman projection onto the subspace induced by the constraint of the barycenter with respect to the unconstrained full Bregman divergence.
We demonstrate the significance of curved Bregman divergences with two examples: (1) symmetrized Bregman divergences and (2) the Kullback-Leibler divergence between circular complex normal distributions.
We then consider monotonic embeddings to define representational curved Bregman divergences and show that the \alpha-divergences are representational curved Bregman divergences with respect to \alpha-embeddings of the probability simplex into the positive measure cone.
As an application, we report an efficient method to calculate the intersection of a finite set of \alpha-divergence spheres.

› Tangent Groupoid and Information Geometry - Zelin Yi, Jun Zhang

Abstract: For a smooth manifold M, the tangent groupoid « glues » the set M \ times M with TM as two underlying pieces in smooth transition from one to the other. We show that any contrast function defined on M \times M naturally leads to a Riemannian metric and a pair of conjugate connections that are objects (so-called « statistical structure ») defined for sections of TM. This is achieved through smooth « extension » of the contrast function and its anti-symmetrized version on M \times M to, respectively, a quadratic and a cubic function on TM. We recovered the standard formulae \ cite{Eguchi1983,Eguchi1985, Eguchi1992,Blaesild1991} linking contrast functions to statistical structure through differentiation of the former by two (to obtain the metric) and three (to obtain the connections) vector fields.

› Two types of matching priors for non-regular statistical models - Masaki Yoshioka, Fuyuhiko Tanaka

Abstract: In Bayesian statistics, the selection of noninformative priors is a crucial issue. There have been discussions on theoretical justification and problems for the Jeffreys prior, as well as alternative objective priors. Among them, we will focus on the two types of matching priors consistent with frequency theory: the probability matching priors and the moment matching priors. In particular, there is no clear relationship between these two matching priors on non-regular statistical models, even though they have similar objectives.

Considering information geometry on a one-sided truncated exponential family, a typical example of non-regular statistical models, we obtain the result that the Lie derivative along one vector field provides the conditions for the probability and moment matching priors. Note that this Lie derivative does not appear in regular models. This result promotes a unified understanding of probability and moment matching priors on non-regular models. Further, we discuss the relationship between the probability and moment matching priors and the \alpha-parallel priors.

› Hyperbolic decomposition of Dirichlet distance for ARMA models - Jaehyung Choi

Abstract: We investigate the hyperbolic decomposition of the Dirichlet norm and distance between autoregressive moving average (ARMA) models. Beginning with the K\ »ahler information geometry of linear systems in the Hardy space and weighted Hardy spaces, we demonstrate that the Dirichlet norm and distance of ARMA models, corresponding to the mutual information between the past and future, are decomposed into functions of the hyperbolic distance between the poles and zeros of the ARMA models.

STATISTICAL MANIFOLDS AND HESSIAN INFORMATION GEOMETRY - MICHEL NGUIFFO BOYOM, STÉPHANE PUECHMOREL

› On Invariant Conjugate Symmetric Statistical Structures on the Space of Zero-Mean Multivariate Normal Distributions - Hikozo Kobayashi, Takayuki Okuda

Abstract: By the results of Furuhata–Inoguchi–Kobayashi [Inf. Geom. (2021)] and

Kobayashi–Ohno [Osaka Math. J. (2025)], the Amari–Chentsov \alpha-connections on the space \mathcal{N} of all n-variate normal distributions are uniquely characterized by the invariance under the transitive action of the affine transformation group among all conjugate symmetric statistical connections with respect to the Fisher metric. In this paper, we investigate the Amari–Chentsov \alpha-connections on the submanifold \mathcal{N}_0 consisting of zero-mean n-variate normal distributions. It is known that \mathcal{N}_0 admits a natural transitive action of the general linear group GL(n,\mathbb{R}). We establish a one-to-one correspondence between the set of GL(n,\mathbb{R})-invariant conjugate symmetric statistical connections on \mathcal{N}_0 with respect to the Fisher metric and the space of homogeneous cubic real symmetric polynomials in n variables. As a consequence, if n \geq 2, we show that the Amari–Chentsov \alpha-connections on \mathcal{N}_0 are not uniquely characterized by the invariance under the GL(n,\mathbb{R})-action among all conjugate symmetric statistical connections with respect to the Fisher metric. Furthermore, we show that any invariant statistical structure on a Riemannian symmetric space is necessarily conjugate symmetric.

› Bi-forms Approach to Potential Functions in Information Geometry - Marco Pacelli, Florio Ciaglia, Guiseppe Marmo, Luca Schiavone, Alessandro Zampini

Abstract: Contrast functions play a fundamental role in information geometry, providing a means for generating the geometric structures of a statistical manifold: a pseudo-Riemannian metric and a pair of torsion-free affine connections. However, conventional contrast-based approaches become insufficient in settings where torsion is naturally present, such as quantum information geometry. This work introduces contrast bi-forms, a generalisation of contrast functions that systematically encode metric and connection data, allowing arbitrary affine connections regardless of torsion. It will be shown that they provide a unified framework for statistical potentials, offering new insights into the inverse problem in information geometry. As an application, we explore teleparallel manifolds, where torsion is intrinsic to the geometry, demonstrating how bi-forms naturally accommodate these structures.

› A Foliation by Escort Distributions of Exponential Families and Extended Divergence - Keiko Uohashi

Abstract: We investigate a foliation by deformed escort distributions for the transition of q-parameters, not for a fixed q-parameter.
In particular,.this study considers a natural foliation of dualistic structures of escort distributions of exponential families from the information geometrical point of view.
We then propose a decomposition of an extended divergence on the foliation, which is an analogue of the previously proposed one for discrete escort distributions.

› Flat F manifolds on Statistical manifolds of Hyperboloid type - Guilherme Feitosa de Almeida

Abstract: This paper explores hyperboloid models as statistical manifolds through the framework of flat F-manifolds. We show that these models admit a flat F-manifold structure, offering an alternative to the Fisher information metric. This new perspective deepens the geomet- ric understanding of probabilistic models and opens pathways to more efficient and interpretable methods in machine learning and statistical inference.

› Maximum likelihood estimation for the λ-exponential family - Xiwei Tian, Ting-Kam Leonard Wong, Jiaowen Yang, Jun Zhang

Abstract: IThe λ-exponential family generalizes the standard exponential family via a generalized convex duality motivated by optimal transport. It is the constant-curvature analogue of the exponential family from the information-geometric point of view, but the development of computational methodologies is still in an early stage. In this paper, we propose a fixed point iteration for maximum likelihood estimation under i.i.d. sampling, and prove using the duality that the likelihood is monotone along the iterations. We illustrate the algorithm with the q-Gaussian distribution and the Dirichlet perturbation.

