

Foundations of Mathematics

Math 300 Sections 902, 905

YOUR NAME

Second Homework:

Due 7 September 2020

Note: \mathbb{R} is the real numbers, \mathbb{Q} is the rational numbers, \mathbb{Z} is the integers, and \mathbb{N} is the natural numbers (which begin with 0).

- Rewrite the following statements in the form “if P , then Q ”.
 - “One, if by land”.
 - “Candor implies equality”.
 - “Pepperoni only if pizza”.
 - “Inattentive when bored”.
 - “Slapstick is sufficient for comedy”.
 - “Quiet is necessary for sleep”.
- Rewrite “If the function f is differentiable, then it is continuous” in each of the six forms of the previous problem.
- Express the following statements in the form “If P then Q ”. For example, “A Hexagon has six sides” becomes “If H is a hexagon, then H has six sides”.
 - An integer is odd or even.
 - All positive real numbers have a square root.
 - All angles of of an equilateral triangle are equal.
 - 1 is the smallest positive integer.
 - When the product of two integers is even, then both integers are even.
- Fill out the truth table for the expressions $P \wedge Q$, $\neg P$, $\neg Q$, $\neg(P \wedge Q)$, $\neg P \wedge \neg Q$, $\neg P \vee \neg Q$, and $(\neg P \wedge \neg Q) \vee Q$:

P	Q	$P \wedge Q$	$\neg P$	$\neg Q$	$\neg(P \wedge Q)$	$\neg P \wedge \neg Q$	$\neg P \vee \neg Q$	$(\neg P \wedge \neg Q) \vee Q$
T	T							
T	F							
F	T							
F	F							

- Which of the following expressions are tautologies? Which are contradictions?
 - $(P \rightarrow Q) \vee (Q \rightarrow P)$
 - $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
 - $P \rightarrow (Q \rightarrow P)$
 - $(P \wedge Q) \wedge (Q \rightarrow \neg P)$

6. Rewrite “Friendship is necessary and sufficient for happiness” in nine additional, equivalent ways.
7. Suppose that each of the following statements is true.

- Ibukun is in middle school.
- Ibukun got 90 on her German test or Adeola got 90 on her German test.
- If Adeola got 90 on her German test, then Ibukun is not in middle school.

If possible, determine the truth values of each of the following statements. Explain your reasoning.

- (a) Ibukun got 90 on her German test.
- (b) Adeola got 90 on her German test.
- (c) Either Adeola or Ibukun did not get 90 on the German test.

8. For statements P , Q , and R :

- (a) Show that $[(P \rightarrow Q) \wedge P] \rightarrow Q$ is a tautology. **Note:** In logic, this is an important rule of logic called *modus ponens*.
- (b) Show that $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow (P \rightarrow R)$ is a tautology. **Note:** In logic, this is an important rule of logic called *sylogism*.
- (c) Give example of a valid syllogism involving Socrates. Give example of a false syllogism involving Socrates.

9. Fill out a truth table (with $8 = 2^3$ rows) for the two expressions $(P \vee Q) \wedge (P \vee R)$ and $P \vee (Q \wedge R)$. What do you observe?

10. Write the converse and contrapositive of the following conditional statements:

- (a) If it rains, then the grass is wet.
- (b) $\alpha^2 = 25$ if $\alpha = 5$.
- (c) The integer a is odd only if $3a$ is odd.
- (d) “Inattentive when bored”.
- (e) “Quiet is necessary for sleep”.
- (f) “Pepperoni is necessary for Pizza”.

11. Give the contrapositive and converse of each of the implications in Problem 1. Write them in the form “If P , then Q ”.

12. Write a useful negation of each of the following statements. Do not leave a negation as the prefix of a statement. For example, the negation of “I will water my garden and pick basil” is “I will not water my garden or I will not pick basil”.

- (a) You will walk or take the bus.
- (b) Knowledge is necessary for truth.
- (c) That was Country and Western.
- (d) That was Country or Western.
- (e) If you wash the dishes or put away the laundry, you can have some chocolate.
- (f) Hard work is necessary for success.

13. Let a , b , and c be integers. Consider the following conditional statement:

If a divides bc , then a divides b or a divides c .

Which of the following statements have the same meaning as this conditional statement, and which are negations of this conditional statement:

- (a) If a divides b or a divides c , then a divides bc .
- (b) If a does not divide b or a does not divide c , then a does not divide bc .
- (c) a divides bc , a does not divide b , and a does not divide c .
- (d) If a does not divide b and a does not divide c , then a does not divide bc .
- (e) a does not divide bc or a divides b or a divides c .
- (f) If a divides bc and a does not divide c , then a divides b .
- (g) If a divides bc or a does not divide b , then a divides c .

14. Use the roster method to specify the elements in each of the following sets and then write a sentence in English describing the set.

- (a) $\{x \in \mathbb{R} \mid x^2 - 2x - 4 = 0\}$.
- (b) $\{n \in \mathbb{Z} \mid n^2 < 27\}$.
- (c) $\{n \in \mathbb{N} \mid n^2 < 27\}$.
- (d) $\{x \in \mathbb{Q} \mid x^2 - 2x - 4 = 0\}$.

15. Use set builder notation to specify the following sets.

- (a) The set of all natural numbers with square at least 15.
- (b) The set of all odd integers.
- (c) The set of all real numbers at most 10 whose square exceeds 3.
- (d) The set of positive rational numbers.