MIDTERM EXAM

ISyE6420
Fall 2022

Released October 20, 12:00am – due October 23, 11:59pm. This exam is not proctored and not time limited except the due date. Late submissions will not be accepted.

Use of all course materials is allowed. Internet search and direct communication with others that violate Georgia Tech Academic Integrity Rules are not permitted.

Please show necessary work to get full credit. The exam must be typed in word/latex/RMarkdown and submitted as a pdf file. Please include the WinBugs/R/Python/Matlab codes as separate files.

Name ________________________________

Problem	1	2	3	Total
Score	/40	/30	/30	/100

1. An automobile piston manufacturing company has two plants in North America. Plant A produces pistons of which 2% are defective, while plant B produces about 10% defective pistons. An automobile manufacturer receives 60% of the pistons from plant A and 40% from plant B. They inspect the pistons before assembly. Answer the following questions.

1. Postulate Beta prior distributions for the defective rate for the two plants: plant A: $p \sim Beta(\alpha_0, \beta_0)$ and plant B: $p \sim Beta(\alpha_1, \beta_1)$. Assume $\alpha_0 + \beta_0 = \alpha_1 + \beta_1 = 100$.

2. During inspection of 200 pistons by the automobile manufacturer, 15 pistons are found to be defective. Find the posterior distribution of the defective rate (p).

3. Find the 95% equi-tailed credible interval for p.

4. Find the 95% HPD credible set for p.

2. Consider the Bayesian model

$$\begin{aligned} y|\theta_1, \theta_2 &\sim N(\theta_1 + \theta_2, 1), \\ \theta_i &\sim^{iid} N(0, 1), \quad i = 1, 2. \end{aligned}$$

Suppose $y = 1$ is observed. Then, find the marginal posterior distributions of θ_1 and θ_2.

3. Consider the following Bayesian hierarchical model:

$$\begin{aligned} y_i|\theta &\sim^{iid} N(\theta, 1), \quad i = 1, \ldots, 10, \\ \theta|\tau^2 &\sim N(0, \tau^2), \\ \tau^2 &\sim Inv - Gamma(1, 1). \end{aligned}$$

The following data were observed: $y = \{3.15, 0.97, -2.01, 0.38, -1.06, 1.60, 0.76, -1.03, -0.56, -2.83\}$. Use Gibbs sampling to sample from the posterior distribution of θ (generate 100,000 samples and use 1,000 samples as burn-in) and answer the following:

1. Plot the posterior density of θ.

2. Find the posterior mean of θ.

3. Find 95% equi-tailed credible interval of θ.

Note: the density of a $Inv - Gamma(a, b)$ is given by $b^a/\Gamma(a)x^{-a-1}e^{-b/x}$.