高数基础班 (3)

常考题型举例: 1.极限概念、性质、存在准则, 2.求极限方法举例 (基本极限;等价代换;有理运算)

P16-P25

主讲 武忠祥 教授



你就慢了



常考题型与典型例题

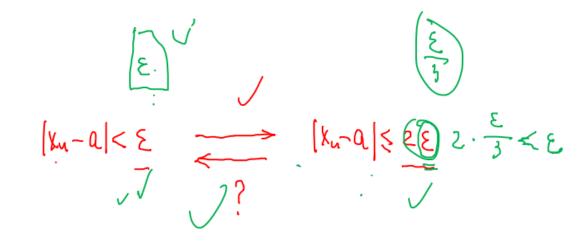
- 1)极限的概念、性质及存在准则
- 2) 求极限
- 3) 无穷小量阶的比较

(一) 极限的概念、性质及存在准则

【例14】(1999年2)"对任意给定的 $\varepsilon \in (0,1)$, 总存在正数 N,

当 n > N 时,恒有 $|x_n - a| \le 2\varepsilon$ 是数列 $\{x_n\}$ 收敛于 a 为 (A) 充分条件但非必要条件;

- (B) 必要条件但非充分条件.
- 充分必要条件.
 - (D) 既非充分条件又非必要条件.



【例15】(2015年3)设
$$\{x_n\}$$
 是数列,下列命题中不正确的是

$$\lim_{n\to\infty} x_n = a, \quad \text{II} \quad \lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} x_{2n+1} = a. \quad \checkmark$$

(B) 若
$$\lim_{n\to\infty} x_n - a$$
, 於 $\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} x_{2n+1} = a$.

(B) 若 $\lim_{n\to\infty} x_2 = \lim_{n\to\infty} x_{2n+1} = a$, 则 $\lim_{n\to\infty} x_n = a$,

(C) 若
$$\lim_{n\to\infty} x_n = a$$
, 則 $\lim_{n\to\infty} x_{3n+1} = a$.

(D) 若 $\lim_{n\to\infty} x_{3n} = \lim_{n\to\infty} x_{3n+1} = a$, 則 $\lim_{n\to\infty} x_n = a$, 以 $\lim_{n\to\infty} x_n = a$, 以 $\lim_{n\to\infty} x_n = a$, 以 $\lim_{n\to\infty} x_n = a$, $\lim_{n\to\infty} x_n = a$,

【例16】(1993年3) 当
$$x \to 0$$
 时, 变量 $\frac{1}{x^2} \sin \frac{1}{x}$ 是 ()。

(A) 无穷小 (B) 无穷大

【解】由于对任意给定的
$$M > 0$$
 及 $\delta > 0$,总存在
$$(x_n) = \frac{1}{2n\pi + \frac{\pi}{2}}, y_n = \frac{1}{2n\pi}, \rightarrow 0$$

使得 $0 < x_n < \delta$, $0 < y_n < \delta$, 此时

$$\left| \frac{1}{x_n^2} \sin x_n \right| = \left(2n\pi + \frac{\pi}{2} \right)^2 > M, \qquad \frac{1}{y_n^2} \sin \frac{1}{y_n} = 0$$

(二) 求极限

常用的求极限方法 (8种)

方法1 利用基本极限求极限

方法2 利用等价无穷小代换求极限

方法3 利用有理运算法则求极限

方法4 利用洛必达法则求极限

方法5 利用泰勒公式求极限

方法6 利用夹逼原理求极限

方法7 利用单调有界准则求极限

方法8 利用定积分定义求极限

方法1 利用基本极限求极限

1) 常用的基本极限

$$\lim_{x\to 0} \frac{\sin x}{x} = 1;$$
 $\lim_{x\to 0} \frac{1}{x} = e;$
 $\lim_{x\to 0} \frac{1}{x} = \ln a;$
 $\lim_{n\to \infty} \frac{a^x - 1}{x} = \ln a;$
 $\lim_{n\to \infty} \frac{a^x - 1}{b_m x^m} + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m} = \begin{cases} \frac{a_n}{b_m}, & n = m, \\ 0, & n < m, \\ \infty, & n > m. \end{cases}$

$$\lim_{n\to\infty} x^{n} = \begin{cases} 0, & |x| < 1, & (-1)^{N} \\ 0, & |x| > 1, & |x| = 1 \end{cases}$$

$$\lim_{n\to\infty} x^{n} = \begin{cases} 0, & |x| < 1, & (-1)^{N} \\ 0, & |x| > 1, & |x| = 1 \end{cases}$$

$$\lim_{n\to\infty} e^{nx} = \begin{cases} 0, & |x| < 0, & |x| < 0, \\ +\infty, & |x| > 0 \end{cases}$$

$$oxed{1^{\infty}}$$
 型极限常用结论

若
$$\lim \alpha(x) = 0$$
, $\lim \beta(x) = \infty$, 且 $\lim \alpha(x)\beta(x) = A$

