Bayesian Statistics

A Review of Necessary Probability

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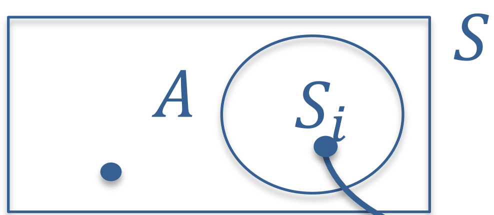
Events and Probabilities with Example Circuit Problem

Before We Begin…

In this section:

- Events, sure and impossible events, unions, intersections, complements
- Probabilities of events and their combinations
- Example: Circuit

Events & Probabilities

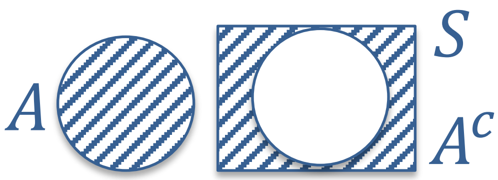


S – sample space
(set of all outcomes in an experiment)

A – event, set of outcomes

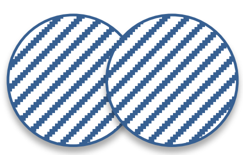
S_i - outcomes

Venn's diagrams



Complement

A^c - outcomes NOT in A



$A \cup B$ – union of A and B
(outcomes in A OR B)

$A \cap B, AB$ – intersection of A and B
(outcomes in A AND B)

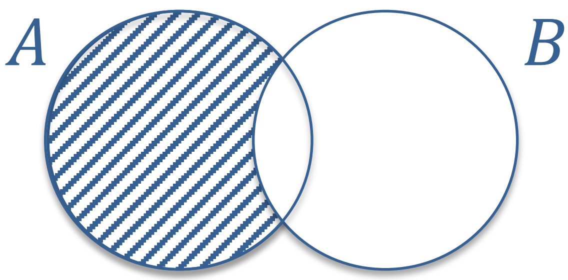
Events & Probabilities

S – sure event

$\emptyset \equiv S^c$ - impossible event

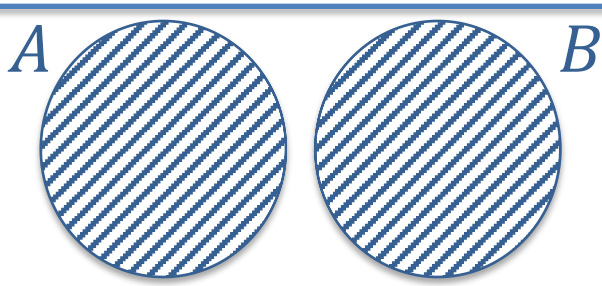
$S \equiv$ all outcomes

$\emptyset \equiv$ no outcomes



$A \backslash \mathrm{B}$ – difference of events

(outcomes in A, but not in B)



A, B exclusive (non-overlapping)

(no common outcomes)

Events & Probabilities

Probabilities – normed measures of events

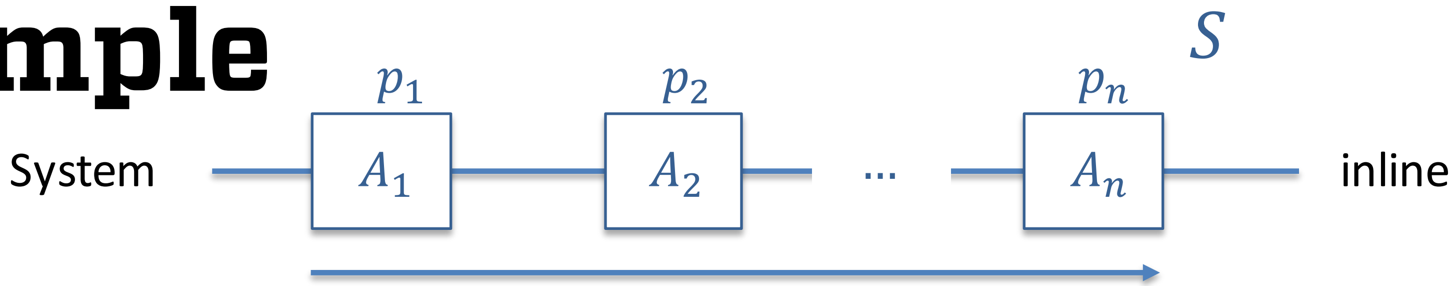
$$P(S) = 1$$
$$P(\emptyset) = 0$$
$$0 \leq P(A) \leq 1$$

