FT-preperties

I. Linearity

A. Reference

Linearity, wikipedia

B. Definition

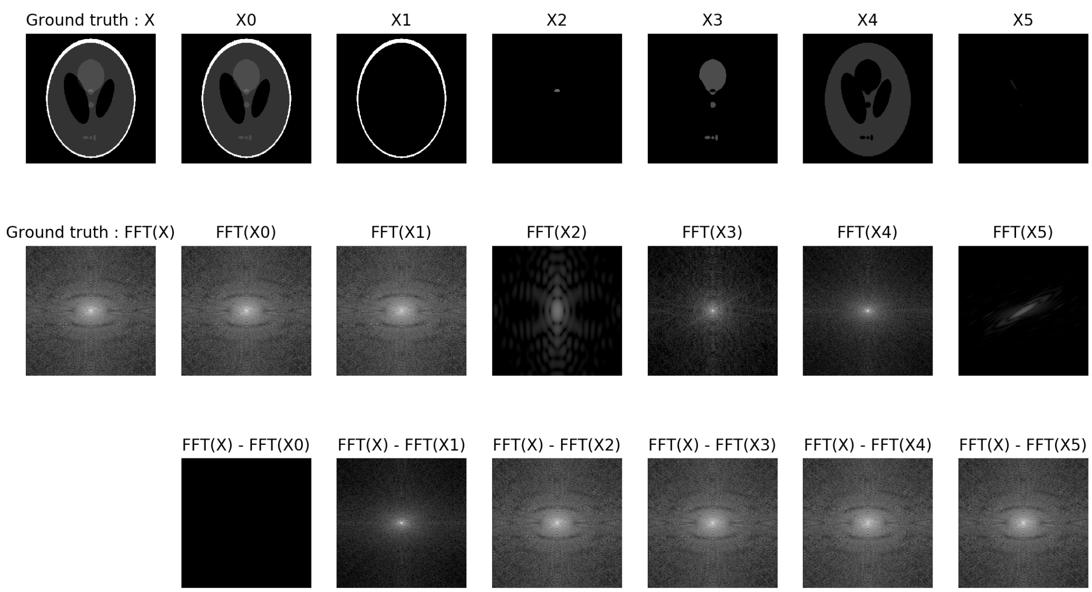
For any complex numbers $a \in \mathbb{C}$ and $b \in \mathbb{C}$,

$$h(x) = a * f(x) + b * g(x) \quad \leftarrow \mathcal{F} \text{ (Fourier transform) } \rightarrow \quad \hat{h}(\xi) = a \cdot \hat{f}(\xi) + b \cdot \hat{g}(\xi)$$

