Foundations of Mathematics Math 300 Sections 902, 905

YOUR NAME

First Homework: 31 August 2020

1. Which of the following sentences are statements?
   (a) $3^2 + 4^2 = 5^2$.
   (b) $a^2 + b^2 = c^2$.
   (c) There exist integers a, b, and c such that $a^2 + b^2 = c^2$.
   (d) If $x^2 = 4$, then $x = 2$.
   (e) For each real number x, if $x^2 = 4$, then $x = 2$.
   (f) For each real number t , $\sin^2 t + \cos^2 t = 1$.
   (g) If n is a prime number, then n^2 has three positive factors.
   (h) $\sin x < \sin(\pi/4)$.
   (i) Every rectangle is a parallelogram.
2. Identify the hypothesis and the conclusion for each of the following conditional statements.
   (a) If a is an irrational number and b is an irrational number, then $a \cdot b$ is an irrational number.
   (b) If $p \neq 2$ and p is an even number, then p is not prime.
3. Determine whether each of the following conditional statements is true or false.
   (a) If $10 < 7$, then $3 = 4$.
   (b) If $7 < 10$, then $3 = 4$.
   (c) If $10 < 7$, then $3 + 5 = 8$.
   (d) If $7 < 10$, then $3 + 5 = 8$.
4. Give a valid definition of an odd integer.
5. Consider the following statement: "If m is an odd integer, then $m+1$ is an even integer."
   (a) Construct a know-show table for a proof of this statement.
   (b) Write a proof of this statement in paragraph form.
6. Consider the following "proof" that if m and n are even, then $m+n$ is even:

   We know that $n = 2t$ and $m = 2t$, so $m+n = 2t + 2t = 4t$. Therefore $m+n$ is even.
   (a) Criticize (discuss its shortcomings).
   (b) Construct a know-show table for a correct proof of this statement.
   (c) Write a correct proof of this statement in paragraph form.

7. Consider the following statement:

   "If m is an even integer and n is an integer, then mn is an even integer."

   (a) Construct a know-show table for a proof of this statement.

   (b) Write a proof of this statement in paragraph form.

8. Which of the following statement are true and which are false? Justify your conclusions.

   (a) If a, b, and c are integers, then $ab + ac$ is an even integer.

   (b) If a, b, and c are integers with both b and c odd integers, then $ab+ac$ is an even integer.

9. An integer n is a **type 0 integer** if there is an integer a such that $n = 3a$. An integer n is a **type 1 integer** if there is an integer a such that $n = 3a + 1$. An integer n is a **type 2 integer** if there is an integer a such that $n = 3a + 2$.

   (a) Give examples of at least four different integers that are type 1.

   (b) Give examples of at least four different integers that are type 2.

   (c) Multiplying pairs of integers from the first part, what do you believe about the truth value of the following statement:

   If m and n are both type 1 integers, then $m \cdot n$ is a type 1 integer.

10. Using the definitions from the previous exercise to help write a proof (in paragraph form) of the following statements.

    (a) If m and n are both type 1 integers, then $m + n$ is a type 2 integer.

    (b) If m and n are both type 2 integers, then $m + n$ is a type 1 integer.

For numbers 5, 6, and 7, you have three options for the know-show tables: (1) Create them in LaTeX (see the example below, and note the cell with two rows), or make the table by hand, take a picture, and (2) follow the instructions for including an image in the LaTeX file given in comments above, or else (3) convert the images to .pdf and merge them with the .pdf for this file.

Here is the know-show table on page 20 of the book, for the assertion: "If x and y are odd integers, then $x \cdot y$ is an odd integer."

Step	**Statement**	**Reason**
P	x and y are odd integers.	Hypothesis
$P1$	There exist integers m and n such that $x = 2m + 1$ and $y = 2n + 1$	Definition of an odd integer.
$P2$	$xy = (2m + 1)(2n + 1)$	Substitution
$P3$	$xy = 4mn + 2m + 2n + 1$	Algebra
$P4$	$xy = 2(2mn + m + n) + 1$	Algebra
$Q1$	There exists an integer q such that $xy = 2q + 1$	Use $q = 2mn + m + n$
Q	$x \cdot y$ is an odd integer	Definition of an odd integer