Second Homework Assignment for Math 151H
Due September 13

Factor the following polynomials as much as you can, using integer coefficients.

1. $x^2 - 1$.

2. $x^3 - 1$.

3. $x^2 - a^2$.

4. $x^3 - a^3$.

5. $x^4 - a^4$.

6. $x^5 - a^5$.

7. $x^6 - a^6$.

8. $x^7 - a^7$.

9. $x^8 - a^8$.

10. Find a formula for the sum and prove it.

$$\sum_{i=1}^{n}(2i - 1) \;=\; 1 + 3 + 5 + \cdots + (2n - 1).$$

The coefficient $\binom{n}{i} := \frac{n!}{i!(n-i)!}$ of $a^{n-i}b^i$ in the expansion $(a+b)^n$ is called a *binomial coefficient*. That is, $(a+b)^n$ equals

$$\binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n.$$

11. Show that $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$.

12. When $n = 7$, we noticed that all except the first and last binomial coefficient was divisible by 7. Can you explain this using the formula?

13. What about binomial coefficients $\binom{p}{i}$ when p is a prime number?

14. Use induction to show that
$$1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$

15. What is the simplest function? Why?

16. What is the second-simplest type of function? Explain.

17. If f and g are functions, what is $f + g$?

18. If f and g are functions, what is $f \cdot g$?

19. What is a polynomial function?

20. What is the domain of a polynomial function?

21. What is a rational function?

22. What is the domian of a rational function?

23. Does there exist a rational function with non-constant denominator and domain all of $\mathbb{R}$?

24. What is π?

25. What is π? (This is not a typo. There are two answers.)

26. Define $\sin x$ in terms of the unit circle.

27. Define $\cos x$ in terms of the unit circle.

28. What is $\tan x$?

29. What is $\sec x$?

30. What is $\cot x$?

31. What is $\csc x$?

32. Show that $\sin(-x) = -\sin x$.

33. Show that $\cos(-x) = \cos x$.

34. Show that $\sin(y + z) = \sin y \cos z + \cos y \sin z$.

35. Derive a formula for $\tan(2\omega)$.

36. Show that $\sin(\frac{\pi}{2} - \alpha) = \cos \alpha$.

37. Give the numerical values of $\sin \frac{\pi}{3}$, $\cos \frac{\pi}{3}$, $\tan \frac{\pi}{3}$, $\sec \frac{\pi}{3}$, $\cot \frac{\pi}{3}$, and $\csc \frac{\pi}{3}$ as radicals.

38. Give the numerical values of $\sin 0$, $\cos 0$, $\tan 0$, $\sec 0$, $\cot 0$, and $\csc 0$ as radicals.

39. Give the numerical values of $\sin \frac{\pi}{2}$, $\cos \frac{\pi}{2}$, $\tan \frac{\pi}{2}$, $\sec \frac{\pi}{2}$, $\cot \frac{\pi}{2}$, and $\csc \frac{\pi}{2}$ as radicals.

40. Give the numerical values of $\sin(-\frac{\pi}{6})$, $\cos(-\frac{\pi}{6})$, $\tan(-\frac{\pi}{6})$, $\sec(-\frac{\pi}{6})$, $\cot(-\frac{\pi}{6})$, and $\csc(-\frac{\pi}{6})$ as radicals.

41. Show that $\dfrac{\sec a - 1}{\sec a + 1} + \dfrac{\cos a - 1}{\cos a + 1} = 0$.

42. Show that $1 + \cot^2(\pi/2 - x) = \dfrac{1}{\sin^2(\pi/2 - x)}$.

43. Show that $\dfrac{\sin \beta}{\csc \beta} + \dfrac{\cos \beta}{\sec \beta} = 1$.

44. Show that $\sec^4 \theta - \sec^2 \theta = \dfrac{1}{\cot^4 \theta} + \dfrac{1}{\cot^2 \theta}$.

45. Show that $\tan 3\beta = \dfrac{3\tan \beta - \tan^3 \beta}{1 - 3\tan^2 \beta}$.

46. Show that $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$.

47. Show that $\cot(x/2) = \dfrac{1 + \cos x}{\sin x}$.

48. Show that $\csc y \sec y = 2 \csc 2y$.