

Software Foundations of Security and  
Privacy (15-316, spring 2017)  
**Lecture 12: Information Flow (2)**

**Jean Yang**

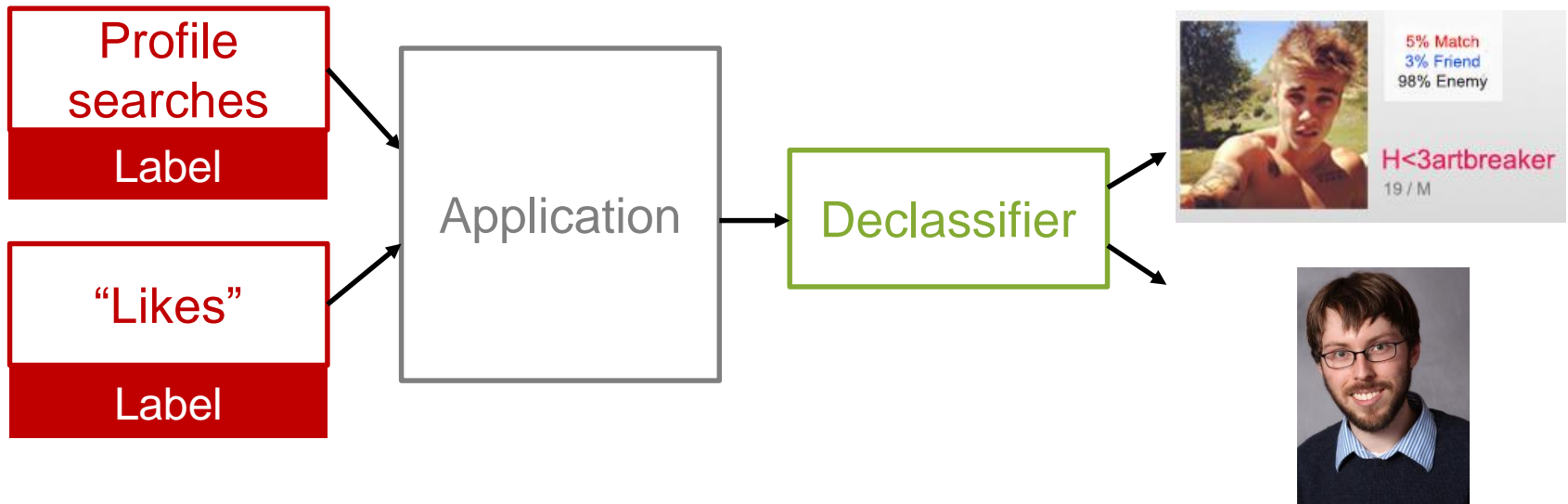
jyang2@andrew.cmu.edu

# Last Class

- Motivation for why we need more than access control.
- Process-based decentralized information flow control.
- How we can make reference monitors for information flow, even though it's not a safety property.

# Recall DIFC

Model for controlling information flow in systems with *mutual distrust* and *decentralized authority*. Sensitive data is *labelled* and can be *declassified* in a decentralized way.



# Problems with Dynamic DIFC

- Non-trivial runtime overheads.
- Required to be conservative, because can only make reference monitors for safety properties.
- Conservative requires us to have all these trusted declassifications all over the place.

# What We Want

- ✓ Fine-grained information flow analysis that gives us non-interference.
- ✓ As little run-time overhead as possible.
- ✓ A way to get some static guarantees before we run our programs.

Information flow types give us all of this!



# **Part One: High-Level Introduction to Language- Level Information Flow Control**

# Information Flow in Java with Jif

[Myers]

- Jif augments Java types with labels that are *statically* checked\*.
  - `int {Alice:Bob} x;`
  - `Object {L} o;`
- Subtyping with the  $\subseteq$  lattice order determines how differently-labeled values should be combined.
- Type inference allows programmers to omit types.

\* Over the years, there has been work to insert additional dynamic checks.

# Hello Labels, My Old Friend

Slide from Vitaly Schmatikov.

