Problem 1

Answer to the problem goes here.

1. Problem 1 part 1 answer here:

H_1: E_6 – works; H_2: E_6 – fails

H_1, H_2 are hypotheses

Define probability of fail as q

$$P(S|H_1) = (1 - q_1q_5)(1 - q_2q_3q_4q_7) = (1 - 0.5 * 0.1)(1 - 0.6 * 0.9 * 0.4 * 0.3) \\ = 0.88844$$

$$P(S|H_2) = 1 - [1 - p_1(1 - q_2q_3q_4)] * [(1 - p_5p_7)] \\ = 1 - [1 - 0.5 * (1 - 0.6 * 0.9 * 0.4)] * (1 - 0.9 * 0.7) = 0.77504$$

$$P(S) = P(S|H_1) * P(H_1) + P(S|H_2) * P(H_2) = 0.88844 * 0.8 + 0.77504 * 0.2 = 0.86576$$

2. Problem 1 part 2 answer here:

$$P(E_6|S) = \frac{P(E_6 \cap S)}{P(S)} = \frac{P(S|E_6) * P(E_6)}{P(S)} = \frac{0.88844 * 0.8}{0.86576} = 0.8209573$$

Problem 2

1. Problem 2 part 1 answer here:

Define H_1, H_2 be the two batches.

In the first draw: $P(H_1) = P(H_2) = 0.5$

$$P(H_1|A) = \frac{[P(A|H_1)P(H_1)]}{P(A)} = \frac{P(A|H_1)P(H_1)}{P(A|H_1)P(H_1) + P(A|H_2)P(H_2)} \\ = \frac{0.95 * 0.5}{0.95 * 0.5 + 0.9 * 0.5} = 0.5135135$$

$$P(H_2|A) = \frac{[P(A|H_2)P(H_2)]}{P(A)} = \frac{P(A|H_2)P(H_2)}{P(A|H_1)P(H_1) + P(A|H_2)P(H_2)}$$
$$= \frac{0.9 * 0.5}{0.95 * 0.5 + 0.9 * 0.5} = 0.4864865$$

In the second draw, the probability of non-conforming given the first try is conforming:

$$P(A^c) = P(A^c|H_1) * P(H_1) + P(A^c|H_2) * P(H_2)$$
$$= 0.05 * 0.5135135 + 0.1 * 0.4864865 = 0.07432432$$

2. Problem 2 part 2 answer here:

$$P(H_2|A^c) = \frac{P(A^c \cap H_2)}{P(A^c)} = \frac{[P(A^c|H_2) * P(H_2)]}{P(A^c)} = \frac{0.1 * 0.4864865}{0.07432432} = 0.6545455$$

Problem 3

1. Problem 3 part 1 answer here:

H0: A, B, C, D work at same time
H1: A, B, C works, D fail
H2: A, B, D works, C fail
H3: A, C, D works, B fail
H4: B, C, D works, A fail
W is machine will work

$P(H0) = p^3 * 0.5$
$P(H1) = p^3 * 0.5$
$P(H2) = p^2 * 0.5 * q$
$P(H3) = p^2 * 0.5 * q$
$P(H4) = p^2 * 0.5 * q$

$$P(W) = P(H0) + P(H1) + P(H2) + P(H3) + P(H4)$$
$$= p^3 * 0.5 + p^3 * 0.5 + 3 * p^2 * 0.5 * q$$
$$P(W^c) = 1 - P(W)$$

When $p = 0.8, P(W^c) = 1 - P(W) = 1 - 0.704 = 0.296$

2. Problem 3 part 2 answer here:

$$P(D^c|W^c) = \frac{P(D^c \cap W^c)}{P(W^c)} = \frac{P(D^c) * P(W^c|D^c)}{P(W^c)} =$$
$$\frac{0.5 * \left(\binom{3}{1}qp^2 + \binom{3}{2}q^2p + q^3\right)}{(1 - p^3 * 0.5 + p^3 * 0.5 + 3 * p^2 * 0.5 * q)} = \frac{0.5 * (3qp^2 + 3q^2p + q^3)}{(1 - p^3 + 1.5p^2q)}$$
$$= \frac{1.5qp^2 + 1.5q^2p + 0.5q^3}{(1 - p^3 - 1.5p^2q)}$$