Machine Learning Loss Functions

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1 Supervised learning

In a supervised learning scheme, our goal is finding an optimal generator G constructed by trainable parameters θ_g and the optimal generator G induces a **minimum value of loss function** $\mathcal{L}(G)$ as expressed in Eq. 1.

$$G^* = \arg\min_G \mathcal{L}(G). \tag{1}$$

1.1 L1 Loss (= Mean Absolute Error Loss; MAE Loss)

$$\mathcal{L}_{L1}(G) = \mathbb{E}_{x,y}[|y - G(x;\theta_g)|], \tag{2}$$

where G is generator, and θ_g is trainable parameters such as convolution kernel (ω) and bias(b). x and y are input and target data, respectively.

1.2 L2 loss (= Mean Squared Error Loss; MSE Loss)

$$\mathcal{L}_{L2}(G) = \mathbb{E}_{x,y}[||y - G(x;\theta_g)||_2^2], \tag{3}$$

where G is generator, and θ_g is trainable parameters such as convolution kernel (ω) and bias(b). x and y are input and target data, respectively.

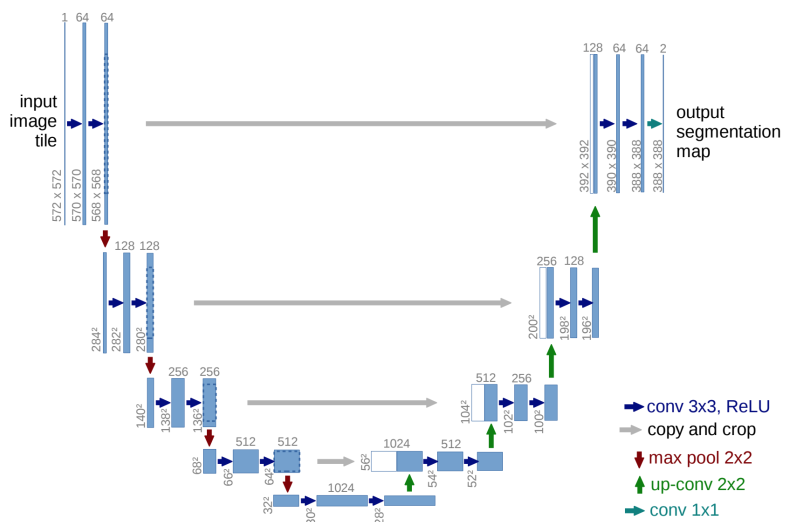

Fig. 1. U-net architecture (example for 32x32 pixels in the lowest resolution). Each blue box corresponds to a multi-channel feature map. The number of channels is denoted on top of the box. The x-y-size is provided at the lower left edge of the box. White boxes represent copied feature maps. The arrows denote the different operations.

Figure 1: U-Net [1] is one of examples for supervised learning.

2 Unsupervised learning

In a unsupervised learning scheme, our goal is finding an optimal generator G and discriminator D constructed by trainable parameters θ_g and θ_D, respectively. The optimal generator G induces a minimum value of loss function $\mathcal{L}(G)$, but the optimal discriminator D induces a maximum value of loss. The optimization problem related with between generator G and discriminator D is called by **minimax game** as expressed in Eq. 4.

$$G^*, D^* = \arg\min_G \max_D \mathcal{L}(G, D). \tag{4}$$

2.1 Generative Adversarial Network (GAN) [2, 3]

$$\begin{aligned} \mathcal{L}_{GAN}(G, D) &= \mathbb{E}_{x\sim p_{data}(x)}[\log D(x;\theta_d)] \\ &+ \mathbb{E}_{z\sim p_z(z)}[\log(1 - D(G(z;\theta_g);\theta_d))], \end{aligned} \tag{5}$$

where G and D are generator and discriminator, respectively, and its θ_g and θ_d are trainable parameters such as convolution kernel (ω) and bias(b). z and y are input (Gaussian and/or normal noise) and target (image) data, respectively, and its $p_{data}(x)$ and $p_z(z)$ are data distributions.