- Confidentiality constraints: who may read it?
  - $\{\text{Alice: Bob, Eve}\}$  label means that Alice owns this data, and Bob and Eve are permitted to read it
  - $\{\text{Alice: Charles; Bob: Charles}\}$  label means that Alice and Bob own this data but only Charles can read it
- Integrity constraints: who may write it?
  - $\{\text{Alice ? Bob}\}$  label means that Alice owns this data, and Bob is permitted to change it



# Labels and Flow

```
int {Alice:Bob} x;
```

```
int {Alice:Bob, Charles} y;
```

```
x = y; // Okay, because policy on x is stronger
```

```
y = x; // Bad, because policy on y is weaker
```

- Each owner can specify an independent policy.
- Code running with owner authority can *declassify* data by adding more permissions.
- When a value is read from a slot, it acquires the slot's label.

# What About Combining Values?

```
int {Alice:Bob} x;
```

```
int {Alice:Bob, Charles} y;
```

```
int {??} z;
```

```
z = x + y;
```

**Q:** What label does  $z$  need in order for this flow to be allowed?

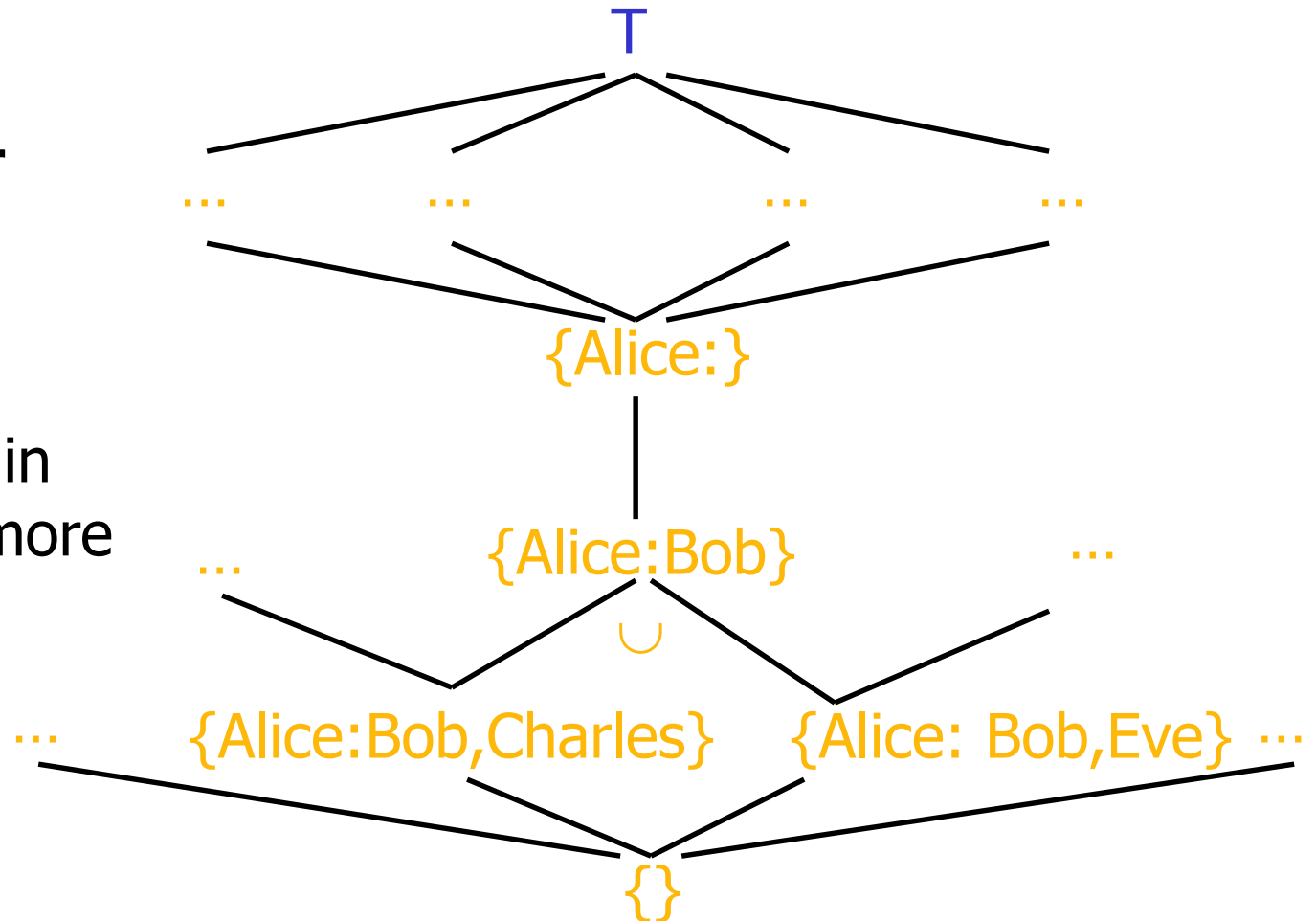
**A:** What label does  $z$  need in order for this flow to be allowed?

