Algebra II Winter 2021 1 February
Frank Sottile
Problem on Third Homework for Frank

Write your answers neatly, in complete sentences, and prove all assertions. Revise your work before handing it in, and submit a .pdf created from a LaTeX source to Gradescope. Correct and crisp proofs are greatly appreciated; oftentimes your work can be shortened and made clearer.

Due Monday 8 February.

For Frank to grade:

1. Let R be a commutative ring and A an R-module. An element a of A is *torsion* if its annihilator $\{r \in R \mid r.a = 0\}$ is not the zero ideal. Prove that if R is an integral domain, then the set $T(A)$ of torsion elements of A is a submodule.

   Give an example of a module over a commutative ring whose set of torsion elements does not form a submodule.
