# COMP9318: Data Warehousing and Data Mining

— L2: Data Warehousing and OLAP —

- Why and What are Data Warehouses?

# Data Analysis Problems

- The same data found in many different systems
  - Example: customer data across different departments
  - The same concept is defined differently
- Heterogeneous sources
  - Relational DBMS, OnLine Transaction Processing (OLTP)
  - Unstructured data in files (e.g., MS Excel) and documents (e.g., MS Word)

# Data Analysis Problems (Cont'd)

- Data is suited for operational systems
  - Accounting,billing,etc.
  - Do not support analysis across business functions
- Data quality is bad
  - Missing data, imprecise data, different use of systems
- Data are "volatile"
  - Data deleted in operational systems (6months)
  - Data change over time – no historical information

# Solution: Data Warehouse

- Defined in many different ways, but not rigorously.
  - A decision support database that is maintained separately from the organization's operational database
  - Support information processing by providing a solid platform of consolidated, historical data for analysis.
- "A data warehouse is a subject-oriented, integrated, time-variant, and nonvolatile collection of data in support of management's decision-making process."—W. H. Inmon
- Data warehousing:
  - The process of constructing and using data warehouses

# Data Warehouse—Subject-Oriented

- Organized around major subjects, such as customer, product, sales.
- Focusing on the modeling and analysis of data for decision makers, not on daily operations or transaction processing.
- Provide a simple and concise view around particular subject issues by excluding data that are not useful in the decision support process.

# Data Warehouse—Integrated

- Constructed by integrating multiple, heterogeneous data sources
  - relational databases, flat files, on-line transaction records
- Data cleaning and data integration techniques are applied.
  - Ensure consistency in naming conventions, encoding structures, attribute measures, etc. among different data sources
    - E.g., Hotel price: currency, tax, breakfast covered, etc.
  - When data is moved to the warehouse, it is converted.

# Data Warehouse—Time Variant

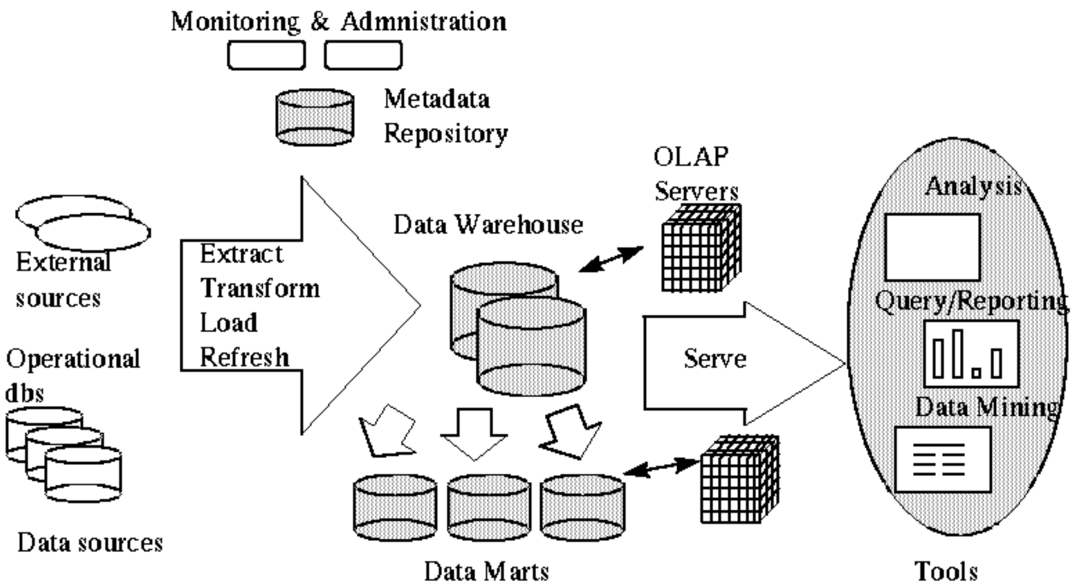
- The time horizon for the data warehouse is significantly longer than that of operational systems.
  - Operational database: current value data.
  - Data warehouse data: provide information from a historical perspective (e.g., past 5-10 years)
- Every key structure in the data warehouse
  - Contains an element of time, explicitly or implicitly
  - But the key of operational data may or may not contain "time element".

# Data Warehouse—Non-Volatile

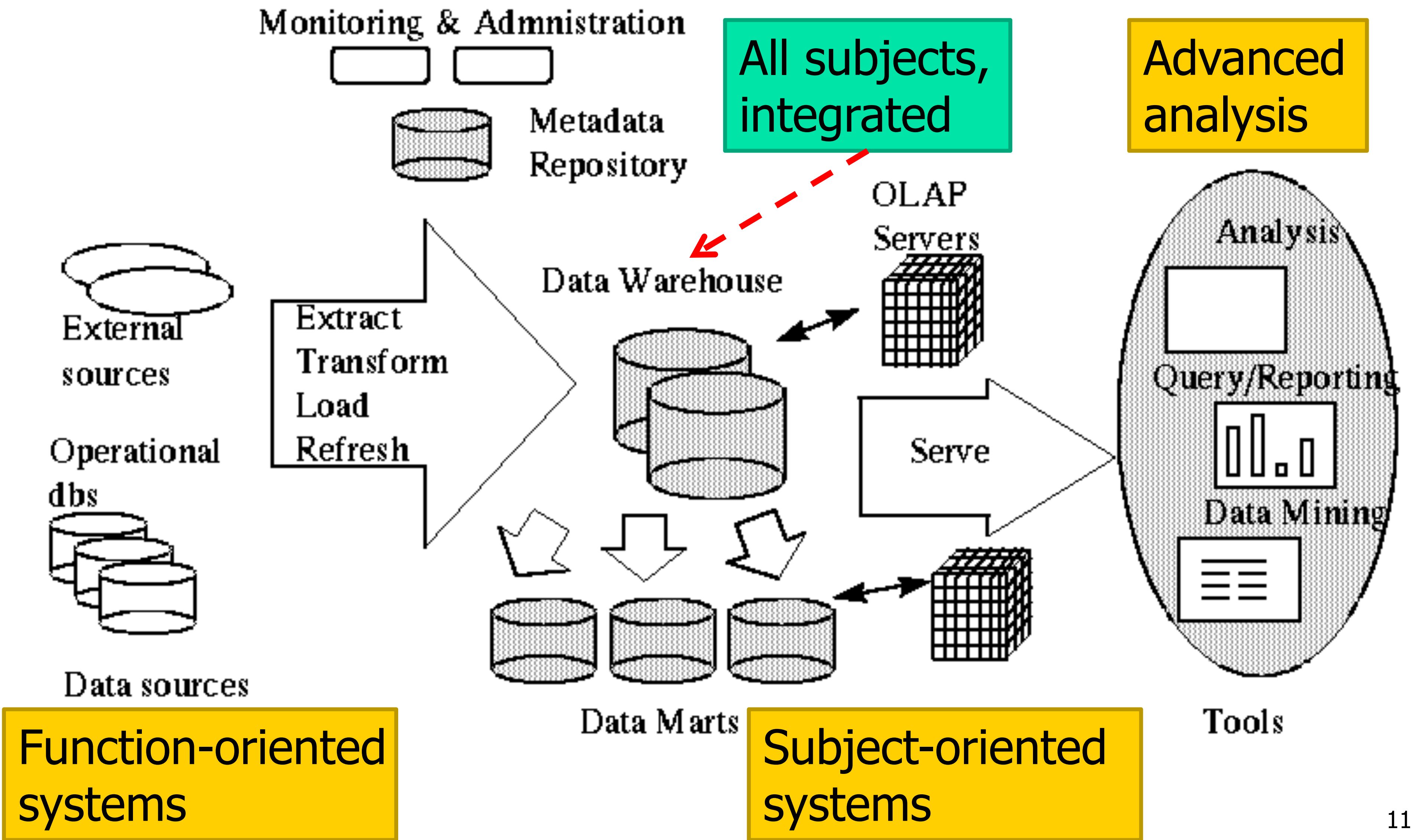
1. A physically separate store of data transformed from the operational environment.
2. Operational update of data does not occur in the data warehouse environment.
   - Does not require transaction processing, recovery, and concurrency control mechanisms
   - Requires only two operations in data accessing:
     - *initial loading of data* and *access of data*.

# Data Warehouse Architecture

- Extract data from operational data sources
  - clean, transform
- Bulk load/refresh
  - warehouse is offline
- OLAP-server provides multidimensional view
- Multidimensional-olap (Essbase, oracle express)
- Relational-olap (Redbrick, Informix, Sybase, SQL server)



# Data Warehouse Architecture



# Why Separate Data Warehouse?

- High performance for both systems
  - DBMS— tuned for OLTP: access methods, indexing, concurrency control, recovery
  - Warehouse—tuned for OLAP: complex OLAP queries, multidimensional view, consolidation.
- Different functions and different data:
  - missing data: Decision support requires historical data which operational DBs do not typically maintain
  - data consolidation: DS requires consolidation (aggregation, summarization) of data from heterogeneous sources
  - data quality: different sources typically use inconsistent data representations, codes and formats which have to be reconciled

# Why OLAP Servers?

- Different workload:
  - OLTP (on-line transaction processing)
    - Major task of traditional relational DBMS
    - Day-to-day operations: purchasing, inventory, banking, manufacturing, payroll, registration, accounting, etc.
  - OLAP (on-line analytical processing)
    - Major task of data warehouse system
    - Data analysis and decision making
- Queries hard/infeasible for OLTP, e.g.,
  - Which **week** we have the largest sales?
  - Does the sales of **dairy products** increase over time?
  - Generate a **spread sheet** of total sales by state and by year.
- Difficult to represent these queries by using SQL ← Why?

# OLTP vs. OLAP

| | **OLTP** | **OLAP** |
|---|---|---|
| **users** | clerk, IT professional | knowledge worker |
| **function** | day to day operations | decision support |
| **DB design** | application-oriented | subject-oriented |
| **data** | current, up-to-date<br>detailed, flat relational<br>isolated | historical,<br>summarized, multidimensional<br>integrated, consolidated |
| **usage** | repetitive | ad-hoc |
| **access** | read/write<br>index/hash on prim. key | lots of scans |
| **unit of work** | short, simple transaction | complex query |
| **# records accessed** | tens | millions |
| **#users** | thousands | hundreds |
| **DB size** | 100MB-GB | 100GB-TB |
| **metric** | transaction throughput | query throughput, response |

# Comparisons

| | Databases | Data Warehouses |
|---|---|---|
| Purpose | Many purposes; Flexible and general | One purpose: Data analysis |
| Conceptual Model | ER | Multidimensional |
| Logical Model | (Normalized) Relational Model | (Denormalized) Star schema / Data cube/cuboids |
| Physical Model | Relational Tables | ROLAP: Relational tables<br>MOLAP: Multidimensional arrays |
| Query Language | SQL (hard for analytical queries) | MDX (easier for analytical queries) |
| Query Processing | B+-tree/hash indexes, Multiple join optimization, Materialized views | Bitmap/Join indexes, Star join, Materialized data cube |

- The Multidimensional Model

# The Multidimensional Model

- A data warehouse is based on a multidimensional data model which views data in the form of a data cube, which is a multidimensional generalization of 2D spread sheet.
- Key concepts:
  - **Facts**: the subject it models
    - Typically transactions in this course; other types includes snapshots, etc.
    - Measures: numbers that can be aggregated
    - Dimensions: context of the measure
  - Hierarchies:
    - Provide contexts of different granularities (aka. grains)

提供不同
的粒度

- Goals for dimensional modeling:
  - Surround facts with as much relevant context (dimensions) as possible ← Why?