# Label Lattice

Slide from Vitaly Schmatikov.

$\subseteq$  order  
 $\cup$  join

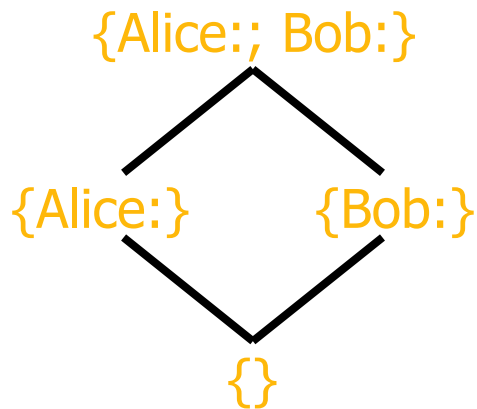
Labels higher in  
the lattice are more  
restrictive



# Challenge: Implicit Flows

[Zdancewic]

Slide from Vitaly Schmatikov.



PC label

```
int{Alice:} a;  
int{Bob:} b;
```

...



```
if (a > 0) then {
```

$\{\} \cup \{Alice:\} = \{Alice:\}$

```
  b = 4;
```

```
}
```

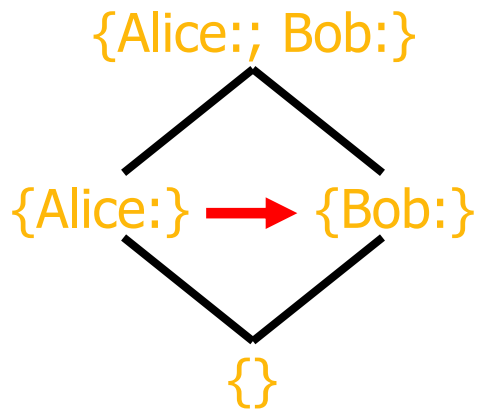


This assignment leaks  
information contained in  
program counter (PC)

# Challenge: Implicit Flows

[Zdancewic]

Slide from Vitaly Schmatikov.



PC label

```
int{Alice:} a;  
int{Bob:} b;
```

...



```
if (a > 0) then {
```

$\{\} \cup \{Alice:\} = \{Alice:\}$

```
  b = 4;
```

```
}
```

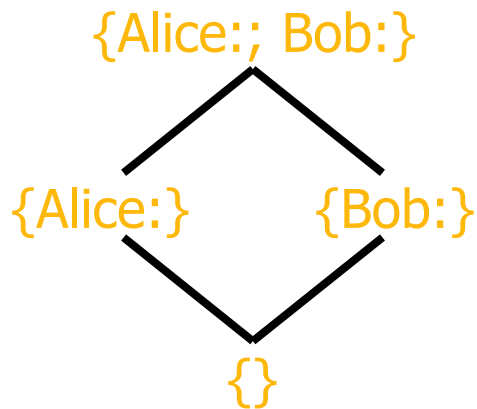


To assign to variable  
with label  $X$ , must have  
 $PC \subseteq X$

# Challenge: Implicit Flows

[Zdancewic]

Slide from Vitaly Schmatikov.



PC label

```
int{Alice:} a;  
int{Bob:} b;
```

...



```
if (a > 0) then {
```

$\{\} \cup \{Alice:\} = \{Alice:\}$

```
f(4);
```

```
}
```



Effects inside function  
can leak information  
about program counter



## Part Two: Formalizing the Security Lattice

# Security Lattice

Slide from Matt Fredrikson.