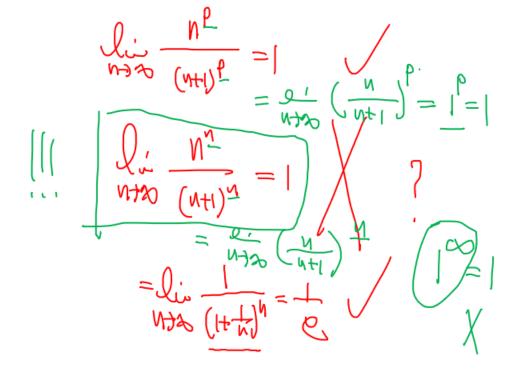
则
$$\lim_{x \to \infty} (1 + \alpha(x))^{\beta(x)} = e^A$$

1)写标准形式 原式 =
$$\lim_{x \to \infty} [1 + \alpha(x)]^{\beta(x)}$$
;

2)求极限
$$\lim \alpha(x)\beta(x) = A;$$

$$\sqrt{3}$$
)写结果 原式 = e^A .

【例17】
$$\lim_{n\to\infty}\frac{n^{\frac{n+1}{2}}}{(n+1)^n}\sin\frac{1}{n}$$



【例18】极限
$$\lim_{x\to\infty} \left(\frac{x^2 + x^2}{(x-a)(x+b)}\right)^x =$$

$$(A) 1 \qquad (B) e \qquad (C) e^{a-b} \qquad (D) e^{b-a}$$

$$(A1) = \lim_{x \to \infty} \left(\frac{x^2}{(x-a)(x+b)} \right)^x \stackrel{\text{in}}{=} \lim_{x \to \infty} \left(\frac{x}{x-a} \right)^x \left(\frac{x}{x+b} \right)^x$$

$$= \lim_{x \to \infty} \left(1 - \frac{a}{x} \right)^{-x} \left(1 + \frac{b}{x} \right)^{-x}$$

【例18】极限im
$$\left(\frac{x^2}{(x-a)(x+b)}\right)^2 =$$

$$\left(A\right) \frac{1}{2} \quad \left(B\right) e \quad \left(C\right) e^{a-b} \quad \left(D\right) e^{b-a}$$
【解2】排除法

 $\lim_{x \to \infty} \left(\frac{x}{x + b} \right)^{x} = e^{-b}$

【例19】
$$\lim_{n\to\infty} \left(\frac{\sqrt[n]{a} + \sqrt[n]{b} + \sqrt[n]{c}}{3}\right)^n$$
,其中 $a > 0, b > 0, c > 0$.

[解] 原式 =
$$\lim_{n \to \infty} \left[1 + \frac{\sqrt{a} + \sqrt{b} + \sqrt{c} - 3}{3} \right]^n - \left[\frac{\sqrt{a} + \sqrt{b} + \sqrt{c} - 3}{3} \right]^n - \left[\frac{\sqrt{a} + \sqrt{b} + \sqrt{c} - 3}{3} \right]^n - \left[\frac{\sqrt{a} + \sqrt{b} + \sqrt{c} - 3}{3} \right]^n - \left[\frac{\sqrt{a} - 1}{3} \right]^n - \left[\frac{\sqrt{a} - 1}$$

原式
$$=e^{\ln \sqrt[3]{abc}}=\sqrt[3]{abc}$$

 $= \ln \sqrt[3]{abc}$

利用等价无穷小

a)乘除关系可以换

若
$$\alpha \sim \alpha_1, \beta \sim \beta_1$$
, 则

$$\lim \frac{\alpha}{} = \lim \frac{\alpha}{}$$

 $\lim \frac{\alpha}{\beta} = \lim \frac{\alpha_1}{\beta} = \lim \frac{\alpha}{\beta} = \lim \frac{\alpha_1}{\beta}$

减关系在一定条件下可以换

$$\frac{\alpha}{\alpha} = \lim_{\alpha \to 0} \frac{\alpha}{\alpha} = \lim_{\alpha \to 0} \frac{\alpha}{\alpha}$$

