

Pre-Selección Nacional de Informática (PSNIC)

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1. Graph

1.1. Articulations Points

```
#include <cstdio>
#include <vector>
#include <stack>

#define RANG 1000010
using namespace std;

vector <int> A[RANG];
stack <int> Q;
int v, a, nod, newn, t, DT[RANG], LOW[RANG];
bool MK[RANG];

void AP (int nod) {
    DT[nod] = LOW[nod] = ++t;
    for (vector <int>::iterator newn = A[nod].begin(); newn != A[nod].end();
newn++) {
        if (!LOW[*newn]) {
            AP (*newn);
            LOW[nod] = min (LOW[nod], LOW[*newn]);
            if (!MK[nod] && (DT[nod] != 1 && DT[nod] <= LOW[*newn]) ||
(DT[nod] == 1 && DT[*newn] > 2)) {
                MK[nod] = true;
                Q.push (nod);
            }
        }
        else
            LOW[nod] = min (LOW[nod], DT[*newn]);
    }
}

main() {
    freopen ("AP.in", "r", stdin);
    freopen ("AP.ou", "w", stdout);

    scanf ("%d %d", &v, &a);
    for (int i = 0; i < a; i++) {
        scanf ("%d %d", &nod, &newn);
        A[nod].push_back (newn);
        A[newn].push_back (nod);
    }

    AP (1);

    while (!Q.empty()) {
        printf ("%d\n", Q.top());
        Q.pop();
    }
}
```

1.2. Bridges

```
#include <cstdio>
#include <vector>
#include <stack>

#define RANG 1000010
using namespace std;

struct tri {
    int nod, newn;
    bool marc;

    int nextn (int *a) {
        if (nod == *a)
            return newn;
        return nod;
    }
} A[RANG];

struct two {
    int nod, newn;
};

stack <two> Q;
vector <int> ID[RANG];
int v, a, nod, newn, t, DT[RANG], LOW[RANG];

void Bridges (int nod) {
    DT[nod] = LOW[nod] = ++t;
    for (vector <int>::iterator id = ID[nod].begin(); id != ID[nod].end(); id++) {
        int newn = A[*id].nextn (&nod);
        if (!LOW[newn]) {
            A[*id].marc = true;
            Bridges (newn);
            LOW[nod] <?= LOW[newn];
            if (DT[nod] < LOW[newn])
                Q.push ((two) {nod, newn});
        }
        else
            if (!A[*id].marc)
                LOW[nod] <?= DT[newn];
    }
}

main() {
    freopen ("Bridges.in", "r", stdin);
    freopen ("Bridges.ou", "w", stdout);
    scanf ("%d %d", &v, &a);
    for (int i = 0; i < a; i++) {
        scanf ("%d %d", &nod, &newn);
        A[i] = (tri) {nod, newn, false};
        ID[nod].push_back (i);
        ID[newn].push_back (i);
    }
    Bridges (1);
    while (!Q.empty()) {
        printf ("%d %d\n", Q.top().nod, Q.top().newn);
        Q.pop();
    }
}
```

1.3. Strong Connected Components

```
#include <cstdio>
#include <vector>
#include <stack>

#define RANG 1000010
using namespace std;

vector <int> A[RANG];
stack <int> Q;
int v, a, t, nod, newn, LOW[RANG], DT[RANG];
bool MK[RANG];

void SCC (int nod) {
    DT[nod] = LOW[nod] = ++t;
    Q.push (nod);
    for (vector <int>::iterator newn = A[nod].begin(); newn != A[nod].end();
newn++) {
        if (!LOW[*newn]) {
            SCC (*newn);
            LOW[nod] = min (LOW[nod], LOW[*newn]);
        }
        else
            if (!MK[*newn])
                LOW[nod] = min (LOW[nod], DT[*newn]);
    }

    if (LOW[nod] == DT[nod]) {
        while (Q.top() != nod) {
            printf ("%d ", Q.top());
            MK[Q.top()] = true;
            Q.pop();
        }
        printf ("%d\n", Q.top());
        MK[nod] = true;
        Q.pop();
    }
}

main() {
    freopen ("SCC.in", "r", stdin);
    freopen ("SCC.ou", "w", stdout);
    scanf ("%d %d", &v, &a);
    for (int i = 1; i <= a; i++) {
        scanf ("%d %d", &nod, &newn);
        A[nod].push_back (newn);
    }
    for (int i = 1; i <= v; i++)
        if (!LOW[i])
            SCC (i);
}
```

