Finance Data Science

Lecture 1: Overview

Laurent El Ghaoui

MFE 230P, Summer 2017
MFE Program
Haas School of Business
UC Berkeley

6/5/2017

Outline

Outline

Big data in finance

Big data market[1]

- Growth to 200$ Bn in 2020.
- 15% of it is in finance.

"Fintech" is a fast-growing industry, some of it is using Big Data as a key component.

- key areas so far: marketplace lending, next-generation payments and blockchain technology.
- emerging trends: specialized data sources & processing (satellite), robo-advisors, insurance tech.

[1] *Source:* Big Data and AI Strategies: Machine Learning and Alternative Data Approach to Investing. Marko Kolanovic & Rajesh T. Krishnamachari, JP Morgan report, May 2017.

Role of data science

- *Descriptive* (unsupervised learning): "understand data"
  clustering, factor analysis, filling missing data, outliers removal
- *Predictive* : "forecast the future"
  regression, classification, & deep learning approaches to those
- *Prescriptive* : "make investment decisions"
  portfolio optimization, control & reinforcement learning for investment planning / decision

Currently a lot of discussion is around the first two (the "machine learning" part), and the last is mostly mentioned in the context of robotics (*e.g.*, self-driving cars). This course makes the case that a lot is to be gained from a comprehensive view where all three components are included.

Sources of data

- *structured:* company data, commercial transactions, credit card, order book data, balance sheets, etc.
- *unstructured:* text (press releases, news, blogs, EDGAR, etc), graphs, satellite images, traffic data, earnings calls transcripts, videos, etc.

In practice, we may not have as much *relevant* data as often touted.

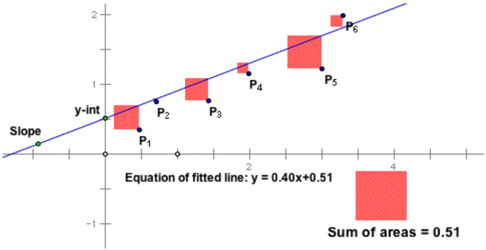
Outline

The four axes

- Unsupervised learning: “understand market structure”
- Supervised learning: “predict sentiment”
- Deep learning: “learn features” in data
- Optimization & reinforcement learning: “learn trades”

Example

Least-squares regression



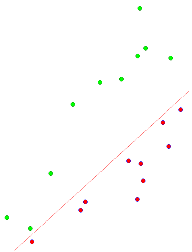
$$\min_w \|X^T w - y\|_2$$

where

- $X = [x_1, \ldots, x_m]$ is a $n \times m$ matrix of data points ($x_i \in \mathbf{R}^n$);
- y is a response vector;
- $\|z\|_2 := \sqrt{z_1^2 + \ldots + z_m^2}$ is the l_2 (*i.e.*, Euclidean) norm of a vector $z \in \mathbf{R}^m$.
- Many variants (with *e.g.*, constraints) exist (more on this later).
- Perhaps the most popular / useful optimization problem.

Example

Linear classification



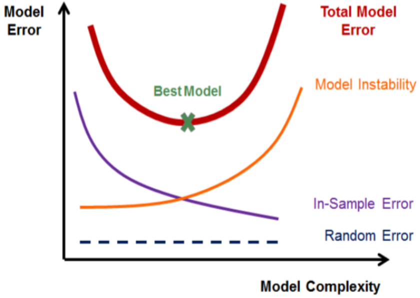
$$\min_{w,b} \sum_{i=1}^{m} \max(0, 1 - y_i(w^T x_i + b))$$

where

- $X = [x_1, \ldots, x_m]$ is a $n \times m$ matrix of data points ($x_i \in \mathbf{R}^n$);
- $y \in \{-1, 1\}$ is a *binary* response vector.
- A new data point is classified as $\hat{y}(x) = \mathbf{sign}(w^T x + b)$.

- Many variants (with *e.g.*, constraints) exist (more on this later).
- Very useful for *e.g.* sentiment analysis.

