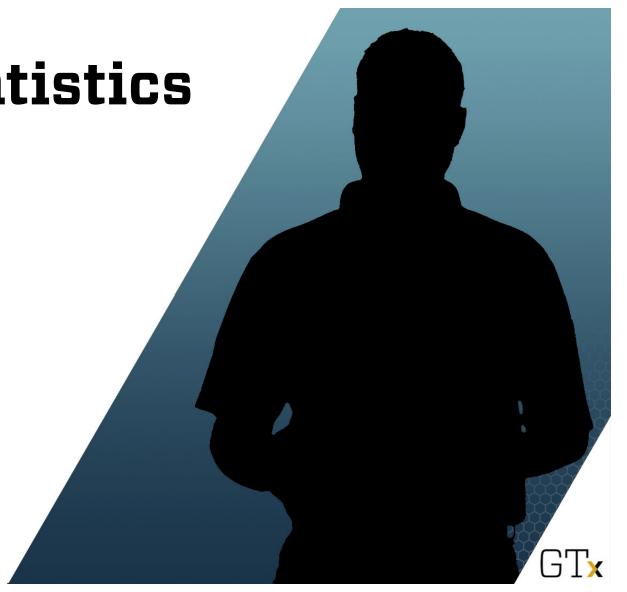
Bayesian StatisticsBayes Formula

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From Prior to Posterior and Bayesian Learning with Examples of Manufacturing Bayes, Bridged Circuit, & Two-headed Coin





From Prior to Posterior Probabilities

Interested in $P(H_i|A)$ for some hypothesis H_i .

By Total Probability, know P(A)

know, given $P(H_i) \rightarrow A$ happen $\rightarrow P(H_i|A)$



Bayesian Learning

$$\underbrace{P(H_i)}_{\text{prior}} \rightarrow P(H_i|A) = \underbrace{\frac{P(A|H_i)}{P(A)}}_{\text{posterior}} \times P(H_i)$$
prob.
$$\underbrace{P(H_i)}_{\text{prob.}} \rightarrow P(H_i|A) = \underbrace{\frac{P(A|H_i)}{P(A)}}_{\text{posterior}} \times P(H_i)$$

This is Bayes formula

Learning?

Prior to experiment where A occurs or does not occur \rightarrow we believe P(H).

After the experiment \rightarrow we update probability of hypothesis H to P(H|A) by Bayes formula.



Ex. Manufacturing Bayes, cont.

- The selected item was found conforming.
 What is the probability that it was produced on Machine 1?
- $P(H_1) = 0.3$ Prior to selection

 A conforming

$$P(H_1|A) = \frac{P(A|H_1)P(H_1)}{P(A)} = \frac{0.94 \times 0.3}{0.951}$$
$$= \boxed{0.2965 < 0.3}$$



Ex. Bridged Circuit

 The circuit S works, what is the probability that element A₅ works as well?

$$P(H_1) = P(A_5 \text{ works}) = 0.6$$

$$P(H_1|S) = \frac{P(S|H_1)P(H_1)}{P(S)}$$

$$= \frac{0.8536 \times 0.6}{0.8315} = 0.6159$$



Ex. Two-headed Coin

In a box there are N coins, N-1 fair and one is two-headed.

A coin is selected from a box and flipped k times. In all k flips it came heads-up.

What is the probability that the two-headed coin was selected?

A – coin lands heads up k times in k flips

 H_1 : fair coin is selected,

 H_2 : two-headed coin is selected



$$P(H_1) = \frac{N-1}{N}, \qquad P(H_2) = \frac{1}{N}$$

$$P(A|H_1) = \underbrace{\frac{1}{2} \times \frac{1}{2} \dots \frac{1}{2}}_{k} = \underbrace{\frac{1}{2^k}}_{k}$$

$$P(A|H_2) = 1$$

$$P(A) = \frac{N-1}{N} \times \frac{1}{2^k} + \frac{1}{N} \times 1 = \frac{N-1+2^k}{2^k N}$$

•
$$P(H_2|A) = \frac{\frac{1}{N} \times 1}{\frac{N-1+2^k}{2^k N}} = \frac{2^k}{(N-1)+2^k}$$

$$\begin{cases} N = 1,000,000 \\ k = 20 \end{cases} P(H_2) = \frac{1}{1,000,000}, P(H_2|A) = 0.5119$$

$$k = 40$$
 $P(H_2) = \frac{1}{1,000,000}, P(H_2|A) = 0.999999995$



Summary