A *security lattice* is a five-tuple  $(SC, \leq, \sqcup, \sqcap, \perp)$  where:

- $SC$  is a set of security classes
- $\leq$  is a *partial order* on  $SC$
- $s_1 \sqcup s_2$  is the *least upper bound* of  $s_1$  and  $s_2$ ,  $s_1, s_2 \leq s_1 \sqcup s_2$ , and  $\forall s \in SC. s_1, s_2 \leq s \Rightarrow s_1 \sqcup s_2 \leq s$
- $s_1 \sqcap s_2$  is the *greatest lower bound* of  $s_1$  and  $s_2$ ,  $s_1 \sqcap s_2 \leq s_1, s_2$ , and  $\forall s \in SC. s \leq s_1, s_2 \Rightarrow s \leq s_1 \sqcap s_2$
- $\perp$  is the least element of  $SC$



# A Simple Lattice for Secrecy

Slide from Matt Fredrikson.



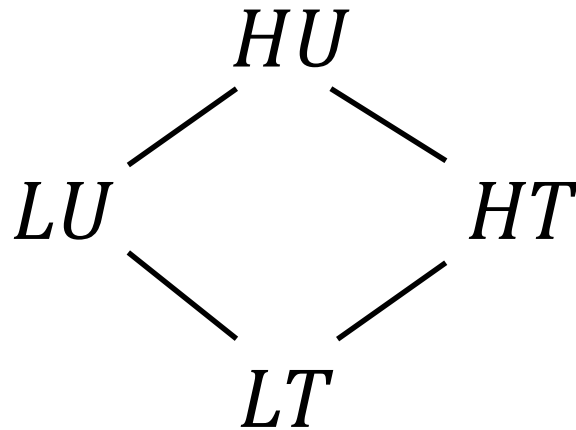
Policy: no high-security flows to low-variables.

- $H$  is “high” and  $L$  is “low”
- $L \leq H$ ,  $\neg(H \leq L)$ , and  $L$  is  $\perp$

The partial order  $\leq$  means “can flow to.”

# Secrecy and Integrity

Slide from Matt Fredrikson.



Policy: no high flows to low, no untrusted flows to trusted

- $H$  is “high,”  $L$  is “low,”  $U$  is “untrusted,” and  $T$  is “trusted”
- $T \leq U, \neg(U \leq T)$



## **Part Three: A Type System for Information Flow**

# A Simple Imperative Language

Slide from Matt Fredrikson.

## Arithmetic expressions

$$a \in AExp ::= n \in \mathbb{Z} \mid x \in \text{Var} \\ \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 \times a_2$$

## Boolean expressions

$$b \in BExp ::= \mathbf{T} \mid \mathbf{F} \mid \neg b \mid b_1 \wedge b_2 \mid a_1 = a_2 \mid a_1 \leq a_2$$

## Commands

$$c \in Com ::= \mathbf{skip} \mid x := a \mid c_1; c_2 \\ \mid \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2 \\ \mid \mathbf{while } b \mathbf{ do } c$$

# Expression Evaluation

Slide from Matt Fredrikson.

States are mappings  $\sigma: \mathbf{Var} \mapsto \mathbb{Z}$

Expression evaluation happens with the *big-step relation*  $\langle \sigma, a \rangle \Downarrow n$

$$\overline{\langle \sigma, n \rangle \Downarrow n}$$

$$\overline{\langle \sigma, x \rangle \Downarrow \sigma(x)}$$

$$\frac{\langle \sigma, a_1 \rangle \Downarrow n_1 \quad \langle \sigma, a_2 \rangle \Downarrow n_2 \quad n = n_1 \mathbf{op} n_2}{\langle \sigma, a_1 \mathbf{op} a_2 \rangle \Downarrow n}$$

# Command Evaluation

Slide from Matt Fredrikson.