› Rényi partial orders for BISO channels - Christoph Hirche

Abstract: A fundamental question in information theory is to quantify the loss of information under a noisy channel. Partial orders are typical tools to that end, however, they are often also challenging to evaluate. For the special class of binary input symmetric output (BISO) channels, Geng et al. showed that among channels with the same capacity, the binary symmetric channel (BSC) and binary erasure channel (BEC) are extremal with respect to the more capable order. Here we extend on this result by considering partial orders based on Renyi mutual information. We establish the extremality of the BSC and BEC in this setting with respect to the generalized Renyi capacity. In the process, we also generalize the needed tools and introduce alpha-Lorenz curves.

› The Fisher-Rao distance between finite energy signals - Franck Florin

Abstract: This paper addresses the observation of finite energy signals in noise and the estimation of their parameters, based on a geometric science of information approach. The parameters define the coordinate system of a statistical manifold. On this manifold, the Fisher-Rao distance characterizes the statistical dissimilarity between observations of two signals represented by their respective parameters. This work proposes a representation of finite energy signal observations and investigates the possibility of obtaining closed-form expressions for the Fisher-Rao distance. We derive the expressions for the Christoffel symbols and the tensorial equations of the geodesics. This leads to geodesic equations expressed as second-order differential equations. We show that the tensor differential equations can be transformed into matrix equations. These equations depend on the parametric model but simplify to only two vectorial equations, which combine the magnitude and phase of the signal and their gradients with respect to the parameters. These equations lead to closed-form expressions of the Fisher-Rao distance in certain cases. We study the example of observing an attenuated signal with a known magnitude spectrum and unknown phase spectrum and calculate the Fisher-Rao distance. We demonstrate that the finite energy signal manifold corresponds to the manifold of the Gaussian distribution with a known covariance matrix, and that the manifold of known magnitude spectrum signals is a submanifold. We compute closed-form expressions of the Fisher-Rao distances and show that the submanifold is non-geodesic, indicating that the Fisher-Rao distance measured within the submanifold is greater than in the full manifold. The results show that prior knowledge of the magnitude spectrum provides an advantage for signal phase parameter estimation when the difference in phase spectrum between the signals varies significantly throughout the bandwidth.

› Statistical models built on sub-exponential random variables - Barbara Trivellato, Paola Siri

Abstract: Results on nonparametric exponential models are presented by exploiting the notion of sub-exponential random variable. Applications of these models to exponential utility maximization problems are also highlighted.

› Production of labelled foliations - Michel Boyom

Abstract: Production of labelled foliations

› Coherent States on the Statistical Manifold - Carlos Alcalde

Abstract: Statistical states are introduced as coherent states seen as probability amplitudes in the Koopman representation. In Hamiltonian dynamical systems they can be studied as Hilbert bundles over a symplectic manifolds. The duality in Bochner's theorem: probability measures ↔ functions of positive type is interpreted a statistical states and measurements. We present Hamiltonian dynamics on the statistical manifold by representations of the symplectic algebra acting on coherent time series.

APPLIED GEOMETRY-INFORMED MACHINE LEARNING (SESSION 2) - PIERRE-YVES LAGRAVE, SANTIAGO VALASCO-FORERO AND TEODORA PETRISOR

› Space filling positionality and the Spiroformer - Pablo Suarez-serrato, Miguel Angel Evangelista Alvarado, Miguel Maurin

Abstract: Transformers excel when dealing with sequential data.
Generalizing transformer models to geometric domains, such as manifolds, we encounter the problem of not having a well-defined global order.
We propose a solution with attention heads following a space-filling curve.
As a first experimental example, we present the Spiroformer, a transformer that follows a polar spiral on the 2-sphere.

› Image Recognition via Vaisman–Neifeld's Geometry - Noemie Combe

Abstract: We introduce a new approach to the reconstruction of hidden structures from incomplete data, unifying techniques from geometric integration and topological analysis within the pioneering frameworks of Vaisman and Neifeld. Our method transcends traditional iterative schemes by employing a refined geometric decomposition of configuration spaces into invariant foliations and moment maps, thereby resolving the intrinsic ambiguities of underdetermined inverse problems.
By synergistically combining Vaisman's deep insights into symmetry and Neifeld's analytic methodologies, we establish a robust, noise-resistant paradigm that not only ensures computational tractability but also fundamentally redefines the landscape of reconstruction in imaging and structural analysis. This framework paves the way for transformative applications across diverse scientific domains, heralding a new era in the synthesis of geometry and topology for inverse problem solving.
Unlike conventional telemetry systems, this solution delivers unparalleled flexibility and scalability, as the number of nodes in the network can be expanded

as needed. The system's physical layer capabilities enable long-range, high-data-rate communication, while the unique network layer algorithm ensures reliable relaying capabilities.
This paper details the practical implementation of the telemetry system within the D328 Deutsche Aircraft flight test campaign, highlighting its advantages over traditional solutions. Key use cases include data acquisition and relay between multiple airborne systems and ground stations, demonstrating the system's potential to significantly enhance range and reliability in demanding environments. By reducing dependency on direct line-of-sight, this approach paves the way for more robust and efficient telemetry operations in future aerospace applications.

› The Stick Model for Distance Geometry - Antonio Mucherino

Abstract: The Distance Geometry Problem (DGP) asks whether a simple weighted undirected graph G can be realized in the Euclidean space so that the distances between embedded vertices correspond to the edge weights. The DGP is a rich and active research field, with many important applications. Several approaches to the DGP are based on the idea of directly placing the vertices of G in space. Our model uses a completely new approach: we focus our attention on the edges, and not on the vertices, and we attempt placing in space the "sticks" that can be associated to each edge of the graph. Sticks have fixed length (hence they always satisfy all distance constraints), and they admit three total degrees of freedom (position of one vertex, plus the stick orientation) in 2D. The automatic satisfaction of all DGP constraints comes at the cost of possibly having several distinct positions associated to the same vertex, potentially a different one for every stick where each vertex is involved. Therefore, we formulate a problem consisting in finding stick configurations where all vertices involved in multiple sticks can find a unique position in space, implying in turn the definition of a valid realization for the original DGP. We initially focus the attention on DGPs where the information on the stick orientations is a priori given, so that to formulate a convex quadratic optimization problem with linear constraints. For the general case, we propose a heuristic which solves, at each iteration, an instance of the quadratic problem.

