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// Hopcroft - Karp  $O(M \cdot \sqrt{N})$ 
const int MAXV = 1001;
const int MAXV1 = 2*MAXV;
int N, M;
vector<int> ady[MAXV];
int D[MAXV1], Mx[MAXV], My[MAXV];

bool BFS(){
    int u, v, i, e;
    queue<int> cola;
    bool f = 0;
    for (i = 0; i < N+M; i++) D[i] = 0;
    for (i = 0; i < N; i++)
        if (Mx[i] == -1) cola.push(i);
    while (!cola.empty()) {
        u = cola.front(); cola.pop();
        for (e = ady[u].size()-1; e >= 0; e--) {
            v = ady[u][e];
            if (D[v + N]) continue;
            D[v + N] = D[u] + 1;
            if (My[v] != -1) {
                D[My[v]] = D[v + N] + 1;
                cola.push(My[v]);
            }
            else f = 1;
        }
    }
    return f;
}

int DFS(int u) {
    for (int v, e = ady[u].size()-1; e >= 0; e--) {
        v = ady[u][e];
        if (D[v+N] != D[u]+1) continue;
        D[v+N] = 0;
        if (My[v] == -1 || DFS(My[v])) {
            Mx[u] = v; My[v] = u;
            return 1;
        }
    }
    return 0;
}

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int Hopcroft_Karp(){
    int i, flow = 0;
    for (i = max(N,M); i >= 0; i--)
        Mx[i] = My[i] = -1;
    while (BFS())
        for (i = 0; i < N; i++)
            if (Mx[i] == -1 && DFS(i))
                ++flow;
    return flow;
}

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// Heavy Light Descomposition
int N, M;
vector<int> V[MN];
vector<int> G[MN];
vector<bool> L[MN];
/// cant- la cantidad de nodos
/// pos- la pos. donde aparece
/// nn- el nod en el cual aparece
/// pd- el link con el padre full superior
/// G-Dp
/// L-lazy
int cant[MN], pos[MN], nn[MN], pd[MN];

void Dfs(int nod, int pad){
    int t = V[nod].size(), newn;
    if (t == 1 && nod != 1){
        pos[nod] = 0;
        nn[nod] = nod;
        cant[nod] = 1;
        pd[nod] = pad;
        return;
    }
    int mej = nod;
    for (int i = 0; i < t; i++){
        newn = V[nod][i];
        if (newn == pad) continue;
        Dfs (newn, nod);
        if (cant[mej] < cant[nn[newn]])
            mej = nn[newn];
    }
    pos[nod] = cant[mej];
    cant[mej] ++;
    nn[nod] = mej;
    pd[mej] = pad;
}

typedef pair<int, int> par;
typedef pair<int, par> tri;
typedef vector<tri> vt;
typedef vector<par> vp;
/// me da el recorrido desde a hasta b en vector<tri>
/// f posicion s.f in, s.f fin

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vt rec(int a, int b) {
    vp A1, B1;
    A1.clear(), B1.clear();
    for (int i = a; i != -1; i = pd[nn[i]])
        A1.push_back(par(nn[i], pos[i]));
    for (int i = b; i != -1; i = pd[nn[i]])
        B1.push_back(par(nn[i], pos[i]));
    vt C1;
    C1.clear();
    reverse(A1.begin(), A1.end());
    reverse(B1.begin(), B1.end());
    int t = 0;
    while (t < A1.size() && t < B1.size() && A1[t] == B1[t])
        t ++;
    if (t >= A1.size() || t >= B1.size() || (t < B1.size() &&
    t < A1.size() && A1[t].first != B1[t].first))
        t --;
    if ((t < A1.size() && t < B1.size()) && A1[t].first ==
    B1[t].first){
        C1.push_back(tri(A1[t].first, par(min(A1[t].second,
        B1[t].second), max(A1[t].second, B1[t].second))));
        t ++;
    }
    for (int i = t; i < A1.size(); i++){
        C1.push_back (tri(A1[i].first, par(A1[i].second,
        cant[A1[i].first] - 1)));
    }
    for( int i = t; i < B1.size(); i++){
        C1.push_back (tri(B1[i].first, par(B1[i].second,
        cant[B1[i].first] - 1)));
    }
    return C1;
}

void havy_light() {
    Dfs (1, -1); // root
    for (int i = 1; i <= N; i++) /// rellenar con 4*cant
        if(cant[i]){
            G[i] = vector<int> (cant[i]*4, 0);
            L[i] = vector<bool> (cant[i]*4, false);
            G[i][1] = cant[i], L[i][1] = true;
        }
}

