1

# FT-preperties

#### I. LINEARITY

# A. Reference

Linearity, wikipedia

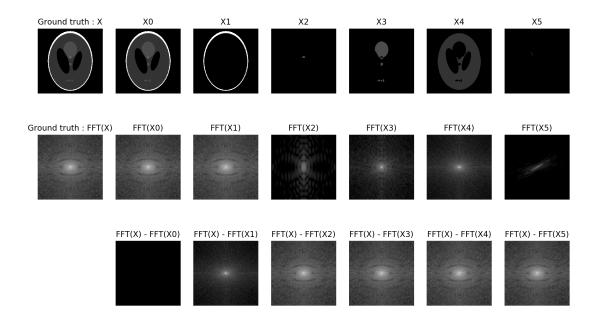
# B. Definition

For any complex numbers  $a \in \mathbb{C}$  and  $b \in \mathbb{C}$ ,

$$h(x) = a * f(x) + b * g(x) \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \hat{h}(\xi) = a \cdot \hat{f}(\xi) + b \cdot \hat{g}(\xi)$$

# C. Execution

Run demo\_fourier\_properties\_1\_linearity.m



#### II. SHIFT IN SPATIAL DOMAIN

# A. Reference

Shift in Spatial domain, wikipedia

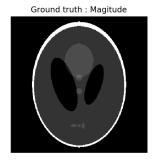
# B. Definition

For any real number  $x_0 \in \mathbb{R}$ ,

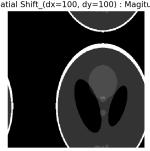
$$h(x) = f(x - x_0) \quad \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \quad \hat{h}(\xi) = e^{-2\pi i x_0 \xi} \hat{f}(\xi)$$

# C. Execution

Run demo\_fourier\_properties\_2\_shift\_i\_spatial\_domain.m



Spatial Shift\_(dx=100, dy=100) : Magitude

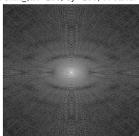








 $Spatial \ Shift\_(dx=100, \ dy=100): Phase \qquad Spatial \ Shift\_(dx=100, \ dy=100): Fourier \ domain$ 



#### III. SHIFT IN FOURIER DOMAIN

# A. Reference

Shift in Fourier domain, wikipedia

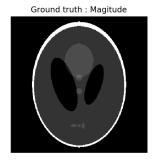
# B. Definition

For any real number  $\xi_0 \in \mathbb{R}$ ,

$$h(x) = e^{2\pi i x \xi_0} f(x) \quad \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \quad \hat{h}(\xi) = \hat{f}(\xi - \xi_0)$$

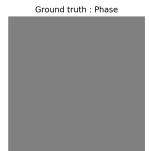
# C. Execution

 $Run\ demo\_fourier\_properties\_3\_shift\_i\_Fourier\_domain.m$ 



Spatial Shift\_(dx=100, dy=100) : Magitude

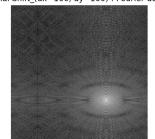




 $Spatial \ Shift\_(dx=100, \ dy=100): Phase \qquad Spatial \ Shift\_(dx=100, \ dy=100): Fourier \ domain$ 







#### IV. CONVOLUTION THEOREM

# A. Reference

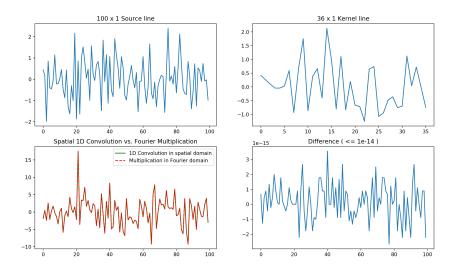
Convolution theorem, wikipedia

# B. Definition

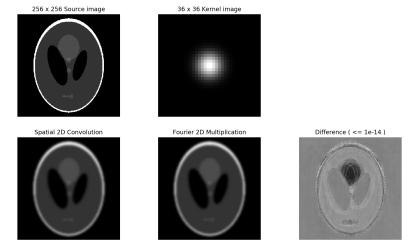
$$h(x) = (f * g)(x) = \int_{-\infty}^{\infty} f(y)g(x-y)dy \quad \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \quad \hat{h}(\xi) = \hat{f}(\xi) \cdot \hat{g}(\xi)$$

#### C. Execution

Run demo\_fourier\_properties\_4\_1d\_convolution\_vs\_multiplication.m Run demo\_fourier\_properties\_5\_2d\_convolution\_vs\_multiplication.m



(a) 1D example



(b) 2D example