› Generating random hyperfractal cities - Geoffrey Deperle, Philippe Jacquet

Abstract: This paper focuses on the challenge of interactively modeling street networks. In order to provide usable datasets for artificial intelligence applications, it is often necessary to generate random cities with adjustable parameters, such as the spatial extent of the city and its traffic distribution. Several models have been developed for this purpose, including the hyperfractal model introduced by Philippe Jacquet. This model offers a significant advantage as it accounts not only for the fractal geometry of urban structures but also for the statistical distribution of traffic within a city. In this work, we extend the simple fractal model, which is particularly useful for describing small cities or individual districts, by constructing random cities based on a tiling structure over which hyperfractals are distributed. This approach enables the connection of multiple hyperfractal districts, providing a more comprehensive urban representation.
Furthermore, we demonstrate how this decomposition can be used to segment a city into distinct districts through fractal

analysis. Finally, we present tools for the numerical generation of random cities following this model.

› Shape Theory via the Atiyah–Molino Reconstruction and Deformations? - Noemie Combe

Abstract: Reconstruction problems lie at the very heart of both mathematics and science, posing the enigmatic challenge: How does one resurrect a hidden structure from the shards of incomplete, fragmented, or distorted data? In this paper, we introduce a new approach that harnesses the profound insights of the Vaisman Atiyah–Molino framework. In stark contrast to conventional methods that depend on persistent homology, our approach exploits the concept of the Vaisman centroid—an intrinsic invariant that encapsulates the averaged geometry of a data set—to resolve the inherent ambiguities of inverse problems. In the present paper, we focus on the theory and applications of the Vaisman centroid, offering an innovative perspective for Topological Data Analysis that eschews persistent homology in favor of a unified geometric paradigm. The subsequent paper will extend these ideas to a full reconstruction scheme via the Atiyah–Molino framework. Our method not only provides a robust and computationally tractable framework for the recovery of hidden structures but also opens new avenues for the analysis of high-dimensional and noisy data across the mathematical sciences.

DIVERGENCES IN STATISTICS AND MACHINE LEARNING - MICHEL BRONIATOWSKI AND WOLFGANG STUMMER

› Minimum of Divergences with Relaxation: a Hilbertian Alternative to Duality Approach - Valérie Girardin, Pierre Maréchal

Abstract: Generalized moment problems –called feature moments in the area of machine learning– are here considered with and without relaxation. The solution is the minimum of phi-divergences, according to an extended Maximum Entropy Principle. Inference from sampled data constraints leads to balance the divergence by a relaxation term.

In the literature, the form of the minimizing solution is obtained by resorting to Fenchel's duality theorem or the method of Lagrange multipliers. An alternative method, resorting to Hilbert spaces, presented here, yields a necessary and sufficient condition under second order assumptions. It is based on a decomposition via a nested procedure of the relaxed problem into two successive problems, one of which without relaxation.

› Relationship between H\»{o}lder Divergence and Functional Density Power Divergence: Intersection and Generalization - Masahiro Kobayashi

Abstract: In this study, we discuss the relationship between two families of density-power-based divergences with functional degrees of freedom—the H\ »{o}lder divergence and the functional density power divergence (FDPD)—based on their intersection and generalization.
These divergence families include the density power divergence and the \gamma-divergence as special cases.
First, we prove that the intersection of the H\ »{o}lder divergence and the FDPD is limited to a general divergence family introduced by Jones et al. (Biometrika, 2001).
Subsequently, motivated by the fact that H\ »{o}lder's inequality is used in the

proofs of nonnegativity for both the H\ »{o}lder divergence and the FDPD, we define a generalized divergence family, referred to as the \xi-H\ »{o}lder divergence.
The nonnegativity of the \xi-H\ »{o}lder divergence is established through a combination of the inequalities used to prove the nonnegativity of the H\ »{o}lder divergence and the FDPD.
Furthermore, we derive an inequality between the composite scoring rules corresponding to different FDPDs based on the \xi-H\ »{o}lder divergence.
Finally, we prove that imposing the mathematical structure of the H\ »{o}lder score on a composite scoring rule results in the \xi-H\ »{o}lder divergence.

› Bayesian-like estimation with unnormalized model - Takashi Takenouchi

Abstract: Parameter estimation of probabilistic models for discrete variables is often infeasible due to the calculation of the normalization constant required to ensure the model represents a valid probability distribution, and various approaches have been developed to resolve this problem.
In this paper, we consider a computationally feasible estimator for discrete probabilistic models based on a concept of empirical localization.
Furthermore, we propose a computationally feasible estimator similar to the MAP estimator in Bayesian estimation by extending the above estimator.

› Some smooth divergences for ell1-approximations - Pierre Bertrand, Wolfgang Stummer

Abstract: For some smooth special case of generalized phi-divergences as well as of new divergences (called scaled shift divergences), we derive approximations of the omnipresent (weighted) ell1-distance and (weighted) ell1-norm.

› A Connection Between Learning to Reject and Bhattacharyya Divergences - Alexander Soen

Abstract: Learning to reject provide a learning paradigm which allows for our models to abstain from making predictions. One way to learn the rejector is to learn an ideal marginal distribution (w.r.t. the input domain) — which characterizes a hypothetical best marginal distribution — and compares it to the true marginal distribution via a density ratio. In this paper, we consider learning a joint ideal distribution over both inputs and labels; and develop a link between rejection and thresholding different statistical divergences. We further find that when one considers a variant of the log-loss, the rejector obtained by considering the joint ideal distribution corresponds to the thresholding of the skewed Bhattacharyya divergence between class-probabilities. This is in contrast to the marginal case — that is equivalent to a typical characterization of optimal rejection, Chow's Rule — which corresponds to a thresholding of the Kullback-Leibler divergence. In general, we find that rejecting via a Bhattacharyya divergence is less aggressive than Chow's Rule.

GEOMETRIC LEARNING AND DIFFERENTIAL INVARIANTS ON HOMOGENEOUS SPACES - REMCO DUITS, ERIK BEKKERS

› Global Positioning on Earth – Mireille Boutin, Rob Eggermont, Gregor Kemper

Abstract: Contrary to popular belief, the global positioning problem on earth may have more than one solutions even if the user position is restricted to a sphere. With 3 satellites, we show that there can be up to 4 solutions on a sphere. With 4 or more satellites, we show that, for any pair of points on a sphere, there is a family of hyperboloids of revolution such that if the satellites are placed on one sheet of one of these hyperboloid, then the global positioning problem has both points as solutions. We give solution methods that yield the correct number of solutions on/near a sphere.

