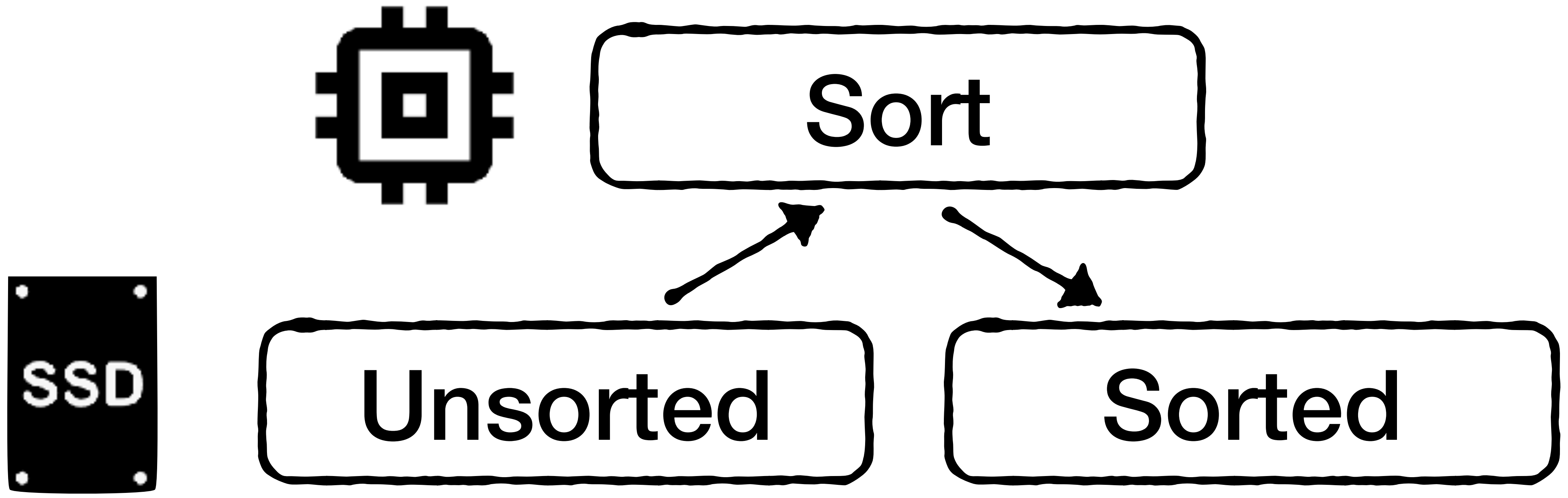
Analyzing I/O for External Sort

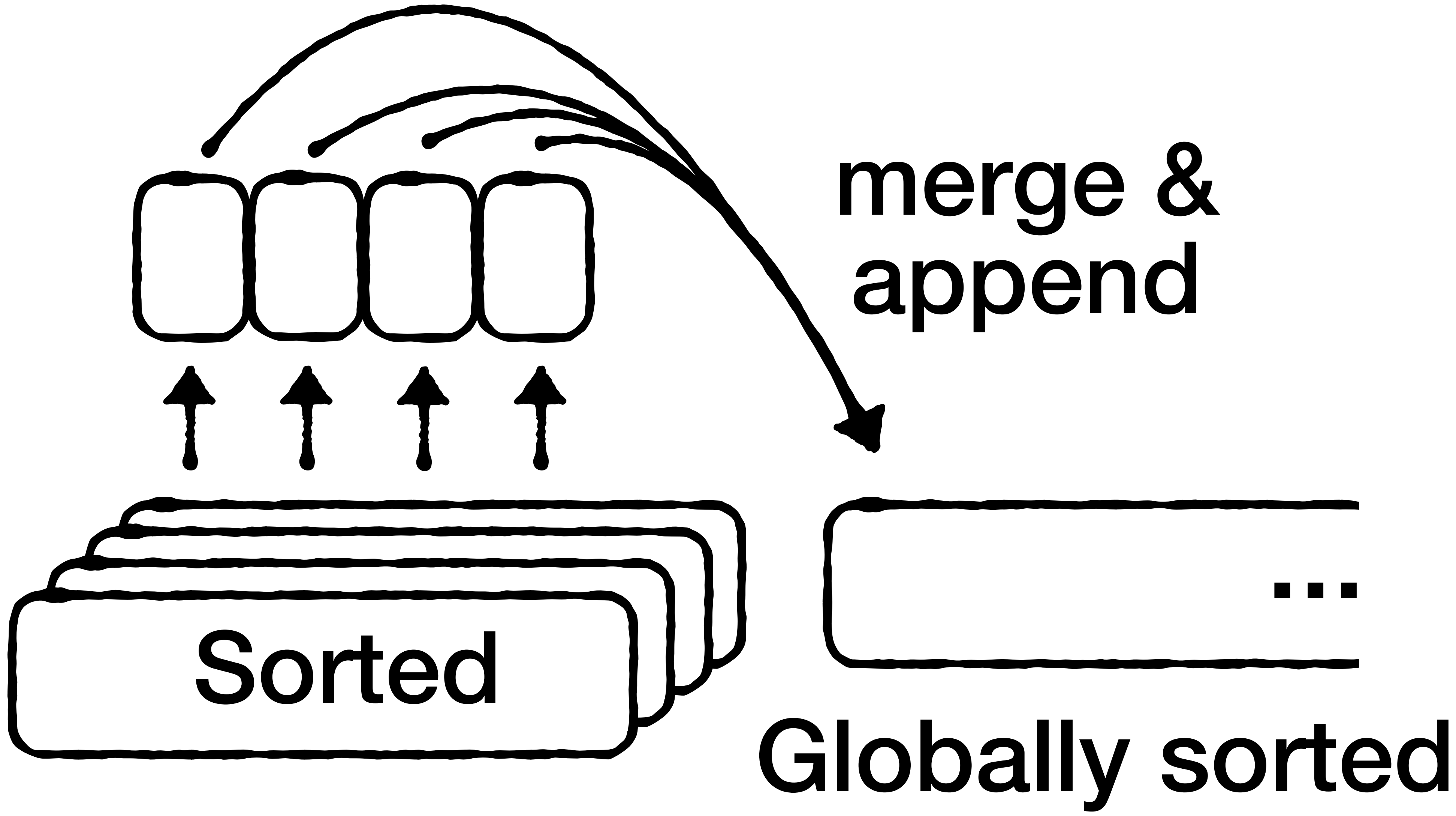
Partitioning Phase

N/B I/Os