若 $\alpha \sim \alpha_1, \beta \sim \beta_1$, 且 $\lim \frac{\alpha_1}{\rho} = A \neq 1$. 则 $\alpha - \beta \sim \alpha_1$

若 $\alpha \sim \alpha_1, \beta \sim \beta_1$, 且 $\lim \frac{\alpha_1}{\beta_1} = A \neq -1$. 则 $\alpha + \beta \sim \alpha_1 + \beta_1$.

$$\alpha$$
 α

 $0 \in X$

~ X + X = 2 X

(2) 常用的等价无穷小: 当 $x \to 0$ 时

$$- \lim_{x \to \infty} x \sim \sin x \sim \tan x \sim \arctan x \sim \ln(1+x) \sim e^{x} - 1;$$

$$= \frac{a^{x} - 1 \sim x \ln a}{x}, \qquad (1+x)^{\alpha} - 1 \sim \alpha x, \qquad 1 - \cos x \sim \frac{1}{2}x^{2}$$

$$= \frac{1}{6}x^{3}$$

$$= \frac{1}{6}x^{3}$$

$$= \frac{1}{6}x^{3}$$

$$= \frac{1}{6}x^{3}$$

$$= \frac{1}{6}x^{3}$$

$$= \frac{1}{3}x^{3}$$

$$= \frac{1}{$$

【例20】(2016年3) 已知函数
$$f(x)$$
 满足
$$\lim_{x\to 0} \frac{\sqrt{1+f(x)\sin 2x}-1}{e^{3x}-1} = 2, \quad \text{则 } \lim_{x\to 0} f(x) = \underline{\qquad}.$$
 【解】由
$$\lim_{x\to 0} \frac{\sqrt{1+f(x)\sin 2x}-1}{e^{3x}-1} = 2 \quad \text{及 } \lim_{x\to 0} (e^{3x}-1) = 0 \quad \text{知},$$

$$\boxplus \lim_{x \to 0} \frac{\sqrt{1+f(x)\sin 2x-1}}{e^{3x}-1} = 2 \quad \not \boxtimes \lim_{x \to 0} (e^{3x})$$

$$\lim_{x \to 0} f(x) \sin 2x = 0$$

$$\int_{1}^{\infty} \frac{1}{2} f(x) \sin 2x = 1$$

$$\lim_{x \to 0} \frac{\sqrt{1 + f(x)\sin 2x} - 1}{e^{3x} - 1} = \lim_{x \to 0} \frac{\frac{1}{2}f(x)\sin 2x}{3x}$$

$$\iiint_{x\to 0} \frac{\sqrt{1+f(x)\sin 2x}-1}{\underbrace{e^{3x}-1}} = \lim_{x\to 0} \frac{\frac{1}{2}f(x)\sin 2x}{\underbrace{3x}}$$

$$\lim_{x \to 0} \frac{\sqrt{1 + f(x) \sin 2x - 1}}{e^{3x} - 1} = \lim_{x \to 0} \frac{2^{x}}{3x}$$

$$\frac{1}{2} f(x) \cdot 2x$$

 $\lim f(x) = 6$.

$$\frac{e^{3x}-1}{2} \qquad \frac{3x}{x\to 0} \qquad \frac{3x}{2}$$

$$= \lim_{x\to 0} \frac{1}{2}f(x)\cdot 2x$$

$$\frac{e^{3x}-1}{e^{3x}-1} = \lim_{x\to 0} \frac{2}{3x}$$

$$\frac{1}{2}f(x)\cdot 2x$$

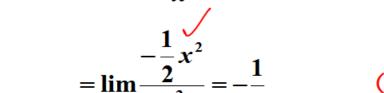
【例21】(2015年, 1)
$$\lim_{x\to 0} \frac{\ln(\cos x)}{x^2} = \frac{-0}{x\to 0}$$
 【解1】原式 $=\lim_{x\to 0} \frac{\ln[1+(\cos x-1)]}{x^2}$ 【从) [計(火-1)]

$$\frac{1}{x \to 0} = \lim_{x \to 0} \frac{\cos x - 1}{x^2}$$

$$\lim_{x \to 0} \frac{\cos x - 1}{x^2}$$

$$\Gamma = \lim_{x \to 0} \frac{1}{x^2}$$

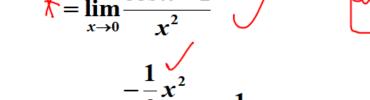
$$-\frac{1}{x^2}$$



sin x

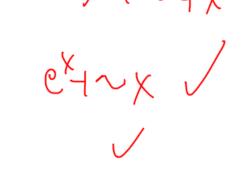
【解3】原式 = lim ln(cos) - ln(t)

