Loewner partial ordering $\preceq$

$$P \preceq Q$$

if and only if $Q - P$ is positive semi-definite:

$$\forall x, \quad x^\top (Q-P)x \geq 0$$

$$x' \leq x, y' \leq y \Rightarrow M(x,y) \leq M(x',y')$$

$$x' \leq x, y' \leq y \Rightarrow G(x',y') = \sqrt{x'y'} \leq G(x,y) = \sqrt{xy}$$

$$\mathrm{LogEuclideanMean}(X,Y) = \exp\left(\frac{\log X + \log Y}{2}\right),$$

$$G(X,Y) = X^{\frac{1}{2}}\,(X^{-\frac{1}{2}}\,Y\,X^{-\frac{1}{2}})^{\frac{1}{2}}\,X^{\frac{1}{2}}$$

$M(X,Y)$ is said operator monotone

$$X' \preceq X, Y' \preceq Y \Rightarrow M(X',Y') \preceq M(X,Y)$$