# Supermarket Example

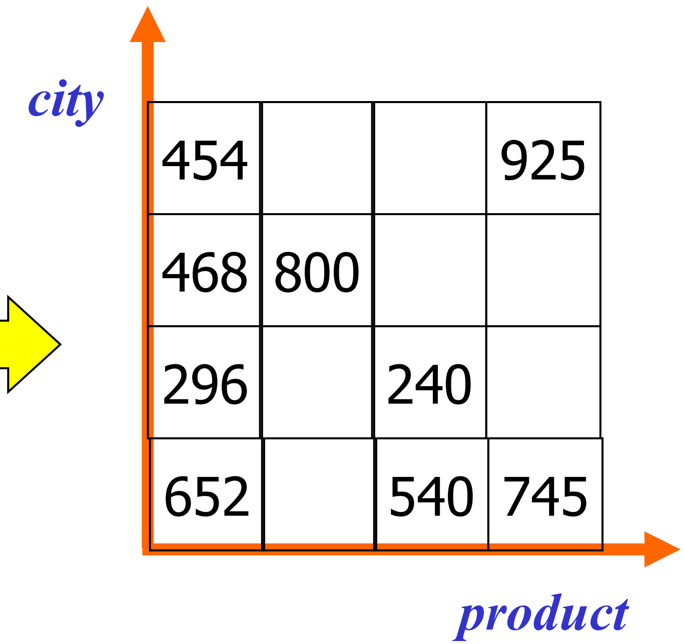
- Subject: analyze total sales and profits
- Fact: Each Sales **Transaction**
  - Measure: Dollars_Sold, Amount_Sold, Cost
  - Calculated Measure: Profit
- Dimensions:
  - Store
  - Product
  - Time

# Visualizing the Cubes

- A valid instance of the model is a data cube

使用聚合函数可以把p1到p4相加获得粒度grain更大的数据

| total Sales | | product | | | |
|---|---|---|---|---|---|
| | | p1 | p2 | p3 | p4 |
| city | NY | $454 | - | - | $925 |
| | LA | $468 | $800 | - | - |
| | SD | $296 | - | $240 | - |
| | SF | $652 | - | $540 | $745 |



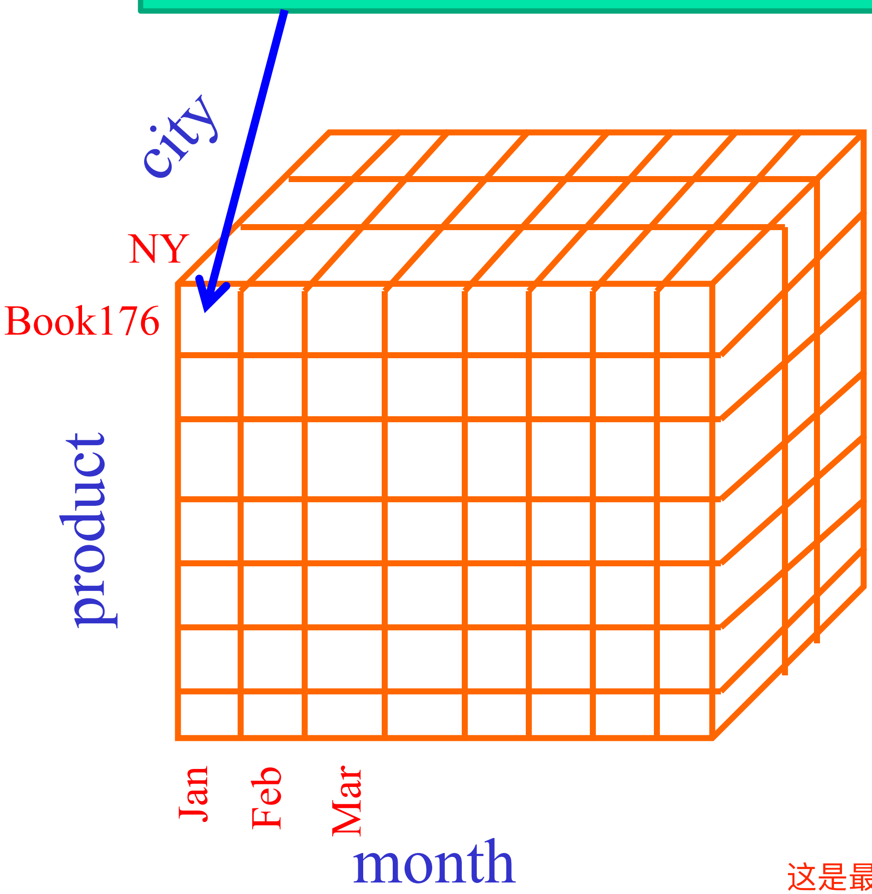

**Concepts**: cell, fact (=non-empty cell), measure, dimensions

Q: How to generalize it to 3D?

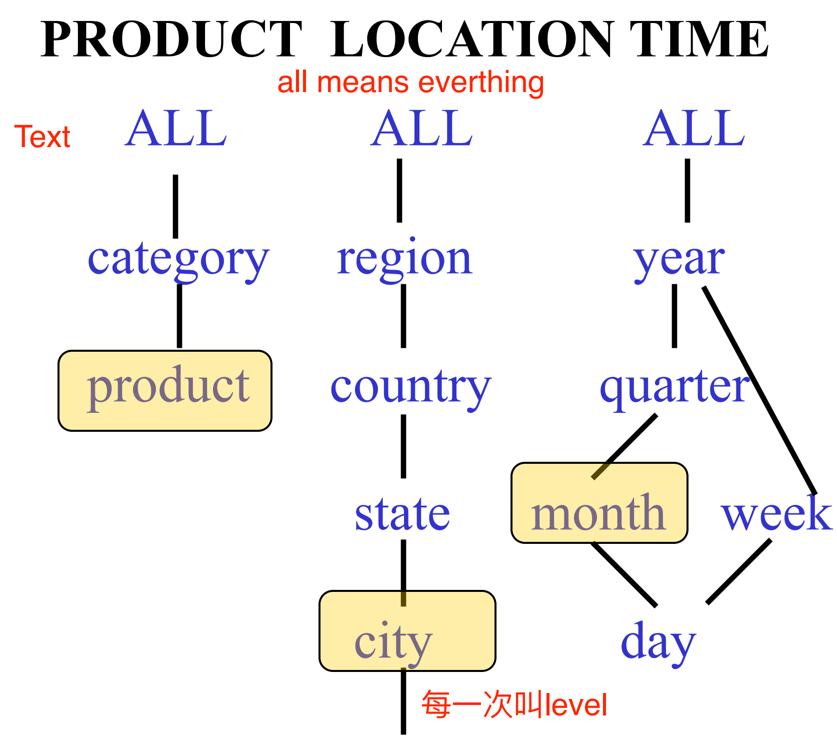
# 3D Cube and Hierarchies

**Concepts**: hierarchy (a tree of dimension values), level

Sales of book176 in NY in Jan can be found in this cell



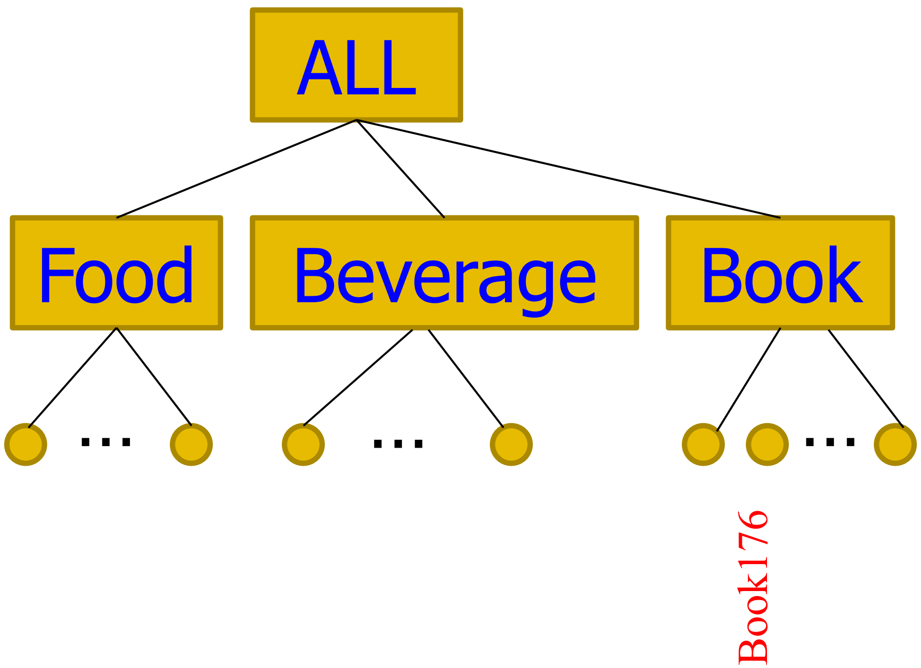

**DIMENSIONS**



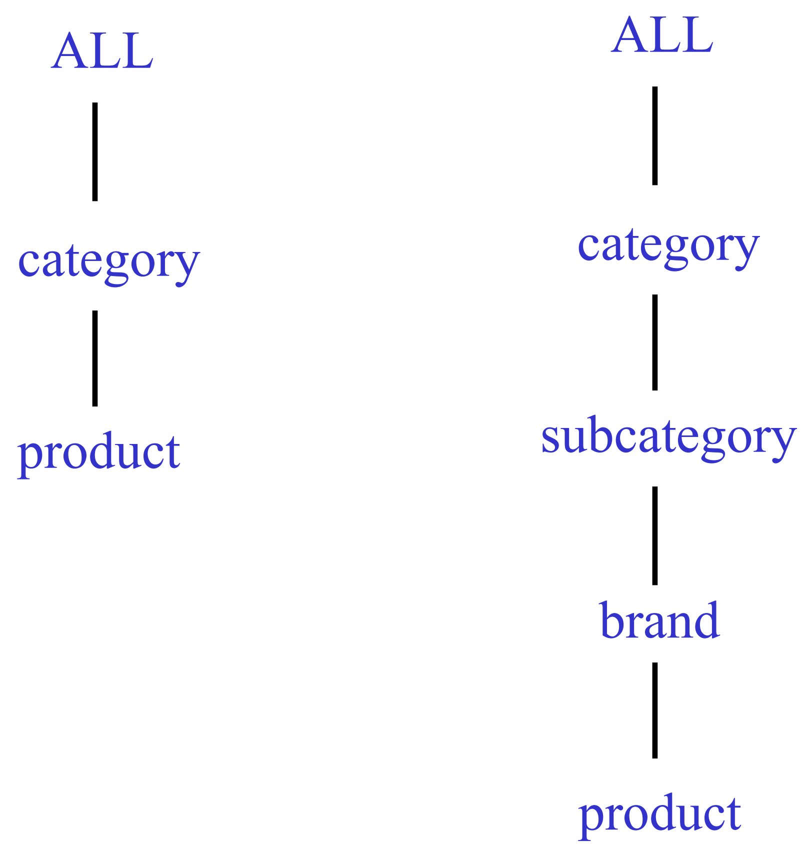

这是最细的粒度在这个数据
结构中fine granularity

# Hierarchies

**Concepts**: hierarchy (a tree of dimension values), level

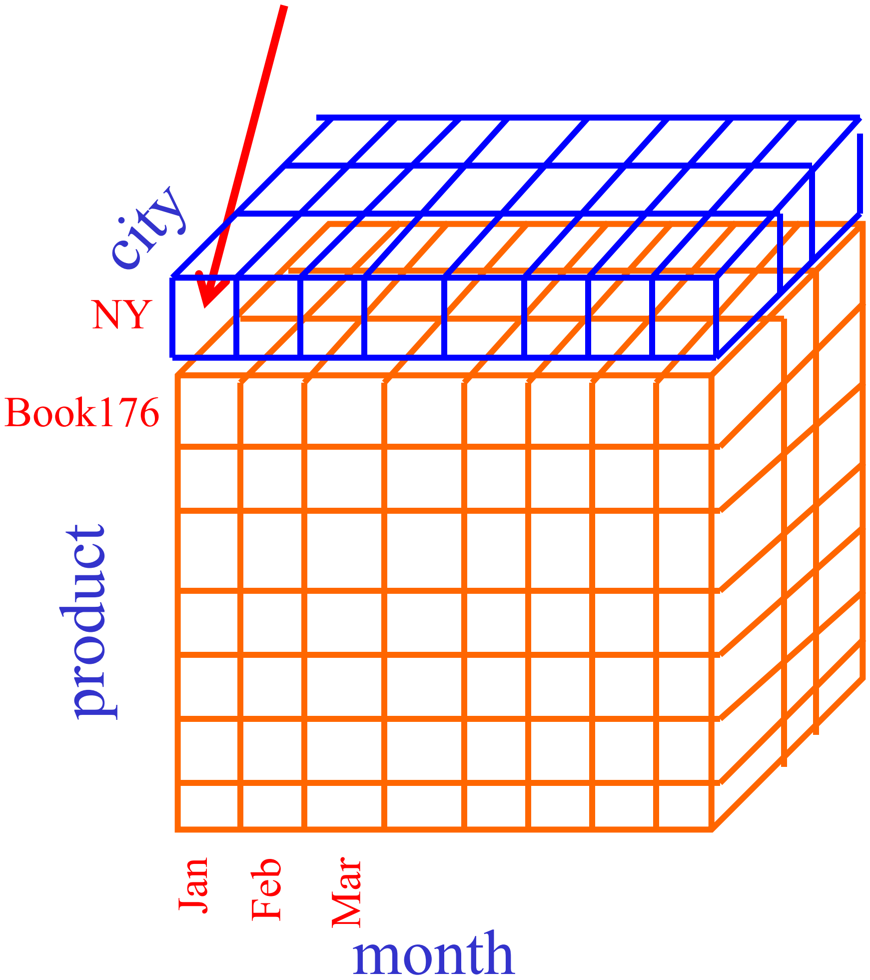


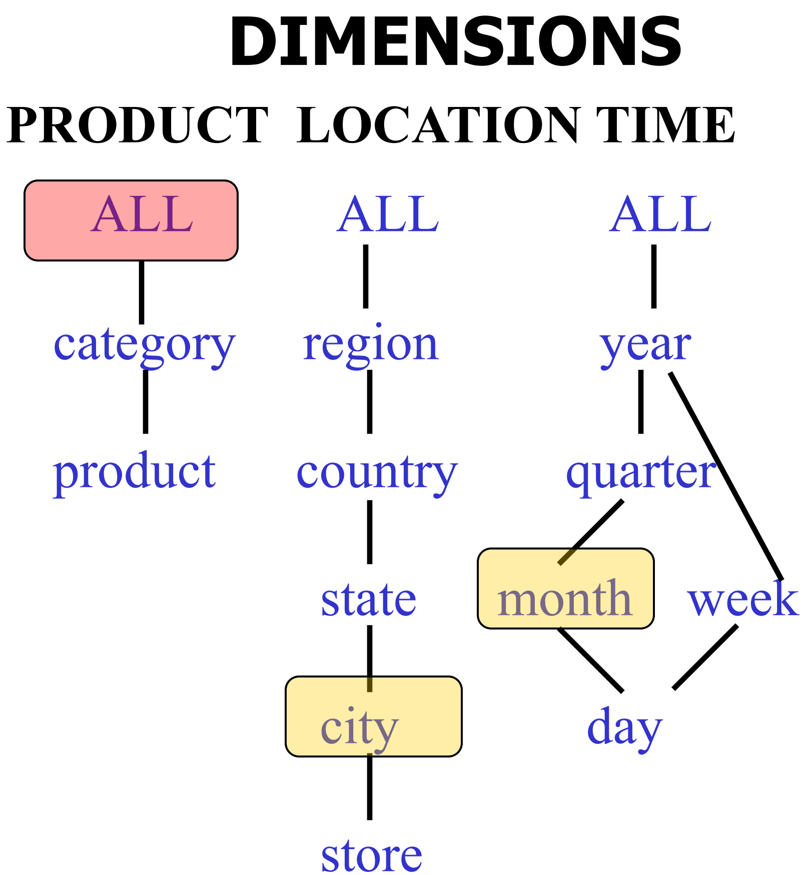

Which design is better? Why?



