

YOUR NAME

Second Homework:

6 September 2022

Use English when possible. Answers should not just be symbols.

- For a real number r , define $A_r := \{r^2\}$, B_r to be the closed interval $[r-1, r+2]$, and C_r to be the interval (r, ∞) . For $S = \{1, 2, 4\}$ determine each of the following

$$\bigcup_{s \in S} A_s \text{ and } \bigcap_{s \in S} A_s \text{ and } \bigcup_{s \in S} B_s \text{ and } \bigcap_{s \in S} B_s \text{ and } \bigcup_{s \in S} C_s \text{ and } \bigcap_{s \in S} C_s.$$

- Determine each of the following:

$$\bigcup_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right) \text{ and } \bigcap_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right) \text{ and } \bigcup_{n=1}^{\infty} \left(\frac{n-1}{n}, \frac{n+1}{n}\right) \text{ and } \bigcap_{n=1}^{\infty} \left(\frac{n-1}{n}, \frac{n+1}{n}\right)$$

- Which of the following sets are partitions of $A = \{a, b, c, d, e, f, g\}$? For each collection of subsets that is not a partition of A , explain your answer.

(a) $S_1 = \{\{a, c, e, g\}, \{b, f\}, \{d, d\}\}$

(b) $S_2 = \{\{a, b, c, d\}, \{e, f\}\}$

(c) $S_3 = \{A\}$

(d) $S_4 = \{\{a\}, \emptyset, \{b, c, d\}, \{e, f, g\}\}$

(e) $S_5 = \{\{a, c, d\}, \{b, g\}, \{e\}, \{b, f\}\}$

- Give an example of a partition of \mathbb{N} into three subsets.

- For $A = \{1, 2\}$ and $B = \{\emptyset\}$ determine $\mathcal{P}(A \times B)$.

- (a) For a set A with $|A| = 2$, what is the largest possible value of $|A \cap \mathcal{P}(A)|$?

- (b) What is the largest possible value of $|A \cap \mathcal{P}(A)|$ if $|A| = 3$?

- (c) This should suggest another question to you. What is the answer to that question? (Do also give that question).

- Consider the sets A , B , C , and D below. Which of the following statements are true? Give an explanation for each false statement.

$$A = \{1, 4, 7, 10, 13, 16, \dots\} \quad B = \{n \in \mathbb{Z} : n \text{ is prime and } n \neq 2\}$$

$$C = \{n \in \mathbb{Z} : n \text{ is odd}\} \quad D = \{1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}$$

(a) $25 \in A$ (b) $33 \in D$ (c) $22 \notin A \cup D$ (d) $B \subseteq C$

(e) $\emptyset \in C \cap D$ (f) $53 \notin B$ (g) $144 \in D$.

- Give a negation of each of the following statements.

- (a) At least two of my library books are overdue.

- (b) One of my two friends misplaced his homework assignment.

- (c) No one expected that to happen.

- (d) It is not often that my instructor teaches that course.

- (e) It is surprising that two students received the same exam score.

9. Which of the following sentences are statements?

- (a) $3^2 + 4^2 = 5^2$.
- (b) $a^2 + b^2 = c^2$.
- (c) There exist integers a , b , and c such that $a^2 + b^2 = c^2$.
- (d) If $x^2 = 4$, then $x = 2$.
- (e) For each real number x , if $x^2 = 4$, then $x = 2$.
- (f) For each real number t , $\sin^2 t + \cos^2 t = 1$.
- (g) If n is a prime number, then n^2 has three positive factors.
- (h) $\sin x < \sin(\pi/4)$.
- (i) Every parallelogram is a rectangle.

Of those that are statements, which are true?

10. Let $P(n)$ be the open sentence “ n and $n + 2$ are primes” where n ranges over the natural numbers \mathbb{N} . Find six positive integers n for which $P(n)$ is true.
11. Fill out the truth table for the expressions $P \wedge Q$, $\sim P$, $\sim Q$, $\sim(P \wedge Q)$, $\sim P \wedge \sim Q$, $\sim P \vee \sim Q$, and $(\sim P \wedge \sim Q) \vee Q$:

P	Q	$P \wedge Q$	$\sim P$	$\sim Q$	$\sim(P \wedge Q)$	$\sim P \wedge \sim Q$	$\sim P \vee \sim Q$	$(\sim P \wedge \sim Q) \vee Q$
T	T							
T	F							
F	T							
F	F							

12. Identify the hypothesis and the conclusion for each of the following conditional statements.
- (a) If a is an irrational number and b is an irrational number, then $a \cdot b$ is an irrational number.
 - (b) If $p \neq 2$ and p is an even number, then p is not prime.
13. Rewrite the following statements in the form “if P , then Q ”.
- (a) “One, if by land”.
 - (b) “Candor implies equality”.
 - (c) “Pepperoni only if pizza”.
 - (d) “Inattentive when bored”.
 - (e) “Slapstick is sufficient for comedy”.
 - (f) “Quiet is necessary for sleep”.
14. Rewrite “If the function f is differentiable, then it is continuous” in each of the six forms of the previous problem.
15. For statements P and Q , construct the truth table for $(P \Rightarrow Q) \Rightarrow \sim P$.