Tenth Homework: Due in recitation: Thursday 9 November 2023

1. Recall the transformations from polar to rectilinear coordinates, $x = x(\rho, \phi, \theta) = \rho \sin\phi \cos\theta$, $y = y(\rho, \phi, \theta) = \rho \sin\phi \sin\theta$, and $z = z(\rho, \phi, \theta) = \rho \cos\phi$.

   Let $\nabla_{\rho,\phi,\theta}$ denote the gradient with respect to the spherical coordinates (e.g. vector of partial derivatives with respect to ρ, ϕ, and θ).

   Compute the vector triple product $(\nabla_{\rho,\phi,\theta}\, x \times \nabla_{\rho,\phi,\theta}\, y) \cdot \nabla_{\rho,\phi,\theta}\, z$ and compare it to the spherical volume element.

2. What are the surfaces with the following equations?

   (a) $\rho \sin\phi = 2$ (b) $\rho^2(\sin^2\phi \;-\; 4\cos^2\theta) = 1$ (c) $\rho^2 - 6\rho + 8 = 0\,.$

3. Write the following equations in both cylindrical and polar coordinates.

   (a) $x^2 + y^2 = 2z$ (b) $z = x^2 - y^2$ (c) $x^2 + y^2 - z^2 = 16\,.$

4. In class, we computed the volume of a four-dimensional balls of radius a, using its 2-dimensional cross sections over a disc in the pane of radius a. (The cross sections were themselves discs.)

   Redo this yourself.

   One may try to use the 1-dimensional cross sections of the four ball over its (equitorial) 3-ball, using spherical coordinates. Set this up, think abut it, but do not try to solve it.

   Write a paragraph comparing these two approaches, including what you are (trying to) do.

5. Consider a five-dimensional ball B of radius a. Observe that its cross sections over the disc of radius a in the x, y-plane are 3-dimensional balls (of varying radii). Similarly, its cross setions over the 3-dimensional ball of radius a (say, in the x, y, z-coordinate 3-plane) are discs of varying radii.

   Set up two different integrals for the volume of the five-dimensional ball illustrating these approaches and solve both. You should get the same answer.

6. Evaluate $\iiint_E x^2 dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 2$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

7. Find the centroid of a solid with constsnt mass density bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

8. Evaluate $\iiint_E x e^{(x^2+y^2+z^2)^2} dV$, whre E is the solid that lies between the spheres of radius 1 and 2, respectively, in the positive octant.

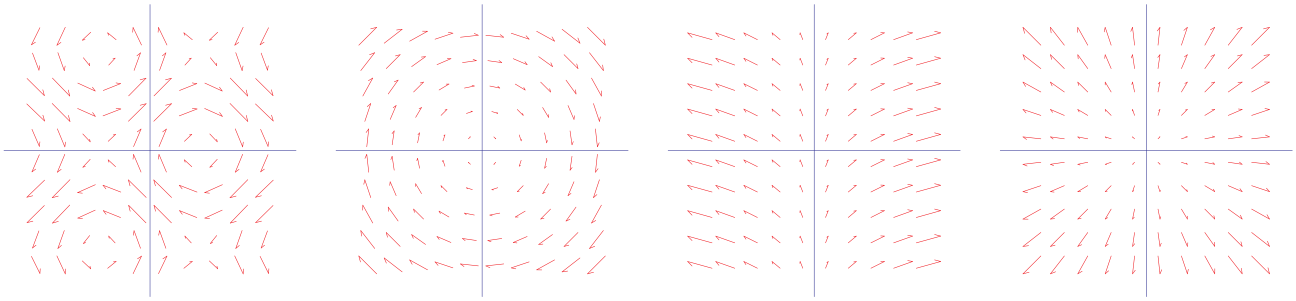
9. Find the mass of a solid hemisphere of radius a if the density at a point is proportional to the distane of that point to the centre of the base.

10. Evaluate the line integral $\int_C y\, ds$, where C is parametrized by $x = t^3$, $y = t^2$, for $0 \leq t \leq 1$.

11. Evaluate the line integral $\int_C xy^2\, ds$, where C is the right half of the circle of radius 4. What about the same integral over the top half of that circle?

Please Turn Over

12. Which of the following vector fields are conservative



(a) (b) (c) (d)

13. Evaluate $\int_C yz\,dy \ + \ xy\,dz$, where C is the curve with parametrizzation $x = \sqrt{t}$, $y = t$, $z = t^2$, and $0 \leq t \leq 1$.

14. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x^2\mathbf{i} + xy\mathbf{j} + z^2\mathbf{k}$, $\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + t^2\mathbf{k}$, and $0 \leq t \leq \pi/2$.

15. Find the mass an centre of mass of a thin wire in the shape of a quarter circle $x^2 + y^2 = r^2$ in the positive quadrant if the mass density function is $\rho(x, y) = x + y$.