Algebra II Winter 2021 — 1 March

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Make-up Homework

Write your answers neatly, in complete sentences, and prove all assertions. Start each problem on a new page (this makes it easier in Gradescope). Revise your work before handing it in, and submit a .pdf created from a LaTeX source to Gradescope. Correct and crisp proofs are greatly appreciated; oftentimes your work can be shortened and made clearer.

These are problems that eluded most of you, and for which CJ may have been overly lenient towards alloting partial credit. Frank is giving you a second chance, and he will mark these.

Due Monday 8 March.

1. Let R be a ring with no zero divisors such that for all $r, s \in R$, there are $a, b \in R$, not both zero, such that $ar + bs = 0$. Show that if $R = M \oplus N$ as R-modules, then one of M or N is the 0-module, $\{0\}$. Use this to show that R has the invariant dimension property.

2. Suppose that M is an R-module and that for $i = 1, 2$, we have short exact sequences $0 \to N_i \to P_i \to M \to 0$ with P_1 and P_2 projective. Show that $P_1 \oplus N_2 \simeq P_2 \oplus N_1$ as R-modules.