Big-step relation  $\langle \sigma_1, c \rangle \Downarrow \sigma_2$

$$\frac{\langle \sigma, a \rangle \Downarrow n}{\langle \sigma, x := a \rangle \Downarrow \sigma[x \mapsto n]}$$

$$\frac{\langle \sigma_1, c \rangle \Downarrow \sigma_2}{\langle \sigma, \mathbf{skip}; c \rangle \Downarrow \sigma_2}$$

$$\frac{\langle \sigma_1, c_1 \rangle \Downarrow \sigma'_1 \quad \langle \sigma'_1, c_2 \rangle \Downarrow \sigma_2}{\langle \sigma_1, c_1; c_2 \rangle \Downarrow \sigma_2}$$

$$\frac{\langle \sigma, b \rangle \Downarrow \mathbf{T} \quad \langle \sigma, c_1 \rangle \Downarrow \sigma_2}{\langle \sigma, \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2 \rangle \Downarrow \sigma_2}$$

$$\frac{\langle \sigma, b \rangle \Downarrow \mathbf{F} \quad \langle \sigma, c_2 \rangle \Downarrow \sigma_2}{\langle \sigma, \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2 \rangle \Downarrow \sigma_2}$$

$$\frac{\langle \sigma, b \rangle \Downarrow \mathbf{F}}{\langle \sigma, \mathbf{while } b \mathbf{ do } c \rangle \Downarrow \sigma}$$

$$\frac{\langle \sigma_1, b \rangle \Downarrow \mathbf{T} \quad \langle \sigma_1, c \rangle \Downarrow \sigma'_1 \quad \langle \sigma'_1, \mathbf{while } b \mathbf{ do } c \rangle \Downarrow \sigma_2}{\langle \sigma_1, \mathbf{while } b \mathbf{ do } c \rangle \Downarrow \sigma_2}$$

# Type Environment $\Gamma$

Slide from Matt Fredrikson.

Let  $L = (SC, \leq, \sqcup, \sqcap, \perp)$  be a security lattice. A *type environment*  $\Gamma: \mathbf{Var} \mapsto SC$  for a program  $c$  maps each variable in  $c$  to a label.

Additionally,  $\Gamma$  contains an additional mapping for the program counter label **pc**.

- $\Gamma \vdash e: \ell$  means expression  $e$  has label  $\ell$  under  $\Gamma$
- $\Gamma \vdash c$  means  $c$  is well-typed under  $\Gamma$
- Environment  $(\Gamma, x :: \ell)$  gives  $x$  type  $\ell$ , preserves rest of  $\Gamma$

# Goal: Noninterference

Slide from Matt Fredrikson.

## State Equivalence

Abbreviated as

$$\sigma_1 \approx_\ell \sigma_2$$

Two states  $\sigma_1, \sigma_2$  are  $\ell$ -equivalent to an observer of class  $\ell \in SC$  under  $\Gamma$ , written  $\sigma_1 \approx_{\ell, \Gamma} \sigma_2$  if and only if

$$\forall x \in \mathbf{Var}. \Gamma(x) \leq \ell \Rightarrow \sigma_1(x) = \sigma_2(x)$$

## Noninterference

A program  $c$  satisfies noninterference at class  $\ell$  under  $\Gamma$  if  $\ell$ -equivalent initial states lead to  $\ell$ -equivalent final states:

$$\forall \sigma_1, \sigma_2. \sigma_1 \approx_\ell \sigma_2 \wedge \langle \sigma_1, c \rangle \Downarrow \sigma'_1 \wedge \langle \sigma_2, c \rangle \Downarrow \sigma'_2 \Rightarrow \sigma'_1 \approx_\ell \sigma'_2$$

Initial states are  
state equivalent

Final states are  
state equivalent



# Typing Rules: Expressions

Slide from Matt Fredrikson.

$$\begin{array}{c}
 \text{Var } \frac{}{\Gamma \vdash x : \Gamma(x)} \quad \text{Int } \frac{}{\Gamma \vdash n : \perp} \quad \text{True } \frac{}{\Gamma \vdash \mathbf{T} : \perp} \quad \text{False } \frac{}{\Gamma \vdash \mathbf{F} : \perp} \\
 \\
 \text{Bin } \frac{\Gamma \vdash a_1 : \ell_1 \quad \Gamma \vdash a_2 : \ell_2}{\Gamma \vdash a_1 \text{ op } a_2 : \ell_1 \sqcup \ell_2}
 \end{array}$$