【解2】原式=lim



【例22】(2009年. 3)
$$\lim_{x\to 0} \frac{e - e^{\cos x}}{\sqrt[3]{1 + x^2 - 1}} = \frac{1}{\sqrt[3]{1 + x^2 - 1}}.$$
【解1】原式 =
$$\lim_{x\to 0} \frac{e^{\cos x}}{\sqrt[3]{1 + x^2 - 1}} = \frac{1}{\sqrt[3]{1 + x^2 - 1}}.$$

[解2] 原式 =
$$\lim_{x \to 0} \frac{e^1 - e^{\cos x}}{1 - x^2} = 3 \lim_{x \to 0} \frac{e^{\frac{x}{3}}(1 - \cos x)}{x^2} = 3e \lim_{x \to 0} \frac{e$$



$$= \lim_{x \to 0} \frac{1}{x^2}$$

$$= \lim_{x \to 0} \frac{\cos x - 1}{3x^2}$$
(等价无穷小代換)

【例23】 (2006年2) 求极限
$$\lim_{x\to 0} \frac{1}{x^3} \left[\left(\frac{2 + \cos x}{3} \right)^x - 1 \right]$$
【解2】原式 $= \lim_{x\to 0} \frac{1}{x^3} \left[\left(1 + \frac{\cos x - 1}{3} \right)^x - 1 \right]$
 $x(\cos x - 1)$

【例24】 求极限
$$\lim_{x\to 0} \frac{\arcsin x - \sin x}{\arctan x - \tan x}$$
.
$$\int_{-\frac{1}{2}}^{-\frac{1}{2}} \int_{-\frac{1}{2}}^{-\frac{1}{2}} \int_{-\frac$$

【例25】(2009年2) 求极限
$$\lim_{x\to 0} \frac{(1-\cos x)[x-\ln(1+\tan x)]}{\frac{\sin^4 x}}$$
.

【解1】原式 = $\lim_{x\to 0} \frac{\frac{1}{2}x^2[x-\ln(1+\tan x)]}{\frac{x^4}{2}} = \frac{1}{2}\lim_{x\to 0} \frac{x-\ln(1+\tan x)}{x^2}$

$$= \frac{1}{2} \lim_{x \to 0} \frac{[x - \tan x] - [\ln(1 + \tan x) - \tan x]}{x^2}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{[x - \tan x] - [\ln(1 + \tan x) - \tan x]}{x^2}$$

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$$= \frac{1}{2} \lim_{x \to 0} \frac{[x - \tan x] - [\ln(1 + \tan x) - \tan x]}{x^2}$$

$$\frac{1}{2} \lim_{x \to 0} \frac{\left[-\frac{1}{3} x^{3} \right] - \left[-\frac{1}{2} \tan^{2} x \right]}{x^{2}}$$

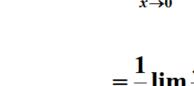
$$= \frac{1}{2} (0 + \frac{1}{2}) = \frac{1}{4}$$

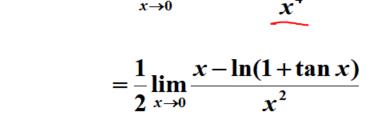
$$\frac{1}{2} (0 + \frac{1}{2}) = \frac{1}{4}$$

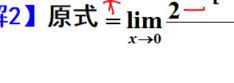
【例25】(2009年2)求极限
$$\lim_{x\to 0} \frac{(1-\cos x)[x-\ln(1+\tan x)]}{\sin^4 x}$$
.

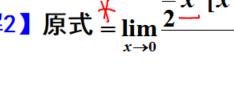
【解2】原式 $= \lim_{x\to 0} \frac{1}{2} x^2 [x-\ln(1+\tan x)]$

$$[\mathbf{m}^2]$$
 原式 $= \lim_{x \to 0} \frac{1}{2} \underline{x}^2$

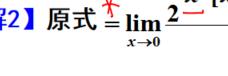


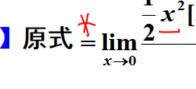


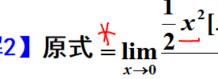




 $4 x \rightarrow 0$





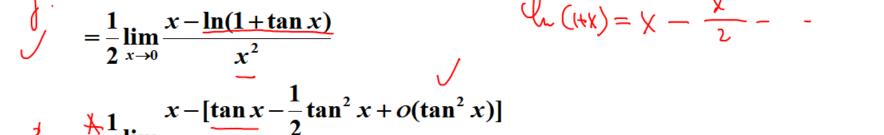


【例25】(2009年2) 求极限
$$\lim_{x\to 0} \frac{(1-\cos x)[x-\ln(1+\tan x)]}{\sin^4 x}$$
.