Events & Probabilities

- Useful mnemonic thinking
  „one layer of paint“

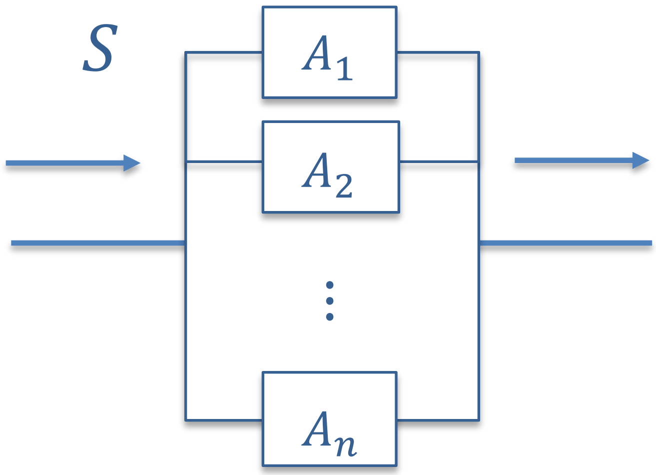
- A, B exclusive
  $P(A \cup B) = P(A) + P(B)$
- A^c is exclusive with A,
  $S = A \cup A^c$ and $P(S) = 1 \Rightarrow$
  $P(A^c) = 1 - P(A)$
- A, B arbitrary
  $$P(A \cup B) = P(A) + P(B) - P(AB)$$
- A, B independent $\Rightarrow$ $P(AB) = P(A)P(B)$
  $\boxed{\overset{\text{def}}{\Longleftrightarrow}}$

Example



$$p_i = P(A_i \text{ works}), \qquad A_1, \dots, A_n \text{ independent}$$

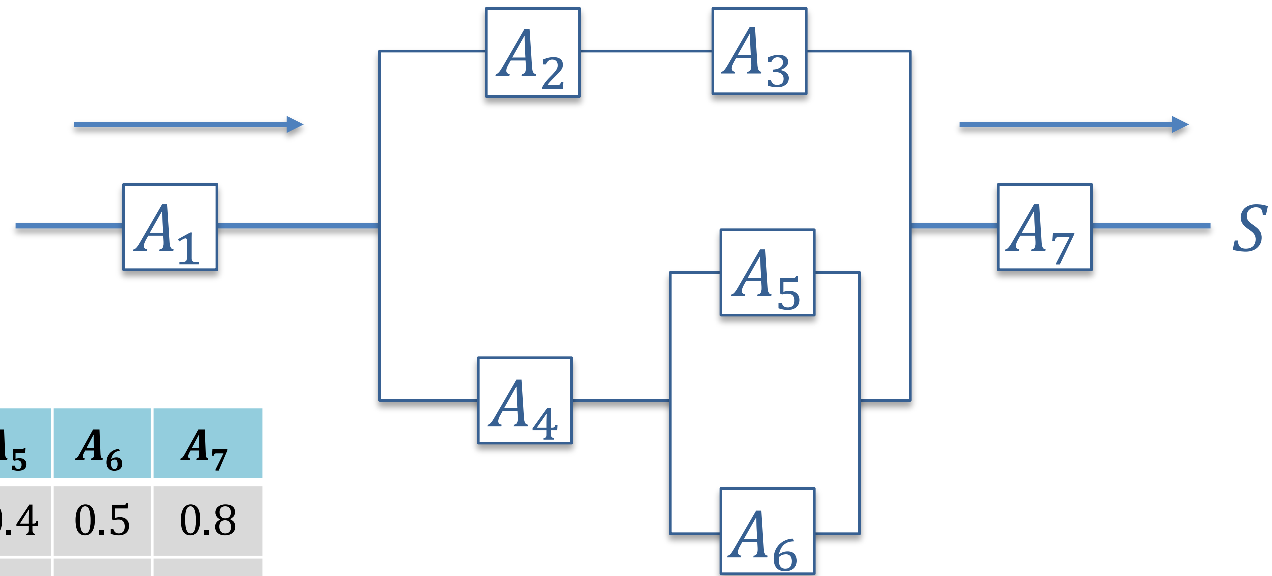
$$p_S = P(S \text{ works}) = P(A_1 A_2 \dots A_n) = [\text{indep}]$$
$$= p_1 p_2 \dots p_n$$



$$P(S \text{ works}) = P(A_1 \cup A_2 \cup \dots \cup A_n)$$
$$= 1 - P(A_1^c A_2^c A_3^c \dots A_n^c) = [\text{indep}]$$
$$= 1 - q_1 q_2 \dots q_n$$

$$q_i = P(A_i^c) \quad p_i + q_i = 1$$

Example



Comp	A_1	A_2	A_3	A_4	A_5	A_6	A_7
work	0.9	0.5	0.3	0.1	0.4	0.5	0.8
fail	0.1	0.5	0.7	0.9	0.6	0.5	0.2

$S_1 = A_2 A_3$ $p_{S_1} = 0.5\times 0.3 = 0.15$ $q_{S_1} = 1 - 0.15 = 0.85$

$S_2 = A_5 \cup A_6$ $q_{S_2} = 0.6\times 0.5 = 0.3$ $p_{S_2} = 1 - 0.3 = 0.7$

$S_3 = A_4 S_2$ $p_{S_3} = 0.1\times 0.7 = 0.07$ $q_{S_3} = 1 - 0.07 = 0.93$

$S_4 = S_1 \cup S_3$ $q_{S_4} = 0.85\times 0.93 = 0.7905$ $p_{S_4} = 1 - 0.7905 = 0.2095$

$S = A_1 S_4 A_7$ $\boxed{p_S = 0.9\times 0.2095\times 0.8 = 0.15084}$ $q_S = 0.84916$

GTx

Summary


GTx