CONTINUATION PASSING STYLE

After CPS conversion, we will resolutely use continuations for everything. This can be seen as a way of making control flow explicit. There are results saying that the output of CPS conversion is invariant under interpretation as pass-by-name or pass-by-value, though we will not go into those results in this class. CPS conversion gives us named intermediate results. Thirdly, we reify control-flow as data. The first two of these three properties are commonly called "monadic form."

1. IL-CPS

We first must define the target language for this transformation. Notably, we split terms into two syntatic classes; *expressions* and *values*. One may think of expressions as values that are computed and then thrown away.

We may formalize this intuition as follows:

$$\begin{aligned} v ::= {} & x \\ & \mid \lambda x : \tau . e \\ & \mid \texttt{pack } [c, v] \texttt{ as } \exists \alpha : k.\tau \\ & \mid \langle v_1, \ldots v_n \rangle \end{aligned}$$

$$\begin{aligned} e ::= {} & vv \\ & \mid \texttt{unpack } [\alpha, x] = v \texttt{ in } e \\ & \mid \texttt{let } x = \pi_i v \texttt{ in } e \\ & \mid \texttt{let } x = v \texttt{ in } e \\ & \mid \texttt{halt} \end{aligned}$$

IL-CPS has the following typing rules:

$$\frac{\Gamma \vdash \tau : T \qquad \Gamma, x : \tau \vdash e : 0}{\Gamma \vdash \lambda x : \tau . e : \tau \to 0} \qquad \frac{\Gamma \vdash v_1 : \neg\tau \qquad \Gamma \vdash v_2 : \tau}{\Gamma \vdash v_1 v_2 : 0}$$

$$\frac{\Gamma \vdash c : k \qquad \Gamma \vdash v : [c/\alpha]\tau \qquad \Gamma, \alpha : k \vdash \tau : T}{\Gamma \vdash \texttt{pack } [c, v] \texttt{ as } \exists \alpha : k.\tau : \exists \alpha : k.\tau}$$

$$\frac{\Gamma \vdash v : \exists \alpha : k.\tau \qquad \Gamma, \alpha : k, x : \tau \vdash e : 0}{\Gamma \vdash \texttt{unpack } [\alpha, x] = v \texttt{ in } e : 0} \qquad \frac{\Gamma \vdash v_i : \tau_i \qquad (\text{for } i = 1 \ldots n)}{\Gamma \vdash \langle v_1, \ldots, v_n \rangle : \times[\tau_1, \ldots, \tau_n]}$$

$$\frac{\Gamma \vdash v : \times[\tau_1, \ldots, \tau_n]}{\Gamma \vdash \texttt{let } x = \pi_i v \texttt{ in } e : 0} \qquad \frac{\Gamma \vdash v : \tau \qquad \Gamma, x : \tau \vdash e : 0}{\Gamma \vdash \texttt{let } x = v \texttt{ in } e : 0} \qquad \frac{}{\Gamma \vdash \texttt{halt} : 0}$$

$$\frac{\Gamma \vdash \tau : T}{\Gamma \vdash \neg\tau : T}$$

Note that in constructive logic, the proposition "$\tau \to 0$" is exactly $\neg\tau$. So we may perhaps cloyingly say that continuations are negation.

A careful reader may notice our usual sleight of hand in the `unpack` rule: the α's mentioned are all asserted to be equal.

2. CPS Conversion Compiler Pass