

Homework 1

ISyE 6420

Fall 2022

1. Circuit. A circuit S consisting of seven independent elements E_1, \dots, E_7 is connected

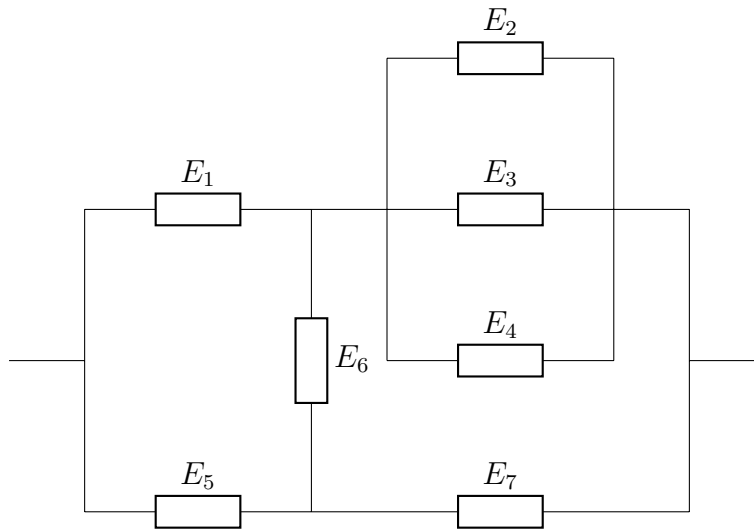


Figure 1: Circuit S with seven independent elements

as in Figure 1. The elements are operational during time interval T with probabilities

	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of working (p)	0.5	0.4	0.1	0.6	0.9	0.8	0.7

- Find the probability that the circuit is operational during time interval T .
- If the circuit was found operational at the time T , what is the probability that the element E_6 was operational.

2. Two Batches. There are two batches of the same product. In batch 1 5% of products are non-conforming. Batch 2 contains 10% non-conforming products. A batch is selected at random and one randomly selected product from that batch is inspected. The inspected product was found conforming and was returned back to its batch.

- What is the probability that the second product, randomly selected from the same batch, is actually found non-conforming?

(b) Assuming the second product selected is non-conforming, what is the probability it came from batch 2?

Hint. This problem uses both Bayes' rule and the law of total probability. The two hypotheses concern the type of batch. For the first draw the hypotheses are equally likely (the batch is selected at random), but for the second draw, the probabilities of hypotheses are updated by the information on the result of the first draw via Bayes rule. Updated probabilities of hypotheses are then used in the total probability formula for the second draw.

3. Machine. A machine has four independent components, three of which fail with probability $q = 1 - p$, and one with probability $1/2$. The machine is operational as long as at least three components are working.

(a) What is the probability that the machine will fail? Evaluate this probability for $p = 0.8$.

(b) If the machine failed, what is the probability that the component which fails with probability $1/2$ actually failed?