Honors Multivariate Calculus

Math 221H Section 201

Tenth Homework:

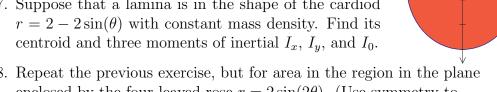
Due in recitation: Thursday 2 November 2023

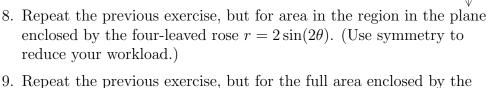
1. Evaluate the following improper integrals.

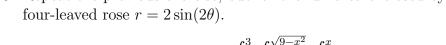
(a)
$$\int_{-\infty}^{\infty} x^2 e^{-x^2} dx$$

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 and (b) $\int_{-\infty}^{\infty} \sqrt{x} e^{-x} dx$.

- 2. Electric charge is distributed over the disk $x^2 + y^2 \le 2$ with charge density $\sigma(x,y) = 1 + x^2 + y^2$. Find the total charge on the disk.
- 3. A lamina (thin sheet of material) occupies the region inside the circle $x^2 + y^2 = 2y$ but outside the unit circle $x^2 + y^2 = 1$. Find its center of mass if the density at any point is inversely proportional to its distance to the origin.
- 4. A lamina has shape that part of the disk $x^2 + y^2 \le 4$ that lies in the first quadrant. Suppose that its mass density is proportional to the square of the distance to the origin, $\rho(x,y) = k(x^2 + y^2)$. Find its center of mass.
- 5. Suppose that a lamina is in the shape of the rectangle $D = \{(x,y) \mid 0 \le x \le 2, 0 \le y \le 3\}$ with mass density $\rho(x,y) = \pi y$. Find its center of mass.
- 6. Find the three moments of inertial I_x , I_y , and I_0 of the lamina from each of the previous two problems.
- 7. Suppose that a lamina is in the shape of the cardiod centroid and three moments of inertial I_x , I_y , and I_0 .

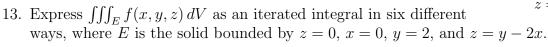








- 11. Compute $\iiint_E xy \, dV$ where E is the tetrahedron with vertices (0,0,0), (3,0,0), (0,2,0), (0,0,1).
- 12. Compute $\iiint_E xy \, dV$ where E is the solid bounded by the parabolic cylinder $y = x^2$ and the planes x = z, x = y, and z = 0.



14. The figure shows the region of integration for the integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z) \, dy \, dz \, dx \, .$$

Rewrite this as an equivalent interated integral in the five other ways.