1.4. Eulerian Circuit Or Path

```
#include <cstdio>
#include <vector>
#include <queue>

#define RANG 1000010
```

```

using namespace std;

struct tri {
    int nod, newn;
    bool marc;

    int nextn (int *x) {
        if (*x == nod)
            return newn;
        return nod;
    }
} A[RANG];

vector <int> ID[RANG];
queue <int> Q;
int v, a, nod, newn, impar, start = 1, G[RANG];

void Euler (int nod) {
    for (vector <int>::iterator id = ID[nod].begin(); id != ID[nod].end(); id++) {
        if (!A[*id].marc) {
            A[*id].marc = true;
            Euler (A[*id].nextn (&nod));
        }
        Q.push (nod);
    }
}

main() {
    freopen ("Euler.in", "r", stdin);
    freopen ("Euler.ou", "w", stdout);

    scanf ("%d %d", &v, &a);
    for (int i = 1; i <= a; i++) {
        scanf ("%d %d", &nod, &newn);
        ID[nod].push_back (i);
        ID[newn].push_back (i);
        A[i] = (tri) {nod, newn, false};
        G[nod]++;
        G[newn]++;
    }

    for (int i = 1; i <= v; i++)
        if (G[i] % 2) {
            impar++;
            start = i;
            if (impar > 2) {
                printf ("The Graph is not Eulerian\n");
                return 0;
            }
        }

    Euler (start);

    if (!impar)
        printf ("Eulerian Circuit\n");
    else
        printf ("Eulerian Path\n");
    for (; !Q.empty(); Q.pop())
        printf ("%d ", Q.front());
}

```

1.5. Floyd Warshall

```
#include <cstdio>
#include <algorithm>

#define RANG 310
using namespace std;

int v, a, q, nod, newn, cost, A[RANG][RANG];

main () {
    freopen ("FW.in", "r", stdin);
    freopen ("FW.ou", "w", stdout);

    memset (A, 63, sizeof (A));
    scanf ("%d %d", &v, &a);
    for (int i = 0; i < a; i++) {
        scanf ("%d %d %d", &nod, &newn, &cost);
        A[nod][newn] <?= cost;
    }

    for (int i = 1; i <= v; i++)
        for (int j = 1; j <= v; j++)
            for (int k = 1; k <= v; k++)
                A[i][j] <?= A[i][k] + A[k][j];

    scanf ("%d", &q);
    while (q--) {
        scanf ("%d %d", &nod, &newn);
        printf ("%d\n", A[nod][newn]);
    }
}
```

1.6. Lowest Common Ancestor

```
#include <cstdio>
#include <vector>
#include <cmath>

#define RANG 100010
using namespace std;

vector <int> A[RANG];
int v, a, q, nod, newn, maxlog, LV[RANG], T[RANG][20];

void DFS (int nod, int lv) {
    LV[nod] = lv;
    maxlog = (int) log2 (lv);
    for (int i = 1; i <= maxlog; i++)
        T[nod][i] = T[T[nod][i - 1]][i - 1];

    for (vector <int>::iterator newn = A[nod].begin(); newn != A[nod].end(); newn++)
        DFS (*newn, lv + 1);
}

int search (int *nod, int *newn) {
    if (LV[*nod] < LV[*newn])
        swap (nod, newn);
}
```

```

maxlog = (int) log2 (LV[*nod]);
for (int i = maxlog; i >= 0; i--)
    if (LV[*nod] - (1 << i) >= LV[*newn])
        *nod = T[*nod][i];
    if (*nod == *newn)
        return *nod;

maxlog = (int) log2 (LV[*nod]);
for (int i = maxlog; i >= 0; i--)
    if (T[*nod][i] != T[*newn][i] && T[*nod][i]) {
        *nod = T[*nod][i];
        *newn = T[*newn][i];
    }
return T[*nod][0];
}

main () {
    freopen ("LCA.in", "r", stdin);
    freopen ("LCA.ou", "w", stdout);

    scanf ("%d %d", &v, &a);
    for (int i = 0; i < a; i++) {
        scanf ("%d %d", &nod, &newn);
        A[nod].push_back (newn);
        T[newn][0] = nod;
    }

    DFS (1 , 1);

    scanf ("%d", &q);
    while (q--) {
        scanf ("%d %d", &nod, &newn);
        printf ("%d\n", search (&nod, &newn));
    }
}