# The (city, moth) Cuboid

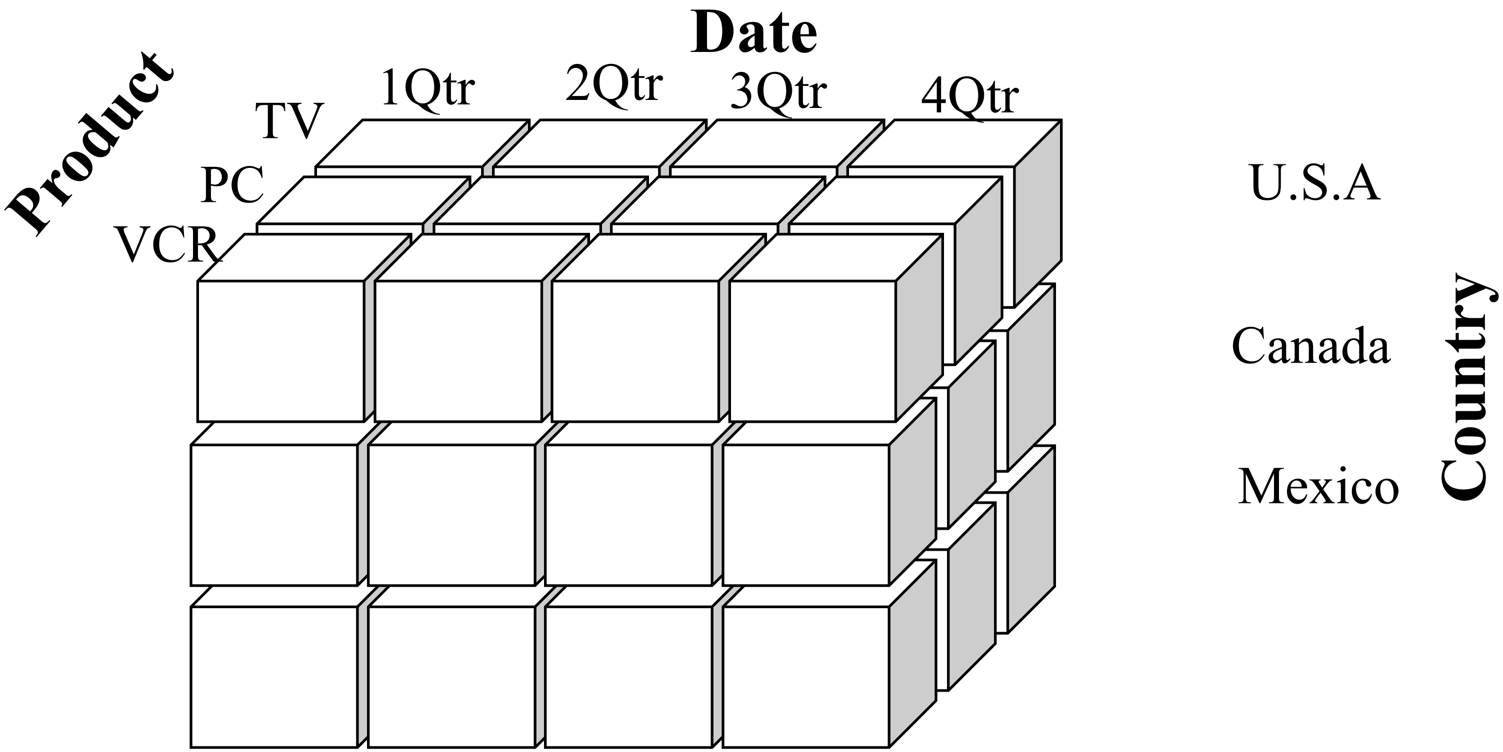
Sales of ALL_PROD in NY in Jan



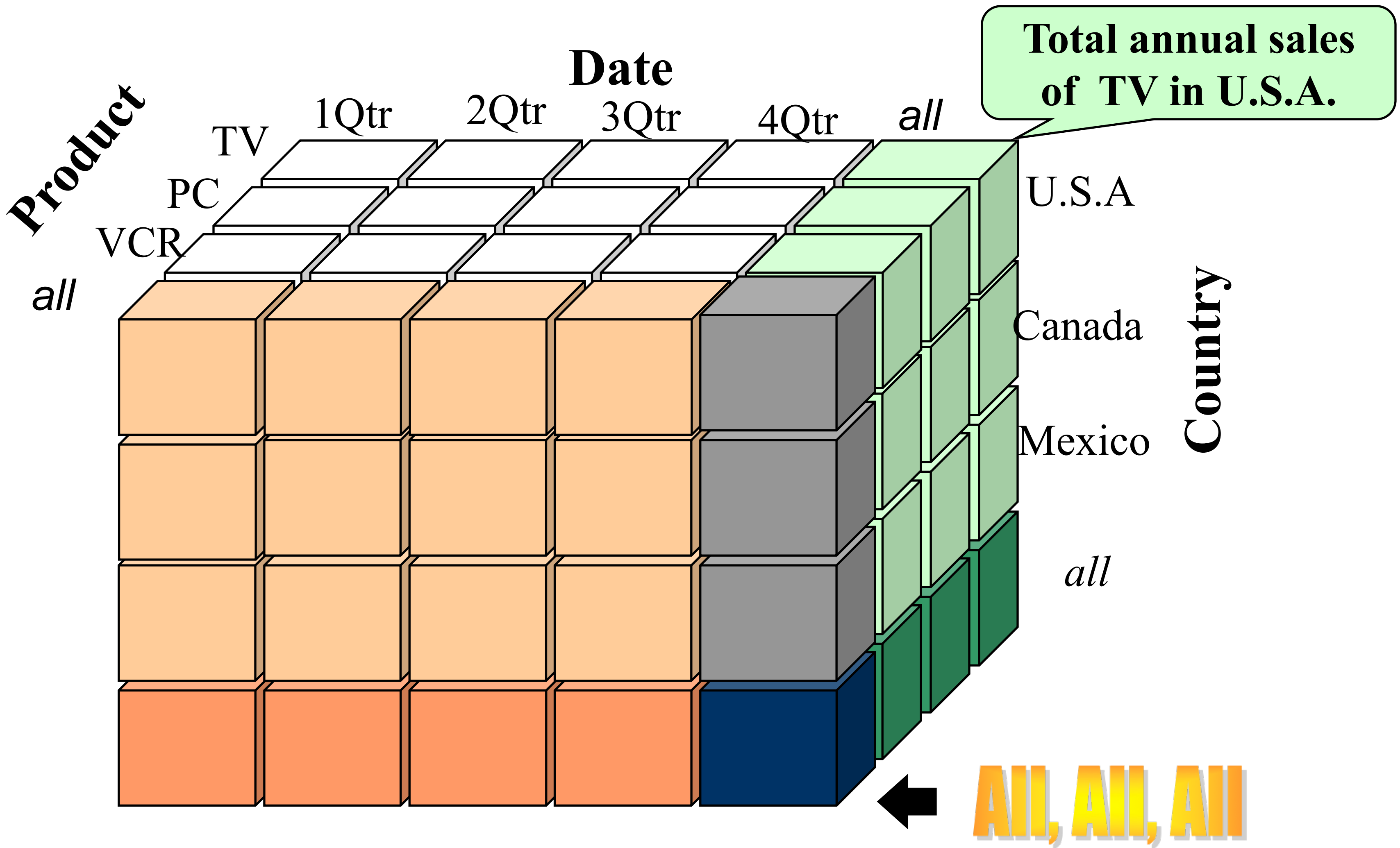
# All the Cuboids

Assume: no other non-ALL levels on all dimensions.

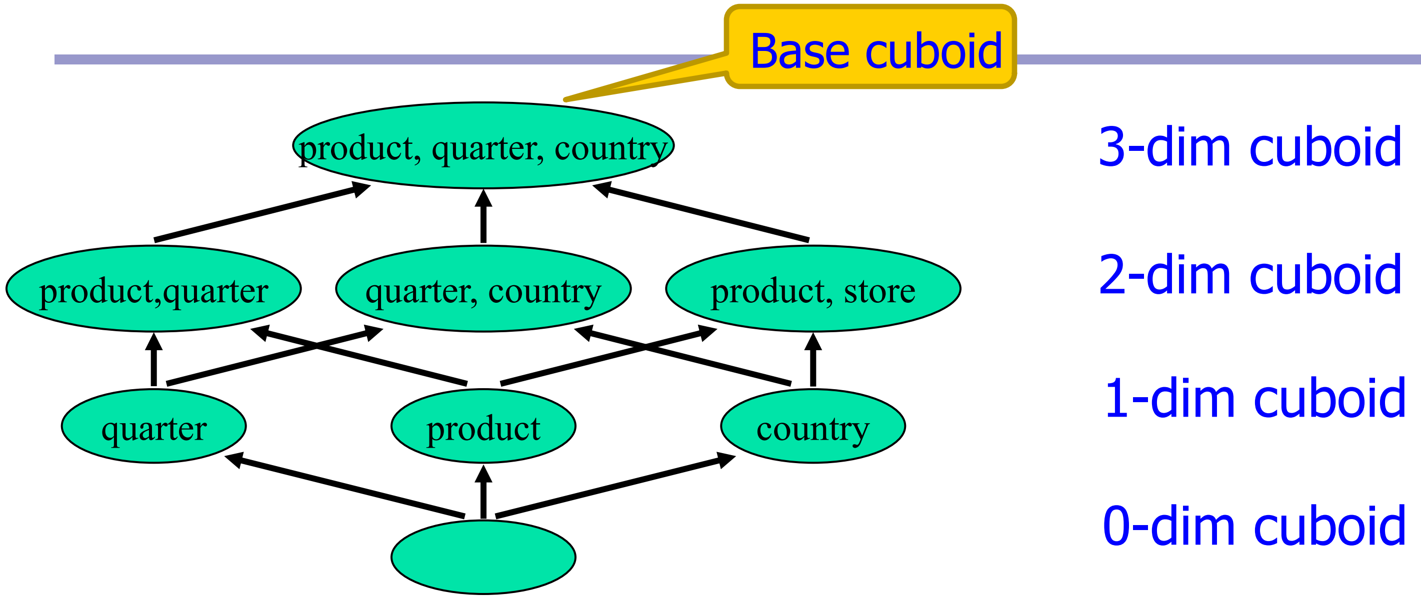


# All the Cuboids /2

Assume: no other non-ALL levels on all dimensions.



# Lattice of the cuboids




- n-dim cube can be represented as ($D_1$, $D_2$, …, $D_d$), where $D_i$ is the set of allowed values on the i-th dimension.
  - if $D_i$ = $L_i$ (a particular level), then Di = all descendant dimension values of $L_i$.
  - ALL can be omitted and hence reduces the effective dimensionality
- A complete cube of d-dimensions consists of $\prod_{i=1}^{d}(n_i + 1)$ cuboids, where $n_i$ is the number of levels (excluding ALL) on i-th dimension.
  - They collectively form a lattice.

# Properties of Operations

- All operations are closed under the multidimensional model
  - i.e., both input and output of an operation is a cube
- So that they can be composed

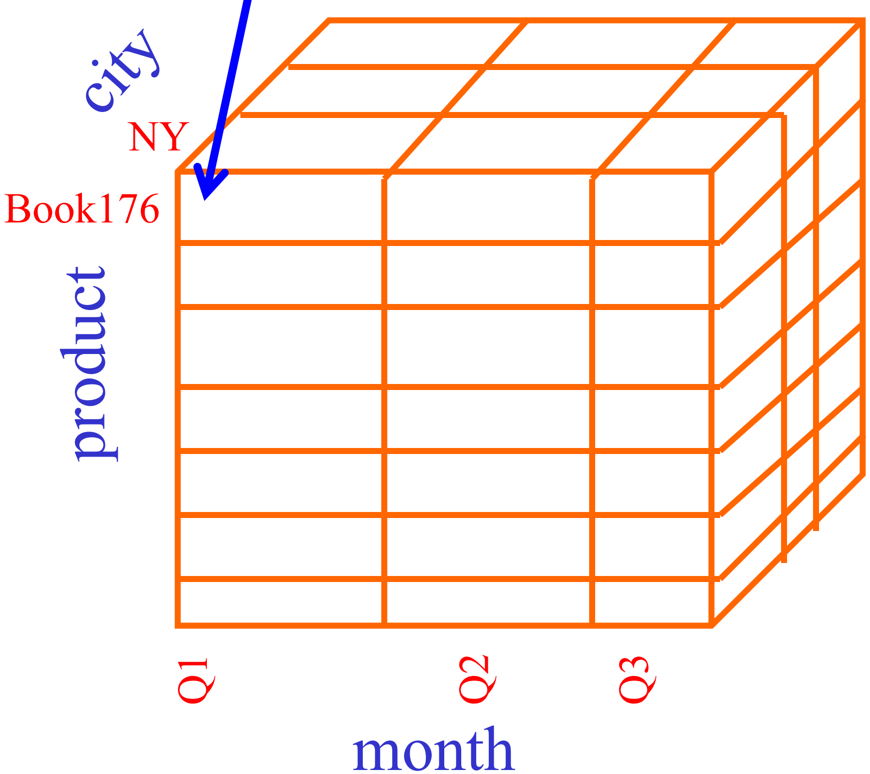
Q: What's the analogy in the Relational Model?

