$$(\hat{\theta}_{\mathrm{MLE}}, l_x(\hat{\theta}_{\mathrm{MLE}}))$$

$$\kappa = \frac{1}{r} = I(\hat{\theta}_{\mathrm{MLE}})$$

$$Var[\hat{\theta}_{\mathrm{MLE}}] \text{ when } n \to \infty$$

$$Fisher information small: \\ \text{Likelihood curvature is small (flat peak)} \\ \text{Variance is large, low accuracy}$$

$$Fisher information large: \\ \text{Likelihood curvature is large (sharp peak)} \\ \text{Variance is small, good accuracy}$$

Taylor 2nd-order with
$$l'(\hat{\theta}_{\text{MLE}}) = 0$$
: $l_x(\theta) \approx l_x(\hat{\theta}_{\text{MLE}}) + \frac{1}{2}(\theta - \hat{\theta}_{\text{MLE}})^2 l''(\hat{\theta}_{\text{MLE}})$
 $-l''(\hat{\theta}_{\text{MLE}}) = -E_{\hat{\theta}_{\text{MLE}}}[l''(\theta)] = I(\hat{\theta}_{\text{MLE}}) \Rightarrow l_x(\theta) \approx l_x(\hat{\theta}_{\text{MLE}}) - \frac{1}{2}(\theta - \hat{\theta}_{\text{MLE}})^2 I(\hat{\theta}_{\text{MLE}})$