› Analysis and Computation of Geodesic Distances on Reductive Homogeneous Spaces - Remco Duits, Gijs Bellaard, Barbara Tumpach

Abstract: Many geometric machine learning and image analysis applications, require a left-invariant metric on the 5D homogeneous space of 3D positions and orientations SE(3)/SO(2). This is done in Equivariant Neural Networks (G-CNNs), or in PDE-Based Group Convolutional Neural Networks (PDE-G-CNNs), where the Riemannian metric enters in multilayer perceptrons, message passing, and max-pooling over Riemannian balls. In PDE-G-CNNs it is proposed to take the minimum left-invariant Riemannian distance over the fiber in SE(3)/SO(2), whereas in G-CNNs and in many geometric image processing methods an efficient SO(2)-conjugation invariant section is advocated. The conjecture rises whether that computationally much more efficient section indeed always selects distance minimizers over the fibers. We show that this conjecture does NOT hold in general, and in the logarithmic norm approximation setting used in practice we analyze the small (and sometimes vanishing) differences. We first prove that the minimal distance section is reached by minimal horizontal geodesics with constant momentum and zero acceleration along the fibers, and we generalize this result to (reductive) homogeneous spaces with legal metrics and commutative structure groups.

› Universal Collection of Euclidean Invariants between Pairs of Position-Orientations - Gijs Bellaard, Bart M. N. Smets, Remco Duits

Abstract: Euclidean E(3) equivariant neural networks that employ scalar fields on position-orientation space M(3) have been effectively applied to tasks such as predicting molecular dynamics and properties.
To perform equivariant convolutional-like operations in these architectures one needs Euclidean invariant kernels on M(3) x M(3).
In practice, a handcrafted collection of invariants is selected, and this collection is then fed into multilayer perceptrons to parametrize the kernels.
We rigorously describe an optimal collection of 4 smooth scalar invariants on the whole of M(3) x M(3).
With optimal we mean that the collection is independent and universal, meaning that all invariants are pertinent, and any invariant kernel is a function of them.
We evaluate two collections of invariants, one universal and one not, using the PONITA neural network architecture.
Our experiments show that using a collection of invariants that is universal positively impacts the accuracy of PONITA significantly.

› Roto-Translation Invariant Metrics on Position-Orientation Space - Gijs Bellaard, Bart M. N. Smets

Abstract: Riemannian metrics on the position-orientation space M(3) that are roto-translation group SE(3) invariant play a key role in image analysis tasks like enhancement, denoising, and segmentation. These metrics enable roto-translation

equivariant algorithms, with the associated Riemannian distance often used in implementation.
However, computing the Riemannian distance is costly, which makes it unsuitable in situations where constant recomputation is needed.
We propose the mav (minimal angular velocity) distance, defined as the Riemannian length of a geometrically meaningful curve, as a practical alternative.
We see an application of the mav distance in geometric deep learning.
Namely, neural networks architectures such as PONITA, relies on geometric invariants to create their roto-translation equivariant model.
The mav distance offers a trainable invariant, with the parameters that determine the Riemannian metric acting as learnable weights.
In this paper we:
1) classify and parametrize all SE(3) invariant metrics on M(3),
2) describes how to efficiently calculate the mav distance,
and 3) investigate if including the mav distance within PONITA can positively impact its accuracy in predicting molecular properties.

› Group Morphology Fixed Points on Homogenous Spaces for Deep Learning Equivariant Networks - Jesus Angulo

Abstract: This paper explores the theoretical integration of mathematical morphology with deep learning, specifically focusing on creating neural network layers that inherently produce fixed points through iterative application of operators. It investigates how principles from mathematical morphology, such as idempotence and convergence of operators in complete lattices, can be leveraged to design efficient and stable deep learning architectures. The work extends these concepts to group-equivariant operators on homogeneous spaces, aiming to build nonlinear iterative layers in deep convolutional neural networks that respect symmetries present in the data. By examining group convolutions, dilations and erosions, the paper lays theoretical groundwork for constructing equivariant fixed-point layers using either maxplus group operations or group convolutions.

› Flow Matching on Lie Groups - Finn Sherry, Bart Smets

Abstract: Flow Matching (FM) is a recent generative modelling technique by Lipman et al. (2022): we aim to learn how to sample from distribution X1 by flowing samples from some distribution X0 that is easy to sample from.
The key trick is that this flow field can be trained while conditioning on the end point in X1: given an end point, simply move along a straight line segment to the end point.
However, straight line segments are only well-defined on Euclidean space. Consequently, Chen et al. (2023) generalised the method to FM on Riemannian manifolds, replacing line segments with geodesics or their spectral approximations.
We take an alternative point of view: we generalise to FM on Lie groups by instead substituting exponential curves for line segments. This leads to a simple, intrinsic, and fast implementation for many matrix Lie groups, since the required Lie group operations (products, inverses, exponentials, logarithms) are simply given by the corresponding matrix operations.
FM on Lie groups could then be used for generative modelling with data consisting of sets of features (in R^n) and poses (in some Lie group), e.g. the latent codes of Equivariant Neural Fields Wessels et al. (2025).

Poster Session

› QUANTUM GEOMETRY INSIGHTS IN DEEP LEARNING – *Noemie Combe (Max Planck Institute for Maths)**

Abstract: In this paper, we explore the fundamental role of the Monge–Ampère equation in deep learning, particularly in the context of Boltzmann machines and energy-based models. We first review the structure of Boltzmann learning and its relation to free energy minimization. We then establish a connection between optimal transport theory and deep learning, demonstrating how the Monge–Ampère equation governs probability transformations in generative models. Additionally, we provide insights from quantum geometry, showing that the space of covariance matrices arising in the learning process coincides with the Connes–Araki–Haagerup (CAH) cone in von Neumann algebra theory. Furthermore, we introduce an alternative approach based on renormalization group (RG) flow, which, while distinct from the optimal transport perspective, reveals another manifestation of the Monge–Ampère domain in learning dynamics. This dual perspective offers a deeper mathematical understanding of hierarchical feature learning, bridging concepts from statistical mechanics, quantum geometry, and deep learning theory.

› Combinatorial Characterization of Exponential Families of Lumpable Stochastic Matrices – *Shun Watanabe (Tokyo University of Agriculture and Technology), Geoffrey Wolfer (Waseda University)**

Abstract: A Markov chain is called lumpable if it can be aggregated into a simpler form by merging certain states, while still preserving the Markov property in the reduced chain. In this paper, we initiate the problem of characterizing exponential families of lumpable Markov chains with respect to a fixed lumping map, and we provide the first set of necessary and sufficient conditions for this characterization.

› The moduli spaces of left-invariant statistical structures on Lie groups – *Hikozo Kobayashi (Hiroshima University)**

Abstract: In the context of information geometry, the concept known as left-invariant statistical structure on Lie groups is defined by Furuhata–Inoguchi–Kobayashi [Inf.Geom.(2021)]. In this poster presentation, we introduce the notion of the moduli space of left-invariant statistical structures on a Lie group. We study the moduli spaces for three particular Lie groups, each of which has a moduli space of left-invariant Riemannian metrics that is a singleton. As applications, we classify left-invariant conjugate symmetric statistical structures and left-invariant dually flat structures on these three Lie groups. A characterization of the Amari–Chentsov \alpha-connection on the Takano Gaussian space is also given. This talk is based on joint work with Yu Ohno, Takayuki Okuda (Hiroshima University), and Hiroshi Tamaru (Osaka Metropolitan University).