```

1.7. Prim

```

#include <cstdio>
#include <vector>
#include <queue>

#define RANG 100
using namespace std;

typedef pair <int, int> two;
vector <two> A[RANG];
priority_queue <two, vector <two>, greater <two> > Q;
bool M[RANG];
int n, a, newn, cost, nod, sol;

main() {
    freopen ("prim.in", "r", stdin);
    freopen ("prim.out", "w", stdout);

    scanf ("%d %d", &n, &a);
    for (int i = 1; i <= a; i++) {
        scanf ("%d %d %d", &nod, &newn, &cost);
        A[nod].push_back (two (newn, cost));
        A[newn].push_back (two (nod, cost));
    }
}

```

```

        Q.push(two (0, 1));
        while (!Q.empty()) {
            nod = Q.top().second;
            cost = Q.top().first;
            Q.pop();
            if (!M[nod]) {
                M[nod] = true;
                sol += cost;
                for (vector<two>::iterator i = A[nod].begin(); i !=
A[nod].end(); i++)
                    if (!M[i->first])
                        Q.push(two (i->second, newn));
            }
        }

        printf ("%d\n", sol);
    }
}

```

1.8. Kruskal With Disjoin Set

```

#include <cstdio>
#include <algorithm>

#define RANG 100
using namespace std;

int v, a, nod, newn, cost, setnod, setnewn, sol, SET[RANG], R[RANG];
struct tri {
    int nod, newn, cost;

    bool operator < (const tri &p) const {
        return cost < p.cost;
    }
} A[RANG];

void make_set (int i) {
    SET[i] = i;
    R[i] = 1;
}

int find_set (int nod) {
    if (SET[nod] != nod)
        SET[nod] = find_set (SET[nod]);
    return SET[nod];
}

void join_set (int nod, int newn) {
    if (R[nod] > R[newn]) {
        SET[newn] = nod;
        R[nod]++;
    }
    else {
        SET[nod] = newn;
        R[newn]++;
    }
}

main() {
    freopen ("kruskal.in", "r", stdin);
    freopen ("kruskal.out", "w", stdout);
}

```



```

scanf ("%d %d", &v, &a);
for (int i = 0; i < a; i++) {
    scanf ("%d %d %d", &nod, &newn, &cost);
    A[i] = (tri) {nod, newn, cost};
}

sort (A, A + a);
for (int i = 1; i <= v; i++)
    make_set (i);

for (int i = 0; i < a; i++) {
    setnod = find_set (A[i].nod);
    setnewn = find_set (A[i].newn);
    if (setnod != setnewn) {
        sol += A[i].cost;
        join_set (setnod, setnewn);
    }
}

printf ("%d\n", sol);
}

```

2. Data Structure

2.1. Binary Indexed Tree

```

#include <cstdio>

#define RANG 1000010
using namespace std;

struct bit {
    int l, T[RANG];

    void add (int *x, int *n) {
        for (int i = *x; i <= l; i += i & -i)
            T[i] += *n;
    }

    int sum (int *x) {
        int sum = 0;
        for (int i = *x; i; i -= i & -i)
            sum += T[i];
        return sum;
    }

    void update (int *x, int *n) {
        int lastx = *x - 1;
        int sumx = sum (x) - sum (&lastx);
        sumx = *n - sumx;
        add (x, &sumx);
    }
} BIT;
int q, x, n, sol;
char qt;

main() {
    freopen ("BIT.in", "r", stdin);

```

```

    freopen ("BIT.ou", "w", stdout);

    scanf ("%d %d\n", &BIT.l, &q);

    while (q--) {
        scanf ("%c ", &qt);
        if (qt == 'a') {
            scanf ("%d %d\n", &x, &n);
            BIT.add (&x, &n);
            continue;
        }
        if (qt == 'u') {
            scanf ("%d %d\n", &x, &n);
            BIT.update (&x, &n);
            continue;
        }
        scanf ("%d %d\n", &x, &n);
        sol = BIT.sum (&n) - BIT.sum (&(--x));
        printf ("%d\n", sol);
    }
}

