Math 221
Sample problems for a final

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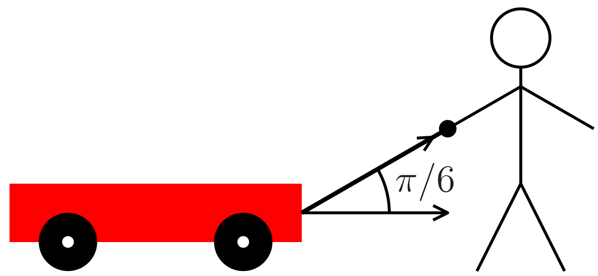
These are 11 of 15 questions that appeared on two versions of a final that I gave in the Fall of 2011. (The remaining four questions had to do with Stokes's and Gauß' Theorem, which we did not cover).
Point totals are in brackets next to each problem.

1. [25] Find the equation of the plane that contains the point $(-1,-3,2)$ and the line

$$x \;=\; -1-2t\,, \qquad y \;=\; 4t\,, \qquad x \;=\; 2+t\,.$$

2. [25] Frank is pulling a wagon a distance of 80 metres along a horizontal path by a constant force of 50 Newtons. If the handle of the wagon makes an angle of $\pi/6$ with the horizontal, how much work did Frank do in pulling the wagon?



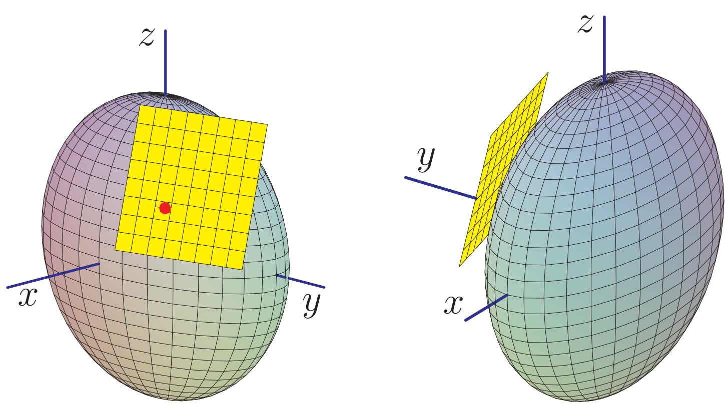
3. [25] The temperature at a point (x,y,z) is given by $T(x,y,z) = 2048\cdot 2^{-2x^2-3y^2-z^2}$, where T is in degrees Kelvin and x,y,z are in metres.

   (a) Find the rate of change of the temperature at the point $P := (1,-1,2)$ in the direction towards the point $(3,-3,2)$.

   (b) In which direction does the temperature increase the fastest at P?

   (c) Find the maximum rate of increase at P.

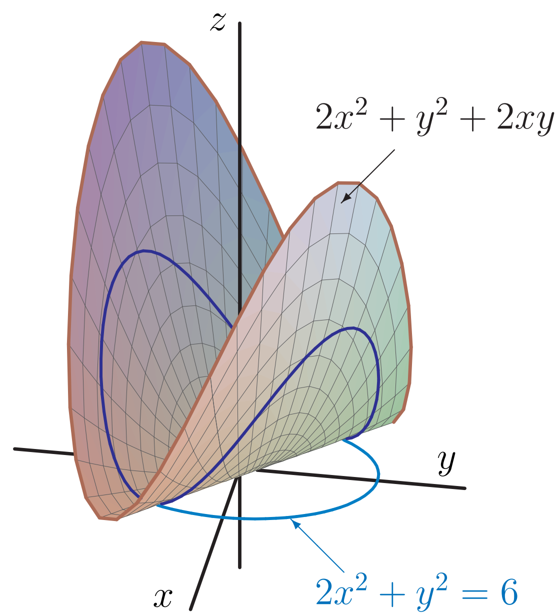
4. [25] Find the equation of the tangent plane to the surface $4x^2+y^2+z^2 = 24$ at the point $(2,2,2)$.