## Example

$$5 \leq 6 + x, \Gamma = x :: H$$

$$\frac{
 \frac{}{\Gamma \vdash 5 : L} \text{Int} \quad
 \frac{
 \frac{}{\Gamma \vdash 6 : L} \text{Int} \quad
 \frac{}{\Gamma \vdash x : H} \text{Var}
 }{\Gamma \vdash 6 + x : H} \text{Bin}
 }{\Gamma \vdash 5 \leq 6 + x : H} \text{Bin}$$

# Typing Rules: Commands

Slide from Matt Fredrikson.

$$\begin{array}{l} \text{Skip} \frac{}{\Gamma \vdash \mathbf{skip}} \\ \text{Asgn} \frac{\Gamma \vdash a : \ell \quad \ell \sqcup \Gamma(\mathbf{pc}) \leq \Gamma(x)}{\Gamma \vdash x := a} \\ \text{Comp} \frac{\Gamma \vdash c_1 \quad \Gamma \vdash c_2}{\Gamma \vdash c_1 ; c_2} \quad \text{While} \frac{\Gamma \vdash b : \ell \quad \ell' = \Gamma(\mathbf{pc}) \sqcup \ell \quad \Gamma, \mathbf{pc} :: \ell' \vdash c}{\Gamma \vdash \mathbf{while } b \mathbf{ do } c} \\ \text{If} \frac{\Gamma \vdash b : \ell \quad \ell' = \Gamma(\mathbf{pc}) \sqcup \ell \quad \Gamma, \mathbf{pc} :: \ell' \vdash c_1 \quad \Gamma, \mathbf{pc} :: \ell' \vdash c_2}{\Gamma \vdash \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2} \end{array}$$

# Command Typing Example

Slide from Matt Fredrikson.

$$\Gamma = p :: H, g :: L, o :: L, pc :: L$$

$$\text{Bin} \frac{\dots}{\Gamma \vdash p = g : H} \quad \text{Int} \frac{}{\Gamma \vdash 1 : L} \quad L \sqcup H \leq \Gamma(o) \quad \text{Asgn} \frac{}{\Gamma, \mathbf{pc} :: H \vdash o := 1}$$

$$\text{If} \frac{\Gamma \vdash p = g : H \quad H = \Gamma(pc) \sqcup H}{\Gamma \vdash \text{if } p = g \text{ then } o := 1 \text{ else } o := 2}$$

## Command Typing Rules

$$\text{Skip} \frac{}{\Gamma \vdash \text{skip}}$$

$$\text{Comp} \frac{\Gamma \vdash c_1 \quad \Gamma \vdash c_2}{\Gamma \vdash c_1 ; c_2}$$

$$\text{Asgn} \frac{\Gamma \vdash a : \ell \quad \ell \sqcup \Gamma(\mathbf{pc}) \leq \Gamma(x)}{\Gamma \vdash x := a}$$

$$\text{While} \frac{\Gamma \vdash b : \ell \quad \ell' = \Gamma(\mathbf{pc}) \sqcup \ell \quad \Gamma, \mathbf{pc} :: \ell' \vdash c}{\Gamma \vdash \text{while } b \text{ do } c}$$

$$\text{If} \frac{\Gamma \vdash b : \ell \quad \ell' = \Gamma(pc) \sqcup \ell \quad \Gamma, \mathbf{pc} :: \ell' \vdash c_1 \quad \Gamma, \mathbf{pc} :: \ell' \vdash c_2}{\Gamma \vdash \text{if } b \text{ then } c_1 \text{ else } c_2}$$

# Command Typing Example

Slide from Matt Fredrikson.

$$\Gamma = p :: H, g :: L, o :: L, pc :: L$$

$$\text{If} \frac{\text{Bin} \frac{\dots}{\Gamma \vdash p=g:H} \quad H = \Gamma(pc) \sqcup H \quad \text{Asgn} \frac{\text{Int} \frac{\dots}{\Gamma \vdash 1:L} \quad L \sqcup H \leq \Gamma(o)}{\Gamma, pc::H \vdash o:=1}}{\Gamma \vdash \text{if } p=g \text{ then } o:=1 \text{ else } o:=2}$$

- Doesn't work.
- Guard raises the **pc** label and Asgn propagates it.
- What about **if**  $p = g$  **then**  $o := 1$  **else**  $o := 1$ ?