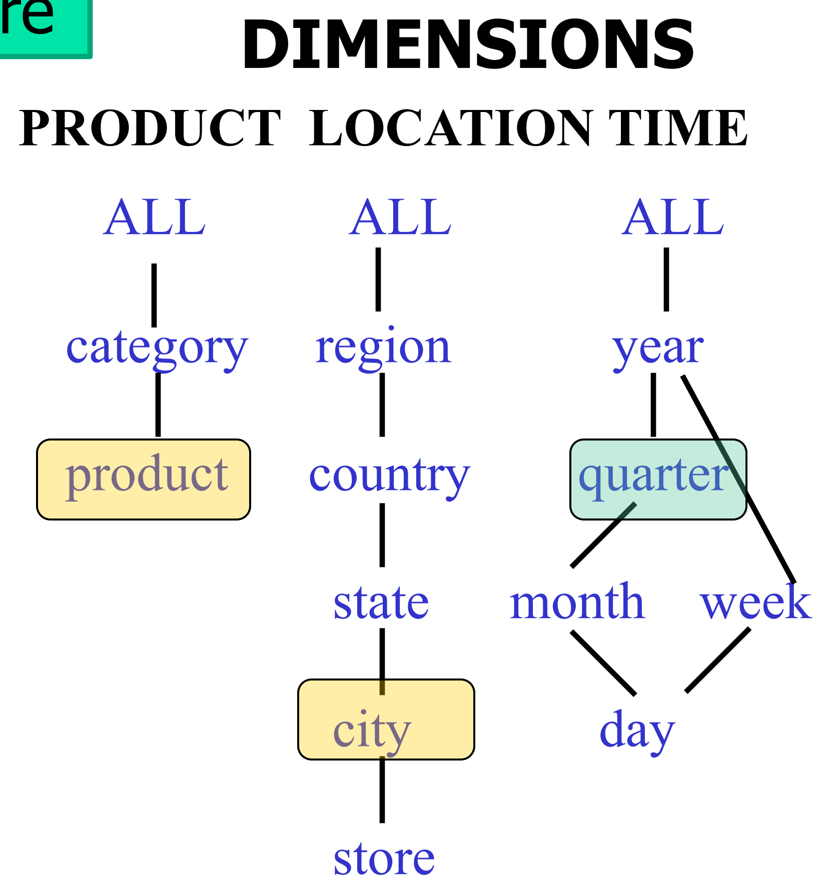
# Common OLAP Operations

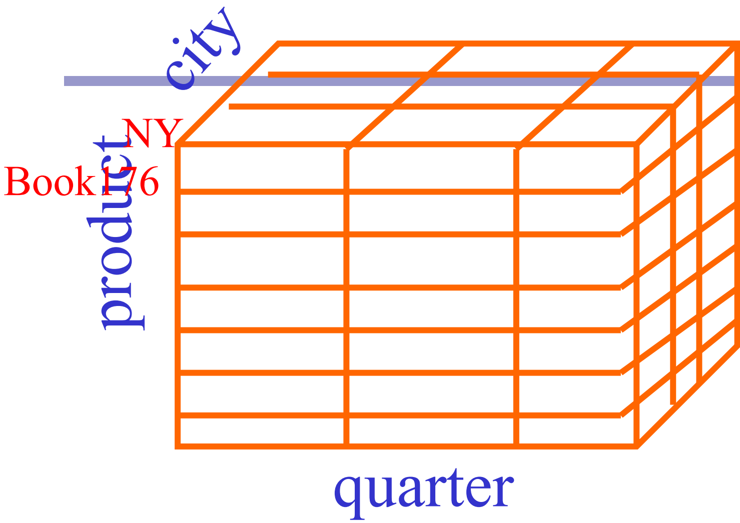
- **<u>Roll-up</u>**: move up the hierarchy

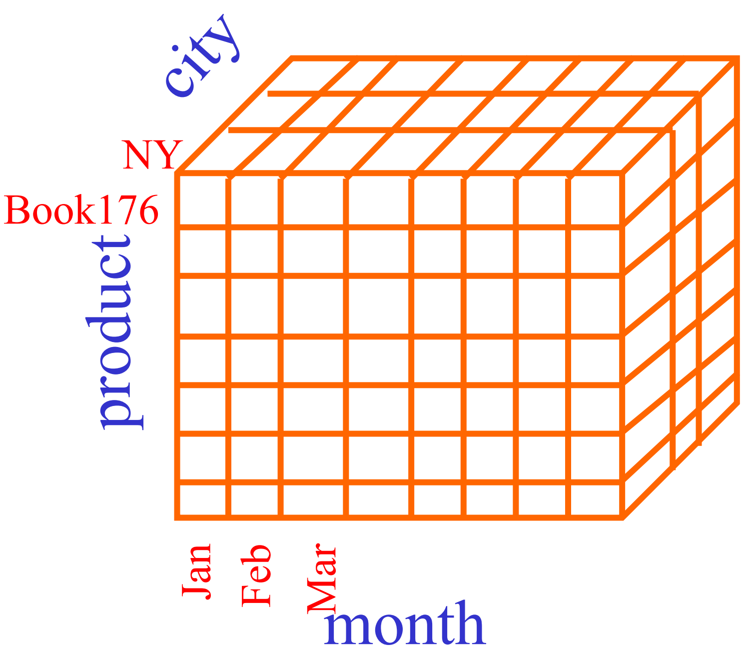
Q: what should be its value?

Sales of book176 in NY in Q1 here

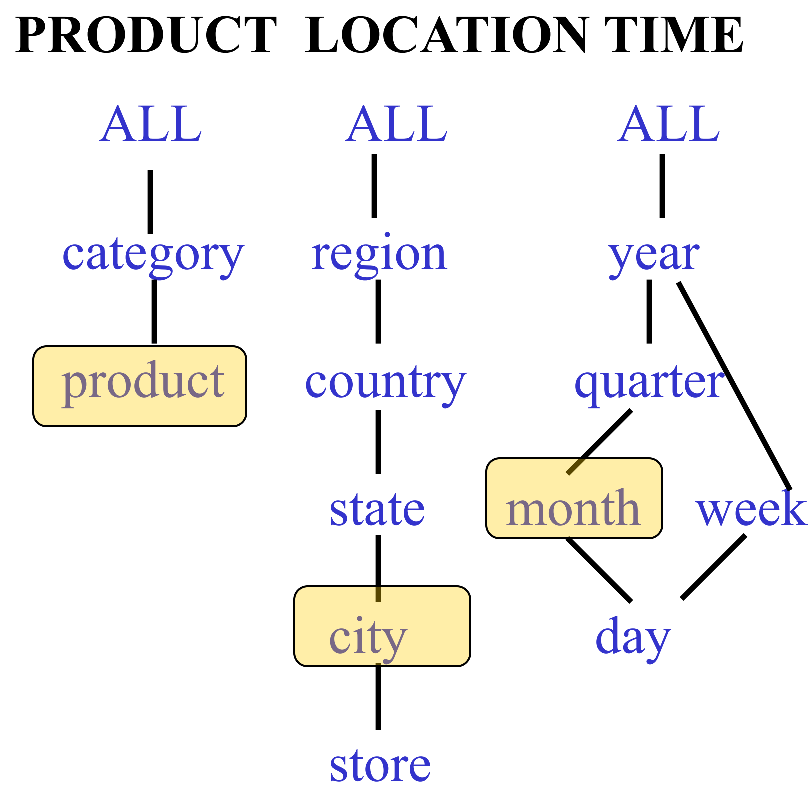



city
NY
Book176
product
quarter


city
NY
Book176
product
Jan
Feb
Mar
month

# DIMENSIONS

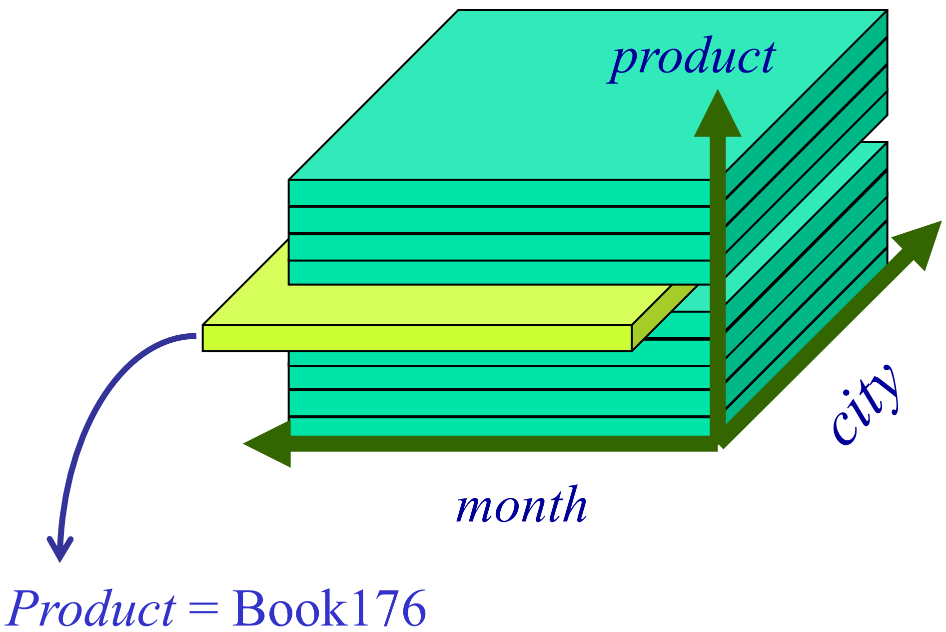

PRODUCT LOCATION TIME
ALL
ALL
ALL
category
region
year
product
country
quarter
state
month
week
city
day
store