G generates a fake sample $\tilde{x} = G(z;\theta_g)$ in $p_{data}(x)$ domain from a noise z in $p_z(z)$ domain. For true data $x \sim p_{data}(x)$ and synthesized data $\tilde{x} = G(z;\theta_g)$, D distinguishes whether a given data belongs to $p_{data}(x)$ domain.

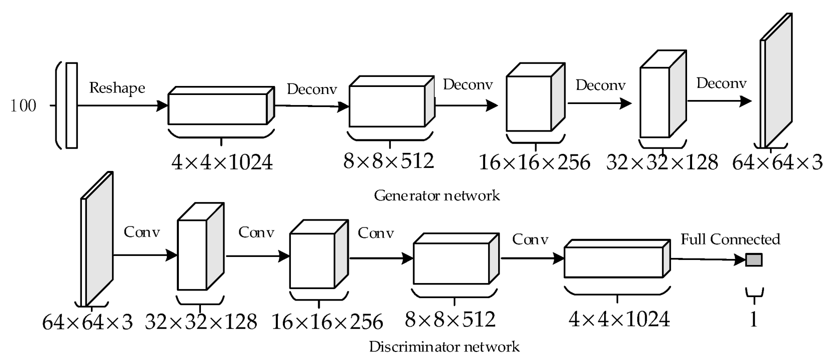

Figure 2: Standard GAN [2, 3]. (top) Generator network architecture (G), and (bottom) Discriminator (D) network architecture.

2.2 pix2pix: Conditional GAN (cGAN) [4]

$$\mathcal{L}_{\text{pix2pix}}(G, D) = \mathcal{L}_{cGAN}(G, D) + \lambda \mathcal{L}_{L1}(G), \tag{6}$$

where $\mathcal{L}_{cGAN}(G, D)$ is an objective function of a conditional GAN and $\mathcal{L}_{L1}(G)$ is an objective function of a L1 loss. λ is hyper-parameter that control the relative importance of the two objectives. $\mathcal{L}_{cGAN}(G, D)$ and $\mathcal{L}_{L1}(G)$ are defined by Eqs. 7 and 8, respectively.

$$\mathcal{L}_{cGAN}(G, D) = \mathbb{E}_{x,y}[\log D(x, y; \theta_d)] + \mathbb{E}_x[\log(1 - D(x, G(x; \theta_g); \theta_d))], \tag{7}$$

$$\mathcal{L}_{L1}(G) = \mathbb{E}_{x,y}[|y - G(x; \theta_g)|], \tag{8}$$

where G and D are generator and discriminator, respectively, and its θ_g and θ_d are trainable parameters such as convolution kernel (ω) and bias(b). x and y are input and target data, respectively.

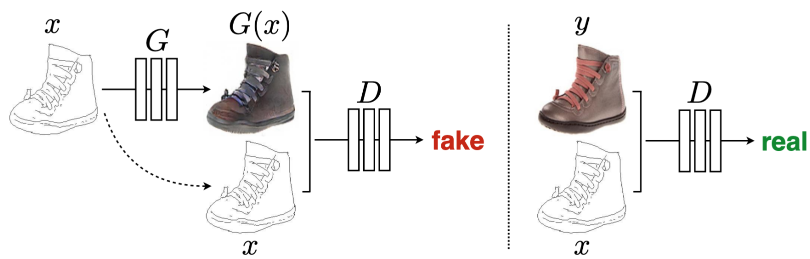

Figure 2: Training a conditional GAN to map edges→photo. The discriminator, D, learns to classify between fake (synthesized by the generator) and real {edge, photo} tuples. The generator, G, learns to fool the discriminator. Unlike an unconditional GAN, both the generator and discriminator observe the input edge map.

Figure 3: pix2pix [4] training scheme. Actually, pix2pix [4] is not an unsupervised learning because they need a paired dataset.

