Algebra Autumn 2013
Frank Sottile
27 August 2013

Homework

Hand in to Frank Thursday 29 August:

1. Let α be a rotation about a point p in the plane and ρ be a reflection in a line through p. What is α^{-1}, geometrically? Show that $\rho\alpha\rho = \alpha^{-1}$. Let σ also be a reflection in a line. What is $\rho\sigma$? What is its order? (There are several cases to consider.)

Hand in to Frank Tuesday 3 September:

2. Let A, B be groups with elements $a \in A$ and $b \in B$. What is the order of the element $(a, b) \in A \times B$?

Hand in for the grader Tuesday 3 September:

3. Show that a group G cannot be the union of two proper subgroups.
4. Show that a finite group G of even order (G has an even number of elements) has a non-identity element a with $a^2 = e$.
5. Suppose that G is a group in which the square of every element is the identity. Prove that G is abelian.
6. Assume that $G = \{e, a, b, c\}$ is a group with four elements and identity e. Suppose that G has no element of order four. Prove that there is a unique group structure for G and deduce that G is abelian.