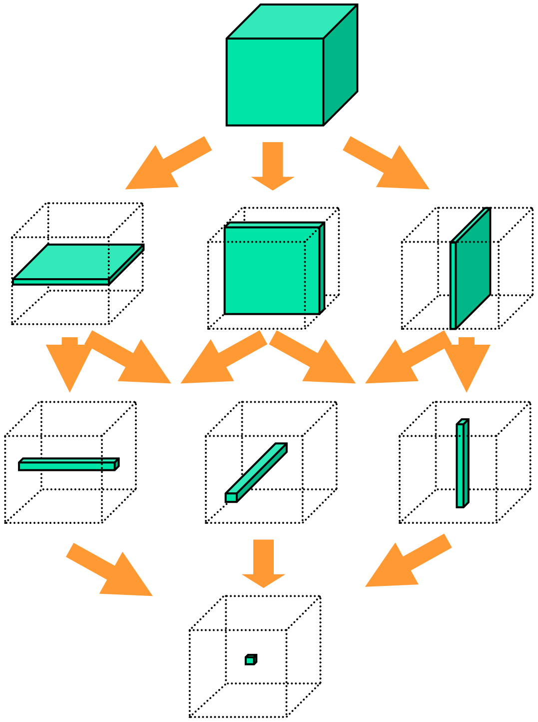
# Common OLAP Operations

- **Drill-down:** move down the hierarchy
  - more fine-grained aggregation

# Slice and Dice Queries

- Slice and Dice: select and project on one or more dimension values

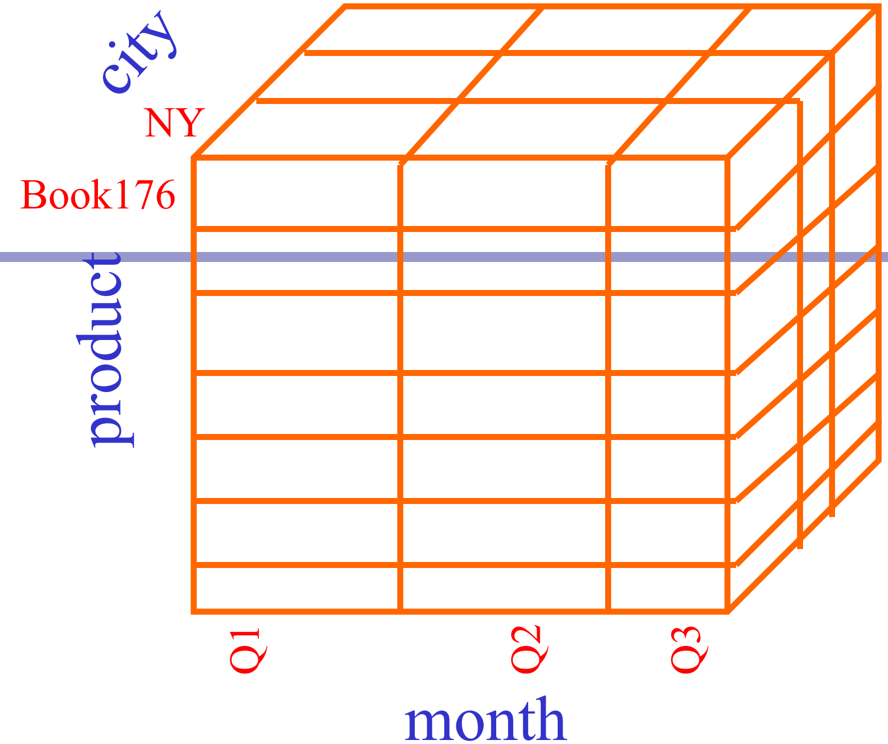


The output cube has smaller dimensionality than the input cube

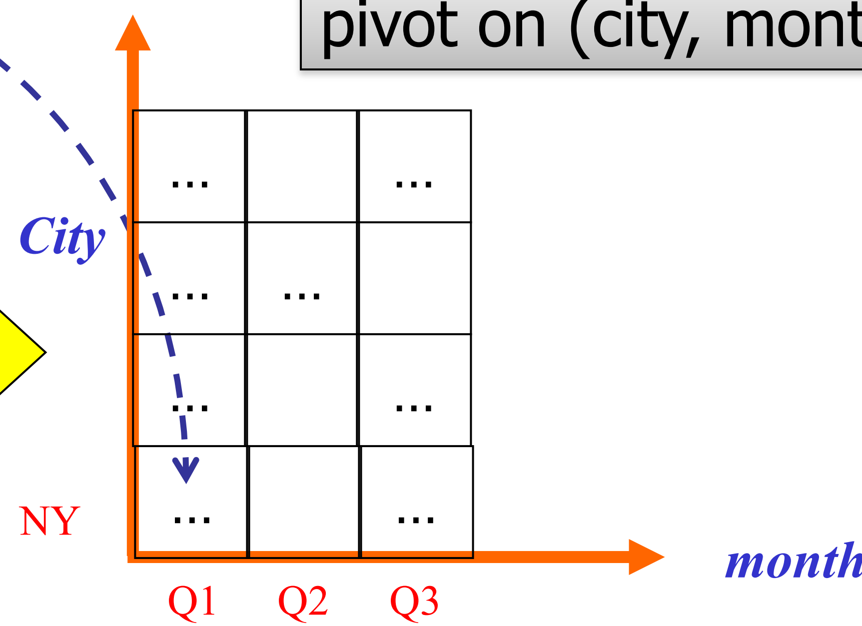
# Pivoting

- <u>Pivoting</u>: aggregate on selected dimensions
  - usually 2 dims (cross-tabulation)




Sales (of all products) in NY in Q1
=sum( ??? )

pivot on (city, month)

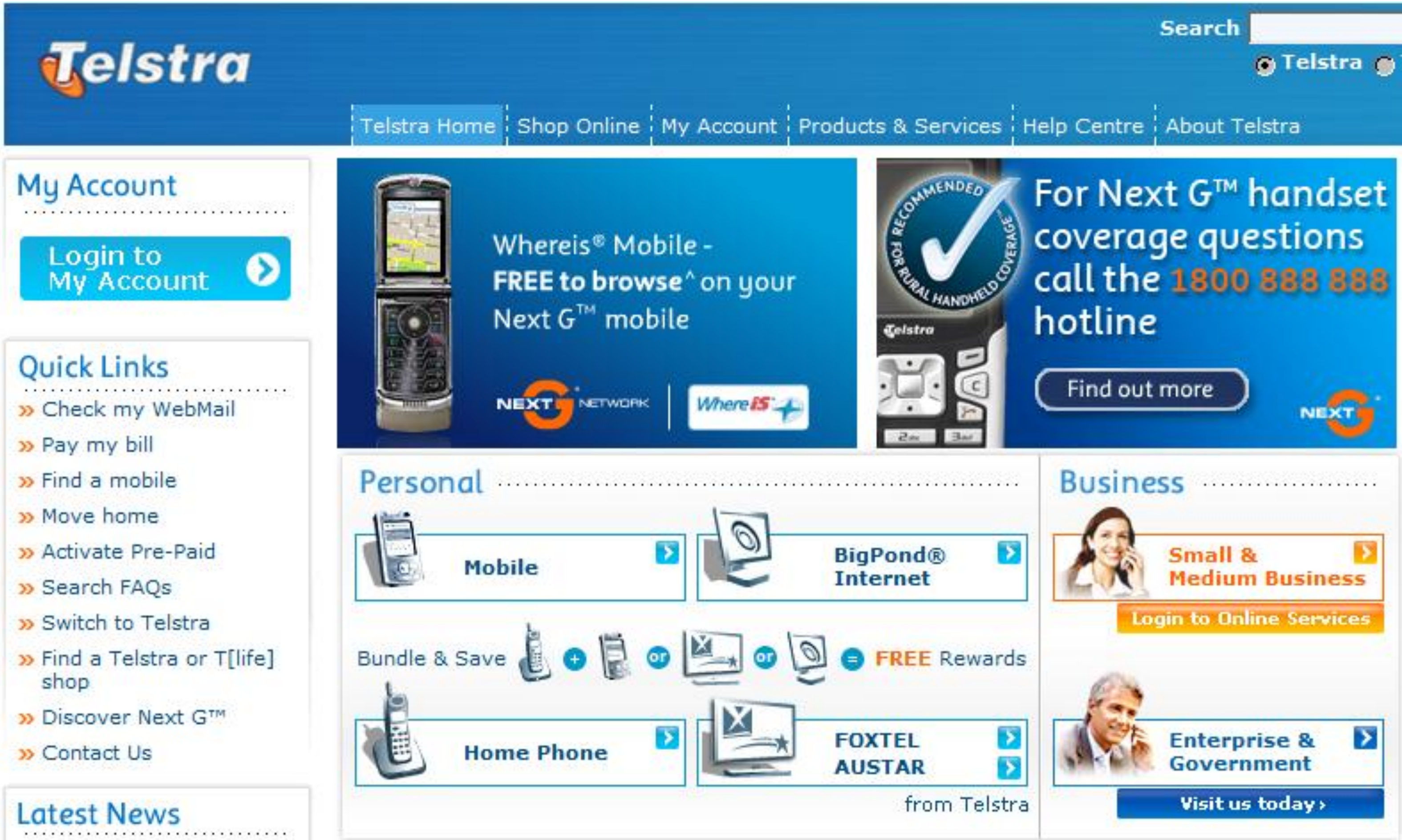


# A Reflective Pause

- Let’s review the definition of data cubes again.

- **Key message**:
  - Disentangle the “object” from its “representation” or “implementation”

# Modeling Exercise 1: Monthly Phone Service Billing




Theme: analyze the income/revenue of Telstra

# Solution

- FACT
- MEASURE
- DIMENSIONS

- The Logical Model

# Logical Models

- Two main approaches:
  - Using relational DB technology:
    - Star schema, Snowflake schema, Fact constellation
  - Using multidimensional technology:
    - Just as multidimensional data cube

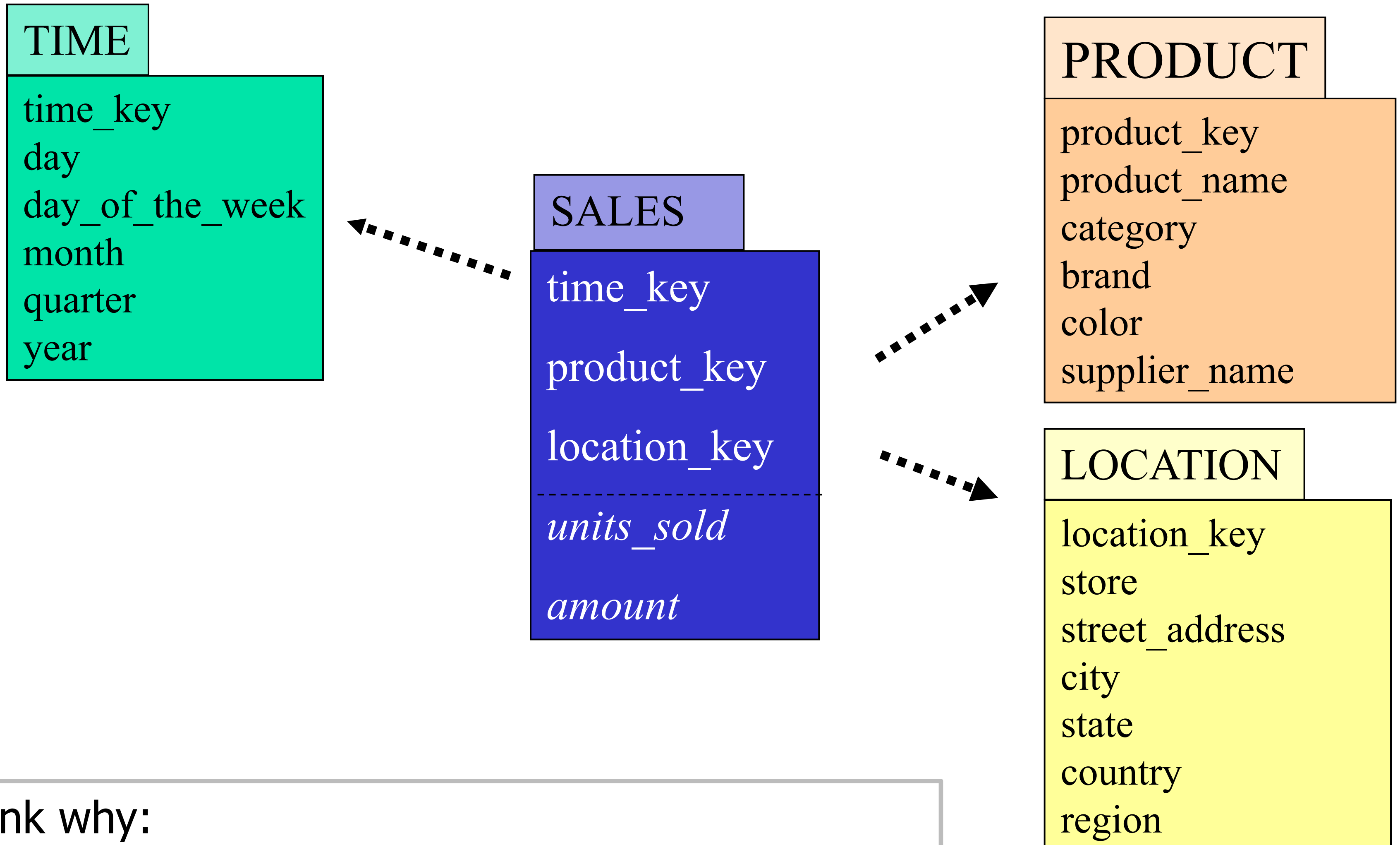
# Universal Schema → Star Schema

- Many data warehouses adopt a star schema to represent the multidimensional model
- Each dimension is represented by a dimension-table
  - LOCATION (location_key, store, street_address, city, state, country, region)
  - dimension tables are not normalized
- Transactions are described through a fact-table
  - each tuple consists of a pointer to each of the dimension-tables (foreign-key) and a list of measures (e.g. sales $$$)

The universal schema for supermarket

| Store | City | State | Prod | Brand | Category | $Sold | #Sold | Cost |
|---|---|---|---|---|---|---|---|---|
| S136 | Syd | NSW | 76Ha | Nestle | Biscuit | 40 | 10 | 18 |
| S173 | Melb | Vic | 76Ha | Nestle | Biscuit | 20 | 5 | 11 |