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// Segment Tree Persistent
const int N = 100000 + 100, LOGN = 20;
const int TOT = 4*N + N*LOGN;
int sum[TOT], L[TOT], R[TOT];
int sz = 1;

int newNode(int s = 0){
    sum[sz] = s;
    return sz++;
}

int build(int b, int e){
    if (b==e) return newNode();
    int mid = (b + e) >> 1;
    int cur = newNode();
    L[cur] = build(b, mid);
    R[cur] = build(mid+1, e);
    return cur;
}

int update(int node, int b, int e, int p){
    if(b == e) return newNode(sum[node] + 1);
    int mid = (b + e) >> 1;
    int cur = newNode();
    if(p <= mid) {
        L[cur] = update(L[node], b, mid, p);
        R[cur] = R[node];
    }
    else {
        R[cur] = update(R[node], mid+1, e, p);
        L[cur] = L[node];
    }
    sum[cur] = sum[L[cur]] + sum[R[cur]];
    return cur;
}

int query(int node1, int node2, int b, int e, int k){
    if(b == e) return b;
    int s = sum[L[node2]] - sum[L[node1]];
    int mid = (b + e) >> 1;
    if(s >= k) return query(L[node1], L[node2], b, mid, k);
    else return query(R[node1], R[node2], mid+1, e, k-s);
}

```

```

int root[N];

int main()
{
    int n, m;
    cin >> n >> m;
    root[0] = build(1, n);
    vector<int> v(n), tmp(n);
    for(int i = 0; i < n; ++i){
        cin >> v[i]; tmp[i] = v[i];
    }
    sort(tmp.begin(), tmp.end());
    tmp.resize(unique(tmp.begin(), tmp.end()) - tmp.begin());
    for(int i = 0; i < n; ++i)
        root[i+1] = update(root[i], 1, n, lower_bound (
            tmp.begin(), tmp.end(), v[i]) - tmp.begin() + 1);
    while(m--){
        int i, j, k;
        cin >> i >> j >> k;
        cout << tmp[query(root[i-1], root[j], 1, n, k)-1];
    }
}

```

// Pollard's Rho Integer Factoring Algorithm

typedef long long ll;

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ll mulmod(ll a, ll b, ll c) { // (a*b)%c, minimizing overflow
    ll x = 0, y = a % c;
    while (b > 0) {
        if (b % 2 == 1)
            x = (x + y) % c;
        y = (y * 2) % c;
        b /= 2;
    }
    return x % c;
}
```

```
ll pollard_rho(ll n) {
    int i = 0, k = 2;
    ll x = 3, y = 3; // random seed = 3, other values possible
    while (1) {
        i++;
        x = (mulmod(x, x, n) + n - 1) % n;
        ll d = __gcd(abs(y - x), n);
        if (d != 1 && d != n) return d;
        if (i == k) y = x, k *= 2;
    }
}
```

```
int main() {
    ll n = 2063512844981574047LL; // n is not a large prime
    ll ans = pollard_rho(n);
    if (ans > n / ans) ans = n / ans;
    cout << ans << ' ' << n / ans;
    // should be: 1112041493 1855607779
    return 0;
}
```

// Floyd's Cycle-Finding Algorithm $O(\mu + \lambda)$ $\mu \rightarrow \mu$, $\lambda \rightarrow \lambda$

typedef pair<int,int> ii;