C. Execution

Run **demo_fourier_properties_1_linearity.py**

D. Results



II. Shift in Spatial domain

A. Reference

Shift in Spatial domain, wikipedia

B. Definition

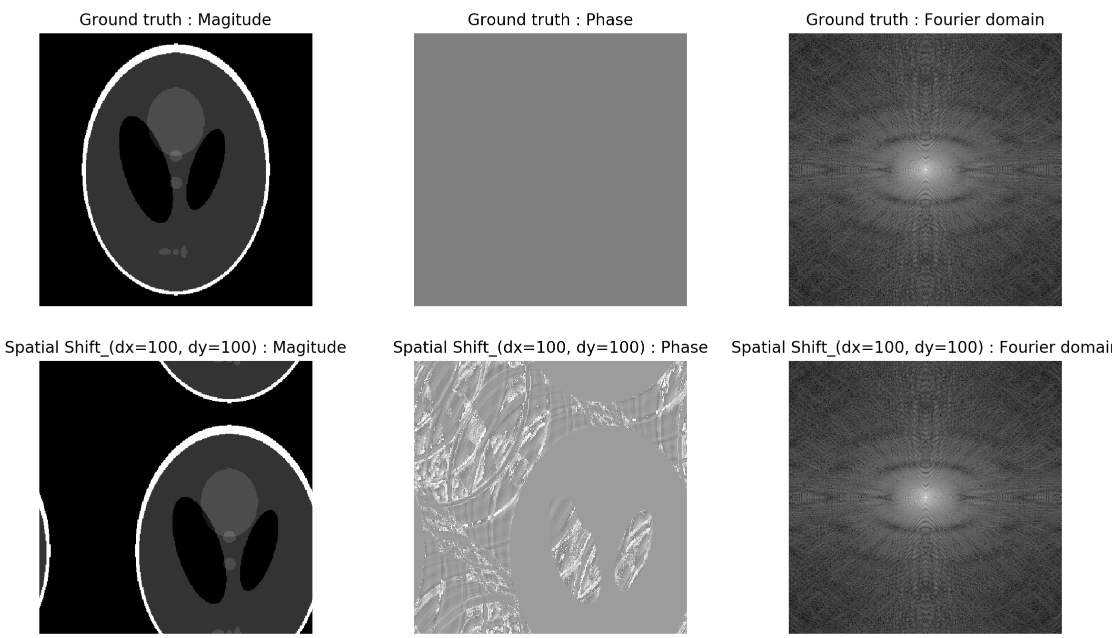
For any real number $x_0 \in \mathbb{R}$,

$$h(x) = f(x - x_0) \quad \leftarrow \mathcal{F} \text{ (Fourier transform) } \rightarrow \quad \hat{h}(\xi) = e^{-2\pi i x_0 \xi} \hat{f}(\xi)$$

C. Execution

Run **demo_fourier_properties_2_shift_i_spatial_domain.py**

D. Results



III. Shift in Fourier domain

A. Reference

Shift in Fourier domain, wikipedia

B. Definition

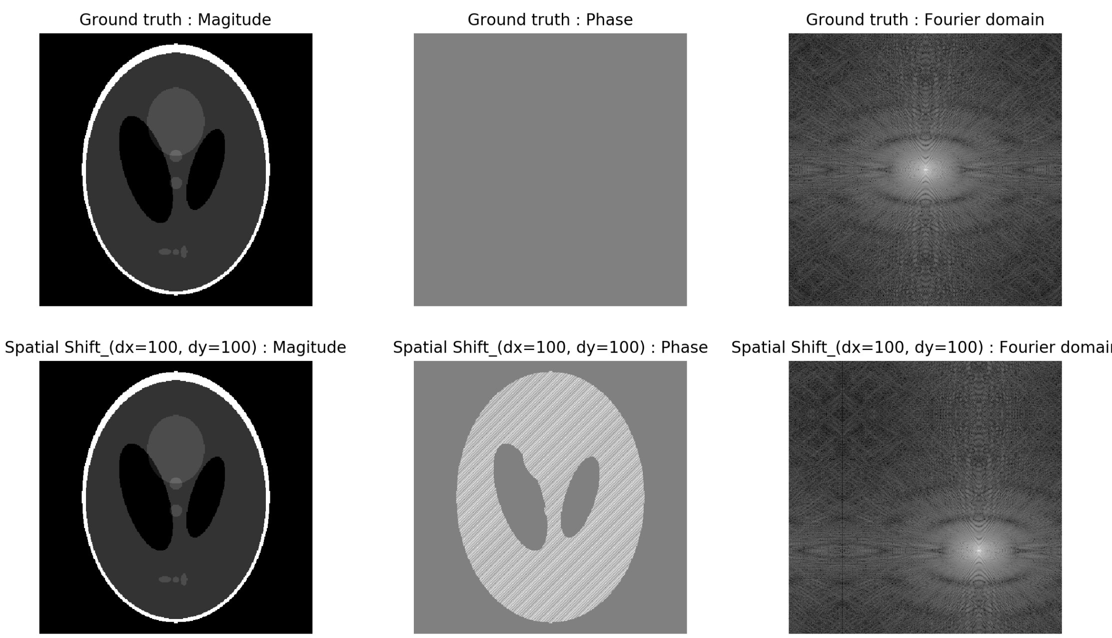
For any real number $\xi_0 \in \mathbb{R}$,

$$h(x) = e^{2\pi i x \xi_0} f(x) \quad \leftarrow \mathcal{F} \text{ (Fourier transform) } \rightarrow \quad \hat{h}(\xi) = \hat{f}(\xi - \xi_0)$$