› Renormalization of circle maps with a flat piece Bertuel TANGUE NDAWA (University of Ngaoundere)

Abstract: We consider order preserving C^3 circle maps with a flat piece, Fibonacci rotation number, and negative Schwarzian derivative where the critical exponents (the degrees of the singularities at the boundary of the flat piece) might be different.In this work, we assume that critical exponents belong to (1,2)^2. As a consequence, the geometry of the wondering (fractal) set of a map our class is degenerate (that is, the scaling ratio goes to zero), and we prove that its renormalization diverges.

› Value of Information in Bayesian Environments

– *Stefan Behringer (U Bielefeld), Roman Belavkin (Middlesex University)*

Abstract: The Value of Information (VoI) framework, as developed by Ruslan Stratonovich, bridges Claude Shannon's information theory with economics, in particular the fields of utility and decision theory. This paper revisits the VoI concept within the Boolean setting well known from hypothesis testing. Then the paper explores examples and results that extend the Bayesian VoI framework beyond the Boolean case.

› Dual Quaternion Variational Autoencoders for Efficient Robotic Motion Learning

– *Aneeqa Mehrab (University of Ferrara), Cinzia Bisi (University of Ferrara), Eduardo Bayro Corrochano (University of technology, Poland), Marcin Kiełczewski (Institute of Automatic Control and Robotics of the Poznan University of technology, Poland)*

Abstract: The research examines how Dual Quaternion Variational Autoencoders (DQVAE) advance robotic mobility by applying them in practice. Quaternion Variational Autoencoders (QVAE) operates normally to monitor rotational movements yet lacks the capability to include translational movement. The development of QVAE into DQVAE enhances its ability to understand motion that combines simultaneous rotation with translation movements.The study aims to develop a Riemann (Study) Manifold structure inside SE(3) latent space for generating optimal end effector route trajectories through geodesics. The proposed solution applies Riemannian manifold geometry to develop flexible and accurate motion learning tasks. We can use this dual quaternion manifold, to find out geodesic for optimal motion planning. It can be used even to avoid dynamic obstacles.

› ECoNet: Rotation Equivariant Contrail Detection Neural Network in Satellite Imagery

– *Edgar Loza (CortAlx Labs), Davide Di Giusto (CortAlx Labs), Vincent Meijer (Delft Univ. of Technology), Teodora Petrisor (CortAlx Labs)*

Abstract: Contrails, aircraft-induced cirrus clouds, likely have a largecontribution on climate change. Their presence can be verified via re-mote sensors (e.g. geostationary satellites), which helps in establishing climate impact calculations. However, contrail detection is a challenging, highly unbalanced computer vision task, similar to medical imagery, where most of the pixels constitute the background. In this work, we propose ECoNet – a rotation-equivariant U-Net exploiting these naturalsymmetries present in satellite imagery. We show that equivariant net-works bring benefits in detection accuracy with less parameters to trainand training time reduced by a factor of 3. We also explore the trans-fer learning on European geostationary satellites to assess its scalability potential.

A Short Proof of Chentsov's Theorem

– Hui-An Shen (University of Bern)

Abstract: Chentsov's Theorem is foundational in information geometry and characterizes the Riemannian metrics satisfying Markov invariance. Amari showed that f-divergences are the unique divergences that satisfy both information monotonicity and decomposibility. In this paper, by relating information monotonicity to Markov invariance, and by generalizing Amari's result, we give a short proof of Chentsov's Theorem. To that end, we introduce two auxiliary theorems. The first auxiliary theorem (Theorem 3) introduces a concept called local-decomposibility (as opposed to decomposibility), and uses it to characterize f-divergences. The second auxiliary theorem (Theorem 4) states that decomposibility is necessary for Markov-invariant Riemannian metrics on probability simplices, the proof of which utilizes a key idea from Campbell's proof for a generalization of Chentsov's Theorem. Finally to complete the proof, we apply an elementary result in Riemannian geometry, where the bilinear form given by the second-order Taylor series of the squared geodesic distance recovers the underlying Riemannian metric. As an application of Chentsov's Theorem and as a concrete example of the interplay between divergences and Riemannian metrics on probability simplices, we introduce a new characterization of the Fisher-Rao distance (Theorem 5).

Symmetry-Loss for Supervised Machine Learning

– Arif Dönmez (IUF – Leibniz Research Institute for Environmental Medicine)

Abstract: Integrating symmetry priors is crucial for robust machine learning. While equivariant neural networks, which enforce symmetry through specialized layer design (e.g., GNNs, E(n)-equivariant networks), represent the dominant paradigm, they can be complex to design for arbitrary groups. This paper proposes an alternative, complementary approach rooted in classical Invariant Theory. Instead of relying on intrinsically equivariant architectures, we introduce a novel symmetry loss function added to the standard supervised objective. This loss penalizes deviations from desired invariance or equivariance properties under a specified group action, effectively regularizing the model towards symmetric solutions without mandating architectural constraints. We formalize this concept, presenting a framework combining standard feature extractors with the symmetry loss, and discuss its theoretical underpinnings. This approach offers flexibility in applying symmetry principles to diverse models. This work presents a promising new direction for leveraging symmetry in supervised learning via loss-based regularization.

Statistical relative curvature tensor fields

– Matsuzoe Hiroshi (Nagoya Institute of Technology)

Abstract: In information geometry, a pair of a primal affine connection and its dual affine connection plays an important role. It is natural to consider tensor fields that reflect the geometry of mutually dual affine connections, not just the curvature tensor field and the torsion tensor field. From this motivation, statistical relative curvature tensor fields and statistical relative torsion tensor fields are introduced.

A geometric approach to Transformer-enabled algebraic computation

- Yota Maeda

Abstract: Recent progress in deep learning applied to symbolic mathematics highlights the importance of dataset quality, as emphasized by Lample and Charton. We revisit the problem of learning Gröbner basis computation using Transformer models. While a dataset generation method tailored to this task was proposed previously, it lacked theoretical assurance regarding its breadth or representativeness.
In this work, we prove that datasets produced by the earlier algorithm are geometrically general, in the sense that they form a Zariski dense subset of the space of generating sets for a fixed ideal. This result suggests that Transformers trained on such data could, in principle, cover a broad variety of Gröbner bases.

