ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Course Nr: EDOC-Equivalence

Title: *Topology of Real Algebraic Varieties*

Professor: *Alex Degtyarev (Bilkent University)*

Credit: *2* ***Exam***: *Oral*

Time: *Wednesday March 19th, 2008 from 2 PM - 4 PM*

Wednesday April 2nd, 2008 from 2 PM to 4 PM
Wednesday April 9th, 2008 from 2 PM to 4 PM
Wednesday April 16th, 2008 from 2 PM to 4 PM
Tuesday April 29th, 2008 from 2 PM to 4 PM
Wednesday April 30th, 2008 from 2 PM to 4 PM

Tuesday May 6th, 2008 from 2 PM to 4 PM
Wednesday, May 7th, 2008 from 2 PM to 4 PM
Wednesday, May 14th, 2008, from 2 PM to 4 PM
Wednesday, May 21st, 2008 from 2 PM to 4 PM

Room: *AAC 006*

Description:

Introduction. Topology of involutions.

– Introduction to the subject; real vs. complex varieties; the basic concept: a real variety is a complex variety with a real structure; relation between the real and complex moduli spaces; codimension of the discriminant and consequences.

– The principal question: classification up to a certain equivalence of the real varieties within a fixed complex deformation class. The strongest relation, equivariant deformation equivalence, requires a sufficiently thorough understanding of the complex moduli space and thus belongs partially to algebraic geometry. The weakest one, homeomorphism of the real parts, can often be solved by relatively simple topological means, provided that the topology of complex varieties is understood. (The existence question remains outside the scope of these notes.)

– Relaxed questions: finiteness and quasi-simplicity. At present, most result still rely on some kind of partial classification.

– Classical and newer tools in topology of involutions: Smith exact sequence and inequality, the concept of (*M* ¡ *d*)-varieties, the experimental fact that it is close to *M*-varieties that usually exhibit interesting topological properties; Hodge decomposition and Petrovsky type inequalities; elementary arithmetic of intersection forms and resulting congruences; equivariant cohomology, localization theorems, Kalinin's spectral sequence, Viro homomorphisms, relation to *M*-varieties.

Topology of surfaces and curves on surfaces.

– A paraphrase and ramification of the general results (mainly, inequalities and congruences) to the case of surfaces.

– Applications to curves on surfaces (via branched coverings), with a brief overview of the simple case of abstract curves.

– A more sophisticated application of the B´ezout theorem/exploiting the small dimension: ramification of the restrictions via explicitly visualized cycles.

– A zoo of known cases.

Special surfaces. Deformations. These lectures will give an account of the deformation study of some special classes of surfaces, each requiring its own sophisticated tools.

– Rational and ruled surfaces. Deformations are understood sufficiently well, and the main problem is to express the data in topological terms.

– $K3$-surfaces, including required rather advanced arithmetic of symmetric bilinear forms.

– Generalizations: symmetric $K3$-surfaces (including Enriques surfaces) and (symmetric) 2-tori (in the latter case, there also is a more traditional approach using the universal covering).

– Applications and ramifications: the total reality conjecture, complex singular curves, etc.

– Elliptic surfaces: real Tate-Shafarevich group, reduction to Jacobian surfaces, partial results on Weierstraß surfaces.

Lausanne, 12 février 2008
TR/tvw