Algebra Autumn 2023
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16 October 2023 Eighth Homework

Write your answers neatly, in complete sentences. I highly recommend recopying your work before handing it in. Correct and crisp proofs are greatly appreciated; oftentimes your work can be shortened and made clearer.

Hand in for the grader Monday 23 October:

32. How many elements of order seven are there in a simple group of order 168?
33. Show that any group of order 200 must contain a normal Sylow p-subgroup, and hence is not simple.
34. Recall that the quaternion group is $Q = \{\pm 1, \pm i, \pm j, \pm k\}$ with $i^2 = j^2 = k^2 = ijk = -1$ and $ij = -ji$, and etc. What is the center $C(Q)$ of Q? Show that $Q/C(Q)$ is abelian.
35. Show that there is a nonabelian subgroup T of $S_3 \times \mathbb{Z}/4\mathbb{Z}$ of order 12 with generators a, b such that $|a| = 6$, $a^3 = b^2$, and $ba = a^{-1}b$.

    Show that any group of order 12 with two generators satisfying these relations is isomorphic to T.
36. Show that the group T of the previous question, A_4, and the dihedral group of symmetries of the regular hexagon are pairwise nonisomorphic.
37. Give a composition series for S_4.
38. Let p be a prime number. How many simple subgroups does $\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$ have?