› Geometry of the Coupled Exponential Family: Simplifying Complexity - *Igor Oliveria, Amenah Al Najafi, and Kenric Nelson Photrek*

Abstract: The geometry of the coupled exponential family is defined to provide a simple separation between nonlinear sources of uncertainty, which define a generalization of the exponential family, and the linear sources of uncertainty, which define the scale of the distributions. The nonlinear statistical coupling (\kappa) measures the shape, nonlinearity, and complexity. The independent equals or Tsallis parameter q measures the number of independent random variables in the same state and is dependent of the coupling, the location shape, and the dimensions. The generalized moments of the coupled exponential distributions, which are a function of the independent equals distribution, are guaranteed to be finite. From this foundation, the information geometric definitions for the coupled entropy and its maximizing distributions the coupled exponential family are derived.
By clearly defining the physical properties of linear uncertainty (scale) and nonlinear uncertainty (shape/coupling/complexity) the coupled entropy is shown to resolve shortcomings in the Rényi and Tsallis entropy functions. Together, the coupled exponential family and coupled entropy function provide an analytical framework for modeling complex systems across diverse domains, from physics and finance to climate and artificial intelligence, exhibiting heavy-tailed phenomena and nonlinear interdependencies. The generalized Fisher metric, its gradient, and the dual connectors are derived, providing the tools for the design of innovative machine learning methods. Applied to the coupled free energy as a cost function for variational inference, algorithms which account for extreme risk via the modeling of of the coupling above one thousand are achievable; despite the variance diverging above one-half. And yet, models with the coupling below one thousandth are shown to improve image perception compared with exponential models.

› Conditional Idempotent Generative Networks (CIGN) - *Niccolo Ronchetti*

Abstract: We propose Conditional Idempotent Generative Networks (CIGN), a novel approach that expands upon Idempotent Generative Networks (IGN) to enable conditional generation. While IGNs offer efficient single-pass generation, they lack the ability to control the content of the generated data. CIGNs address this limitation by incorporating conditioning mechanisms, allowing users to steer the generation process towards specific data samples.
We establish the theoretical foundations for CIGNs, outlining their scope, loss function design, and evaluation metrics.
We then discuss two architectures for implementing CIGNs: channel conditioning

and filter conditioning. Finally, we showcase experimental results on the MNIST dataset, demonstrating the effectiveness of both architectures.

› Hilbert geometry of the symmetric positive-definite bicone - *Jacek Karwowski*

Abstract: The extended Gaussian family is the closure of the Gaussian family obtained by completing the Gaussian family with the counterpart elements induced by degenerate covariance or degenerate precision matrices, or a mix of both degeneracies. The parameter space of the extended Gaussian family forms a symmetric positive semi-definite matrix bicone, i.e. two partial symmetric positive semi-definite matrix cones joined at their bases. We study the Hilbert geometry of such an open bounded convex symmetric positive-definite bicone. We report the closed-form formula for the corresponding Hilbert metric distance and characterise exhaustively its invariance properties. We also touch upon potential applications of this geometry for dealing with extended Gaussian distributions.

› Information Geometry of Exponentiated Gradient: Convergence beyond L-Smoothness - Yara Elshiaty

Abstract: We study the minimization of smooth, possibly nonconvex functions over the positive orthant, a key setting in Poisson inverse problems. Our approach is grounded in information geometry: we interpret the exponentiated gradient (EG) method as Riemannian gradient descent (RGD) on the positive orthant endowed with the Fisher information geometry. In this view, the EG update corresponds to employing the e-Exponential map as a retraction. Leveraging this geometric formulation, we establish global convergence under weak assumptions—without requiring L-smoothness—and prove finite termination of the Riemannian Armijo line search. Numerical experiments, including an accelerated variant, demonstrate EG's practical advantages, such as faster convergence compared with interior-point-based geometry.

Program at glance

DAY 1 - OCTOBER 29TH

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
09:00	**"Conference Opening Session Frédéric Barbaresco & Frank Nielsen (GSI'25 General Chairs)"**		
09:30	**"Topological Deep Learning: Unlocking the Structure of Relational Systems Nina Miolane (UC Santa Barbara, USA)"**		
10:30	**Coffee Break + GSI'25 Posters Session**		
11:00–12:20	"Geometric Statistics (Session 1) Chairmen: Xavier Pennec, Stefan Sommer, Benjamin Eltzner"	"A geometric approach to differential equations (Session 1) Chairman: Javier de Lucas Araujo"	"Lie Group in Learning Distributions & in Filters (Session 1) Chairmen: Eren M. Kiral, Koichi Tojo, Ha Q. Minh"
12:20–12:40	**Lunch break + GSI'25 Posters Session**		
14:00	**"The Lp Fisher-Rao metrics and the alpha-connections Alice Le Brigant (Université Paris 1 Panthéon-Sorbonne, France)"**		
15:00–16:00	"Geometric Statistics (Session 2) Chairmen: Xavier Pennec, Stefan Sommer, Benjamin Eltzner"	"A geometric approach to differential equations (Session 2) Chairman: Bartosz Zawora"	"Lie Group in Learning Distributions & in Filters (Session 2) Chairmen: Eren M. Kiral, Koichi Tojo, Ha Q. Minh"
16:00–16:30	**Coffee Break + GSI'25 Posters Session**		
16:30–17:50	"Neurogeometry Chairmen: Alessandro Sarti, Giovanna Citti, Giovanni Petri"	"New trends in Nonholonomic Systems Chairmen: Manuel de Leon, Leonardo Colombo"	"Learning of Dynamic Processes Chairman: Stéphane Chretien"
18:45	**Conference Group Photo**		
19:00	**Cocktail**		

