Foundations of Mathematics — Math 300 Sections 902, 905

YOUR NAME

Ninth Homework: — Due 11 November 2020

1. [16] Do all parts of Problem 17 in the Exercises for Section 6.3 in the Sundstrom book.

2. [10] Let A and B be sets. Recall the definitions of the identity functions $I_A\colon A \to A$ and $I_B\colon B \to B$: For $a \in A$, $I_A(a) = a$ and for $b \in B$, $I_B(b) = b$.

   Let $f\colon A \to B$ be a function. Prove by a direct computation that $f = f \circ I_A$ and that $f = I_B \circ f$.

3. [10] Let A be a set. Prove that the identity function I_A is a bijection.

4. [15] For each of the following, either give an example of functions $f\colon A \to B$ and $g\colon B \to C$ that satisfy the given properties, or explain why no such example exists.

   (a) The function g is a surjection, but the function $g \circ f$ is not a surjection.
   (b) The function g is an injection, but the function $g \circ f$ is not an injection.
   (c) The function f is not a surjection, but the function $g \circ f$ is a surjection.
   (d) The function g is not a surjection, but the function $g \circ f$ is a surjection.
   (e) The function g is not an injection, but the function $g \circ f$ is a surjection.

5. [17] Let $f\colon A \to B$ and $g\colon B \to A$ be functions. Recall the identity functions $I_A\colon A \to A$ and $I_B\colon B \to B$. Preferably using theorems previously proven in the class (state those that you use), show the following.

   (a) If $g \circ f = I_A$, then f is an injection.
   (b) If $f \circ g = I_B$, then f is a surjection.
   (c) If $g \circ f = I_A$ and $f \circ b = I_B$, then f and g are bijections and $g = f^{-1}$.

6. [12] Let $f\colon S \to T$ be a function, A, B be subsets of S and C, D be subsets of T. For $x \in S$ and $y \in T$, carefully explain what is means to say that

   (a) $y \in f(A \cup B)$.
   (b) $y \in f(A) \cap f(B)$.
   (c) $x \in f^{-1}(C \cap D)$.
   (d) $x \in f^{-1}(C) \cup f^{-1}(D)$.

7. [10] Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = -2x + 1$ and let

   $$A := [2,5] \qquad B := [-1,3] \qquad C := [-2,3] \qquad D := [1,4]$$

   Find each of the following sets:

   (a) $f(A)$
   (b) $f^{-1}(C)$
   (c) $f^{-1}(C \cap D)$
   (d) $f^{-1}(f(B))$
   (e) $f^{-1}(C) \cup f^{-1}(D)$

8. [10] Let $f\colon A \to B$ be a function and $T \subset B$. Prove that $T \supseteq f(f^{-1}(T))$.