```

2.2. Range Min - Max Quering

```

#include <cstdio>
#include <algorithm>
#include <cmath>

#define RANG 1000000
using namespace std;

int n, c, p, q, a, b;
struct two {
    int min, max;
} T[RANG][19];

main() {
    freopen ("RMQ.in", "r", stdin);
    freopen ("RMQ.ou", "w", stdout);

    scanf ("%d", &n);
    for (int i = 1; i <= n; i++) {
        scanf ("%d", &T[i][0].min);
        T[i][0].max = T[i][0].min;
    }

    c = (int) log2 (n);
    a = n;
    for (int j = 1; j <= c; j++) {
        p = 1 << j - 1;
        a -= p;
        for (int i = 1; i <= a; i++) {
            T[i][j].min = min (T[i][j - 1].min, T[i + p][j - 1].min);
            T[i][j].max = max (T[i][j - 1].max, T[i + p][j - 1].max);
        }
    }

    scanf ("%d", &q);
    while (q--) {
        scanf ("%d %d", &a, &b);
        c = (int) log2 (b - a);
    }
}

```

```

        b = b - (1 << c) + 1;
        printf ("%d %d\n", min (T[a][c].min, T[b][c].min), max (T[a][c].max,
T[b][c].max));
    }

    c = (int) log2 (n);
    for (int i = 1; i <= n; i++) {
        for (int j = 0; j <= c; j++)
            printf ("%d//%d ", T[i][j].min, T[i][j].max);
        printf ("\n");
    }
}

```

2.3. Segment Tree

```

#include <cstdio>
#include <algorithm>

#define RANG 1000010
#define oo 1 << 30
using namespace std;

int a, b, q, N[RANG];
char qt;

struct st {
    int l, T[RANG];

    int build (int x, int xend, int lv) {
        if (x == xend)
            return T[lv] = N[x];
        int piv = (x + xend) / 2;
        return T[lv] = min (build (x, piv, lv * 2), build (piv + 1, xend, lv
* 2 + 1));
    }

    int update (int x, int xend, int lv) {
        if (x > a || xend < a)
            return T[lv];
        if (x == xend)
            return N[x];
        int piv = (x + xend) / 2;
        return T[lv] = min (update (x, piv, lv * 2), update (piv + 1, xend, lv *
2 + 1));
    }

    int query (int x, int xend, int lv) {
        if (a > xend || b < x)
            return oo;
        if (a <= x && b >= xend)
            return T[lv];
        int piv = (x + xend) / 2;
        return min (query (x, piv, lv * 2), query (piv + 1, xend, lv * 2 +
1));
    }
} ST;

main () {
    freopen ("ST.in", "r", stdin);
    freopen ("ST.ou", "w", stdout);
}

```

```

scanf ("%d", &ST.l);
for (int i = 1; i <= ST.l; i++)
    scanf ("%d", &N[i]);

ST.build (1, ST.l, 1);

scanf ("%d\n", &q);
while (q--) {
    scanf ("%c %d %d\n", &qt, &a, &b);
    if (qt == 'q')
        printf ("%d\n", ST.query (1, ST.l, 1));
    else {
        N[a] = b;
        ST.update (1, ST.l, 1);
    }
}
}

```

2.4. Trie

```

#include <cstdio>
#include <algorithm>

#define RANG 256
using namespace std;

struct trie {
    bool marc;
    trie *next[RANG];
} TRIE, *P;
int l, k, q, lW;
char W[RANG];

main () {
    freopen ("Trie.in", "r", stdin);
    freopen ("Trie.ou", "w", stdout);

    scanf ("%d", &l);
    for (int i = 0; i < l; i++) {
        scanf ("%s", &W);

        P = &TRIE;
        lW = strlen (W);
        for (int j = 0; j < lW; j++) {
            if (P -> next[W[j]] == NULL) {
                P -> next[W[j]] = new trie();
                P = P -> next[W[j]];
            }
            else
                P = P -> next[W[j]];
        }
        P -> marc = true;
    }

    scanf ("%d", &q);
    while (q--) {
        scanf ("%s", &W);
        P = &TRIE;
        lW = strlen (W);
        for (k = 0; k < lW; k++) {
            if (P -> next[W[k]] == NULL)

```

```

        break;
        P = P -> next[W[k]];
    }

    if (k == lW && P -> marc)
        printf ("YES\n");
    else
        printf ("NO\n");
}
}