【解3】原式 =
$$\lim_{x\to 0} \frac{1}{2}x^2[x-\ln(1+\tan x)]$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{x-\ln(1+\tan x)}{2}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{x-\ln(1+\tan x)}{2}$$



	$2 \xrightarrow{x \to 0}$	x^2	/	
	v	Itan x 1	$\int_{0}^{\infty} (\tan^2 x)$	
1	$\stackrel{\text{lim}}{=} 1$	$\frac{1}{2}$	$\frac{1}{2} \frac{x + O(\tan x)}{1}$	
1	$2 \xrightarrow{x \to 0}$	x	2/	

$$\stackrel{=}{=} \frac{1}{2} \lim_{x \to 0} \frac{x - \left[\tan x - \frac{1}{2} \tan^2 x + o(\tan^2 x)\right]}{x^2}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{x - \tan x}{x^2} + \frac{1}{4} \lim_{x \to 0} \frac{\tan^2 x}{x^2}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{-\frac{1}{3} x^3}{x^2} + \frac{1}{4} \lim_{x \to 0} \frac{x^2}{x^2} = \frac{1}{4}$$

$$\lim(f(x) \pm g(x)) = \lim f(x) \pm \lim g(x)$$

$$\lim(f(x) \cdot g(x)) = \lim f(x) \cdot \lim g(x)$$

$$\lim \left(\frac{f(x)}{g(x)}\right) = \frac{\lim f(x)}{\lim g(x)} \quad (B \neq 0)$$

e fi(x) /8/4.

$$N + N = 2N \times 1$$

$$N + (N) = 0$$

$$1 \times 1 \times 1$$

常用的结论
$$f(x) = A \neq 0 \Rightarrow \lim_{x \to \infty} f(x)g(x) = A\lim_{x \to \infty} g(x);$$
即: 极限非零的因子的极限可先求出来.

2) $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ 存在, $\lim_{x \to \infty} g(x) = 0 \Rightarrow \lim_{x \to \infty} f(x) = 0;$
3) $\lim_{x \to \infty} \frac{f(x)}{g(x)} = A \neq 0$, $\lim_{x \to \infty} f(x) = 0 \Rightarrow \lim_{x \to \infty} g(x) = 0;$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = A \neq 0$$
, $\lim_{x \to \infty} f(x) = 0 \Rightarrow \lim_{x \to \infty} g(x) = 0;$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = A \neq 0$$
, $\lim_{x \to \infty} f(x) = 0 \Rightarrow \lim_{x \to \infty} g(x) = 0;$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = A \neq 0$$
, $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to \infty$

【例26】(2010年3)若
$$\lim_{x\to 0} \left[\frac{1}{x} - \left(\frac{1}{x} - a\right)e^{x}\right] = 1$$
, 则 a 等于 ()

(A) 0 (B) 1 - - (C) 2 (D) 3

= -1 + a

$$1 = \lim_{x \to 0} \left[\frac{1}{x} - \left(\frac{1}{x} - a \right) e^x \right] = \lim_{x \to 0} \left[\frac{1 - e^x}{x} \right] + a \lim_{x \to 0} e^x$$

则
$$a=2$$
 故应选 (C).

【例27】(2018年3)已知实数
$$a,b$$
 满足 $\lim_{x \to +\infty} [(ax + b)e^{\frac{1}{x}} - x] = 2$, 求 a,b .

【解】 $2 = \lim_{x \to +\infty} be^{\frac{1}{x}} + \lim_{x \to +\infty} (axe^{\frac{1}{x}} - x)$

$$= b + \lim_{x \to +\infty} x(ae^{\frac{1}{x}} - 1)$$

$$b + \lim_{x \to +\infty} x(ae^{x} - 1)$$

$$b + \lim_{x \to +\infty} x(e^{\frac{1}{x}} - 1)$$

$$= b + \lim_{x \to +\infty} x(e^{x} - 1)$$

$$= b + \lim_{n \to +\infty} x \cdot \frac{1}{n}$$

= b + 1

故 a=b=1.

