Foundations of Mathematics Math 300H Section 970

YOUR NAME (-1 if you do not put your name here)

First Homework: 30 August 2022

1. Write each of the following sets by listing its elements within braces.

   (a) $A = \{n \in \mathbb{Z} : -4 \leq n \leq 4\}$

   (b) $B = \{n \in \mathbb{Z} : n^2 \leq 7\}$

   (c) $C = \{n \in \mathbb{N} : n^3 < 100\}$

   (d) $D = \{x \in \mathbb{R} : x^3 - x = 0\}$

   (e) $E = \{x \in \mathbb{R} : x^2 + 1 = 0\}$

2. Let $S = \{-10, -9, -8, \ldots, 8, 9, 10\}$. Describe each of the following sets as $\{x \in S : p(x)\}$, where $p(x)$ is some condition on x.

   (a) $A = \{-10, -9, \ldots, -1, 1, \ldots, 9, 10\}$

   (b) $B = \{-10, -9, \ldots, -1, 0\}$

   (c) $C = \{-5, -4, \ldots, 0, 1, \ldots, 7\}$

   (d) $D = \{-10, -9, -8, \ldots, 4, 6, 7, \ldots, 10\}$

3. Give examples of three sets A, B, and C such that

   (a) $A \subseteq B \subsetneq C$

   (b) $A \in B$, $B \in C$, and $A \notin C$

   (c) $A \in B$ and $A \subsetneq C$

   (The sets may be different in each of (a), (b), and (c).)

4. Which of the following sets are equal?

   $A = \{n \in \mathbb{Z} : |n| < 2\}$ $B = \{n \in \mathbb{Z} : n^2 \leq 1\}$

   $C = \{n \in \mathbb{Z} : n^3 = n\}$ $D = \{-1, 0, 1\}$

   $E = \{n \in \mathbb{Z} : n^2 \leq n\}$

5. Let $A = \{\emptyset, \spadesuit, \Psi\}$. Determine which of the following are true or false.

   (a) $\spadesuit \subseteq \mathcal{P}(A)$ (e) $\emptyset \subseteq \mathcal{P}(A)$ (i) $\{\emptyset, \{\spadesuit\}\} \subseteq \mathcal{P}(A)$

   (b) $\Psi \in \mathcal{P}(A)$ (f) $\emptyset \in \mathcal{P}(A)$ (j) $\{\emptyset, \{\spadesuit\}\} \in \mathcal{P}(A)$

   (c) $\{\Psi\} \subseteq \mathcal{P}(A)$ (g) $\{\emptyset\} \subseteq \mathcal{P}(A)$ (k) $A \subseteq \mathcal{P}(A)$

   (d) $\{\spadesuit\} \in \mathcal{P}(A)$ (h) $\{\emptyset\} \in \mathcal{P}(A)$ (l) $A \in \mathcal{P}(A)$

6. Let U be some universal set that contains sets A, B, and C as subsets. Investigate the two sets $A - (B - C)$ and $(A - B) - C$. Are they the same? different? Is one a subset of the other?

   Make a conjecture about their relation, and explain why it holds.

7. Determine whether the following statements are true or false.

   (a) If $\{1\} \in \mathcal{P}(A)$, then $1 \in A$ but $\{1\} \notin A$.

   (b) If A, B, and C are sets such that $A \subsetneq \mathcal{P}(B) \subsetneq C$ and $|A| = 2$, then $|C|$ can be 5, but it cannot be 4.

   (c) If a set B has one more element than a set A, then $\mathcal{P}(B)$ has at least two more elements than $\mathcal{P}(A)$.

   (d) If four sets A, B, C, and D are subsets of $\{1, 2, 3\}$ such that $|A| = |B| = |C| = |D| = 2$, then at least two of these sets are equal.

8. Determine the cardinality of each of the following sets.

   (a) $A = \{1, 2, 3, \{1, 2, 3\}, 4, \{4\}\}$

   (b) $B = \{x \in \mathbb{R} : |x| = -\pi\}$

   (c) $C = \{m \in \mathbb{N} : 2 < m \leq 5\}$

   (d) $D = \{n \in \mathbb{N} : n < 0\}$

   (e) $E = \{k \in \mathbb{N} : 1 \leq k^2 \leq 100\}$

   (f) $F = \{k \in \mathbb{Z} : 1 \leq k^2 \leq 100\}$

9. Given examples of three sets A, B, and C such that $B \neq C$ but $B - A = C - A$.

10. Given an example of two subsets A and B of $\{1, 2, 3\}$ such that all of the following sets are different: $A \cup B$, $A \cup \overline{B}$, $\overline{A} \cup B$, $\overline{A} \cup \overline{B}$, $A \cap B$, $A \cap \overline{B}$, $\overline{A} \cap B$, and $\overline{A} \cap \overline{B}$.