Administrative Issues

- Homework 3 is due by October 9 at 11:59pm ET.
- Homework 4 is due by October 16 at 11:59pm ET.
- Midterm: Oct 20-Oct23
- If you decide to use some programming language for HW3 that is fine. However at the end of the day, you are responsible for reading the documentation associated with that language. Please make sure to enter your probability density functions appropriately!

  Example of this: In this course we say X has Gamma(a, b) distribution if $f(x \mid a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}$ for $x \geq 0$ but Matlab defines the Gamma (a, b) distribution using $f(x \mid a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-x/b}$ for $x \geq 0$. Please be careful about these differences and make sure the function does what you think it does! A Gamma$(2, 4)$ variable in our course is equivalent to a Gamma$(2, 1/4)$ in Matlab.
- Reminder: **No Handwritten Documents are permitted for any submission**

HW3 Guidance

Q1: Maxwell

Find the MLE of Maxwell:

We let $\boldsymbol{y} = (y_1, \ldots, y_n)$ and find the likelihood as

$$\begin{aligned} L(\theta) = \prod_{i=1}^n f(x_i \mid \theta) &= \prod_{i=1}^n \sqrt{\frac{2}{\pi}} \theta^{3/2} y_i^2 \exp\left\{-\theta y_i^2/2\right\} \\ &= \left(\sqrt{\frac{2}{\pi}}\right)^n \theta^{\frac{3}{2}n} \exp\left\{-\frac{\theta}{2} \sum_{i=1}^n y_i^2\right\} \\ &\propto \theta^{3n/2} \exp\left\{-\frac{\theta}{2} \sum_{i=1}^n y_i^2\right\} \end{aligned}$$

We have $\log L(\theta) = (3n/2)\log(\theta) - \frac{\theta}{2} \sum_{i=1}^n y_i^2$

Thus, $\frac{d}{d\theta} \log L(\theta) = (3n/2\theta) - \frac{1}{2} \sum_{i=1}^n y_i^2 = 0$

Rearranging, this gives us $\hat{\theta} = \frac{3n}{\sum_{i=1}^n y_i^2}$

(b) Compute 95% equitailed credible set using your favorite software package. Recall that a credible set for θ of size $1 - \alpha$ is the interval (a, b) defined as follows:

$$P(a \leq \theta \leq b \mid X) = \int_a^b \pi(\theta \mid X) d\theta = 1 - \alpha$$

An equal-tailed credible set has the additional constraint that

$$P(\theta \geq b \mid X) = \frac{\alpha}{2} \text{ and } P(\theta \leq a \mid X) = \frac{\alpha}{2}$$

Q2: Mixture

(a) Should be relatively straightforward. Don't over-complicate things!

(b) One way to find the posterior distribution given the likelihood $X|\theta \sim N(\theta, \sigma^2)$ where σ^2 is fixed, and prior $\theta \sim N(\theta_0, \sigma_0^2)$, is the following:

$$\pi(\theta \mid x) \propto f(x \mid \theta)\pi(\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\theta)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}}$$