5. [25] Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if z is implicitly defined as a function of x and y by the equation

$$xyz = \cos(x+y+z)\,.$$

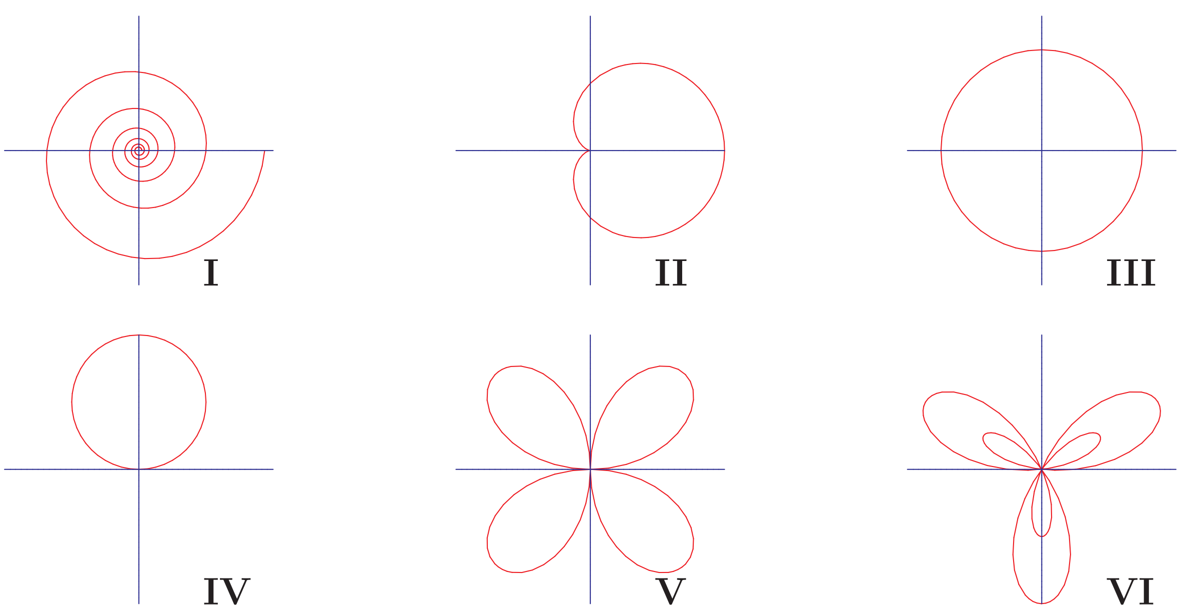
6. [25] Use Lagrange multipliers to find the maximum and minimum values that the function $f(x,y) = x^2 + y^2 + 2xy$ takes on the ellipse $2x^2 + y^2 = 6$.



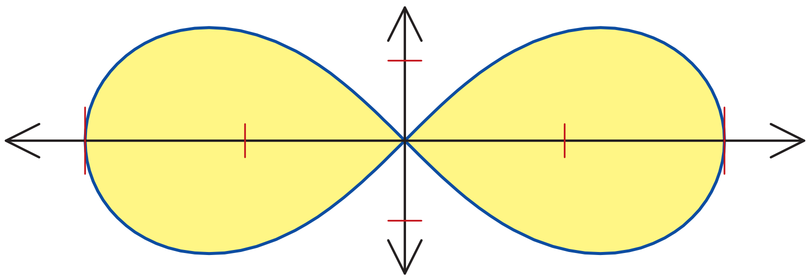
7. [25] Identify the polar curve with its polar equation.