int f(int x);

```
ii floydCycleFinding(int x0) {
    // finding  $k \cdot \mu$ , hare's speed is 2x tortoise's
    int tortoise = f(x0);
    int hare = f(f(x0)); // f(x0) is the node next to x0
    while (tortoise != hare) {
        tortoise = f(tortoise);
        hare = f(f(hare));
    }
    // finding  $\mu$ , hare and tortoise move at the same speed
    int mu = 0;
    hare = x0;
    while (tortoise != hare) {
        tortoise = f(tortoise);
        hare = f(hare);
        mu++;
    }
    // finding  $\lambda$ , hare moves, tortoise stays
    int lambda = 1;
    hare = f(tortoise);
    while (tortoise != hare) {
        hare = f(hare);
        lambda++;
    }
    return ii(mu, lambda);
}
```

```

return factors; }          // if N does not fit in 32-bit integer and is a prime number
// then 'factors' will have to be changed to vector<ll>
// inside int main(), assuming sieve(1000000) has been called before
vi res = primeFactors(2147483647);          // slowest, 2147483647 is a prime
res = primeFactors(136117223861LL);        // slow, 2 large pfactors 104729*1299709
res = primeFactors(142391208960LL);        // faster, 2^10*3^4*5*7^4*11*13

// functions involving prime factors
// numPF(N): Count the number of prime factors of N
ll numPF(ll N) {
    ll PF_idx = 0, PF = primes[PF_idx], ans = 0;
    while (N != 1 && (PF * PF <= N)) {
        while (N % PF == 0) { N /= PF; ans++; }
        PF = primes[++PF_idx];
    }
    if (N != 1) ans++;
    return ans;
}

// numDiffPF(N): count the number of different prime factors of N
ll numDiffPF(ll N) {
    ll PF_idx = 0, PF = primes[PF_idx], ans = 0;
    while (PF * PF <= N) {
        if (N % PF == 0) ans++;          // count this pf only once
        while (N % PF == 0) N /= PF;
        PF = primes[++PF_idx];
    }
    if (N != 1) ans++;
    return ans;
}

// sumPF(N): sum the prime factors of N
ll sumPF(ll N) {
    ll PF_idx = 0, PF = primes[PF_idx], ans = 0;
    while (PF * PF <= N) {
        while (N % PF == 0) { N /= PF; ans += PF; }
        PF = primes[++PF_idx];
    }
    if (N != 1) ans += N;
    return ans;
}

// numDiv(N): count the number of divisors of N
ll numDiv(ll N) {
    ll PF_idx = 0, PF = primes[PF_idx], ans = 1;          // start from ans = 1
    while (N != 1 && (PF * PF <= N)) {
        ll power = 0;          // count the power
        while (N % PF == 0) { N /= PF; power++; }
        ans *= (power + 1);          // according to the formula
        PF = primes[++PF_idx];
    }
    if (N != 1) ans *= 2;          // (last factor has pow = 1, we add 1 to it)
    return ans;
}

```

```

// sumDiv(N): sum the divisors of N
ll sumDiv(ll N) {
    ll PF_idx = 0, PF = primes[PF_idx], ans = 1; // start from ans = 1
    while (N != 1 && (PF * PF <= N)) {
        ll power = 0;
        while (N % PF == 0) { N /= PF; power++; }
        ans *= ((ll)pow((double)PF, power + 1.0) - 1)/(PF - 1); // formula
        PF = primes[++PF_idx];
    }
    if (N != 1) ans *= ((ll)pow((double)N, 2.0) - 1) / (N - 1); // last one
    return ans;
}

// EulerPhi(N): count the number of positive integers < N that are
// relatively prime to N.
ll EulerPhi(ll N) {
    ll PF_idx = 0, PF = primes[PF_idx], ans = N; // start from ans = N
    while (N != 1 && (PF * PF <= N)) {
        if (N % PF == 0) ans -= ans / PF; // only count unique factor
        while (N % PF == 0) N /= PF;
        PF = primes[++PF_idx];
    }
    if (N != 1) ans -= ans / N; // last factor
    return ans;
}

// square matrix exponentiation
#define MAX_N 105 // increase/decrease this value as needed
struct Matrix {
    int mat[MAX_N][MAX_N]; // we will return a 2D array
};

Matrix matMul(Matrix a, Matrix b) { // O(n^3)
    Matrix ans; int i, j, k;
    for (i = 0; i < MAX_N; i++)
        for (j = 0; j < MAX_N; j++)
            for (ans.mat[i][j] = k = 0; k < MAX_N; k++) // if necessary, use
                ans.mat[i][j] += a.mat[i][k] * b.mat[k][j]; // modulo arithmetic
    return ans;
}

Matrix matPow(Matrix base, int p) { // O(n^3 log p)
    Matrix ans; int i, j;
    for (i = 0; i < MAX_N; i++)
        for (j = 0; j < MAX_N; j++)
            ans.mat[i][j] = (i == j); // prepare identity matrix
    while (p) { // iterative version of Divide & Conquer exponentiation
        if (p & 1) ans = matMul(ans, base); // if p is odd (last bit is on)
        base = matMul(base, base); // square the base
        p >>= 1; // divide p by 2
    }
    return ans;
}

```