2.3 CycleGAN [5]

$$\begin{aligned}\mathcal{L}_{\text{cycleGAN}}(G_{X\to Y}, G_{Y\to X}, D_X, D_Y) &= \mathcal{L}_{GAN}(G_{X\to Y}, D_Y) \\ &+ \mathcal{L}_{GAN}(G_{Y\to X}, D_X) \\ &+ \lambda\mathcal{L}_{cyc}(G_{X\to Y}, G_{Y\to X}) \\ &+ \rho\mathcal{L}_{identity}(G_{X\to Y}, G_{Y\to X}), \end{aligned} \tag{9}$$

where $\mathcal{L}_{GAN}(G_{X\to Y}, D_Y)$ and $\mathcal{L}_{GAN}(G_{Y\to X}, D_X)$ are an objective function of a GAN, $\mathcal{L}_{cyc}(G_{X\to Y}, G_{Y\to X})$ is an objective function of a cycle consistency loss and $\mathcal{L}_{identity}(G_{X\to Y}, G_{X\to Y})$ is an objective function of an identity loss. λ and ρ are hyper-parameters that control the relative importance. $\mathcal{L}_{GAN}(G, D)$, $\mathcal{L}_{cyc}(G_{X\to Y}, G_{Y\to X})$, and $\mathcal{L}_{identity}(G_{X\to Y}, G_{Y\to X})$ are defined by Eqs. 10, 11, and 12, respectively.

$$\begin{aligned}\mathcal{L}_{GAN}(G_{X\to Y}, D_Y) &= \mathbb{E}_{y\sim p_{data}(y)}[\log D_Y(y;\theta_d^y)] \\ &+ \mathbb{E}_{x\sim p_{data}(x)}[\log(1 - D_Y(G_{X\to Y}(x;\theta_g^{X\to Y});\theta_d^y))],\end{aligned} \tag{10a}$$

$$\begin{aligned}\mathcal{L}_{GAN}(G_{Y\to X}, D_X) &= \mathbb{E}_{x\sim p_{data}(x)}[\log D_X(x;\theta_d^x)] \\ &+ \mathbb{E}_{y\sim p_{data}(y)}[\log(1 - D_X(G_{Y\to X}(y;\theta_g^{Y\to X});\theta_d^x))],\end{aligned} \tag{10b}$$

$$\begin{aligned}&\mathcal{L}_{cyc}(G_{X\to Y}, G_{Y\to X}) \\ &\quad= \mathbb{E}_{x\sim p_{data}(x)}[|G_{Y\to X}(G_{X\to Y}(x;\theta_g^{X\to Y});\theta_g^{Y\to X}) - x|] \\ &\quad+ \mathbb{E}_{y\sim p_{data}(y)}[|G_{X\to Y}(G_{Y\to X}(y;\theta_g^{Y\to X});\theta_g^{X\to Y}) - y|],\end{aligned} \tag{11}$$

$$\begin{aligned}\mathcal{L}_{identity}(G_{X\to Y}, G_{Y\to X}) &= \mathbb{E}_{y\sim p_{data}(y)}[|G_{Y\to X}(x;\theta_g^{Y\to X}) - x|] \\ &+ \mathbb{E}_{x\sim p_{data}(x)}[|G_{X\to Y}(y;\theta_g^{X\to Y}) - y|],\end{aligned} \tag{12}$$

where G and D are generator and discriminator, respectively, and its θ_g and θ_d are trainable parameters such as convolution kernel (ω) and bias(b). x and y are data for each difference classes, respectively, and $p_{data}(x)$ and $p_{data}(y)$ are its data distributions.

$G_{X\to Y}$ generates a fake sample $\tilde{y} = G_{X\to Y}(x;\theta_g^{X\to Y})$ in $p_{data}(y)$ domain from a true sample x in $p_{data}(x)$ domain, while $G_{Y\to X}$ generates a fake sample $\tilde{x} = G_{Y\to X}(y;\theta_g^{Y\to X})$ in $p_{data}(x)$ domain from a true sample y in $p_{data}(y)$ domain. For true data $x \sim p_{data}(x)$ and synthesized data $\tilde{x} = G_{Y\to X}(y;\theta_g^{Y\to X})$, D_X distinguishes whether a given data belongs to $p_{data}(x)$ domain. On the contrary, true data $y \sim p_{data}(y)$ and synthesized data $\tilde{y} = G_{X\to Y}(x;\theta_g^{X\to Y})$ are classified by D_Y whether a given data belongs to $p_{data}(y)$ domain.