(a) $r = 3/2$ (b) $r = \cos(\theta) - 1$ (c) $r = 2\sin(\theta)$

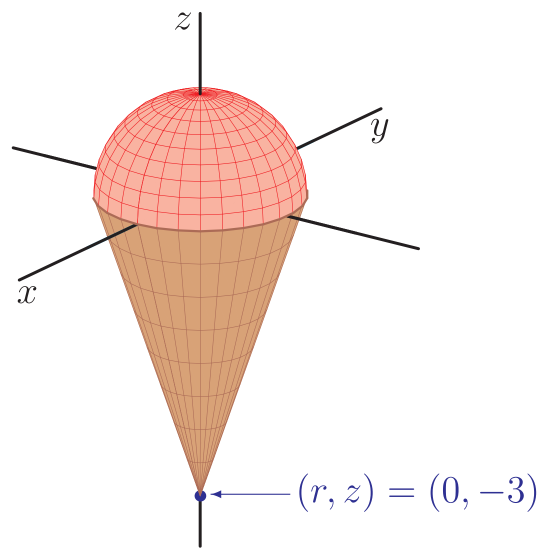
(d) $r = e^{\theta}$ (e) $r = 1 + 3\sin(3\theta)$ (f) $r = 2\sin(2\theta)$



8. Use integration in polar coordinates to find the area enclosed by the lemniscate $r^2 = 4\cos(2\theta)$.

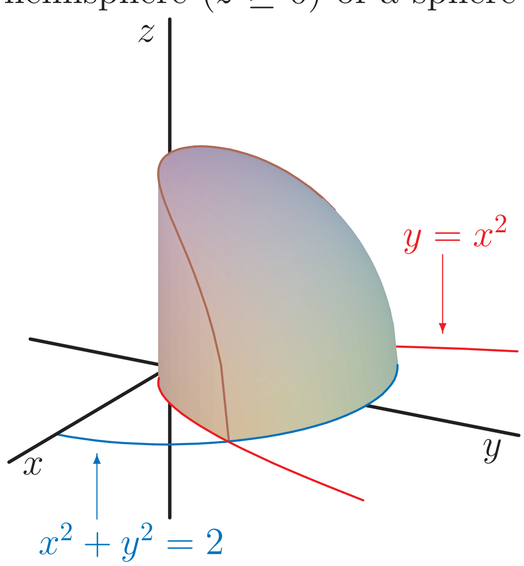


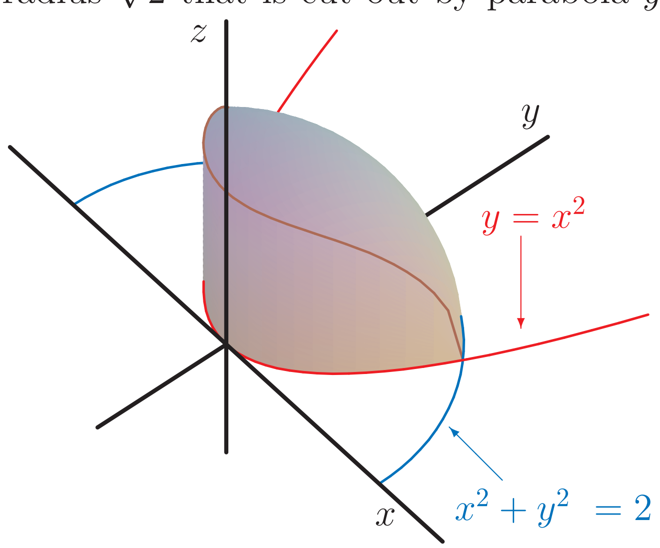
9. [25] Set up, but do not solve, an integral to find the volume of the strawberry ice cream cone. The ice cream is the upper hemisphere of the unit circle, and the cone has height 3.
Use cylindrical coordinates.



[5] Bonus: What is this volume? (Do not use the Calculus!)

10. [25] Set up, but do not solve, an integral to find the volume of that part of the upper hemisphere ($z \geq 0$) of a sphere of radius $\sqrt{2}$ that is cut out by parabola $y = x^2$.





11. [25] Evaluate $\int_C (x^2 + y^2)dx + 2xydy$ where C is positively (anti-clockwise) oriented boundary of the region $0 \leq x \leq 1$ and $2x^2 \leq y \leq 2x$.