Foundations of Mathematics Math 300 Sections 902, 905

YOUR NAME

Seventh Homework: Due 2 November 2020

1. [14] Do all parts of Problem 12 in the Exercises for Section 5.3 in the Sundstrom book.

2. [16] Sketch a picture (with labels) of each of the following in the Cartesian plane $\mathbb{R}^2$ with the usual conventions (the first coordinate is horizontal, while the second is vertical):

   (a) $[0,2]\times[1,3]$ (b) $(0,2)\times(1,3]$

   (c) $[2,3]\times\{1\}$ (d) $\{2\}\times[3,4]$

   (e) $\mathbb{R}\times(2,4)$ (f) $(1,3]\times\mathbb{R}$

   (g) $\mathbb{R}\times\{-1\}$ (h) $\{-1\}\times[1,\infty)$

3. [10] Is the following proposition true or false? Justify your conclusion.

   Let $A$, $B$, and $C$ be sets with $A\neq\emptyset$. If $A\times B=A\times C$, then $B=C$.

   Explain where the assumption $A\neq\emptyset$ is needed. What happens when $A=\emptyset$?

4. [10] For each positive integer $n\in\mathbb{N}$, let $A_n := (-\frac{1}{n}, 1-\frac{1}{n})\subset\mathbb{R}$.

   Determine each of $\bigcap_{k\in\mathbb{N}} A_k$ and $\bigcup\{A_\ell \mid \ell\in\mathbb{N}\}$.

5. [8] Let $\nu$ be the function from $\mathbb{N}$ to $\mathbb{N}$ whose value at a positive integer $n$ is the number of digits in the American English spelling of the number $n$. For example $\nu(0)=4$, as '0' is written zero with four letters. Similarly, $\nu(22)=9$, as twentytwo has nine letters.

   If we restrict the domain of $\nu$ to $\{1,2,\ldots,20\}$, what is its range?

6. [10] A *real function* is one whose domain and codomain are subsets of $\mathbb{R}$. For each of the following real functions, determine their largest possible domain and their range.

   (a) The function $f$ defined by $f(x)=x/(x^2-3x-2)$.

   (b) The function $g$ defined by $g(x)=\ln(1-\cos(x))$.

7. [8] Let $s\colon\mathbb{N}\to\mathbb{N}$ be the function whose value $s(n)$ at a number $n$ is the sum of the distinct natural number divisors of $n$. Compute the values of $s$ on the set $\{1,\ldots,10\}$. Is the function $s$ injective? Is it surjective? Justify your conclusions.

8. [8] Let $d\colon\mathbb{N}\to\mathbb{N}$ be the function whose value $d(n)$ at a number $n$ is the number of distinct natural number divisors of $n$. Compute the values of $d$ on the set $\{1,\ldots,10\}$. Is the function $d$ injective? Is it surjective? Justify your conclusions.

9. [8] Let $h\colon\mathbb{R}\times\mathbb{R}$ be the function defined for $(x,y)\in\mathbb{R}\times\mathbb{R}$ by $h(x,y):=x^2y+3xy-2y+3$. Is the function $h$ injective? Is it surjective? Justify your conclusions.

10. [8] Define $\varphi\colon\mathbb{N}\to\mathbb{Z}$ be the function defined for $n\in\mathbb{N}$ by $f(n)=\dfrac{1+(-1)^n(2n-1)}{4}$. Is the function $\varphi$ injective? Is it surjective? Justify your conclusions.