1

FT-preperties

I. LINEARITY

A. Reference

Linearity, wikipedia

B. Definition

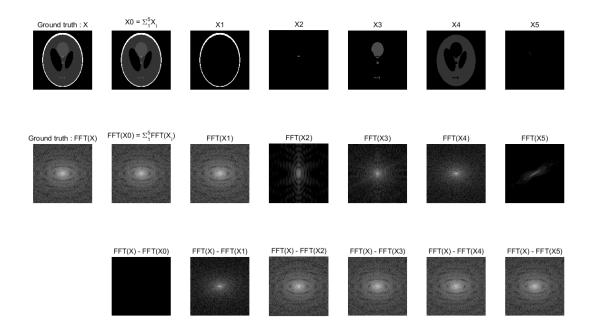
For any complex numbers $a \in \mathbb{C}$ and $b \in \mathbb{C}$,

$$h(x) = a * f(x) + b * g(x) \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \hat{h}(\xi) = a \cdot \hat{f}(\xi) + b \cdot \hat{g}(\xi)$$

C. Execution

Run demo_fourier_properties_1_linearity.m

D. Results



II. SHIFT IN SPATIAL DOMAIN

A. Reference

Shift in Spatial domain, wikipedia

B. Definition

For any real number $x_0 \in \mathbb{R}$,

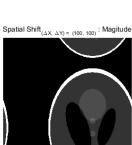
$$h(x) = f(x - x_0) \quad \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \quad \hat{h}(\xi) = e^{-2\pi i x_0 \xi} \hat{f}(\xi)$$

C. Execution

 $Run\ demo_fourier_properties_2_shift_in_spatial_domain.m$

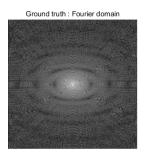
D. Results

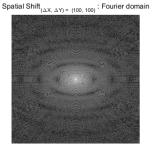












III. SHIFT IN FOURIER DOMAIN

A. Reference

Shift in Fourier domain, wikipedia

B. Definition

For any real number $\xi_0 \in \mathbb{R}$,

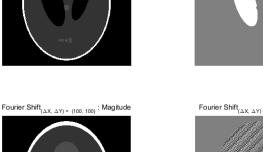
$$h(x) = e^{2\pi i x \xi_0} f(x) \quad \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \quad \hat{h}(\xi) = \hat{f}(\xi - \xi_0)$$

C. Execution

 $Run\ \ demo_fourier_properties_3_shift_in_Fourier_domain.m$

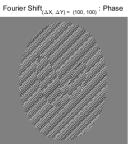
D. Results

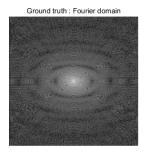


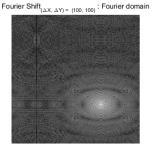




Ground truth : Phase







IV. CONVOLUTION THEOREM

A. Reference

Convolution theorem, wikipedia

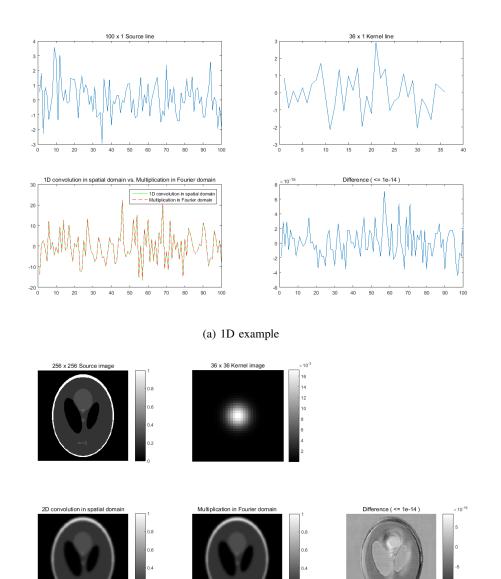
B. Definition

$$h(x) = (f * g)(x) = \int_{-\infty}^{\infty} f(y)g(x - y)dy \quad \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \quad \hat{h}(\xi) = \hat{f}(\xi) \cdot \hat{g}(\xi)$$

C. Execution

Run demo_fourier_properties_4_1d_convolution_vs_multiplication.m Run demo_fourier_properties_5_2d_convolution_vs_multiplication.m

D. Results



(b) 2D example