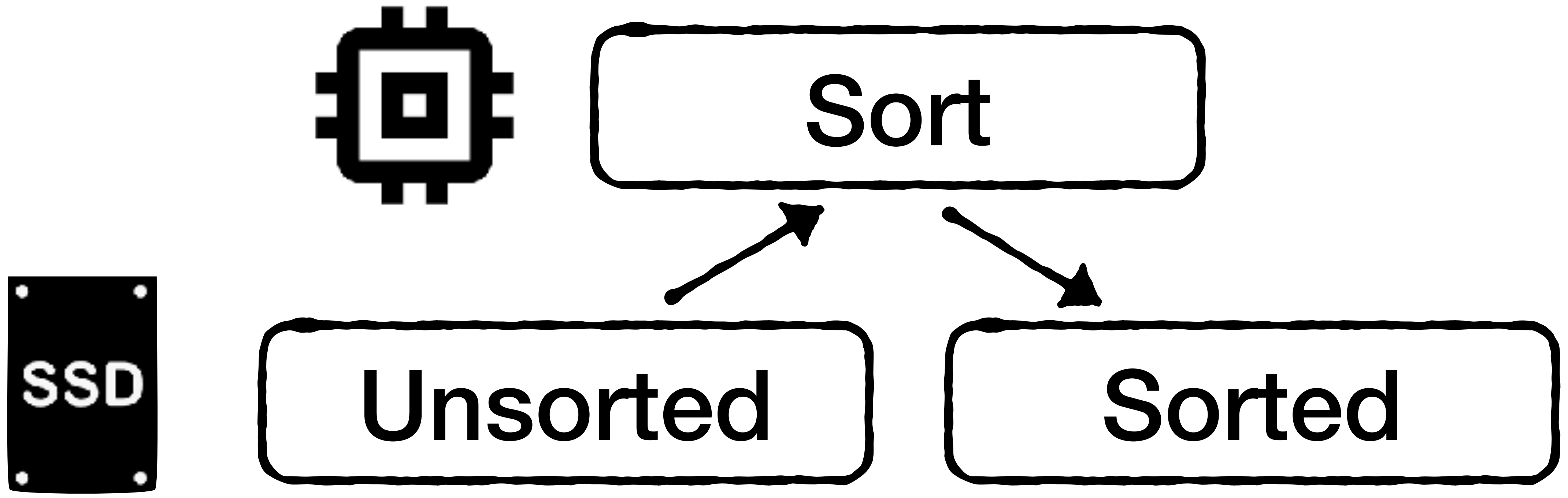
Merging Phase

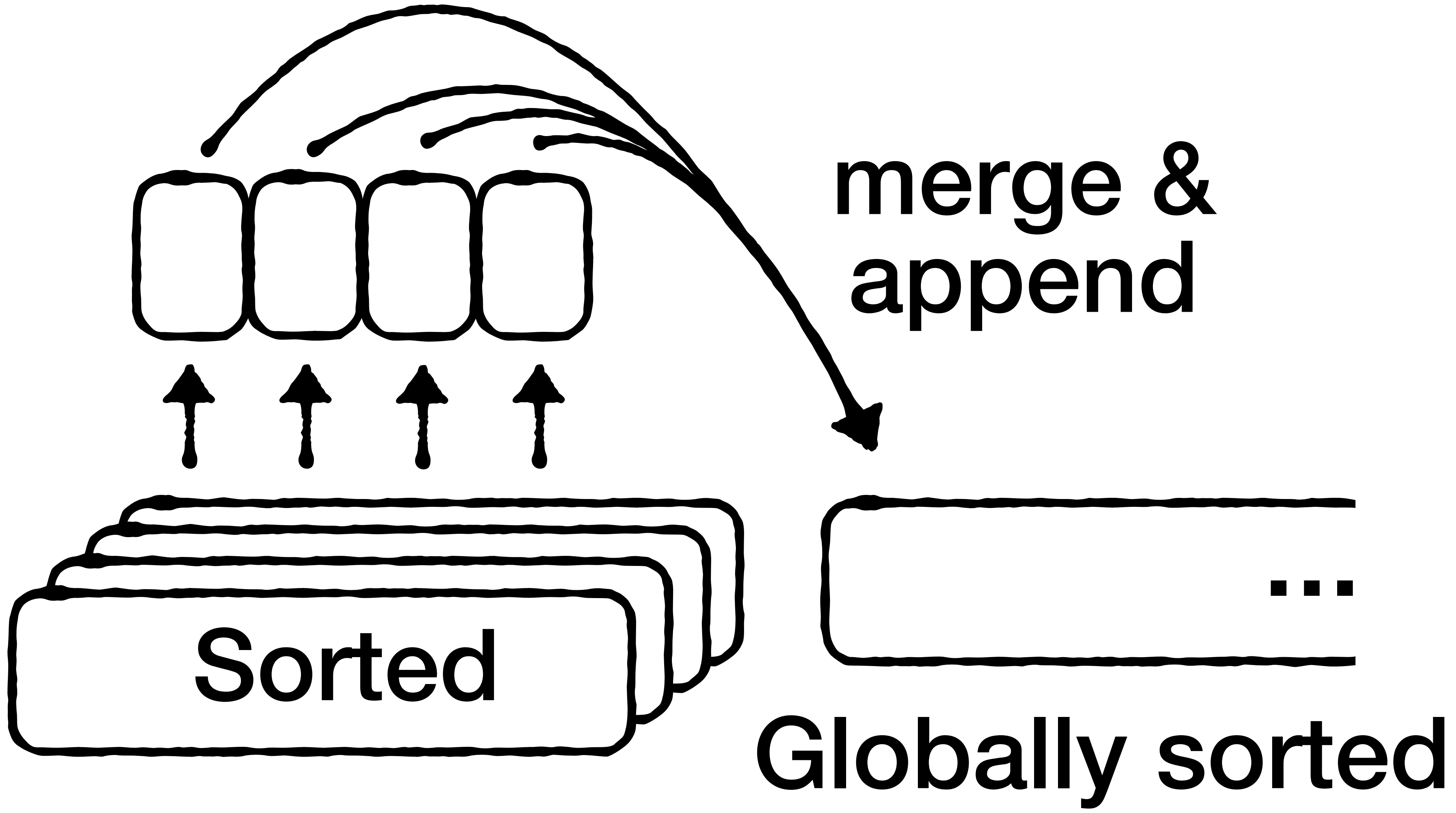
$$N/B * \lceil \log_{M/B}(N/M) \rceil$$



Analyzing I/O for External Sort

Partitioning Phase		Merging Phase		Total
N/B I/Os	+	$N/B * \lceil \log_{M/B}(N/M) \rceil$	=	$\mathbf{O(N/B * \log_{M/B}(N/B))}$