```

3. String Matching

3.1. Knuth Morris Pratt

```

#include <stdio>
#include <cstring>

#define RANG 1000010
using namespace std;

int lA, lB, mf, F[RANG];
char A[RANG], B[RANG];

main() {
    freopen ("KMP.in", "r", stdin);
    freopen ("KMP.ou", "w", stdout);

    scanf ("%s", A + 1);
    scanf ("%s", B + 1);

    lA = strlen (A + 1);
    lB = strlen (B + 1);

    //printf ("0 ");
    for (int i = 2; i <= lA; i++) {
        while (mf > 0 && A[i] != A[mf + 1])
            mf = F[mf];
        if (A[i] == A[mf + 1])
            mf++;
        F[i] = mf;
        //printf ("%d ", F[i]);
    }

    for (int i = 1, mf = 0; i <= lB; i++) {
        while (mf > 0 && A[mf + 1] != B[i])
            mf = F[mf];
        if (A[mf + 1] == B[i])
            mf++;
        if (mf == lA) {
            printf ("%d\n", i - lA + 1);
            mf = F[mf];
        }
    }
}

```

3.2. Suffix Array

```
# include <stdio>
# include <algorithm>
# include <cstring>

using namespace std;

# define MAXN 1010

int N, K, k;
int pos[MAXN], suf[MAXN], T[MAXN], LCP[MAXN];
char word[MAXN];

bool cmp(const int &a, const int &b) {
    if (pos[a] != pos[b])
        return pos[a] < pos[b];
    if (a + K < N && b + K < N)
        return pos[a + K] < pos[b + K];
    return a > b;
}

int main() {
    freopen("suffixarr.in", "r", stdin);
    freopen("suffixarr.out", "w", stdout);

    scanf("%s", &word);
    N = strlen(word);

    for (int i = 0; i < N; i++) {
        suf[i] = i;
        pos[i] = word[i];
    }

    for (K = 0; K < N; K ? K *= 2 : K++) {
        sort(suf, suf + N, cmp);
        for (int i = 1; i < N; i++)
            T[i] = T[i - 1] + cmp(suf[i - 1], suf[i]);
        for (int j = 0; j < N; j++)
            pos[suf[j]] = T[j];
    }

    for (int i = k = 0; i < N; i++) {
        if (pos[i] == N - 1) continue;
        for (int j = suf[pos[i] + 1];
             j + k < N &&
             i + k < N &&
             word[j + k] == word[i + k]; k++);
        LCP[pos[i]] = k;
    }

    for (int i = 0; i < N; i++)
        printf("%d - %s\n", LCP[i], word + suf[i]);
}
```

4. Dynamic

4.1. Edit Distance

```

#include <stdio>
#include <algorithm>

#define RANG 5010
using namespace std;

int lA, lB, maxl, s, T[RANG][RANG];
char A[RANG], B[RANG];

main () {
    freopen ("ED.in", "r", stdin);
    freopen ("ED.ou", "w", stdout);

    scanf ("%s\n %s\n", A + 1, B + 1);

    lA = strlen (A + 1);
    lB = strlen (B + 1);
    maxl = max (lA, lB);
    for (int i = 0; i <= maxl; i++)
        T[i][0] = T[0][i] = i;
    for (int i = 1; i <= lA; i++)
        for (int j = 1; j <= lB; j++) {
            s = 1;
            if (A[i] == B[j])
                s = 0;
            T[i][j] = min (min (T[i][j - 1] + 1, T[i - 1][j] + 1), T[i -
1][j - 1] + s);
        }

    printf ("%d\n", T[lA][lB]);
}

```

4.2. Longest Comun Subsequence

```

#include <stdio>
#include <cstring>

#define RANG 100
using namespace std;

int lA, lB, T[RANG][RANG];
char A[RANG], B[RANG];

main() {
    freopen ("LCS.in", "r", stdin);
    freopen ("LCS.out", "w", stdout);

    scanf ("%s\n", A + 1);
    scanf ("%s", B + 1);

    lA = strlen (A + 1);
    lB = strlen (B + 1);
    for (int i = 1; i <= lB; i++)
        for (int j = 1; j <= lA; j++)
            if (B[i] == A[j])
                T[i][j] = T[i - 1][j - 1] + 1;
            else
                T[i][j] = max (T[i - 1][j], T[i][j - 1]);

    printf ("%d\n", T[lB][lA]);
}

```

4.3. Longest Increasing Or Decreasing Subsequence

```
(<) --> lower_bound
(<=) --> upper_bound
#include <cstdio>
#include <algorithm>

#define RANG 100
using namespace std;

int n, m, up, N[RANG], SOL[RANG], ID[RANG], L[RANG];

void write (int ID) {
    if (ID) {
        write (L[ID]);
        printf ("%d ", N[ID]);
    }
}

main() {
    freopen ("LIS.in", "r", stdin);
    freopen ("LIS.ou", "w", stdout);

    scanf ("%d", &n);
    for (int i = 1; i <= n; i++)
        scanf ("%d", &N[i]);

    for (int i = 1; i <= n; i++) {
        if (SOL[m] <= N[i]) {
            SOL[++m] = N[i];
            ID[m] = i;
            L[i] = ID[m - 1];
        }
        else {
            up = upper_bound (SOL + 1, SOL + m + 1, N[i]) - SOL;
            SOL[up] = N[i];
            ID[up] = i;
            L[i] = ID[up - 1];
        }
    }

    printf ("%d\n", m);
    write (ID[m]);
}
```