# The Star Schema

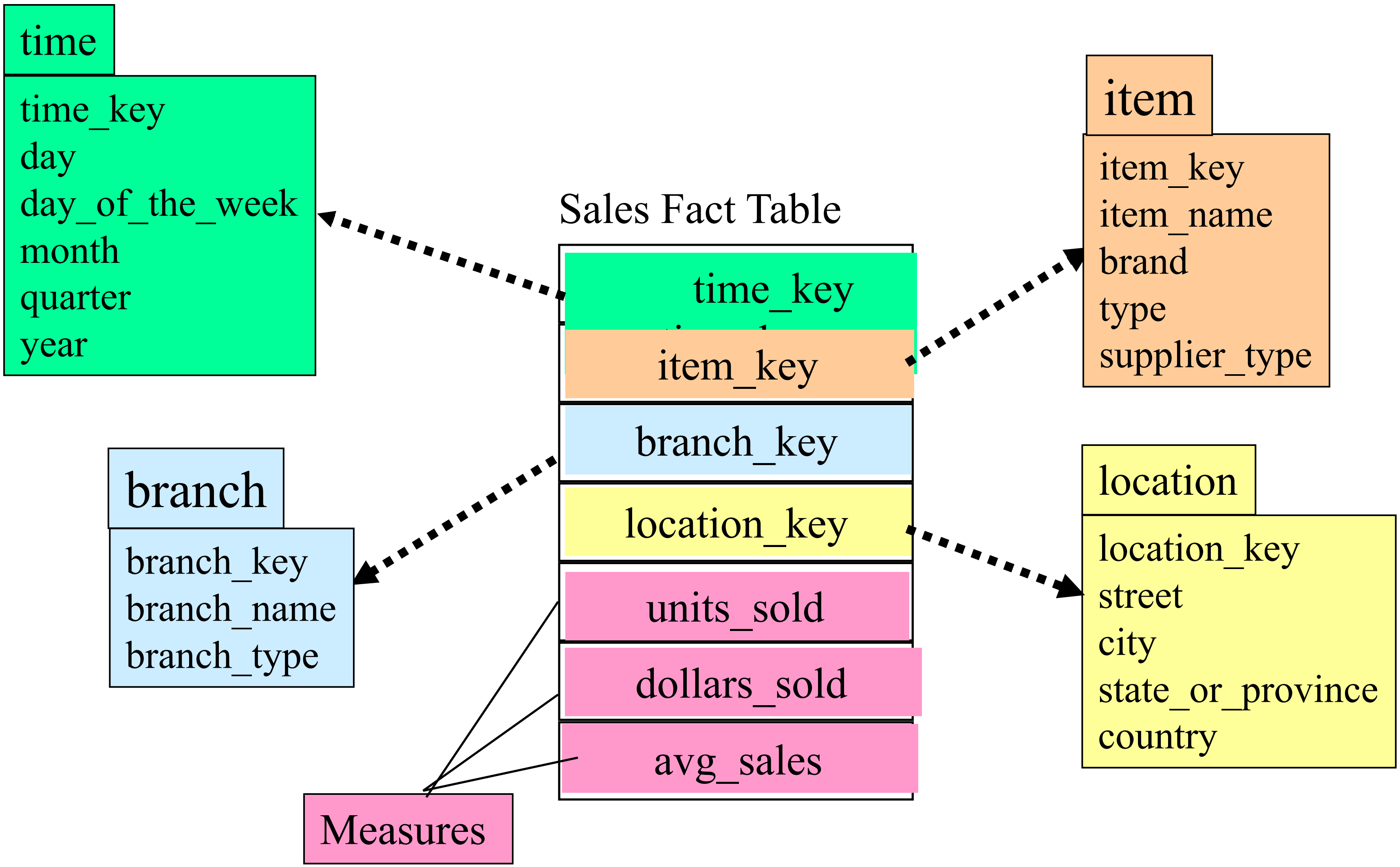



Think why:
(1) Denormalized once from the universal schema
(2) Controlled redundancy

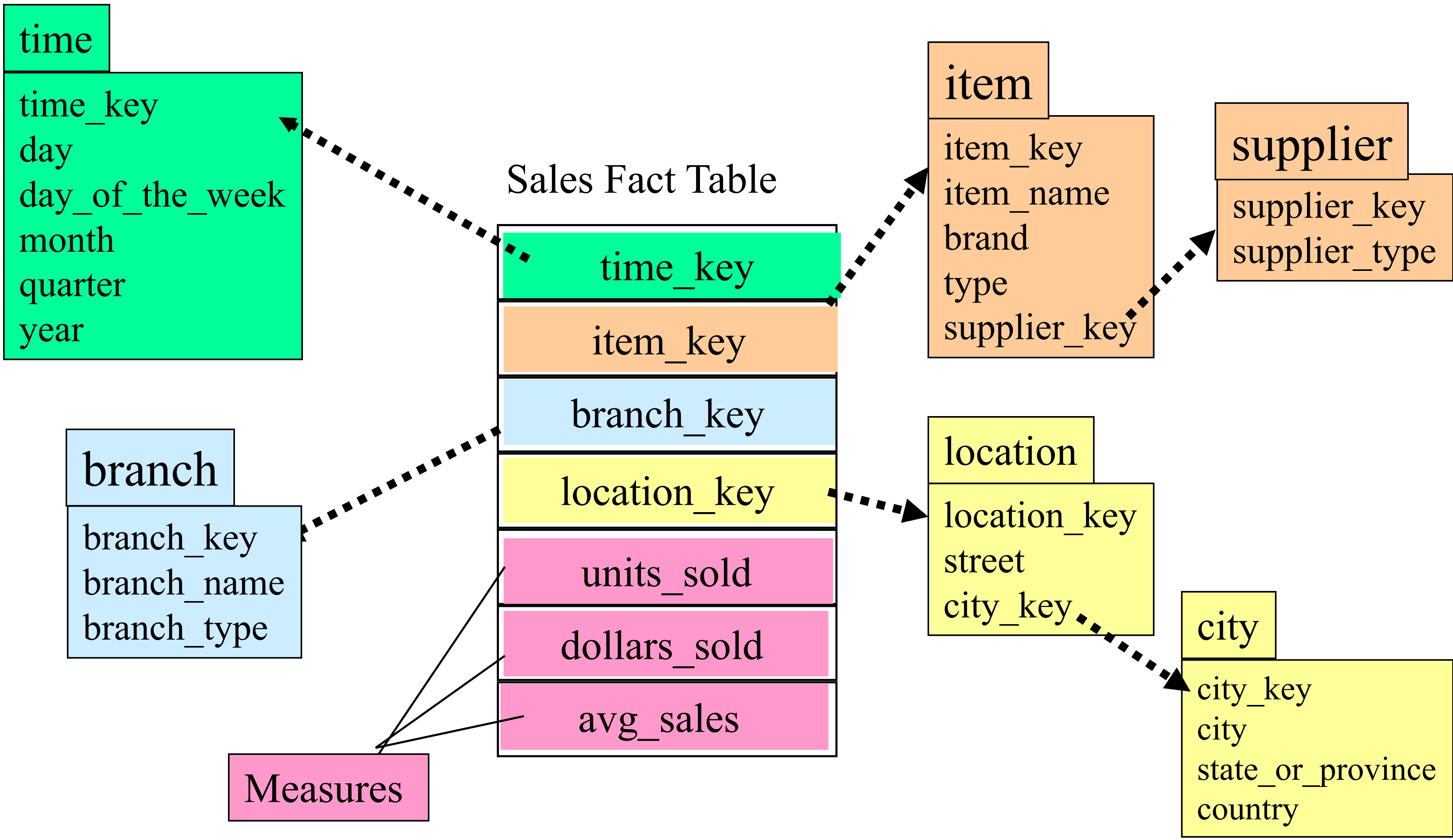
# Typical Models for Data Warehouses

- Modeling data warehouses: dimensions & measures
  - Star schema: A fact table in the middle connected to a set of dimension tables
  - Snowflake schema: A refinement of star schema where some dimensional hierarchy is normalized into a set of smaller dimension tables, forming a shape similar to snowflake
  - Fact constellations: Multiple fact tables share dimension tables, viewed as a collection of stars, therefore called galaxy schema or fact constellation

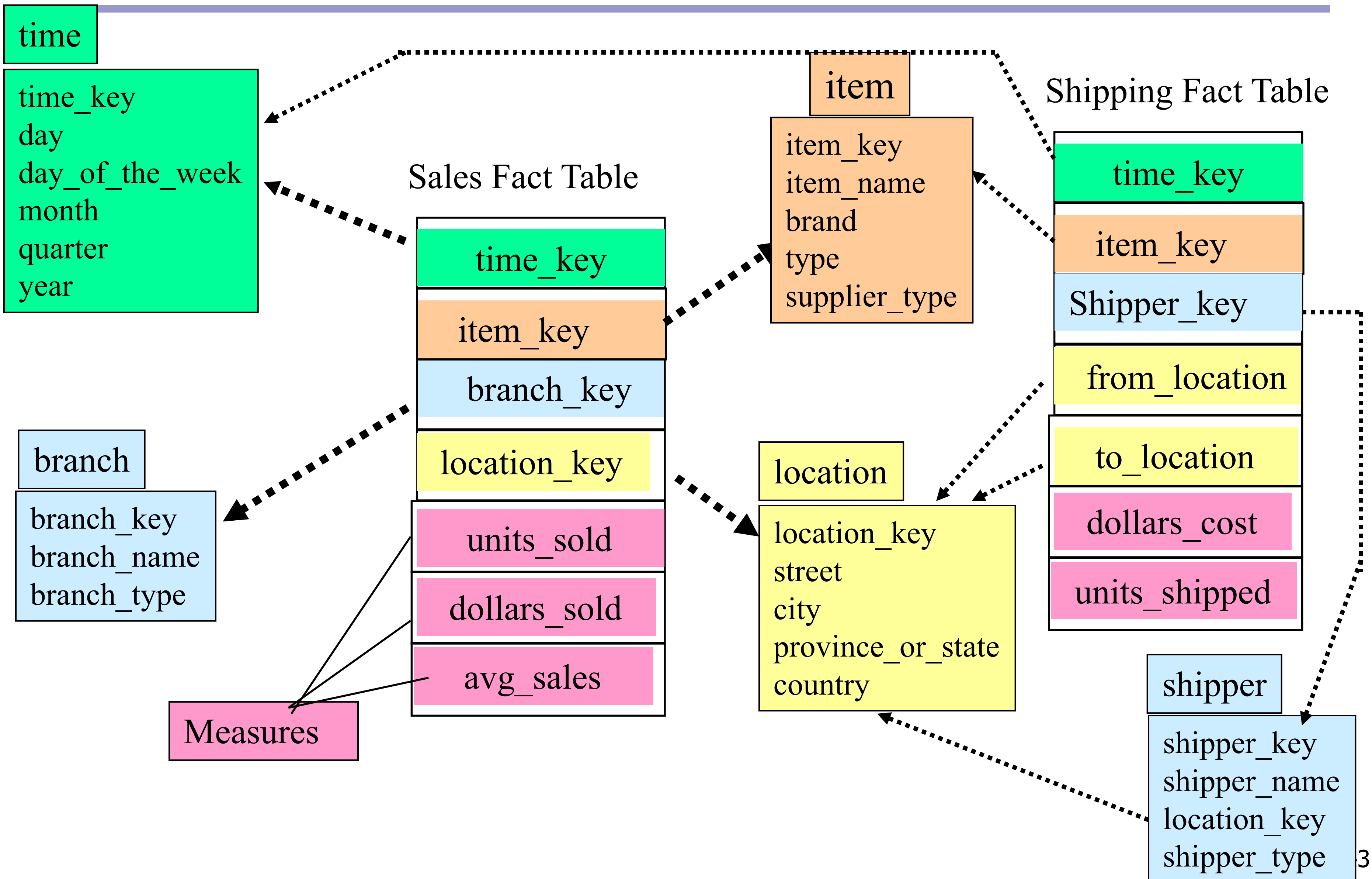
# Example of Star Schema



# Example of Snowflake Schema



# Example of Fact Constellation



# Advantages of Star Schema

- Facts and dimensions are clearly depicted
  - dimension tables are relatively static, data is loaded (append mostly) into fact table(s)
  - easy to comprehend (and write queries)

*"Find total sales per product-category in our stores in Europe"*

```
SELECT PRODUCT.category, SUM(SALES.amount)
FROM     SALES, PRODUCT,LOCATION
WHERE    SALES.product_key = PRODUCT.product_key
AND      SALES.location_key = LOCATION.location_key
AND      LOCATION.region="Europe"
GROUP BY PRODUCT.category
```

Operations: Slice (Loc.Region.Europe) + Pivot (Prod.category)

- Query Language

# Query Language

- Two approaches:
  - Using relational DB technology: SQL (with extensions such as CUBE/PIVOT/UNPIVOT)
  - Using multidimensional technology: MDX

```
SELECT PRODUCT.category,
SUM(SALES.amount)
FROM     SALES, PRODUCT,LOCATION
WHERE    SALES.product_key =
PRODUCT.product_key
AND      SALES.location_key =
LOCATION.location_key
AND      LOCATION.region="Europe"
GROUP BY PRODUCT.category
```

⇨

```
SELECT
 {[PRODUCT].[category]} on ROWS,
 {[MEASURES].[amount]} on COLUMNS
FROM     [SALES]
WHERE    ([LOCATION].[region].[Europe])
```