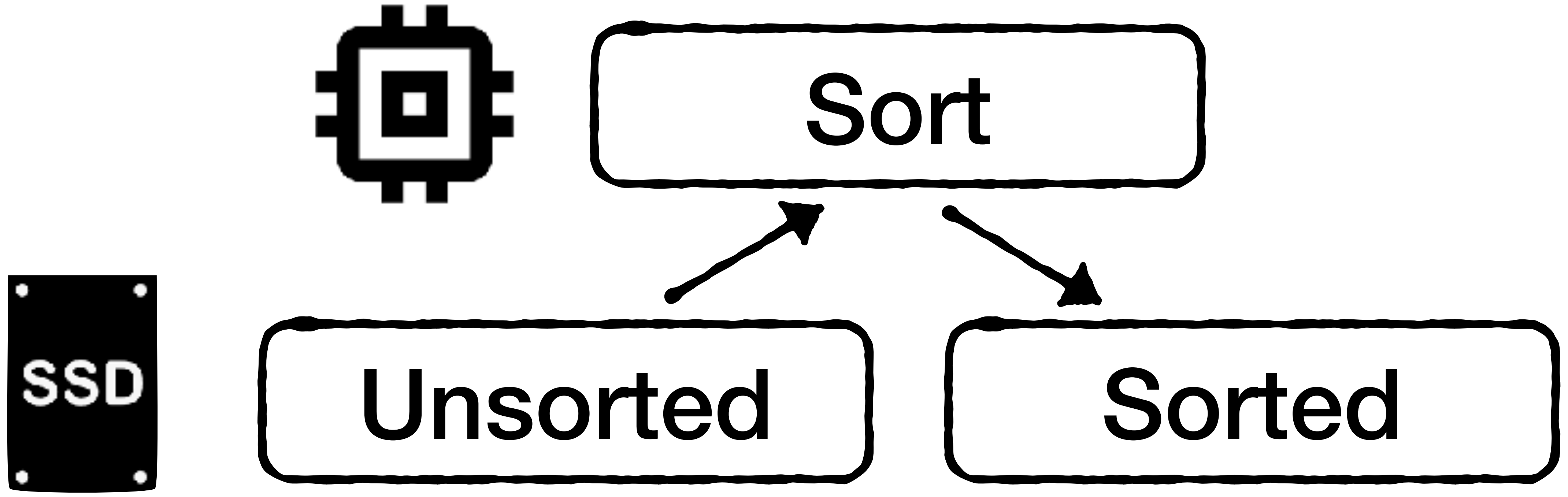


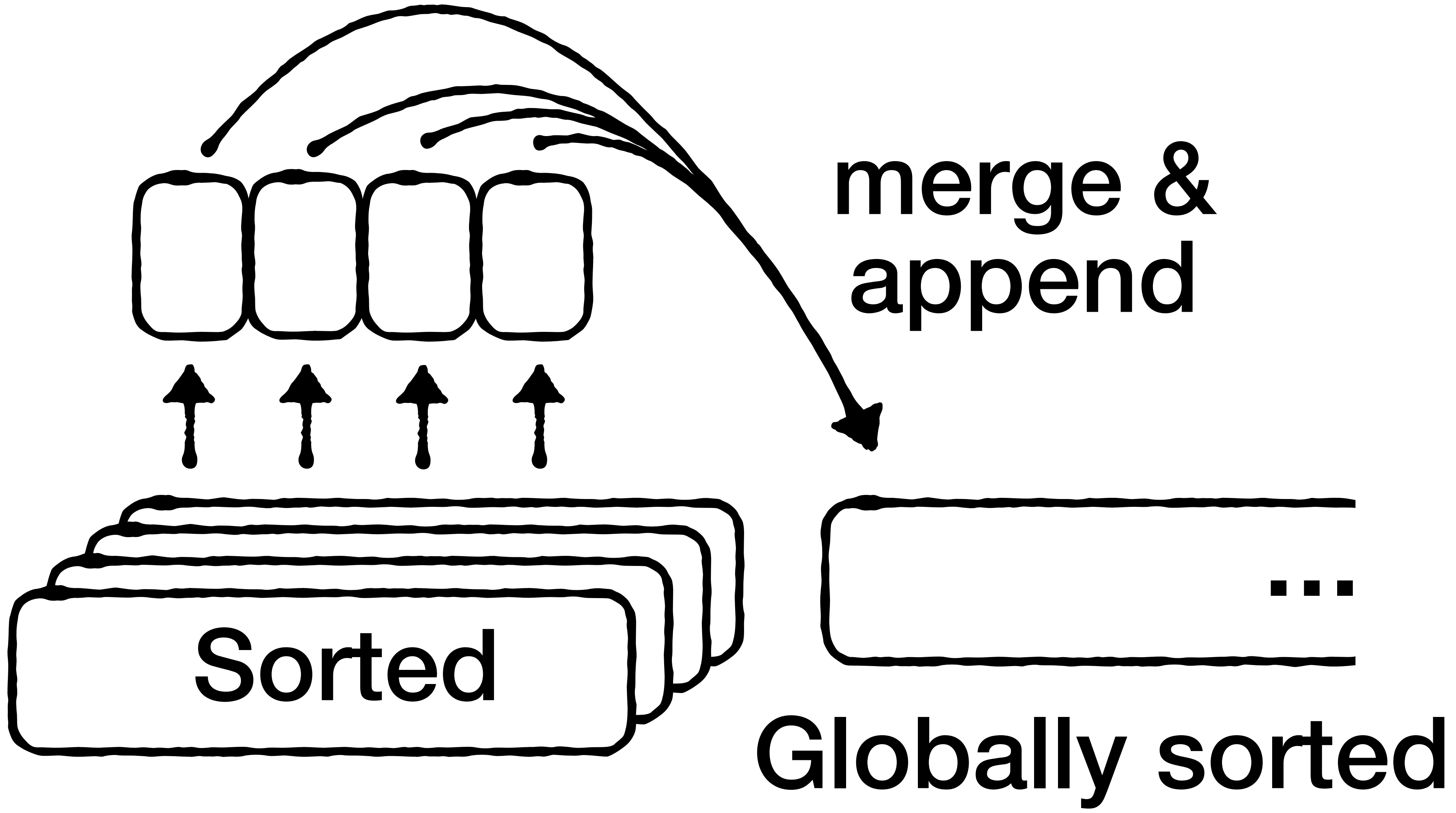


Analyzing I/O for External Sort

$O(N/B * \log_{M/B}(N/B))$

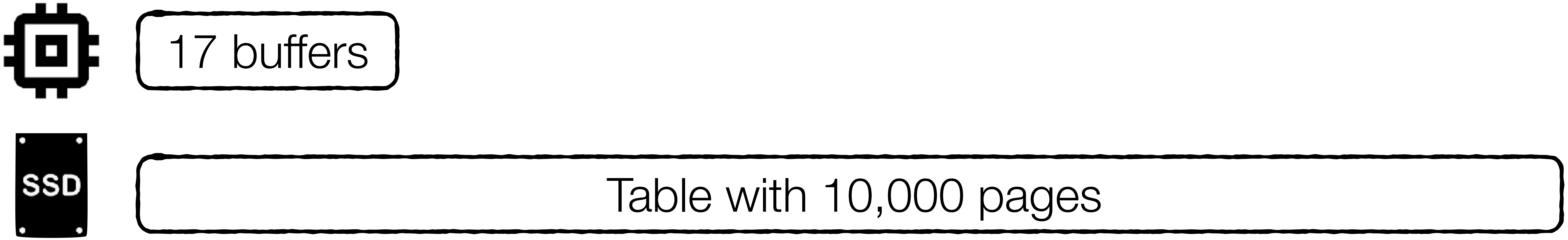
The more common cost expression you'd typically see





Question 1

Suppose you have a file with 10,000 pages and you have 17 buffers in memory. We need to externally sort the file.



Question 1

Suppose you have a file with 10,000 pages and you have 17 buffers in memory. We need to externally sort the file.

(A) How many partitions are created after the first pass?

$10{,}000 / 17 = 589$

(B) How many passes does it take to sort the file completely?

$\lceil 1 \text{ partitioning pass} + \log_{16}(589) \text{ merging passes} \rceil = 4$

(C) How many I/Os are issued in total to sort the file?

$10{,}000 * 4 = 40{,}000$

(D) How many buffers would you need to sort the file in just 2 passes?

$1 + \log_{M-1}(10{,}000/M) = 2$, and solve for M. M = 100

Question 1 - Follow up

Suppose you have a file with 10,000 pages and you have 17 buffers in memory. We need to externally sort the file.

