

## More review problems for Math 151

### You should be able to do all of these

Factor the following polynomials as much as you can, using integer coefficients.

1.  $x^2 - 1$ .
2.  $x^3 - 1$ .
3.  $x^2 - a^2$ .
4.  $x^3 - a^3$ .
5.  $x^4 - a^4$ .
6.  $x^5 - a^5$ .
7.  $x^6 - a^6$ .
8.  $x^7 - a^7$ .
9.  $x^8 - a^8$ .
10. Find a formula for
$$\sum_{i=1}^n (2i - 1) = 1 + 3 + 5 + \cdots + (2n - 1).$$
11. Use induction to show that
$$1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$
12. What is the simplest function? Why?
13. What is the second-simplest type of function? Explain.
14. If  $f$  and  $g$  are functions, what is  $f + g$ ?
15. If  $f$  and  $g$  are functions, what is  $f \cdot g$ ?
16. What is a polynomial function?
17. What is the domain of a polynomial function?
18. What is a rational function?
19. What is the domain of a rational function?
20. Does there exist a rational function with non-constant denominator and domain all of  $\mathbb{R}$ ?
21. What is  $\pi$ ?
22. What is  $\pi$ ? (This is not a typo. There are two answers.)
23. Define  $\sin x$  in terms of the unit circle.
24. Define  $\cos x$  in terms of the unit circle.

25. What is  $\tan x$ ?
26. What is  $\sec x$ ?
27. What is  $\cot x$ ?
28. What is  $\csc x$ ?
29. Show that  $\sin(-x) = -\sin x$ .
30. Show that  $\cos(-x) = \cos x$ .
31. Show that  $\sin(y + z) = \sin y \cos z + \cos y \sin z$ .
32. Derive a formula for  $\tan(2\omega)$ .
33. Show that  $\sin(\frac{\pi}{2} - \alpha) = \cos \alpha$ .
34. Give the numerical values of  $\sin \frac{\pi}{3}$ ,  $\cos \frac{\pi}{3}$ ,  $\tan \frac{\pi}{3}$ ,  $\sec \frac{\pi}{3}$ ,  $\cot \frac{\pi}{3}$ , and  $\csc \frac{\pi}{3}$  as radicals.
35. Give the numerical values of  $\sin 0$ ,  $\cos 0$ ,  $\tan 0$ ,  $\sec 0$ ,  $\cot 0$ , and  $\csc 0$  as radicals.
36. Give the numerical values of  $\sin \frac{\pi}{2}$ ,  $\cos \frac{\pi}{2}$ ,  $\tan \frac{\pi}{2}$ ,  $\sec \frac{\pi}{2}$ ,  $\cot \frac{\pi}{2}$ , and  $\csc \frac{\pi}{2}$  as radicals.
37. Give the numerical values of  $\sin(-\frac{\pi}{6})$ ,  $\cos(-\frac{\pi}{6})$ ,  $\tan(-\frac{\pi}{6})$ ,  $\sec(-\frac{\pi}{6})$ ,  $\cot(-\frac{\pi}{6})$ , and  $\csc(-\frac{\pi}{6})$  as radicals.
38. Show that  $\frac{\sec a - 1}{\sec a + 1} + \frac{\cos a - 1}{\cos a + 1} = 0$ .
39. Show that  $1 + \cot^2(\pi/2 - x) = \frac{1}{\sin^2(\pi/2 - x)}$ .
40. Show that  $\frac{\sin \beta}{\csc \beta} + \frac{\cos \beta}{\sec \beta} = 1$ .
41. Show that  $\sec^4 \theta - \sec^2 \theta = \frac{1}{\cot^4 \theta} + \frac{1}{\cot^2 \theta}$ .
42. Show that  $\tan 3\beta = \frac{3 \tan \beta - \tan^3 \beta}{1 - 3 \tan^2 \beta}$ .
43. Show that  $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$ .
44. Show that  $\cot(x/2) = \frac{1 + \cos x}{\sin x}$ .
45. Show that  $\csc y \sec y = 2 \csc 2y$ .