Honors Multivariate Calculus — Math 221H Section 201

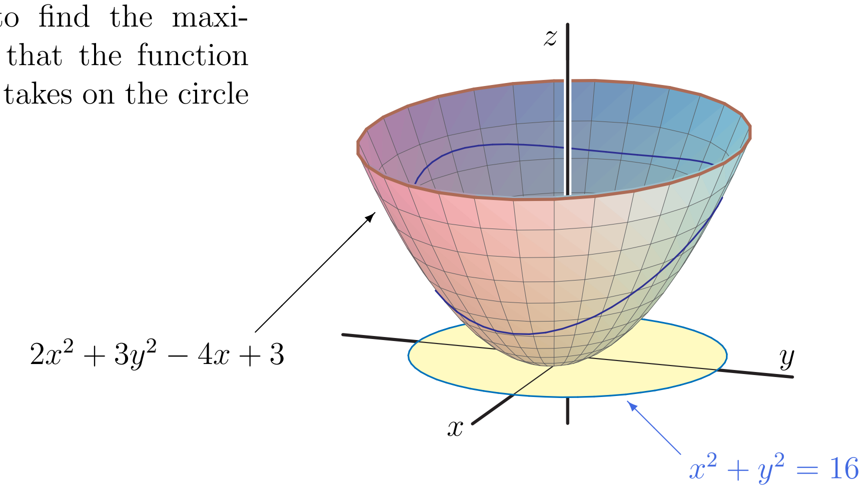
Eighth Homework: Due in recitation: Thursday 19 October 2023

Homework about polynomial optimization using Lagrange multipliers

1. Use Lagrange multipliers to find the point on the paraboloid $z = x^2 + y^2$ that is closest to the point $(1, 2, 0)$.

2. Use the method of Lagrange multipliers to find the points on the surface $x^2y - z^2 + 4 = 0$ that are closest to the origin.

3. Use Lagrange multipliers to find the maximum and minimum values that the function $f(x, y) = 2x^2 + 3y^2 - 4x + 3$ takes on the circle $x^2 + y^2 = 16$.



4. Let a, b, c be positive numbers. Use the method of Lagrange multipliers to find the volume of the largest box in the positive octant with three faces lying in the coordinate planes and one vertex on the plane $x/a + y/b + z/c = 1$.

5. For each of the following, find the maximum of the given function $f(x, y)$ over the bounded set S. Use the methods of critical points to find the maximum and minimum values over the interior of S and the method of Lagrange multipliers to find the maximum and minimum on the boundary of S.

   (a) $f(x, y) = x + y - xy$. $\quad S = \{(x, y) \mid x^2 + y^2 \leq 9\}$.

   (b) $f(x, y) = \frac{x}{1+y^2}$. $\quad S = \{(x, y) \mid x^2/4 + y^2/9 \leq 1\}$.

6. Let $w = x_1 x_2 \cdots x_n$.

   (a) Use Lagrange multipliers to maximize w subject to $x_1 + x_2 + \cdots + x_n = a$, where a is a fixed positive number and the x_i are positive, $x_1, \ldots, x_n > 0$.

   (b) Use this to deduce the *Arithmetic Mean–Geometric Mean Inequality* for positive numbers $a_1, \ldots, a_n$, which is

$$\sqrt[n]{a_1 a_2 \cdots a_n} \;\leq\; \frac{a_1 + a_2 + \ldots + a_n}{n} \,.$$

7.