Suppose we are I/O-bound (CPU is not the bottleneck). Would it be better to employ a 2 page buffer for each node? Answer for disk vs. SSD.

This would entail one additional merging pass over the data:
$\lceil 1$ partitioning pass + $\log_8(589)$ merging passes $\rceil = 5$

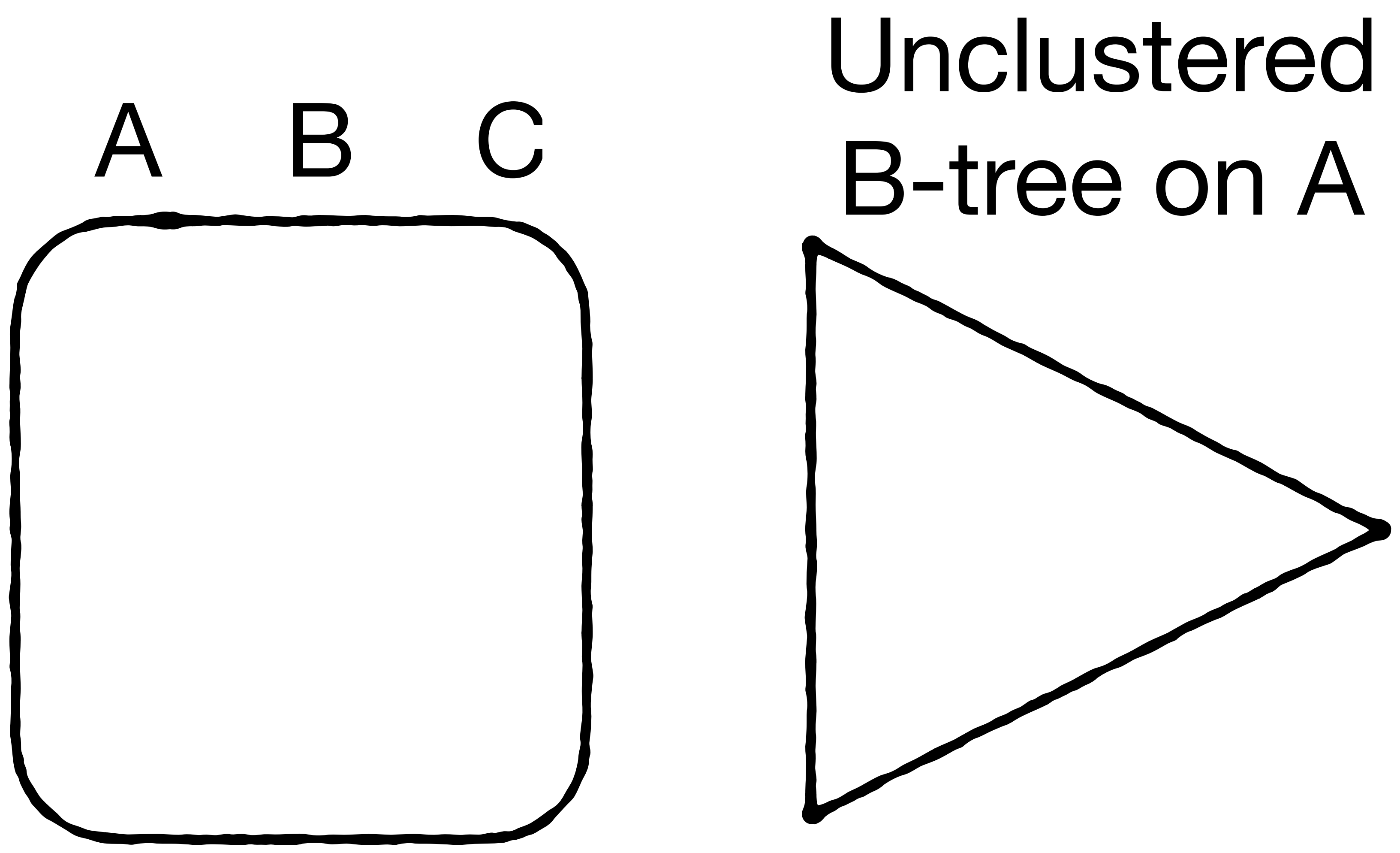
This means we need to do 50,000 read I/Os rather than 40,000 as before.

On SSD, we get a 25% slowdown assuming sequential access costs the same as random access.