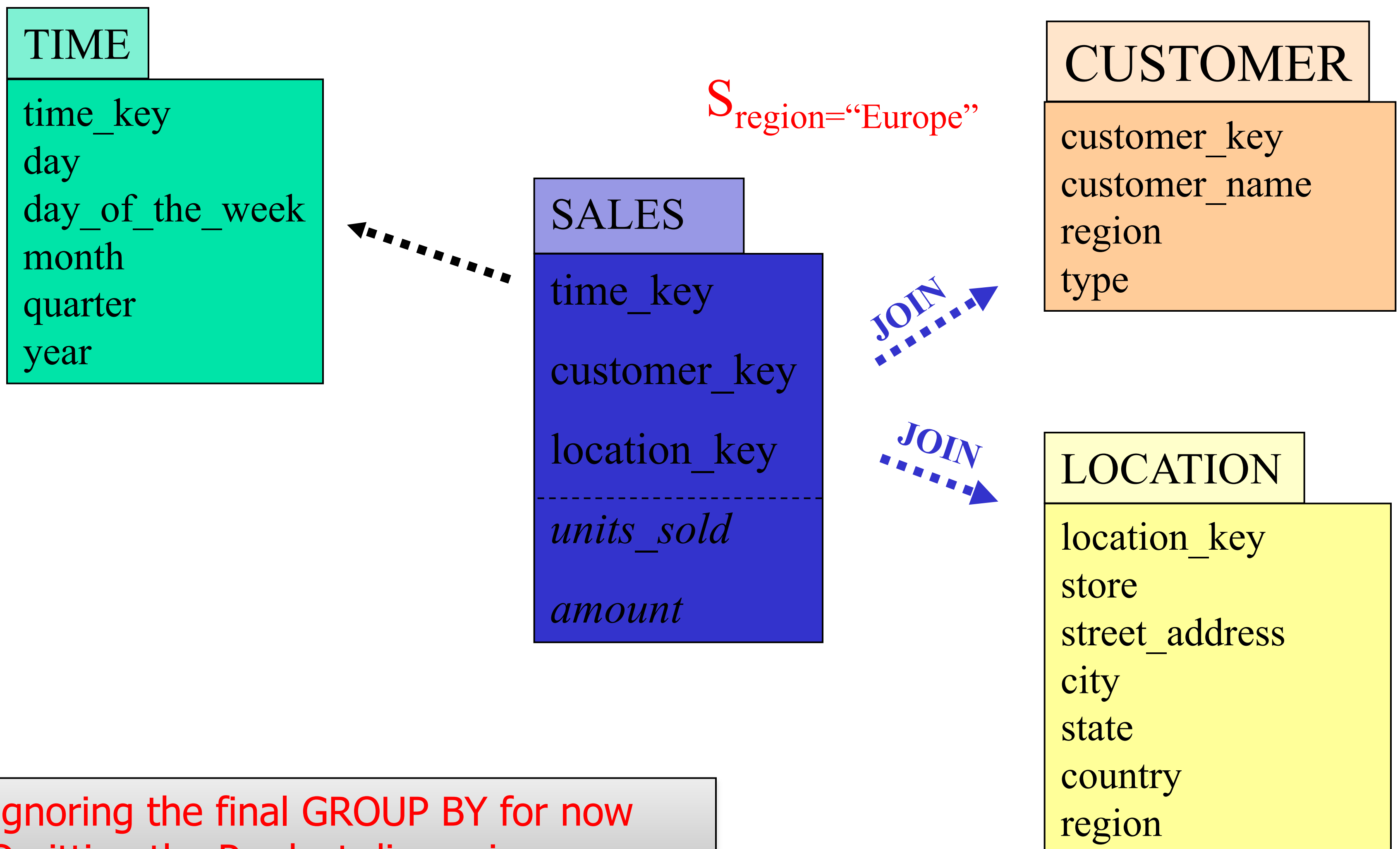
Operations: Slice (Loc.Region.Europe) + Pivot (Prod.category, Measures.amnt)

- Physical Model + Query Processing Techniques

# Physical Model + Query Processing Techniques

- Two main approaches:
  - Using relational DB technology: ROLAP
  - Using multidimensional technology: MOLAP
- Hybrid: HOLAP
  - Base cuboid: ROLAP
  - Other cuboids: MOLAP

# Q1: Selection on low-cardinality attributes




- Ignoring the final GROUP BY for now
- Omitting the Product dimension

# Indexing OLAP Data: Bitmap Index

(1) BI on dimension tables

- Index on an attribute (column) with low distinct values
- Each distinct values, v, is associated with a n-bit vector (n = #rows)
  - The *i*-th bit is set if the *i*-th row of the table has the value v for the indexed column
- Multiple BIs can be efficiently combined to enable optimized scan of the table

**Custom**

| Cust | Region | Type |
|---|---|---|
| C1 | Asia | Retail |
| C2 | Europe | Dealer |
| C3 | Asia | Dealer |
| C4 | America | Retail |
| C5 | Europe | Dealer |

**BI on Customer.Region**

| v | bitmap |
|---|---|
| Asia | 1 0 1 0 0 |
| Europe | 0 1 0 0 1 |
| America | 0 0 0 1 0 |

# Indexing OLAP Data: Bitmap Index /2

(1) Bitmap join index (BI on Fact Table Joined with Dimension tables)

- Conceptually, perform a join, map each dimension value to the bitmap of corresponding fact table rows.

```
-- ORACLE SYNTAX –

CREATE BITMAP INDEX sales_cust_region_bjix
 ON    sales(customer.cust_region)
FROM sales, customer
WHERE sales.cust_id = customers.cust_id;
```

# Indexing OLAP Data: Bitmap Index /3

**Sales**

| time | customer | loc | Sale |
|---|---|---|---|
| 101 | C1 | 100 | 1 |
| 173 | C1 | 200 | 2 |
| 208 | C2 | 100 | 3 |
| 863 | C3 | 200 | 5 |
| 991 | C1 | 100 | 8 |
| 1001 | C2 | 200 | 13 |
| 1966 | C4 | 100 | 21 |
| 2017 | C5 | 200 | 34 |

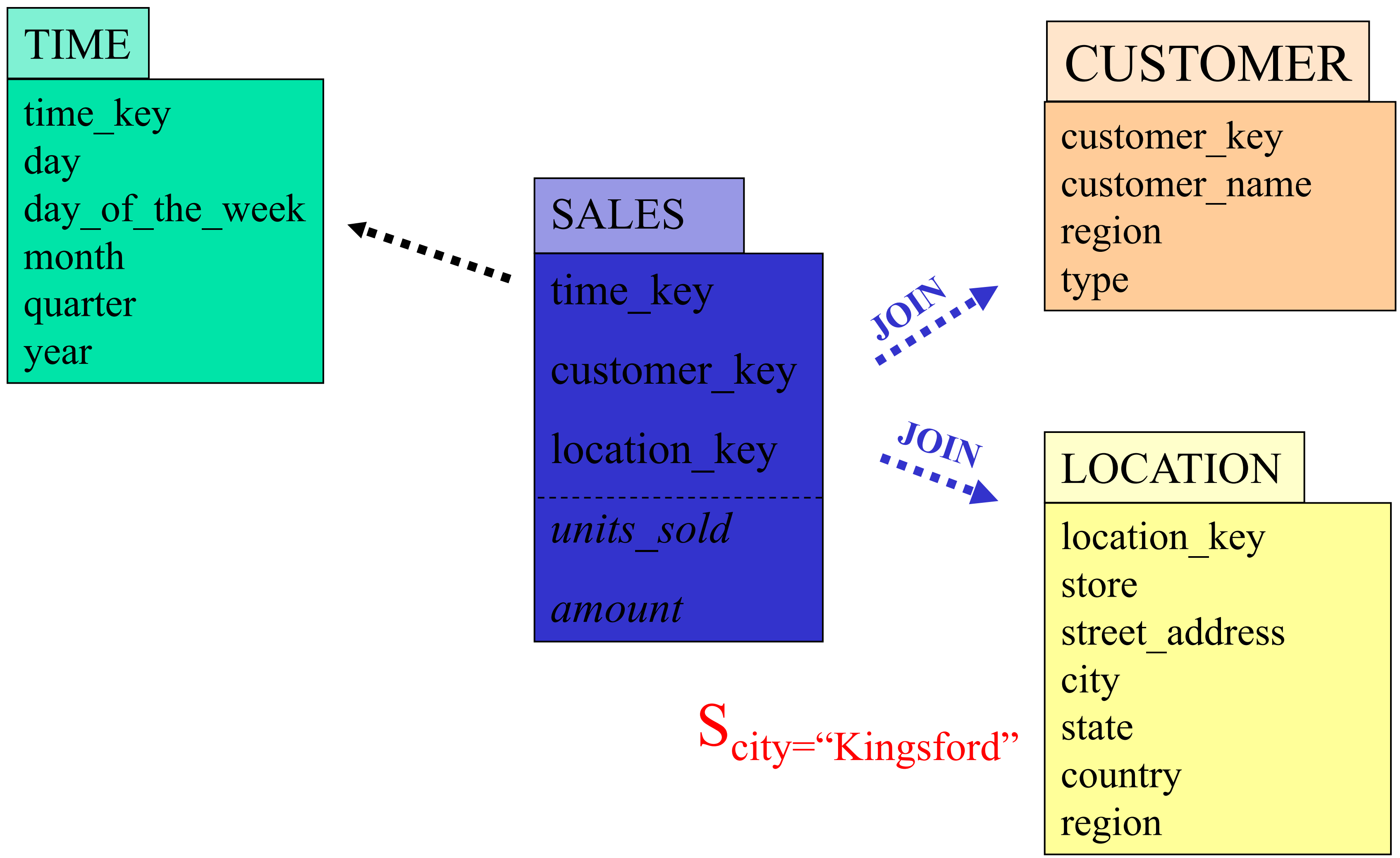
**Customer**

| Cust | Region | Type |
|---|---|---|
| C1 | Asia | Retail |
| C2 | Europe | Dealer |
| C3 | Asia | Dealer |
| C4 | America | Retail |
| C5 | Europe | Dealer |

**BI on Sales(Customer.Region)**

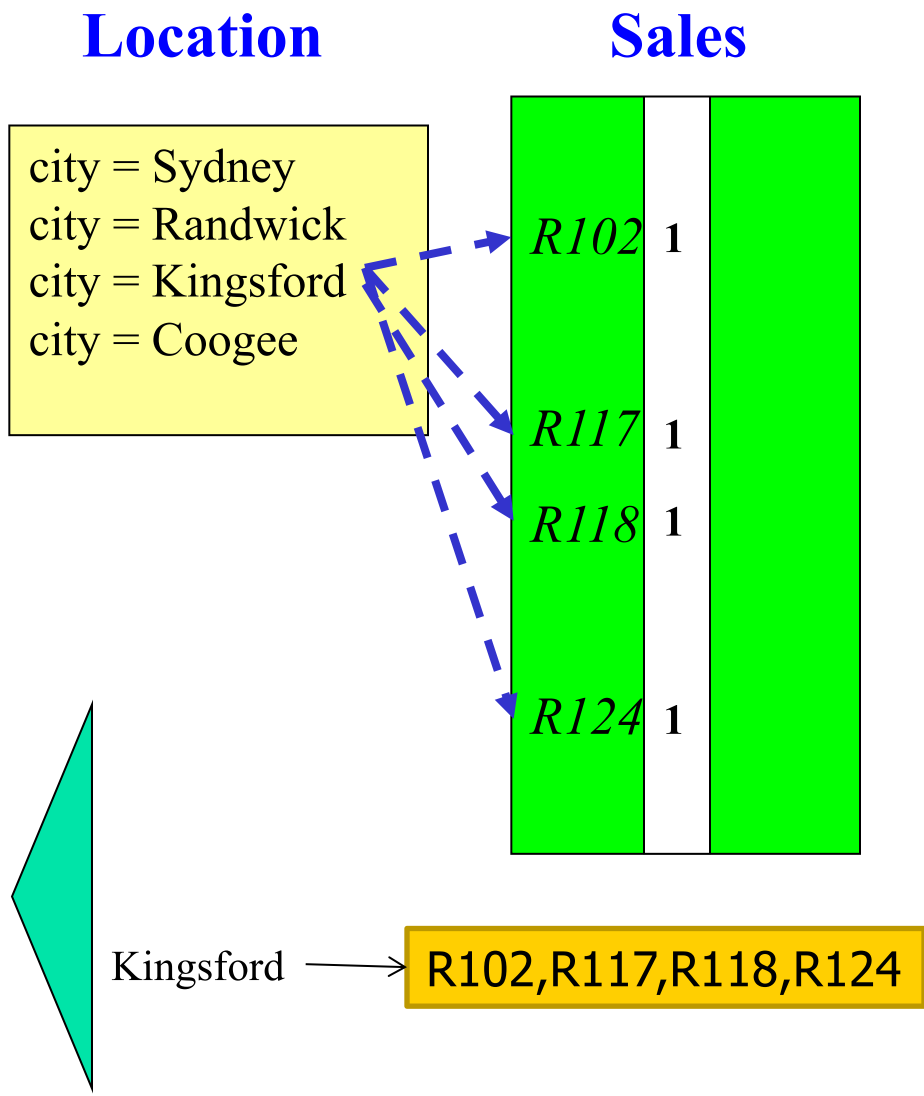
| v | bitmap |
|---|---|
| Asia | 11011000 |
| Europe | 00100101 |
| America | 00000010 |