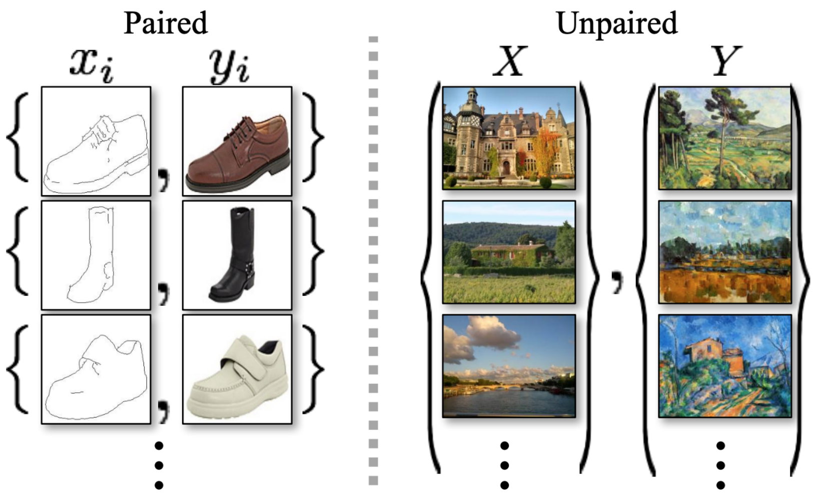

Figure 2: *Paired* training data (left) consists of training examples $\{x_i, y_i\}_{i=1}^N$, where the correspondence between x_i and y_i exists [22]. We instead consider *unpaired* training data (right), consisting of a source set $\{x_i\}_{i=1}^N$ ($x_i \in X$) and a target set $\{y_j\}_{j=1}$ ($y_j \in Y$), with no information provided as to which x_i matches which y_j.

Figure 4: Example of unpaired data distributions $x \sim p_{data}(x)$ and $y \sim p_{data}(y)$.

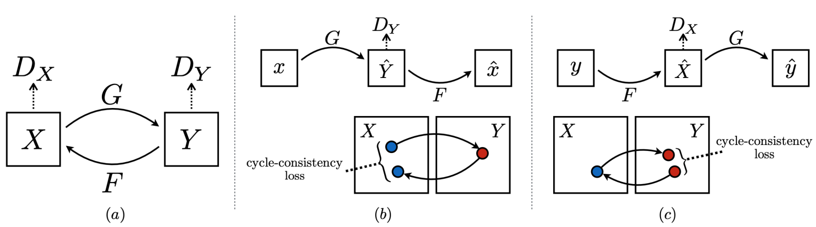

Figure 3: (a) Our model contains two mapping functions $G : X \to Y$ and $F : Y \to X$, and associated adversarial discriminators D_Y and D_X. D_Y encourages G to translate X into outputs indistinguishable from domain Y, and vice versa for D_X and F. To further regularize the mappings, we introduce two *cycle consistency losses* that capture the intuition that if we translate from one domain to the other and back again we should arrive at where we started: (b) forward cycle-consistency loss: $x \to G(x) \to F(G(x)) \approx x$, and (c) backward cycle-consistency loss: $y \to F(y) \to G(F(y)) \approx y$

Figure 5: cyclegan [5] training scheme.

References

[1] Olaf Ronneberger, Philipp Fischer, and Thomas Brox, "U-net: Convolutional networks for biomedical image segmentation," in *International Conference on Medical image computing and computer-assisted intervention.* Springer, 2015, pp. 234–241.

[2] Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio, "Generative adversarial nets," in *Advances in neural information processing systems*, 2014, pp. 2672–2680.

[3] Alec Radford, Luke Metz, and Soumith Chintala, "Unsupervised representation learning with deep convolutional generative adversarial networks," *arXiv preprint arXiv:1511.06434*, 2015.

[4] Phillip Isola, Jun-Yan Zhu, Tinghui Zhou, and Alexei A Efros, "Image-to-image translation with conditional adversarial networks," in *Proceedings of the IEEE conference on computer vision and pattern recognition*, 2017, pp. 1125–1134.

[5] Jun-Yan Zhu, Taesung Park, Phillip Isola, and Alexei A Efros, "Unpaired image-to-image translation using cycle-consistent adversarial networks," in *Proceedings of the IEEE international conference on computer vision*, 2017, pp. 2223–2232.