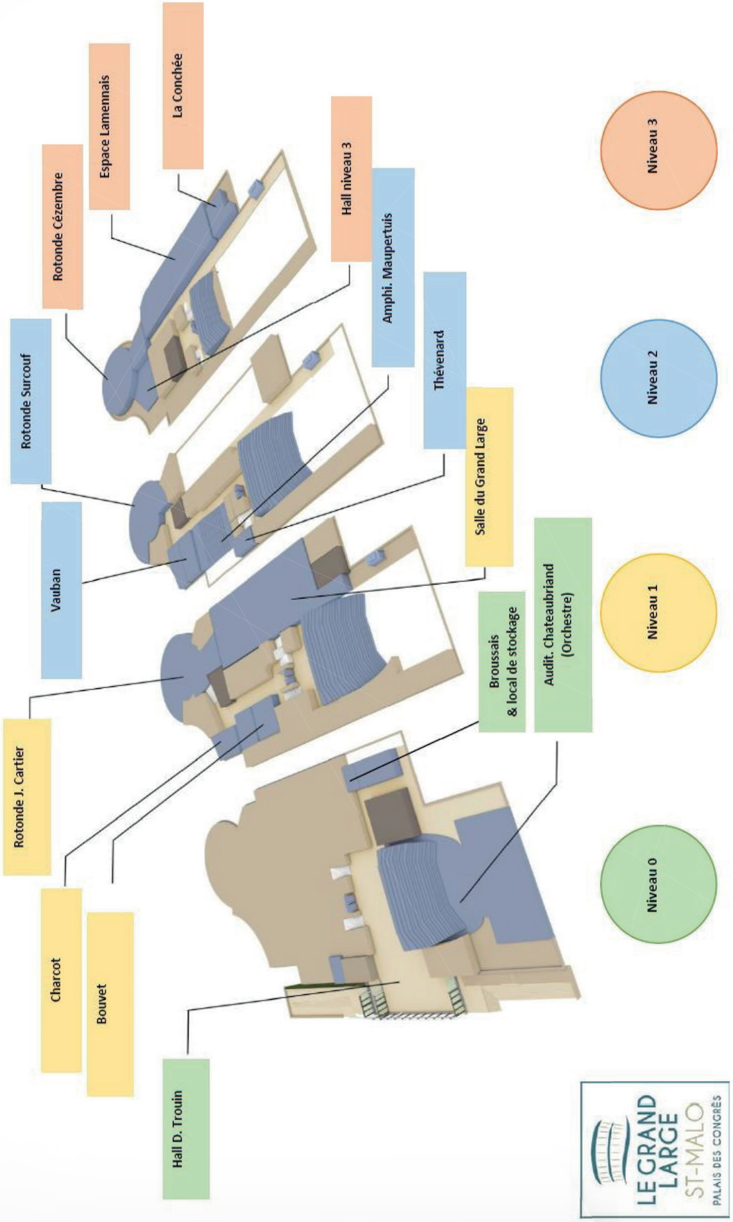
DAY 2 - OCTOBER 30TH

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
09:00	**Metriplectic Dynamics: A Geometrical Framework for Thermodynamically Consistent Dynamical Systems - Philip J. Morrison (The University of Texas at Austin, USA)**		
10:00	"Geometric Methods in Thermodynamics (Session 1) Chairman : François GAY-BALMAZ, Hiroaki YOSHIMURA"	"Stochastic Geometric Dynamics Chairman : Ana Bela CRUZEIRO, Jean-Claude ZAMBRINI, Stefania UGOLINI"	"Optimization and learning on manifolds (Session 1) Chairman : Cyrus MOSTAJERAN, Salem SAID"
11:20	Coffee Break + GS1'25 Posters Session (Chair: Rita FIORESI)		
11:50	"Geometric Methods in Thermodynamics (Session 2) Chairman : François GAY-BALMAZ, Hiroaki YOSHIMURA"	"Classical & Quantum Information, Geometry and Topology Chairman : Florio M. CIAGLIA, FABIO DI COSMO, Pierre BAUDOT and Grégoire SERGEANT-PERTHUIS"	"Optimization and learning on manifolds (Session 2) Chairman : Cyrus MOSTAJERAN, Salem SAID"
13:30	Lunch Break + GS1'25 Posters Session (Chair: Rita FIORESI)		
15:00	**Bicentenary of Thermodynamics and Sadi Carnot's Seminal Work: From Constantin Carathéodory's Contact Geometry Model to Jean-Marie Souriau's Symplectic Foliation Structure - Frédéric BARBARESCO (THALES, France)**		
16:00	Coffee Break + GS1'25 Posters Session (Chair Rita FIORESI)		
16:30 - 18h30	"Information Geometry, Delzant Toric Manifold & Integrable System Chairman : Mathieu MOLITOR, Hajime FUJITA, Daisuke TARAMA and Frédéric BARBARESCO"	"Geometric Green Learning on Groups and Quotient Spaces Chairman : Alice Barbara TUMPACH, Diarra FALL and Levin MAIER"	"Applied Geometry-Informed Machine Learning (Session 1) Chairman : Pierre-Yves LAGRAVE, Santiago VALASCO-FORERO and Teodora PETRISOR"
19:30	Conference Group Photo at Maison du Corsaire		
20:00	Gala Dinner at Maison du Corsaire		

DAY 3 - OCTOBER 31ST

Time	Auditorium Maupertuis	Room Vauban 1	Room Vauban 2
08:30	**A Noncommutative perspective of Graph Neural Networks - Rita FIORESI (University of Bologna, Italy)**		
09:30	"Geometric Mechanics (Session 1) Chairman : Gery de SAXCE, Zdravko TERZE, François DUBOIS"	"Computational Information Geometry and Divergences (session 1) Chairman : Frank NIELSEN and Olivier RIOUL"	"Statistical Manifolds and Hessian information geometry (Session 1) Chairman : Michel NGUIFFO BOYOM, Stéphane PUECHMOREL"
11:10 - 11:40	**Coffee Break + GS1'25 Posters Session**		
11:40	"Geometric Mechanics (Session 2) Chairman : Gery de SAXCE, Zdravko TERZE, François DUBOIS"	"Computational Information Geometry and Divergences (session 2) Chairman : Frank NIELSEN and Olivier RIOUL"	"Applied Geometry-Informed Machine Learning (Session 2) Chairman : Pierre-Yves LAGRAVE, Santiago VALASCO-FORERO and Teodora PETRISOR"
13:20 - 14:50	**Lunch Break + GS1'25 Posters Session**		
14:50	**Extended Variational Learning via Fenchel–Young Losses - Mário A.T. FIGUEIREDO (Universidade de Lisboa, Portugal)**		
15:50	"Divergences in Statistics and Machine Learning Chairman : Michel BRONIATOWSKI and Wolfgang STUMMER"	"Statistical Manifolds and Hessian information geometry (Session 2) Chairman : Michel NGUIFFO BOYOM, Stéphane PUECHMOREL"	"Geometric Learning and Differential Invariants on Homogeneous Spaces Chairman : Remco DUITS, Erik BEKKERS"
17:50	**Closing Session (Paper Awards)**		