On disk, we can do two I/Os at approx. the cost of 1, so the algorithm completes in time needed for 25,000 I/Os. This can lead to a 37% speedup.

Question 2

Consider a table that is too large to fit in memory. We have an unclustered B-tree index over column A. We get the following query: “Select * from table sort by A”. There are two options to process this query: (1) scan the index, or (2) externally sort the file based on column A. What are the costs of these methods, and under which circumstances which you choose each one?



Select * from table sort by A

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(1) An unclustered index requires issuing one I/O into the table for each entry. Retrieving the table in a sorted order therefore takes O(N) I/Os.

(2) External sorting takes $O(N/B * \log_{M/B}(N/B))$ I/Os. It is cheaper as long as $\log_{M/B}(N/B) < B$. This holds true for realistic values N, B and M.

Method (2) is generally better, but if we have large entry sizes (small B), little memory, and/or astronomical data, approach (1) may be better.

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Follow up: how would your answer change if we have a clustered index on column A?

Scanning the clustered index to return sorted data now costs N/B I/Os, which is cheaper than even even a two-pass external sort, which costs 2N/B.

Question 3

You are given M memory and N entries where N >> M >> 3. Why is it a bad idea to use the available memory as virtual memory (e.g., to allocate using 'new' space for M entries, and to use in-memory Quicksort? Use cost models to justify your answer.

With virtual memory, swapping would kick in. Quicksort scans the data sequentially, so we would expect $O(N/B * \log_2 N)$ I/Os as it performs $\log_2 N$ iterations.

In contrast, external sort provides $O(N/B * \log_{M/B}(N/B))$, which dominates.