How to evaluate results



- In supervised learning, we can reserve a part of the available data to test a model trained on the remaining part. There is a trade-off between model complexity and error.
- In unsupervised learning, there is no such "yardstick". One way is to consider the stability of the result with respect to perturbations in data. (More on this later.)

Outline

What is optimization?

Optimization is a field of applied mathematics also known as "mathematical programming".

It is a *language* that allows to describe precisely how a decision should be made.

It includes as special cases:

- Machine learning problems: the decision may be about what prediction rule to use, in order to predict alpha or sentiment;
- Decision problems: Portfolio optimization.

Most machine learning problems can be viewed as a special case of an optimization problem.

- This connection allows to design algorithms (*e.g.*, stochastic gradient) to solve ML problems.
- It allows points to a better understanding of how to design models (*e.g.*, take into account prediction errors within a portfolio optimization problem).

Optimization problem

A standard form

An optimization problem is a problem of the form

$$p^* := \min_x \ f_0(x) \text{ subject to } f_i(x) \leq 0, \quad i = 1, \ldots, m,$$

where

- $x \in \mathbf{R}^n$ is the *decision variable*;
- $f_0 : \mathbf{R}^n \to \mathbf{R}$ is the *objective* (or, *cost*) function;
- $f_i : \mathbf{R}^n \to \mathbf{R}$, $i = 1, \ldots, m$ represent the *constraints*;
- p^* is the *optimal value*.

Often the above is referred to as a “mathematical program” (for historical reasons).

Example

A short-term financing problem

A company faces the following *net cash flow requirements:*

Month	Jan	Feb	Mar	Apr	May	Jun
Net cash flow (in $ k)	−150	−100	200	−200	50	300

Available *sources of funds:*

- Line of credit (max 100k, interest rate 1% per month);
- In any of the first 3 months it can issue 90-day commercial paper bearing a total interest of 2% for the 3-month period;
- Excess funds can be invested at 0.3% per month.

Example

A short-term financing problem: decision problem

Variables :

- Balance on the credit line x_i for month $i = 1, 2, 3, 4, 5$.
- Amount y_i of commercial paper issued ($i = 1, 2, 3$).
- Excess funds z_i for month $i = 1, 2, 3, 4, 5$.
- z_6, the company's wealth in June.

Decision problem:

$$\text{maximize } z_6 \text{ subject to } \begin{cases} \text{Bounds on variables,} \\ \text{Cash-flow balance equations.} \end{cases}$$

Example

A short-term financing problem: constraints

- Non-negativity: $x_i \geq 0$, $i = 1, \ldots, 6$; $z_i \geq 0$, $i = 1, \ldots, 6$; $y_i \geq 0$, $i = 1, 2, 3$.
- Upper bounds on x_i's: $x_i \leq 100$, $i = 1, \ldots, 5$.
- Cash flow balance equations.

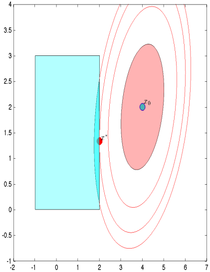
Linear programming formulation:

$$
\begin{array}{ll}
\max\limits_{x,y,z} & z_6 \\
\text{s.t.} & x_1 + y_1 - z_1 = 150, \\
& x_2 + y_2 - 1.01x_1 + 1.003z_1 - z_2 = 100, \\
& x_3 + y_3 - 1.01x_2 + 1.003z_2 - z_3 = -200, \\
& x_4 - 1.02y_1 - 1.01x_3 + 1.003z_3 - z_4 = 200, \\
& x_5 - 1.02y_2 - 1.01x_4 + 1.003z_4 - z_5 = -50, \\
& -1.02y_3 - 1.01x_5 + 1.003z_5 - z_6 = -300, \\
& 100 \geq x_i \geq 0, \quad i = 1, \ldots, 5, \\
& y_i \geq 0, \quad i = 1, 2, 3, \\
& z_i \geq 0, \quad i = 1, \ldots, 6.
\end{array}
$$

Nomenclature

A toy optimization problem

$$\begin{array}{ll} \min_x & 0.9x_1^2 - 0.4x_1x_2 - 0.6x_2^2 - 6.4x_1 - 0.8x_2 \\ \text{s.t.} & -1 \le x_1 \le 2, \quad 0 \le x_2 \le 3. \end{array}$$



- *Feasible set* in light blue.
- 0.1- *suboptimal set* in darker blue.
- *Unconstrained minimizer* : x_0; optimal point: x^*.
- *Level sets* of objective function in red lines.
- A *sub-level set* in red fill.