5. Geometry

5.1. Closest Pair Of Points (Convex Hull)

```
#include <cstdio>
#include <cmath>
#include <algorithm>
```



```

#define RANG 1000010
#define oo 1 << 30
using namespace std;

struct two {
    double x, y;

    bool operator < (const two &p) const {
        if (x != p.x)
            return x < p.x;
        return y < p.y;
    }
} P[RANG], T[RANG];
int l, lim = 1, top;
double dsol = oo;

double cross (const two &pf, const two &p1, const two &p2) {
    double m1 = (p2.y - pf.y) * (p1.x - pf.x);
    double m2 = (p2.x - pf.x) * (p1.y - pf.y);
    return m1 - m2;
}

double dist (const two &p1, const two &p2) {
    return sqrt ((p2.y - p1.y) * (p2.y - p1.y) + (p2.x - p1.x) * (p2.x -
p1.x));
}

main () {
    freopen ("CPP.in", "r", stdin);
    freopen ("CPP.ou", "w", stdout);

    scanf ("%d", &l);
    for (int i = 0; i < l; i++)
        scanf ("%lf %lf", &P[i].x, &P[i].y);

    sort (P, P + l);

    T[++top] = P[0];
    T[++top] = P[1];
    for (int i = 2; i < l; i++) {
        while (top > lim && cross (T[top - 1], T[top], P[i]) < 0)
            top--;
        T[++top] = P[i];
        dsol = min (dsol, dist (T[top - 1], T[top]));
    }

    lim = top;
    T[++top] = T[l - 2];
    T[++top] = T[l - 3];
    for (int i = l - 4; i >= 0; i--) {
        while (top > lim && cross (T[top - 1], T[top], P[i]) < 0)
            top--;
        T[++top] = P[i];
        dsol = min (dsol, dist (T[top - 1], T[top]));
    }

    printf ("%lf\n", dsol);
}

```

5.2. Closest Pair Of Points (Sweep Line)

```

#include <cstdio>
#include <algorithm>
#include <cmath>
#include <set>

#define RANG 1000010
#define oo 1 << 30
using namespace std;

struct two {
    double x, y;
} P[RANG], *last = P;

struct cmp_x {
    bool operator () (const two &p1, const two &p2) const {
        return p1.x < p2.x;
    }
};

struct cmp_y {
    bool operator () (const two &p1, const two &p2) const {
        return p1.y < p2.y;
    }
};

multiset <two, cmp_y> Q;
multiset <two, cmp_y>::iterator lo, hi;
double dsol = oo;
int l;

double dist (const two &p1, const two &p2) {
    return sqrt ((p2.y - p1.y) * (p2.y - p1.y) + (p2.x - p1.x) * (p2.x -
p1.x));
}

main () {
    freopen ("CPP.in", "r", stdin);
    freopen ("CPP.ou", "w", stdout);

    scanf ("%d", &l);
    for (int i = 0; i < l; i++)
        scanf ("%lf %lf", &P[i].x, &P[i].y);

    sort (P, P + l, cmp_x());

    for (two *i = P; i < &P[l]; i++) {
        while (i -> x - last -> x >= dsol)
            Q.erase (Q.find (*last++));

        lo = Q.lower_bound ((two) {i -> x, i -> y - dsol});
        hi = Q.upper_bound ((two) {i -> x, i -> y + dsol});

        for (; lo != hi; lo++)
            dsol = min (dsol, dist (*lo, *i));
        Q.insert (*i);
    }

    printf ("%lf\n", dsol);
}

