Foundations of Mathematics — Math 300H Section 970

Tenth Homework: Due 15 November 2022

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1. Let $A = \{a, b\}$. How many relations are there on A? If possible, do this without listing them.

2. Define a relation R on $\mathbb{N}$ (the strictly positive integers) by aRb if $a/b \in \mathbb{N}$. For $c, d \in \mathbb{N}$, under what conditions do we have $cR^{-1}d$?

3. Let A be a set with four elements, $|A| = 4$. What is the maximum number of elements that a relation R on A can contain such that $R \cap R^{-1} = \emptyset$?

4. Let $A = \{p, q, r, s\}$. How many relations defined on A are reflexive, symmetric, and transitive and contain the ordered pairs (p, q), (q, r), and (r, s)?

5. Let $A = \{a, b, c, d\}$. Give an example of a relation on A that is:

   (a) reflexive and symmetric but not transitive.

   (b) reflexive and transitive but not symmetric.

   (c) symmetric and transitive but not reflexive.

   (d) reflexive but neither symmetric nor transitive.

   (e) symmetric but neither reflexive nor transitive.

   (f) transitive but neither reflexive nor symmetric

6. Let C be the set of all polynomials of degree at most 3. An element f of C can then be expressed as $f = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$. Define the relation R on C by pRq if p and q have a root in common. (For example, $p = (x-1)^2$ and $q = x^2 - 1$ have the root 1 in common so that pRq.) Determine which of the properties: reflexive, symmetric and transitive are possessed by R.

7. Let R be an equivalence relation on $A = \{a, b, c, d, e, f, g\}$ such that aRc, bRf, cRd, and dRg. If there are three distinct equivalence classes resulting from R, then determine these equivalence classes and determine all elements of R.