Other standard forms

Equality constraints. We may single out equality constraints, if any:

$$\min_x \; f_0(x) \text{ subject to } \begin{array}{l} h_i(x) = 0, \quad i = 1, \ldots, p, \\ f_i(x) \leq 0, \quad i = 1, \ldots, m, \end{array}$$

where h_i's are given. Of course, we may reduce the above problem to the standard form above, representing each equality constraint by a pair of inequalities.

Abstract forms. Sometimes, the constraints are described abstractly via a set condition, of the form $x \in \mathcal{X}$ for some subset $\mathcal{X}$ of $\mathbf{R}^n$. The corresponding notation is

$$\min_{x \in \mathcal{X}} \; f_0(x).$$

Minimization vs. maximization

Some problems come in the form of maximization problems. Such problems are readily cast in standard form via the expression

$$\max_{x \in \mathcal{X}} \; f_0(x) = -\min_{x \in \mathcal{X}} \; : \; g_0(x),$$

where $g_0 := -f_0$.

- *Minimization* problems correspond to loss, cost or risk minimization.
- *Maximization* problems typically correspond to utility or return (*e.g.*, on investment) maximization.

Penalization

A trade-off between two objecgives is commonly accomplished via a *penalized* problem:

$$\max_x \; f(x) + \lambda g(x),$$

where f and g represent loss and risk functions, and $\lambda > 0$ is a risk-aversion parameter.

Example: penalized least-squares

$$\min_w \; \|X^T w - y\|_2^2 + \lambda \|w\|_2^2$$

Here, the risk term $\|w\|_2^2$ controls the variance associated with noise in X.

Optimization in Finance

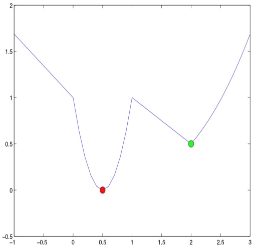
Applications

- Machine learning:
  - Unsupervised learning: Market data analysis, covariance estimation and factor models, matrix completion, clustering.
  - Supervised learning: Model fitting, regression, classification, sentiment analysis.
- Decision-making:
  - Single-period: Portfolio optimization, asset allocation.
  - Multi-period: Portfolio optimization, asset liability management.
- Pricing and arbitrage detection: Static and dynamic.

Outline

Global vs. local minima

The curse of optimization



- Point in red is globally optimal (optimal for short).
- Point in green is only locally optimal.
- In many applications, we are interested in global minima.

Curse of optimization

Optimization algorithms for general problems can be trapped in local minima.

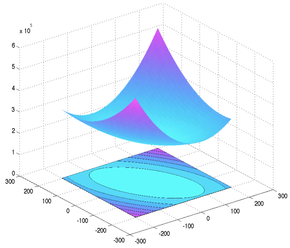
Convex function

Definition

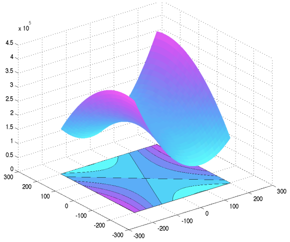
A function $f : \mathbf{R}^n \to \mathbf{R}$ is convex if it satisfies the condition

$$\forall\, x, y \in \mathbf{R}^n, \lambda \in [0,1] \;:\; f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y)$$

Geometrically, the graph of the function is “bowl-shaped”.