## **Part Three: Proving Soundness**

# Soundness

[Volpano, Smith, Irvine '96]

Slide from Matt Fredrikson.

The type system is sound if whenever conditions 1-3 hold for program  $c$  and type environment  $\Gamma$ , then  $c$  has noninterference (i.e., the final states  $\sigma_1' \approx_\ell \sigma_2'$  for any starting states  $\sigma_1 \approx_\ell \sigma_2$ ).

1.  $\Gamma \vdash c$

2.  $\langle \sigma_1, c \rangle \Downarrow \sigma_1', \langle \sigma_2, c \rangle \Downarrow \sigma_2'$

3.  $\sigma_1 \approx_\ell \sigma_2$

# Two Key Lemmas

Slide from Matt Fredrikson.

**Lemma (Simple Security).** Expressions never read variables above their typed class: if  $\Gamma \vdash e : \ell$ , then for every variable  $x$  appearing in  $e$ ,  $\Gamma(x) \leq \ell$ .

**Lemma (Confinement).** Commands never write to variables below **pc**'s typed class: if  $\Gamma \vdash c$ , then for every variable  $x$  assigned in  $c$ ,  $\Gamma(\mathbf{pc}) \leq \Gamma(x)$ .

# Proof: Simple Security

Slide from Matt Fredrikson.

**Lemma (Simple Security).** If  $\Gamma \vdash e : \ell$ , then for every variable  $x$  appearing in  $e$ ,  $\Gamma(x) \leq \ell$ .

Proof by induction on the structure of  $e$  :

- Base cases  $n$ , **T**, and **F** are trivial.
- Base case  $x$ : we have  $\Gamma \vdash x : \ell$ .  
By Var,  $\Gamma(x) = \ell$ , so  $\Gamma(x) \leq \ell$ .
- Case  $e_1 \mathbf{op} e_2$ : by Bin, we have  $\Gamma \vdash e_1 : \ell_1$  and  $\Gamma \vdash e_2 : \ell_2$ . By induction, we have  $\forall x \in e_1. \Gamma(x) \leq \ell_1$  and  $\forall x \in e_2. \Gamma(x) \leq \ell_2$ . Then  $\Gamma(x) \leq \ell_1 \sqcup \ell_2 = \ell$  for all  $e = e_1 \mathbf{op} e_2$ .  $\square$



# Proof: Confinement

Slide from Matt Fredrikson.

**Lemma (Confinement).** if  $\Gamma \vdash c$ , then for every variable  $x$  assigned in  $c$ ,  $\Gamma(\mathbf{pc}) \leq \Gamma(x)$ .

Proof by induction on the structure of  $c$  :

- Base case **skip** is trivial.
- Base case  $x := a$ : we have  $\Gamma \vdash a : \ell$ .  
By Asgn,  $\ell \sqcup \Gamma(\mathbf{pc}) \leq \Gamma(x)$ , so  $\Gamma(\mathbf{pc}) \leq \Gamma(x)$ .
- Case  $c_1; c_2$  follows directly by induction.
- Case **while**  $b$  **do**  $c$ : suppose  $\Gamma \vdash b : \ell$ .  
By While, we have that  $\Gamma, \mathbf{pc} :: (\ell \sqcup \Gamma(\mathbf{pc})) \vdash c$ .  
By induction, we have that  $\forall x \in c. \ell \sqcup \Gamma(\mathbf{pc}) \leq \Gamma(x)$ .  
By  $\leq$ -transitivity,  $\forall x \in c. \Gamma(\mathbf{pc}) \leq \Gamma(x)$ .
- The case for **if** is similar to **while**.  $\square$

# Proof Sketch: Soundness

Slide from Matt Fredrikson.

**Theorem (Soundness).** The type system is sound if whenever conditions 1-3 hold for program  $c$  and type environment  $\Gamma$ , then  $c$  has noninterference (i.e., the final states  $\sigma'_1 \approx_\ell \sigma'_2$  for any starting states  $\sigma_1 \approx_\ell \sigma_2$ ).

