2D Noise Function

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We introduce how to compute the FBM 2D noise term N.

- **Step 1: Split the position into integer and fractional parts**. Let

$$\mathbf{p} = (p_x, p_y), \quad \mathbf{i} = \lfloor \mathbf{p} \rfloor, \quad \mathbf{f} = \{\mathbf{p}\} = \mathbf{p} - \mathbf{i}$$

  Note that $\mathbf{i}$ is the grid cell and $\mathbf{f}$ is the position inside the cell.

- **Step 2: Compute random values at the 4 grid corners**. Using a hash function:

$$h(\mathbf{u}) = \text{frac}\{\sin[\mathbf{u} \cdot (a, b) \cdot M]\}$$

  where $(a, b, M) = (202.5, 112.7, 43758.5453123)$ are three magic numbers (M is a classic number in computer graphics industry, and a, b are related to the current date), and $\text{frac}(\cdot)$ is the fractional part operator, defined as $\text{frac}(x) = x - \lfloor x \rfloor$.

  Then, define:

$$A = h(\mathbf{i}), \quad B = h(\mathbf{i} + (1, 0)), \quad C = h(\mathbf{i} + (0, 1)), \quad D = h(\mathbf{i} + (1, 1))$$

  Note that these four numbers are random in $[0, 1]$.

- **Step 3: Smooth the fractional position.** Use a Hermite smoothstep to make interpolation smooth (0 slope at edges):

$$\mathbf{u} = (u_x, u_y) = \mathbf{f}^2(3 - 2\mathbf{f})$$

- **Step 4: Bilinear interpolation.** The result of 2D noise is:

$$N(\mathbf{p}) = A(1 - u_x)(1 - u_y) + B(u_x)(1 - u_y) + C(1 - u_x)(u_y) + D(u_x)(u_y)$$

Finally, the full noise formula is:

$$N(\mathbf{p}) = \sum_{d_x=0}^{1} \sum_{d_y=0}^{1} \{h[\mathbf{i} + (d_x, d_y)] \cdot [(1 - d_x)(1 - u_x) + d_x u_x] \cdot [(1 - d_y)(1 - u_y) + d_y u_y]\}$$