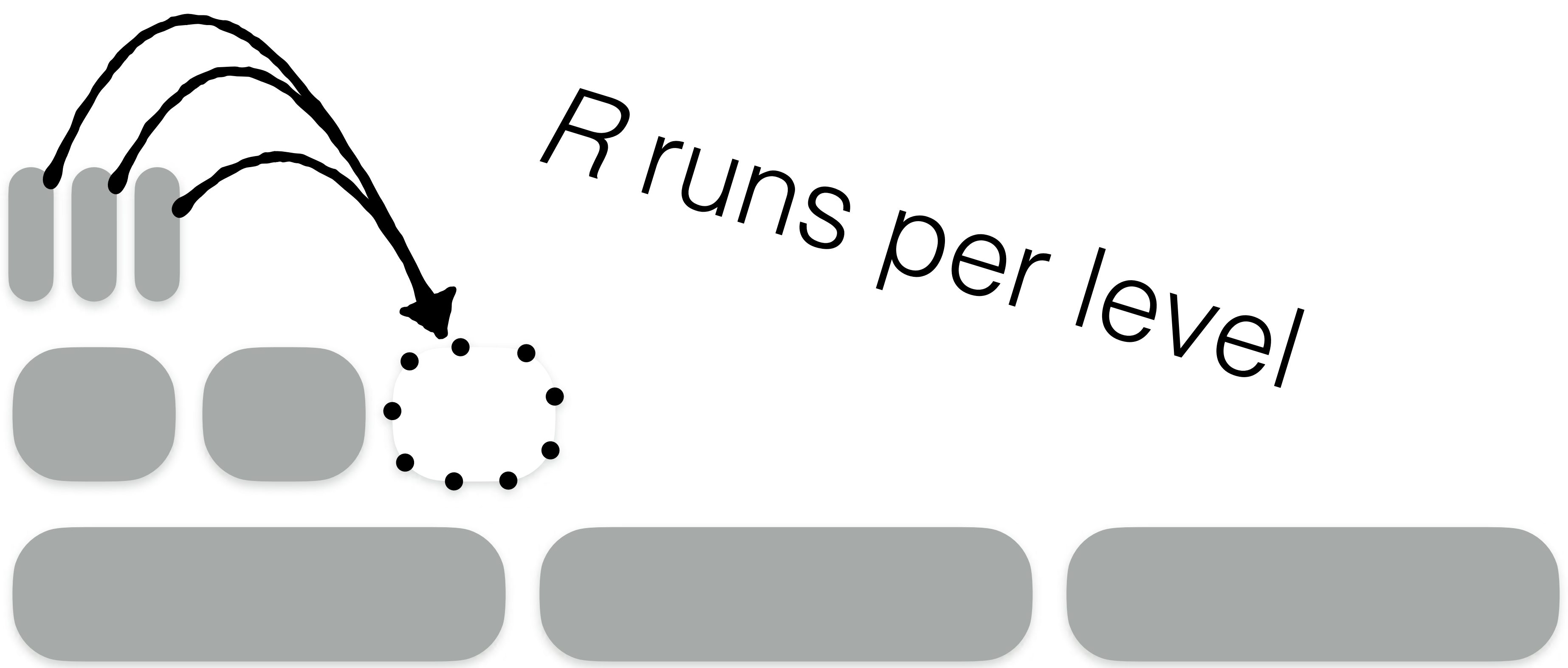
Question 4

Suppose we have a tiered LSM-tree with a size ratio of R. Analyze the CPU costs of compaction. Then propose a technique to improve CPU costs.

Each entry is merged O($\log_T(N/P)$) times across the tree

For each entry we merge, we must check the minimum across O(R) runs.

Total CPU costs: O(N * $\log_T(N/P)$ * R)



Question 4

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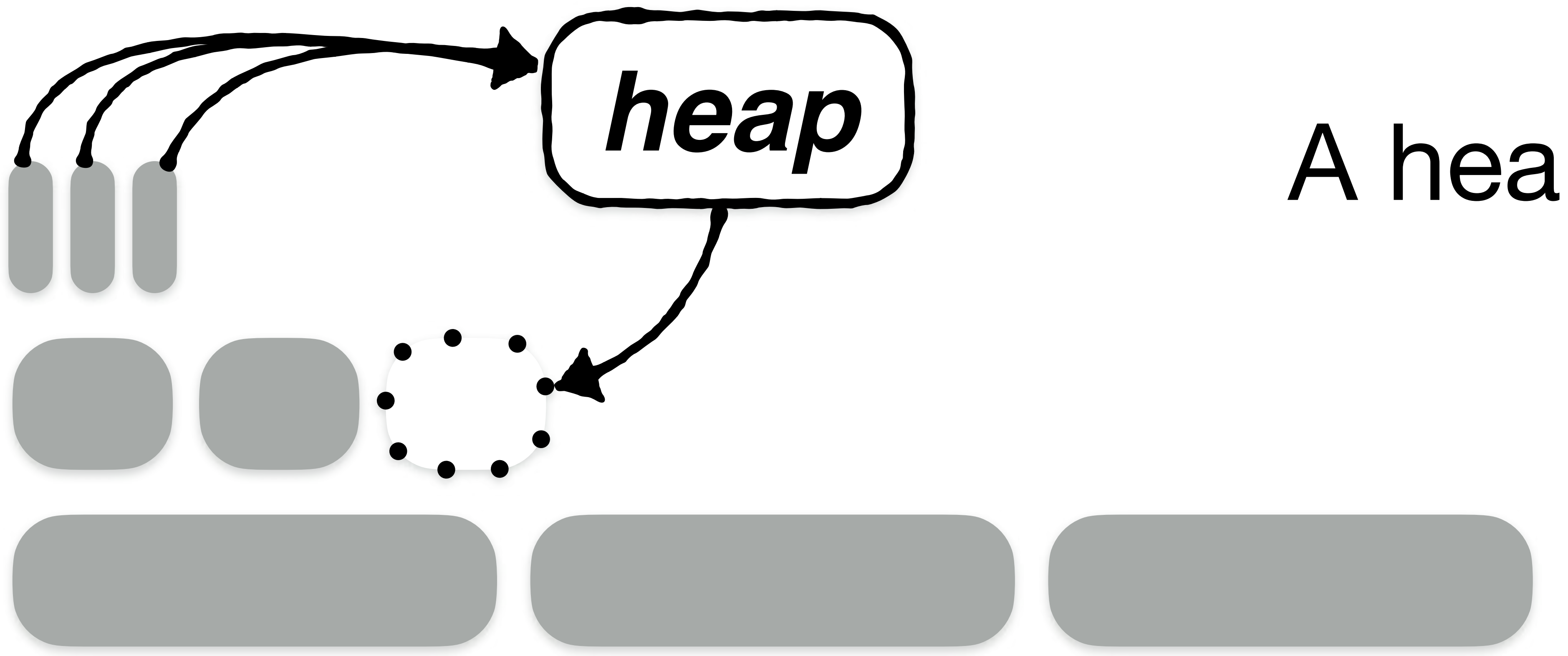

A heap brings this down to $O(N * \log_T(N/P) * \log_2(R))$