```

5.3. Convex Hull (Graham Scan)

```
#include <cstdio>
#include <algorithm>

#define RANG 100010
using namespace std;

double x, y;
int l, top, lim = 1;

struct two {
    double x, y;

    bool operator < (const two &p) const {
        if (x != p.x)
            return x < p.x;
        return y < p.y;
    }
} P[RANG], T[RANG];

double cross (const two &pf, const two &p1, const two &p2) {
    double m1 = (p2.y - pf.y) * (p1.x - pf.x);
    double m2 = (p2.x - pf.x) * (p1.y - pf.y);
    return m1 - m2;
}

main () {
    freopen ("convex_hull.in", "r", stdin);
    freopen ("convex_hull.ou", "w", stdout);

    scanf ("%d", &l);
    for (int i = 0; i < l; i++) {
        scanf ("%lf %lf", &x, &y);
        P[i] = (two) {x, y};
    }

    sort (P, P + l);

    for (int i = 0; i < l; i++) {
        while (top > lim && cross (T[top - 1], T[top], P[i]) <= 0)
            top--;
        T[++top] = P[i];
    }

    lim = top;
    for (int i = l - 1; i >= 0; i--) {
        while (top > lim && cross (T[top - 1], T[top], P[i]) <= 0)
            top--;
        T[++top] = P[i];
    }

    printf ("%d\n", top);
    for (int i = 1; i <= top; i++)
        printf ("%01f %01f\n", T[i].x, T[i].y);
}
```

5.4. Union Area (Segment Tree + Sweep Line)

```
# include <cstdio>
```

```

#include <vector>
#include <algorithm>

using namespace std;

#define MAXN 10010
#define MAXC 30010

struct event {
    int start, lo, hi, flag;
    event(int x, int i, int j, int s) {
        start = x;
        lo = i;
        hi = j;
        flag = s;
    }
    bool operator <(const event &q)
    const {return start < q.start;}
};

int N, last, sol;
int tree[MAXC * 3], cant[MAXC * 3];
vector <event> L;

void update (int n, int lo, int hi, int s, int f, int val) {
    if (lo > f || hi < s || lo > hi)
        return ;

    if (lo >= s && hi <= f)
        tree[n] += val;
    else {
        int mid = (lo + hi)/2;
        update(n * 2, lo, mid, s, f, val);
        update(n * 2 + 1, mid + 1, hi, s, f, val);
    }

    if (!tree[n])
        if (lo == hi)
            cant[n] = 0;
        else cant[n] = cant[n * 2] + cant[n * 2 + 1];
    else cant[n] = (hi - lo) + 1;
}

int main() {
    freopen("unionarea.in", "r", stdin);
    freopen("unionarea.out", "w", stdout);

    scanf("%d", &N);
    for (int i = 1; i <= N; i++) {
        int a, b, c, d;
        scanf("%d%d%d%d", &a, &b, &c, &d);
        if (b > d)
            swap(b, d);
        L.push_back(event(a, b, d - 1, 1));
        L.push_back(event(c, b, d - 1, -1));
    }

    sort(L.begin(), L.end());

    last = L[0].start;
    for (int i = 0; i < L.size(); i++) {
        sol += (L[i].start - last) * cant[1];
        last = L[i].start;
        update(1, 0, MAXC, L[i].lo, L[i].hi, L[i].flag);
    }
}

```

```

    }
    printf("%d\n", sol);
}

```

6. Math

6.1. Gaus Comun Divisor

```

#include <cstdio>
#include <algorithm>

using namespace std;

int a , b;

int GCD (int a, int b) {
    while (a) {
        swap (a, b);
        a %= b;
    }
    return b;
}

main () {
    freopen ("GCD.in", "r", stdin);
    freopen ("GCD.ou", "w", stdout);

    scanf ("%d %d", &a, &b);

    printf ("MCD es %d\n", GCD (a, b));
    printf ("MCM es %d\n", a * b / GCD (a, b));
}

```

6.2. Big Mod (b^e) % m

```

#include <cstdio>

using namespace std;

long long q, b, e, m, sq;

long long square (long long n) {
    return n * n;
}

long long big_mod (int b, int e, int m) {
    if (!e)
        return 1;
    if (e % 2 == 0)
        return square (big_mod (b, e / 2, m)) % m;
    return (b % m * big_mod (b, e - 1, m)) % m;
}

main () {
    scanf ("%d", &q);
}

```

```

        while (q--) {
            scanf ("%I64d %I64d %I64d", &b, &e, &m);
            printf ("%I64d\n", big_mod (b, e, m));
        }
}

