Foundations of Mathematics Autumn 2019 Frank Sottile 9 September 2019

Part of Fourth Homework

Write your answers neatly, in complete sentences. I highly recommend recopying proofs.

Hand in Monday, 16 September with the rest of your homework:

- 1. Determine whether each of the following sentences is a statement, a predicate (open statement), or neither.
 - (a) The Boston Celtics have won 16 NBA championships.
 - (b) The plane is leaving in four minutes.
 - (c) Get a note from your doctor.
 - (d) Is this the best that you can do?
 - (e) Excessive exposure to the sun may cause melanoma.
 - (f) $5 \cdot 2 = 9$.
 - (g) Someone in the room is a murderer.
 - (h) $x^2 + 1 \neq 0$.
 - (i) For every real number x, $x^2 + 1 \neq 0$.
 - (j) The equation for a circle of radius 1 centred at the origin is $x^2 + y^2 = 1$.
 - (k) If m and n are even integers, then mn is odd.
- 2. For each of the following statements, determine if it has any universal or existential quantifiers. If it has universal quantifiers, rewrite it in the form "for all...". If it has existential quantifiers, rewrite it in the form "there exists ... such that ...". Introduce variables where appropriate.
 - (a) The area of a rectangle is its length times its width.
 - (b) A triangle may be equilateral.
 - (c) 8 8 = 0.
 - (d) The sum of an even integer and an odd integer is even.
 - (e) For every even integer, there is an odd integer such that the sum of the two is odd.
 - (f) A function that is continuous on the closed interval [a, b] is integrable on [a, b].
 - (g) A function is continuous on [a, b] whenever it is differentiable on [a, b].
 - (h) A real-valuled function that is continuous at 0 is not necessarily differentiable at 0.
 - (i) All positive real numbers have a square root.
 - (i) The smallest positive integer is 1.
- 3. Write a useful negation of each statement in Exercise 2.