1.  $\Gamma \vdash c$
2.  $\langle \sigma_1, c \rangle \Downarrow \sigma'_1, \langle \sigma_2, c \rangle \Downarrow \sigma'_2$
3.  $\sigma_1 \approx_\ell \sigma_2$

Proof by induction on the derivation of  $\langle \sigma_1, c \rangle \Downarrow \sigma'_1$  :

- Use Simple Security to argue about identical evaluation.
- Use Confinement to argue about  $\ell$ -equivalent updates.

# Example: while

**Theorem (Soundness).** Want following conditions:

1.  $\Gamma \vdash c$
2.  $\langle \sigma_1, c \rangle \Downarrow \sigma'_1, \langle \sigma_2, c \rangle \Downarrow \sigma'_2$
3.  $\sigma_1 \approx_\ell \sigma_2$

Suppose  $\langle \sigma_1, \text{while } b \text{ do } c \rangle \Downarrow \sigma'_1$  and typing ends with:

$$\frac{\Gamma \vdash b : \ell_1 \quad \ell_2 = \Gamma(\text{pc}) \sqcup \ell_1 \quad \Gamma, \text{pc} :: \ell_2 \vdash c}{\text{while } b \text{ do } c}$$

Case  $\ell_2 \leq$   $\frac{\langle \sigma_{1,2}, b \rangle \Downarrow \mathbf{T} \quad \langle \sigma_{1,2}, c \rangle \Downarrow \sigma''_{1,2}}{\langle \sigma_{1,2}, \text{while } b \text{ do } c \rangle \Downarrow \sigma'_{1,2}}$  (low memory):

- By Sim,  $\langle \sigma_{1,2}, \text{while } b \text{ do } c \rangle \Downarrow \sigma'_{1,2}$  for all  $x$  in  $b$ .
- By (3),  $\sigma_1 \approx_\ell \sigma_2$  for all  $x$  in  $b$ , so  $\langle \sigma_1, b \rangle \Downarrow v$  and  $\langle \sigma_2, b \rangle \Downarrow v$ .
- If  $v = \mathbf{F}$ , then  $\sigma_1 = \sigma'_1$  and  $\sigma_2 = \sigma'_2$ . Invoke (3).
- If  $v = \mathbf{T}$ , then  $\sigma''_1 \approx_\ell \sigma''_2$  by induction. Then  $\sigma'_1 \approx_\ell \sigma'_2$  also by induction.

# Example: while

**Theorem (Soundness).** Want following conditions:

1.  $\Gamma \vdash c$
2.  $\langle \sigma_1, c \rangle \Downarrow \sigma'_1, \langle \sigma_2, c \rangle \Downarrow \sigma'_2$
3.  $\sigma_1 \approx_\ell \sigma_2$

Suppose  $\langle \sigma_1, \mathbf{while} \ b \ \mathbf{do} \ c \rangle \Downarrow \sigma'_1$  and typing ends with:

$$\text{While} \frac{\Gamma \vdash b : \ell_1 \quad \ell_2 = \Gamma(\mathbf{pc}) \sqcup \ell_1 \quad \Gamma, \mathbf{pc} :: \ell_2 \vdash c}{\Gamma \vdash \mathbf{while} \ b \ \mathbf{do} \ c}$$

Case  $\ell_2 > \ell$  (condition cannot flow into low memory):

- By Confinement,  $\ell_1 \leq \Gamma(x)$  for all  $x$  assigned in  $c$ .
- For  $x$  assigned in  $c$ ,  $\neg(\Gamma(x) \leq \ell)$ .
- For every  $x$  in  $c$  where  $\Gamma(x) \leq \ell$ ,  $\sigma_{1,2}(x) = \sigma'_{1,2}(x)$ .
- By (3), we have  $\sigma_1 \approx_\ell \sigma'_2$ .  $\square$

# Discussion Questions

- What kinds of guarantees can language-based information flow provide?
- What are the tradeoffs of static information flow analysis?
- This work came *before* the Flume work. Why did people become interested in coarser-grained information flow?