```

6.3. Counting Combinations $C(n, k)$

```

#include <cstdio>
#define RANG 110
using namespace std;

long long q, n, k, T[RANG][RANG];

//O (n)
double comb (long long n, long long k) {
    double comb = 1;
    if (n - k < k)
        k = n - k;
    for (int i = 2; i <= k; i++)
        comb /= i;
    k = n - k;
    for (int i = n; i > k; i--)
        comb *= i;
    return comb + 0.01;
}

//O (k)
double combfast (long long n, long long k) {
    double comb = 1;
    if (n - k < k)
        k = n - k;
    for (int i = 1; i <= k; i++)
        comb = comb * (n - k + i) / i;
    return comb + 0.01;
}

//O (1)
long long pascal_tri (long long n, long long k) {
    return T[n][k];
}

main () {
    scanf ("%d", &q);

    for (int i = 0; i < RANG; i++)
        T[i][0] = T[i][i] = 1;
    for (int i = 1; i < RANG; i++)
        for (int j = 1; j < RANG; j++)
            T[i][j] = T[i - 1][j] + T[i - 1][j - 1];

    while (q--) {
        scanf ("%lld %lld", &n, &k);
        printf ("%0.1f\n", comb (n, k));
        printf ("%0.1f\n", combfast (n, k));
        printf ("%lld\n", pascal_tri (n, k));
    }
}

```

6.4. Exponentiation n^k

```
#include <stdio>
#include <cmath>

using namespace std;

long long q, n, k;

long long square (long long n) {
    return n * n;
}

//O (N)
long long slowexp (long long n, long long k) {
    long long sol = 1;
    for (long long i = 1; i <= k; i++)
        sol *= n;
    return sol;
}

//O (log2 (N))
long long fastexpr (long long n, long long k) {
    if (!k)
        return 1;
    if (k % 2 == 0)
        return square (fastexpr (n, k / 2));
    return n * fastexpr (n, k - 1);
}

//O (log2 (N))
long long fastexpi (long long n, long long k) {
    long long sol = 1;
    while (k) {
        if (k & 1)
            sol *= n;
        n *= n;
        k >>= 1;
    }
    return sol;
}

main () {
    scanf ("%lld", &q);

    while (q--) {
        scanf ("%lld %lld", &n, &k);
        printf ("%lld\n", slowexp (n, k));
        printf ("%lld\n", fastexpr (n, k));
        printf ("%lld\n", fastexpi (n, k));
        printf ("%0.1f\n", pow (n, (double) k));
        printf ("%0.1f\n", exp (log (n) * k));
    }
}
```

6.5. Fórmulas

```
Pi = 4 * (2/3 * 4/3 * 4/5 * 6/5 * ...) = 3,14159265358979
Pi = 2 * acos (0);
```

$$\text{Golden Number} = \frac{\sqrt{5} + 1}{2} = 1,61803398874989$$

Fibonacci Number

$$F(n) = F(n - 1) * \text{Golden Number}$$

Sumatoria de n

$$S(n) = \frac{n * (n + 1)}{2}$$

Números Catalan

$$C(n) = C(n, 2n) - C(n - 1, 2n) = \frac{(2n)!}{n! * (n + 1)!}$$

Variaciones

$$V(n, p) = \frac{n!}{(n - p)!}$$

$$V'(n, p) = n^p;$$

Permutaciones

$$P(n) = n!$$

$$P'(n, n_1, n_2, \dots, n_k) = \frac{n!}{n_1! * n_2! * \dots * n_k!}$$

Donde n es el total de elementos y n(k) la cantidad de repeticiones

Combinaciones

$$C(n, p) = C(n - 1, p - 1) + C(n - 1, p) = \frac{n!}{(n - p)! * p!}$$

Mínimo Común Múltiplo

$$\text{MCM}(a, b) = \frac{a * b}{\text{MCD}(a, b)}$$

Carmichael Numbers >= 3 Primes Factors

561, 1105, 1729, 2465, 2821, 6601, 8911, 10585, 15841, 29341, 41041, 46657, 52633, 62745, 63973

Sumatoria de n^2

$$1^2 + 2^2 + \dots + n^2 = n * (n + 1) * (2n + 1) / 6;$$

Sumatoria de x^n

$$x^0 + x^1 + \dots + x^n = (x^{(n + 1)} - 1) / (x - 1);$$

Sumatoria de los divisores de un número

$$N = p_1^{a_1} + p_2^{a_2} + \dots + p_k^{a_k}$$

sea $p_1 < p_2 < \dots < p_k$ números primos.

$$S(N) = \frac{p_1^{(a_1 + 1)} - 1}{p_1 - 1} * \dots * \frac{p_k^{(a_k + 1)} - 1}{p_k - 1}$$