Administrative Issues

- Homework 2 is due by September 25 at 11:59pm ET.
- Reminder: **No Handwritten Documents are permitted for any submission**

HW2 Guidance

1. **2D Density tasks**

   Marginal distribution for x:

   $$f_X(x) = \int_D f(x,y)dy$$

   Conditional distribution for y given x:

   $$f(y|x) = \frac{f(x,y)}{f_X(x)}$$

2. **Weibull Lifetimes**. From Wikipedia: The likelihood function (often simply called the likelihood) describes the joint probability of the observed data as a function of the parameters of the chosen statistical model.[1] For each specific parameter value θ in the parameter space, the likelihood function $p(\mathbf{X}|\theta)p(\mathbf{X}|\theta)$ therefore assigns a probabilistic prediction to the observed data $\mathbf{X}$. Since it is essentially the product of sampling densities, the likelihood generally encapsulates both the data-generating process as well as the missing-data mechanism that produced the observed sample.

   When you have more than one datapoint, the likelihood is calculated as follows:

   $$L = \prod_{i=1}^{n} f(x_i, y_i)$$

   where $f(x,y)$ is the pdf.

3. Memoryless property of exponential distributions is useful:

   If X is exponential with parameter $\lambda > 0$, then X is a **memoryless** random variable, that is $P(X > x + a | X > a) = P(X > x)$, for $a, x \geq 0$.

Other stuff

Likelihood	Prior	Posterior
$X_i \mid \theta \sim \mathcal{N}(\theta, \sigma^2)$	$\theta \sim \mathcal{N}(\mu, \tau^2)$	$\theta \mid \mathbf{X} \sim \mathcal{N}\left(\frac{\tau^2}{\tau^2+\sigma^2/n}\bar{X} + \frac{\sigma^2/n}{\tau^2+\sigma^2/n}\mu, \frac{\tau^2\sigma^2/n}{\tau^2+\sigma^2/n}\right)$
$X_i \mid \theta \sim \mathcal{B}in(m, \theta)$	$\theta \sim \mathcal{B}e(\alpha, \beta)$	$\theta \mid \mathbf{X} \sim \mathcal{B}e\left(\alpha + \sum_{i=1}^n X_i, \beta + mn - \sum_{i=1}^n X_i\right)$
$X_i \mid \theta \sim \mathcal{P}oi(\theta)$	$\theta \sim \mathcal{G}a(\alpha, \beta)$	$\theta \mid \mathbf{X} \sim \mathcal{G}a\left(\alpha + \sum_{i=1}^n X_i, \beta + n\right)$
$X_i \mid \theta \sim \mathcal{NB}(m, \theta)$	$\theta \sim \mathcal{B}e(\alpha, \beta)$	$\theta \mid \mathbf{X} \sim \mathcal{B}e\left(\alpha + mn, \beta + \sum_{i=1}^n X_i\right)$
$X_i \mid \theta \sim \mathcal{G}a(1/2, 1/(2\theta))$	$\theta \sim \mathcal{IG}(\alpha, \beta)$	$\theta \mid \mathbf{X} \sim \mathcal{IG}\left(\alpha + n/2, \beta + \frac{1}{2}\sum_{i=1}^n X_i\right)$
$X_i \mid \theta \sim \mathcal{U}(0, \theta)$	$\theta \sim \mathcal{P}a(\theta_0, \alpha)$	$\theta \mid \mathbf{X} \sim \mathcal{P}a\left(\max\{\theta_0, X_1, \ldots, X_n\}, \alpha + n\right)$
$X_i \mid \theta \sim \mathcal{N}(\mu, \theta)$	$\theta \sim \mathcal{IG}(\alpha, \beta)$	$\theta \mid \mathbf{X} \sim \mathcal{IG}\left(\alpha + n/2, \beta + \frac{1}{2}\sum_{i=1}^n (X_i - \mu)^2\right)$
$X_i \mid \theta \sim \mathcal{G}a(v, \theta)$	$\theta \sim \mathcal{G}a(\alpha, \beta)$	$\theta \mid \mathbf{X} \sim \mathcal{G}a\left(\alpha + nv, \beta + \sum_{i=1}^n X_i\right)$
$X_i \mid \theta \sim \mathcal{P}a(c, \theta)$	$\theta \sim \mathcal{G}a(\alpha, \beta)$	$\theta \mid \mathbf{X} \sim \mathcal{G}a\left(\alpha + n, \beta + \sum_{i=1}^n \log(X_i/c)\right)$

Note: To use this table correctly, please note that the left side is the likelihood for a single observation. It is **not** the joint likelihood, which needs to be multiplied with the prior to get the posterior.

For example: single observation likelihood Normal $X_i \mid \theta \sim \mathcal{N}(\theta, \sigma^2)$ leads to a joint likelihood of

$$
\begin{aligned}
L(X_1, \ldots, X_n) &= \prod_{i=1}^n f(x_i, y_i) \\
&= \prod_{i=1}^n \frac{1}{\sigma\sqrt{2n}} \exp\left(-\frac{(x_i - \theta)^2}{2\sigma^2}\right) \\
&= \frac{1}{\sigma^n (2n)^{1/2}} \exp\left(-\sum_{i=1}^n \frac{(x_i - \theta)^2}{2\sigma^2}\right)
\end{aligned}
$$