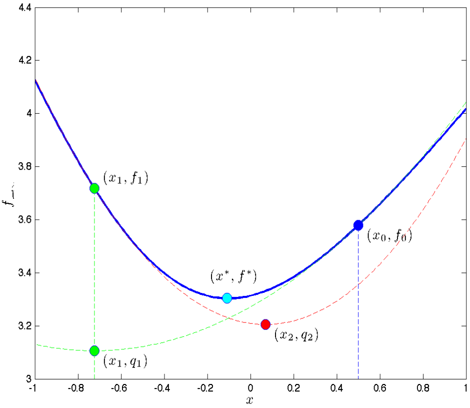
Convex function.



Non-convex function.

Convexity and local minima

When trying to minimize convex functions, specialized algorithms will always converge to a global minimum, irrespective of the starting point, provided some (weak) assumptions on the function hold.



The Newton algorithm.

Convex optimization

Definition

The problem in standard form

$$p^* := \min_x \; f_0(x) \text{ subject to } f_i(x) \leq 0, \quad i = 1, \ldots, m,$$

is convex if the functions $f_0, \ldots, f_m$ are all convex.

Examples:

- Linear programming ($f_0, \ldots, f_m$ affine).
- Quadratic programming (f_0 convex quadratic, $f_1, \ldots, f_m$ affine).
- Second-order cone programming (f_0 linear, f_i's of the form $\|A_i x + b_i\|_2 + c_i^T x + d_i$, for appropriate data A_i, b_i, c_i, d_i).

Software for convex optimization

- Free: CVX [3], Yalmip, Mosek (student version) [1].
- Really free: CVXPY [4] (in development).
- Commercial: Mosek, CPLEX, etc.

CVX syntax for cash-flow problem (assume data is in matrix A, vector b):

```
cvx_begin
variables x(5,1) y(3,1) z(6,1);
minimize( z(6) )
subject to
 A*[x;y;z] == b;
 x >= 0; x <= 100;
 y >= 0;
 z >= 0;
cvx_end
```

Non-convex problems

Examples

- *Boolean/integer optimization:* some variables are constrained to be Boolean or integers. Convex optimization can be used for getting (sometimes) good approximations.
- *Cardinality-constrained problems:* we seek to bound the number of non-zero elements in a vector variable. Convex optimization can be used for getting good approximations.
- *Non-linear programming:* usually non-convex problems with differentiable objective and functions. Algorithms provide only local minima. Includes as special case many machine learning prolems (*e.g.*, neural nets).

Not all non-convex problems are hard!

Does convexity really matter?

In machine learning, convexity may not be a big deal; *e.g.*, ARIMA or neural net models are essentially non-convex, non-linear least-squares. Local minima are not usually an issue: a local minimum is “good enough”.

The main reason: *there are no constraints* in those problems.

When there are constraints, and the problem is not convex, the algorithms may not behave well (*e.g.*, may not find a feasible point, even though there exist one). Thus when it comes to portfolio optimization, convex models should be preferred.

Outline

Course goals

- Introduce you to the main concepts in machine learning and optimization.
- Illustrate the relevance of those concepts in financial engineering.
- Introduce you to novel concepts that have not been fully tested in finance, but offer promise given their successes in other fields (*e.g.*, deep learning for images; robust portfolio optimization).

Course topics

- *Topic 1: Optimization models.* Basic optimization nomenclature, convex functions and sets. Linear and quadratic programming.

Next:

- *Linear algebra background.* Vectors and matrices, scalar product, mean and variance, eigenvalues and singular values, covariance matrices.
- *Unsupervised learning.* Clustering, principal component analysis, covariance matrix estimation, matrix completion, feature engineering.
- *Supervised learning.* Basics of prediction and classification. Least-squares regression, regularization, robust and quantile regression, auto-regressive and other time-series models, extensions.
- *Mean-variance models for portfolio design.* Linear and quadratic programming models. Transaction costs, execution models. Robustness.