C. Execution

Run **demo_fourier_properties_3_shift_i_Fourier_domain.py**

D. Results



IV. Convolution theorem

A. Reference

Convolution theorem, wikipedia

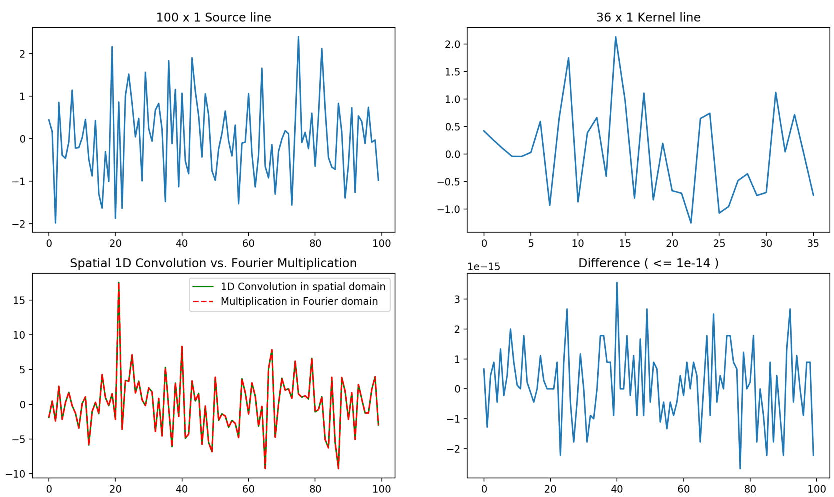
B. Definition

$$h(x) = (f * g)(x) = \int_{-\infty}^{\infty} f(y)g(x-y)dy \quad \leftarrow \mathcal{F} \text{ (Fourier transform)} \rightarrow \quad \hat{h}(\xi) = \hat{f}(\xi) \cdot \hat{g}(\xi)$$

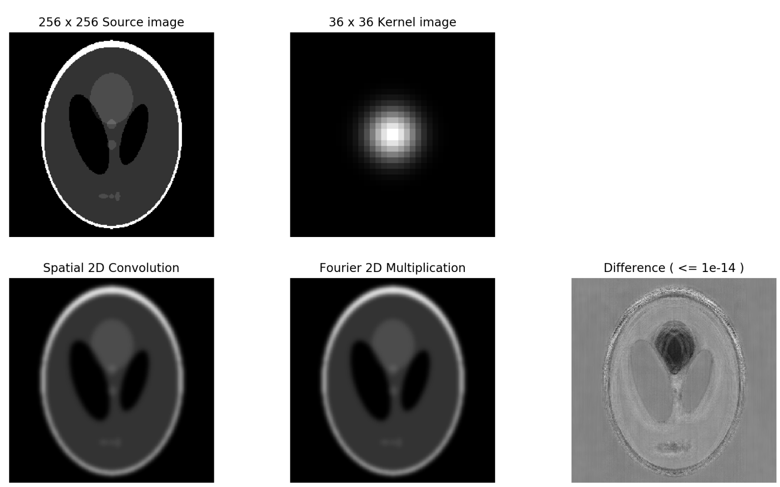
C. Execution

Run **demo_fourier_properties_4_1d_convolution_vs_multiplication.py**
Run **demo_fourier_properties_5_2d_convolution_vs_multiplication.py**

D. Results



(a) 1D example



(b) 2D example