Bayesian Statistics

Bayes Formula

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From Prior to Posterior and Bayesian Learning with Examples of Manufacturing Bayes, Bridged Circuit, & Two-headed Coin

Before We Begin...

From Prior to Posterior Probabilities

Interested in $P(H_i|A)$ for some hypothesis H_i.

By Total Probability, know $P(A)$

$$\rightarrow P(H_i|A) \stackrel{\text{def}}{=} \frac{P(AH_i)}{P(A)}$$

$$= \frac{P(A|H_i)P(H_i)}{P(A)}$$

know, given $P(H_i) \rightarrow A$ happen $\rightarrow P(H_i|A)$

Bayesian Learning

$$\underbrace{P(H_i)}_{\substack{\text{prior}\\\text{prob.}}} \to P(H_i|A) = \underbrace{\frac{P(A|H_i)}{P(A)}}_{\substack{\text{posterior}\\\text{prob.}}} \times P(H_i)$$

This is Bayes formula

- Learning?

  Prior to experiment where A occurs or does not occur → we believe $P(H)$.

  After the experiment → we update probability of hypothesis H to $P(H|A)$ by Bayes formula.

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Ex. Manufacturing Bayes, cont.

- The selected item was found conforming.

  What is the probability that it was produced on Machine 1?

- $P(H_1) = 0.3$ Prior to selection

  A - conforming

  $$P(H_1|A) = \frac{P(A|H_1)P(H_1)}{P(A)} = \frac{0.94 \times 0.3}{0.951}$$

  $$= \boxed{0.2965 < 0.3}$$

Ex. Bridged Circuit

- The circuit S works, what is the probability that element A_5 works as well?

$$P(H_1) = P(A_5 \text{ works}) = 0.6$$

$$P(H_1|S) = \frac{P(S|H_1)P(H_1)}{P(S)}$$

$$= \frac{0.8536 \times 0.6}{0.8315} = \boxed{0.6159}$$

Ex. Two-headed Coin

In a box there are N coins, $N-1$ fair and one is two-headed.

A coin is selected from a box and flipped k times. In all k flips it came heads-up.

What is the probability that the two-headed coin was selected?

A – coin lands heads up k times in k flips

H_1: fair coin is selected,

H_2: two-headed coin is selected

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$$P(H_1) = \frac{N-1}{N}, \qquad P(H_2) = \frac{1}{N}$$

$$P(A|H_1) = \underbrace{\frac{1}{2} \times \frac{1}{2} \dots \frac{1}{2}}_{k} = \frac{1}{2^k}$$

$$P(A|H_2) = 1$$

$$P(A) = \frac{N-1}{N} \times \frac{1}{2^k} + \frac{1}{N} \times 1 = \frac{N-1+2^k}{2^k N}$$

- $P(H_2|A) = \frac{\frac{1}{N}\times 1}{\frac{N-1+2^k}{2^k N}} = \frac{2^k}{(N-1)+2^k}$

$$\left.\begin{matrix} N = 1{,}000{,}000 \\ k = 20 \end{matrix}\right\} \quad P(H_2) = \frac{1}{1{,}000{,}000}, P(H_2|A) = 0.5119$$

$$k = 40 \qquad P(H_2) = \frac{1}{1{,}000{,}000}, P(H_2|A) = 0.999999095$$

Summary


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