Course material and references

- *Lecture slides* (early version posted in advance) on bcourses. Make sure to check out the version posted after lecture.
- *Textbooks:*
  - G.C. Calafiore and L. El Ghaoui. *Optimization Models.* Cambridge, 2014.
    Introductory reference on optimization.
  - G. Cornuejols and R. Tütüncü. *Optimization methods in Finance.* Cambridge, Mathematics, Finance and Risk series, 2007.
    Introductory level with many finance applications.
  - S. Boyd and L. Vandenberghe. *Convex optimization.* Cambridge University Press, 2004.
    In-depth treatment of convex models.
  - *Optimization models.* livebook available at `http://livebooklabs.com/keeppies/c5a5868ce26b8125`.
    A gentler introduction with many applications in engineering, finance, operations research, statistics.
- *Software:* we will rely on matlab and CVX (matlab toolbox for convex optimization, [3]).

References (follow'd)

- T. Hastie, R. Tibshirani, J. Friedman. *The elements of statistical learning* . Springer, 2001.

  Good introduction to the fundamentals of machine learning, from a statistics viewpoint.
- I. Goodfellow , Y. Bengio and A. Courville. Deep learning.

  A reference on this hot topic.

Homeworks

There will be a total of about four homeworks, most of which will require the use of software such as CVX [3], or Mosek [1], all of which have free (student) matlab-based versions.

Topics:

- *Homework 1* : Convexity; clustering.
- *Homework 2* : Factor models, PCA, generalized low-rank models, matrix completion.
- *Homework 3* : Feature engineering. Kernel methods for supervised learning. Regression & classification.
- *Homework 4* : Portfolio optimization and robustness.

Logistics

- *Instructor:* Laurent El Ghaoui (`elghaoui@berkeley.edu`).
- *TA:* Mustafa Eisa (`m.eisa@berkeley.edu`).
- *L.E.G.'s office hours:* W 3-4PM, S276.
- *M.E.'s office hours:* F 4-5PM, S276.
- *Discussion section:* M 3-4PM, F320.
- *Homeworks:* by teams of four max, one HW turned in for each team.
- *Grading:* 60 % homeworks, 40 % final.

How do we communicate?

- Preferred way: bcourses and github (details provided during discussion section today).
- Email to me. Always cc Mustafa!
- or, during OHs.

Outline

Exercise

You have $12,000 to invest at the beginning of the year, and three different funds from which to choose. The municipal bond fund has a 7% yearly return, the local bank's Certificates of Deposit (CDs) have an 8% return, and a high-risk account has an expected (hoped-for) 12% return. To minimize risk, you decide not to invest any more than $2,000 in the high-risk account. For tax reasons, you need to invest at least three times as much in the municipal bonds as in the bank CDs. Denote by x, y, z be the amounts (in thousands) invested in bonds, CDs, and high-risk account, respectively.

Problem: Assuming the year-end yields are as expected, what are the optimal investment amounts for each fund? Solve via CVX.

Outline

References I

E. D. Andersen and K. D. Andersen.
The MOSEK optimization package.

A. Bental, L. El Ghaoui, and A. Nemirovski.
Robust Optimization.
Princeton University Press, 2009.

S. Boyd and M. Grant.
The CVX optimization package, 2010.

S. Boyd and T. Tinoco.
The CVXPY optimization package, 2010.

Stephen Boyd and Lieven Vandenberghe.
Convex optimization.
Cambridge university press, 2004.

Giuseppe Calafiore and Laurent El Ghaoui.
Optimization models.
Cambridge University Press, October 2014.

G. Cornuejols and R. Tütüncü.
Optimization Methods in Finance.
Cambridge University Press, 2007.

Gerard Cornuejols and Reha Tütüncü.
Optimization methods in finance, volume 5.
Cambridge University Press, 2006.

References II

Jerome Friedman, Trevor Hastie, and Robert Tibshirani.
The elements of statistical learning, volume 1.
Springer series in statistics Springer, Berlin, 2001.

Ian Goodfellow, Yoshua Bengio, and Aaron Courville.
Deep Learning.
MIT Press, 2016.
http://www.deeplearningbook.org.