【例28】(2004年3) 若极限
$$\lim_{x\to 0} \frac{\sin x}{e^x - a} (\cos x - b) = 5$$
 ,则 $a = ____$, $b = ____$.

【解】由于
$$\lim_{x\to 0} \frac{\sin x}{e^x - a} (\cos x - b) = \lim_{x\to 0} \frac{\sin x (\cos x - b)}{e^x - a} = \underbrace{5 \neq 0}$$

解】由于
$$\lim_{x\to 0} \frac{\sin x}{e^x - a} (\cos x - b) = \lim_{x\to 0} \frac{\sin x (\cos x - b)}{e^x - a} = \underline{5 \neq 0}$$

且 $\lim_{x\to 0} \sin x (\cos x - b) = 0$,
$$\lim_{x\to 0} (e^x - a) = 0$$
, 即 $a = 1$.

$$\lim_{x \to 0} \sin x(\cos x - b) = 0,$$

$$\lim_{x \to 0} (e^x - a) = 0, \quad \text{III} \quad a = 1.$$

$$\lim_{x \to 0} \frac{\sin x}{e^x - a} (\cos x - b) = \lim_{x \to 0} \frac{\sin x}{e^x - 1} (\cos x - b)$$

$$\lim_{x \to 0} (e^x - a) = 0, \quad \square \qquad a = 1.$$

$$\lim_{x \to 0} \frac{\sin x}{e^x - a} (\cos x - b) = \lim_{x \to 0} \frac{\sin x}{e^x - 1} (\cos x - b)$$

$$= \lim_{x \to 0} \frac{x}{e^x - a} (\cos x - b) = \lim_{x \to 0} \frac{x}{e^x - 1} (\cos x - b)$$

$$\lim_{x \to 0} \frac{\sin x}{e^{x} - a} (\cos x - b) = \lim_{x \to 0} \frac{\sin x}{e^{x} - 1} (\cos x - b)$$

$$= \lim_{x \to 0} \frac{x}{x} (\cos x - b) = 1 - b$$

由 1-b=5 得, b=-4.

【例29】 (1997年2) 求极限
$$\lim_{x \to -\infty} \frac{\sqrt{4x^2 + x - 1} + x + 1}{\sqrt{x^2 + \sin x}}$$
 (一x) $\left[\sqrt{4 + \frac{1}{x} - \frac{1}{x^2} - 1 - \frac{1}{x}}\right]$

【解1】原式 =
$$\lim_{x \to \infty} \frac{(-x)[\sqrt{4 + \frac{1}{x} - \frac{1}{x^2}} - 1 - \frac{1}{x}]}{(-x)\sqrt{1 + \frac{\sin x}{x^2}}}$$

$$=\lim_{x\to-\infty} \frac{\sqrt{4+\frac{1}{x}-\frac{1}{x^2}}}{\sqrt{1+\frac{\sin x}{x^2}}}$$

$$=\lim_{x\to-\infty} \frac{\sqrt{4+\frac{1}{x}-\frac{1}{x^2}-1-\frac{1}{x}}}{\sqrt{1+\frac{\sin x}{x^2}}} = 1$$

 $=2-1\mp0=1$

I 原式 =
$$\lim_{x \to -\infty} \frac{\sqrt{\frac{x + x^2}{x^2}}}{(-x)\sqrt{1 + \frac{\sin x}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{4 + \frac{1}{x} - \frac{1}{x^2}} - 1 - \frac{1}{x}}{\sin x} = 1$$

[解2] 原式 = $\lim_{x \to -\infty} \frac{\sqrt{4x^2 + x - 1}}{\sqrt{x^2 + \sin x}} + \lim_{x \to -\infty} \frac{x}{\sqrt{x^2 + \sin x}} + \lim_{x \to -\infty} \frac{1}{\sqrt{x^2 + \sin x}}$

$$(-x)\sqrt{1 + \frac{\sin x}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{4 + \frac{1}{x} - \frac{1}{x^2} - 1 - \frac{1}{x}}}{\sqrt{1 + \frac{\sin x}{x^2}}} = 1$$

VX2= |X = -X

$$\frac{(-x)\sqrt{1 + \frac{\sin x}{x^2}}}{\sqrt{1 + \frac{\sin x}{x^2}}} = 1$$

$$= \lim_{x \to -\infty} \frac{\sqrt{4 + \frac{1}{x} - \frac{1}{x^2} - 1 - \frac{1}{x}}}{\sqrt{1 + \frac{\sin x}{x^2}}} = 1$$



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