Palais du Grand-Large Map



Room Maupertuis (2nd floor)
Plenary Sessions & Parallel Sessions

Welcome Desk (Ground Floor)
Registration & Badges

Room Vauban 1&2 (2nd floor)
Parallel Sessions

Rotonde Surcouf (2nd floor)
Coffee Breaks & Posters Session

Welcome Cocktail

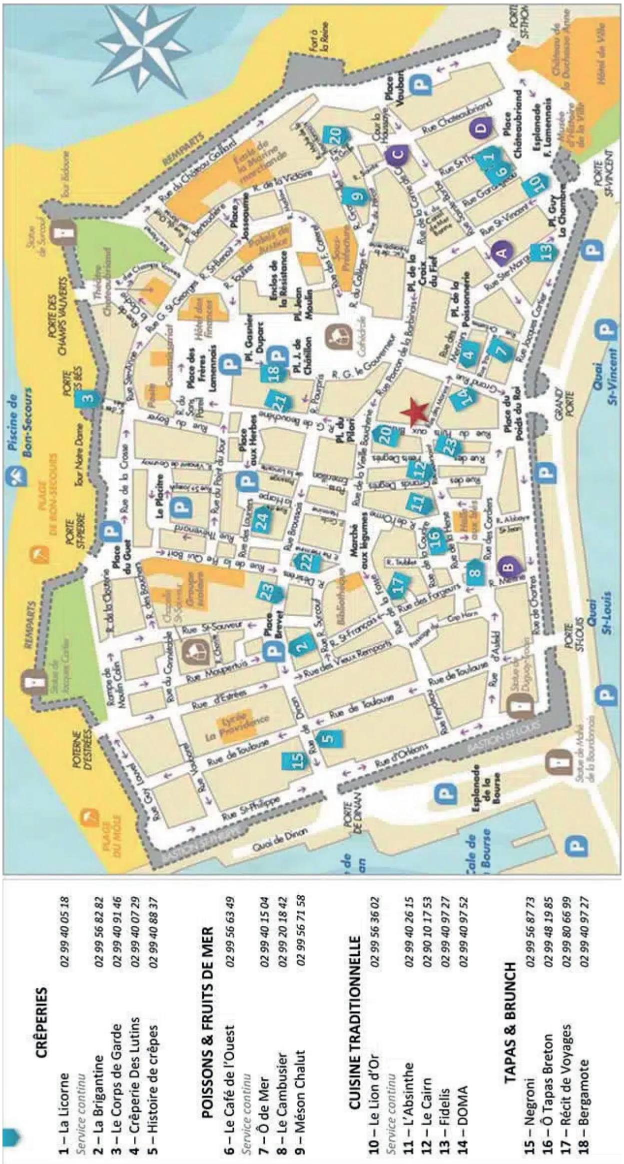
The Welcome Cocktail Reception will take place on October 29th at 19.00pm in La Rotonde Surcouf.
Same floor than Maupertuis Plenary Session Room.

Gala Dinner

The Gala Dinner will take place on October 30th at 20.00 pm in La Demeure de Corsaire. 5 Rue d'Asfeld, 35400 Saint-Malo

The gala dinner is included in the full registrations only.

List of restaurants



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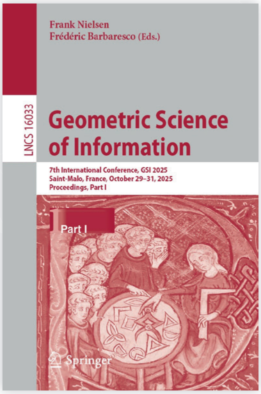
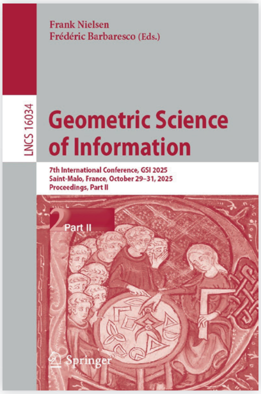
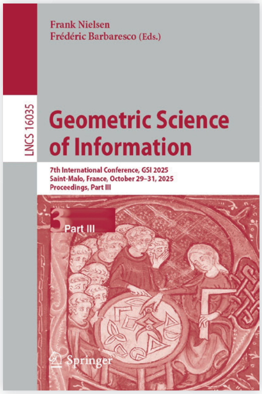
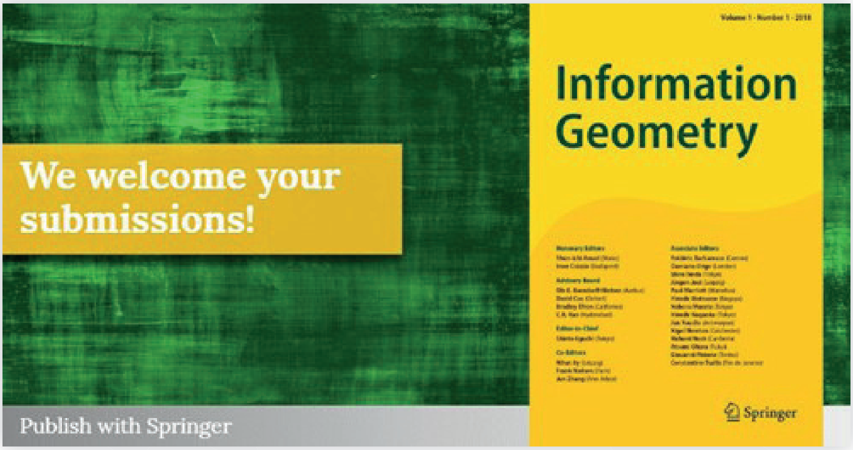






GSI'25 Special Issue INGE



This special issue of Information Geometry (INGE) will include selected refereed papers presented at the 7th international conference on Geometric Science of Information (GSI'25) held in Saint-Malo, France, from 28th to 31st October 2025.

All papers will be refereed according to the high standards of INGE.

Paper Submission: Authors should submit their work to Information Geometry via the online platform: https://link.springer.com/journal/41884/submission-guidelines

Frederic Barbaresco and Frank Nielsen

GSI'25 MDPI Special Issue: Methods from Differential Topology and Differential Geometry in Information Geometry

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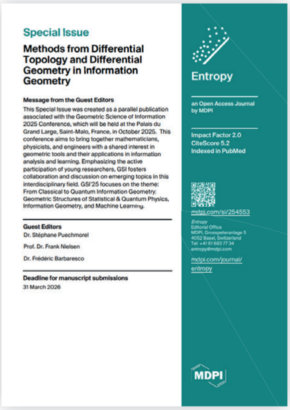

Special Issue

Methods from Differential Topology and Differential Geometry in Information Geometry

Message from the Guest Editors

This Special Issue was created as a parallel publication associated with the Geometric Science of Information 2025 Conference, which will be held at the Palais du Grand Large, Saint-Malo, France, in October 2025. This conference aims to bring together mathematicians, physicists, and engineers with a shared interest in geometric tools and their applications in information analysis and learning. Emphasizing the active participation of young researchers, GSI fosters collaboration and discussion on emerging topics in this interdisciplinary field. GSI'25 focuses on the theme: From Classical to Quantum Information Geometry: Geometric Structures of Statistical & Quantum Physics, Information Geometry, and Machine Learning.

Guest Editors

Dr. Stéphane Puechmorel

Prof. Dr. Frank Nielsen

Dr. Frédéric Barbaresco

Deadline for manuscript submissions

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