# Q2: Selection on high-cardinality attributes

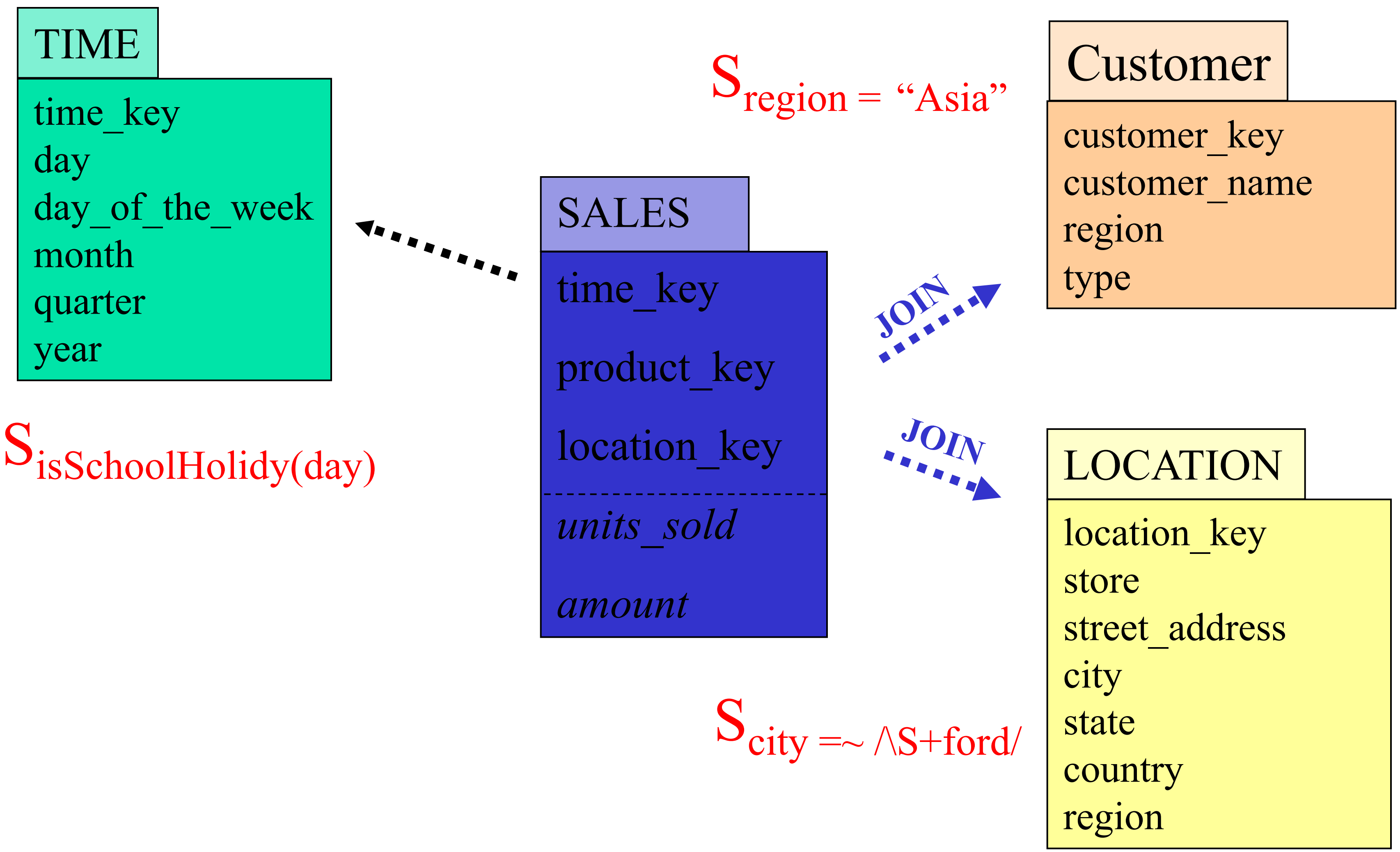


# Indexing OLAP Data: Join Indices

- Join index relates the values of the dimensions of a star schema to rows in the fact table.
  - a join index on *city* maintains for each distinct city a list of ROW-IDs of the tuples recording the sales in the city
- Join indices can span multiple dimensions OR
  - can be implemented as bitmap-indexes (per dimension)
  - use bit-op for multiple-joins



# Q3: Arbitrary selections on Dimensions



# Star Query and Star Join (Cont.)

Usually only part of the dim tables because of the selection predicates

**Time**

| time |
|---|
| 1 |
| 2 |

X

**Customer**

| Cust |
|---|
| 10 |
| 20 |

X

**Loc**

| loc |
|---|
| 100 |
| 200 |

*thousands of tuples*

| time | cust | loc |
|---|---|---|
| 1 | 10 | 100 |
| 1 | 10 | 200 |
| 1 | 20 | 100 |
| 1 | 20 | 200 |
| 2 | 10 | 100 |
| 2 | 10 | 200 |
| 2 | 20 | 100 |
| 2 | 20 | 200 |

**Sales**

*millions of tuples*

| time | cust | loc | sold |
|---|---|---|---|
| 1 | 10 | 100 | 7 |
| 1 | 10 | 150 | 13 |
| 1 | 20 | 150 | 2 |
| 2 | 20 | 200 | 16 |
| … | … | … | … |
| 1000 | 2000 | 500 | 86 |

$$\text{Sales} \rhd\lhd \sigma_1(\text{Time}) \rhd\lhd \sigma_2(\text{Cust}) \rhd\lhd \sigma_3(\text{Loc}) \Rightarrow$$

$$\text{Sales} \rhd\lhd (\sigma_1(\text{Time}) \times \sigma_2(\text{Cust}) \times \sigma_3(\text{Loc}))$$

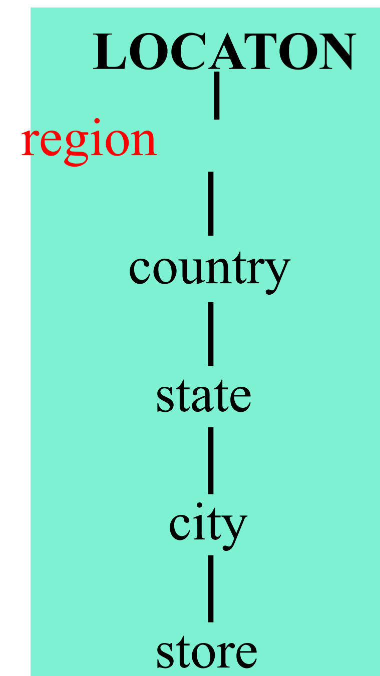
# Q4: Coarse-grain Aggregations

- *"Find total sales per customer type in our stores in Europe"*
  - Join-index will prune ¾ of the data (uniform sales), but the remaining ¼ is still large (several millions transactions)
    - Index is unclustered
- High-level aggregations are expensive!!!!!

⇒Long Query Response Times

⇒Pre-computation is necessary

⇒Pre-computation is most beneficial



# Cuboids = GROUP BYs

- Multidimensional aggregation = selection on corresponding cuboid

$$GB_{(type,\ city)}(\ Sales \bowtie \sigma_1(Time) \bowtie \sigma_2(Cust) \bowtie \sigma_3(Loc))\ )$$

$\sigma_1$ selects some Years, $\sigma_2$ selects some Brands, $\sigma_3$ selects some Cities,

$$GB_{(type,\ city)}(\ \sigma_{1'2'3'}(Cuboid(Year, Type, City))\ )$$

- Materialize some/all of the cuboids
  - A complex decision involving cuboid sizes, query workload, and physical organization

# Two Issues

- How to store the materialized cuboids?
- How to compute the cuboids efficiently?

# CUBE BY in ROLAP

| Sales | | Product | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | ALL |
| Store | 1 | 454 | - | - | 925 | 1379 |
| | 2 | 468 | 800 | - | - | 1268 |
| | 3 | 296 | - | 240 | - | 536 |
| | 4 | 652 | - | 540 | 745 | 1937 |
| | ALL | 1870 | 800 | 780 | 1670 | 5120 |

| Store | Product_key | sum(amout) |
|---|---|---|
| 1 | 1 | 454 |
| 1 | 4 | 925 |
| 2 | 1 | 468 |
| 2 | 2 | 800 |
| 3 | 1 | 296 |
| 3 | 3 | 240 |
| 4 | 1 | 625 |
| 4 | 3 | 240 |
| 4 | 4 | 745 |
| 1 | ALL | 1379 |
| 2 | ALL | 1268 |
| 3 | ALL | 536 |
| 4 | ALL | 1937 |
| ALL | 1 | 1870 |
| ALL | 2 | 800 |
| ALL | 3 | 780 |
| ALL | 4 | 1670 |
| ALL | ALL | 5120 |

4 Group-bys here:
**(store,product)**
**(store)**
**(product)**
**()**

- Need to write 4 queries!!!
- Compute them independently

**SELECT** LOCATION.store, SALES.product_key, SUM (amount)

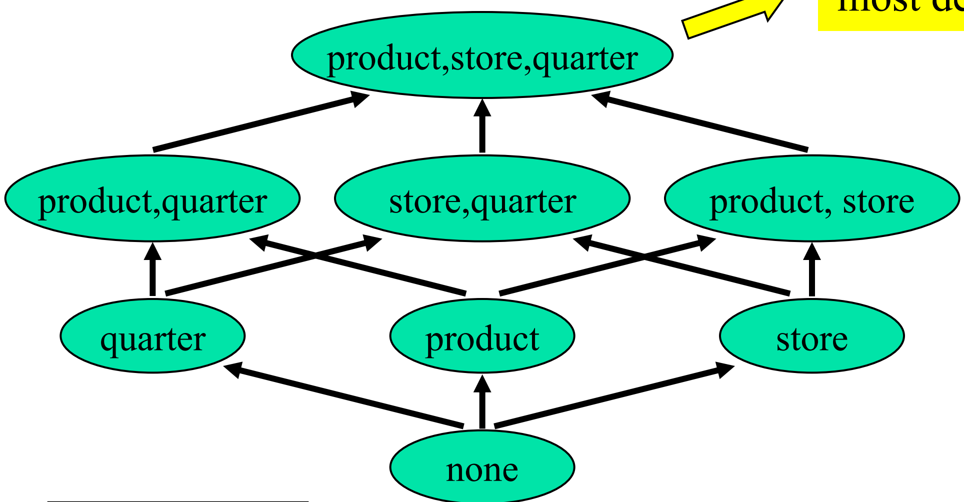
**FROM** SALES, LOCATION

**WHERE** SALES.location_key=LOCATION.location_key

**CUBE BY** SALES.product_key, LOCATION.store

# Top-down Approach

- Model dependencies among the aggregates:




most detailed "view"

can be computed from *view* (product,store,quarter) by summing-up all quarterly sales

**Cuboid AB**

| A | B | M |
|---|---|---|
| 1 | 1 | 10 |
| 1 | 3 | 20 |
| 1 | 2 | 30 |
| 1 | 1 | 40 |
| 2 | 4 | 50 |

**Cuboid A**

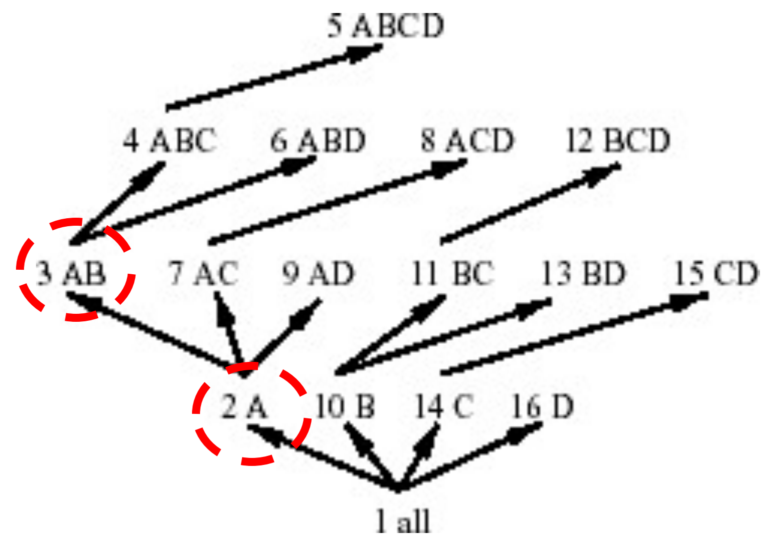
| A | B | M |
|---|---|---|
| 1 | (all) | ??? |
| 2 | (all) | ??? |

**Cuboid B**

| A | B | M |
|---|---|---|
| (all) | 1 | ??? |
| (all) | 2 | ??? |
| (all) | 3 | ??? |
| (all) | 4 | ??? |

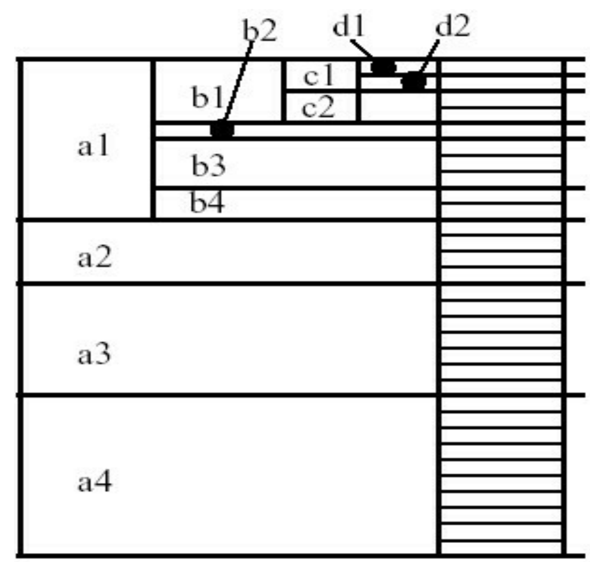
# Bottom-Up Approach (BUC)

- BUC (Beyer & Ramakrishnan, SIGMOD'99)
- Ideas
  - Compute the cube from bottom up
  - Divide-and-conquer
- A simpler recursive version:
  - BUC-SR




| A | B | ... |
|---|---|---|
| 1 | 1 | ... |
| 1 | 3 | ... |
| 1 | 2 | ... |
| 1 | 1 | ... |
| 2 | ... | ... |

...



# Understanding Recursion /1

- Powerful computing/problem-solving techniques
- Examples
  - Factorial:
    - f(n) = 1, if n = 1
    - f(n) = f(n-1) * n, if n ≥ 1
  - Quick sort:
    - Sort([x]) = [x]
    - Sort([x1, …, *pivot*, … xn]) = sort[ys] ++ sort[zs]), where

      ys = [ x | x ***in*** xi, x ≤ *pivot* ]

      zs = [ x | x **<-** xi, x > *pivot* ]

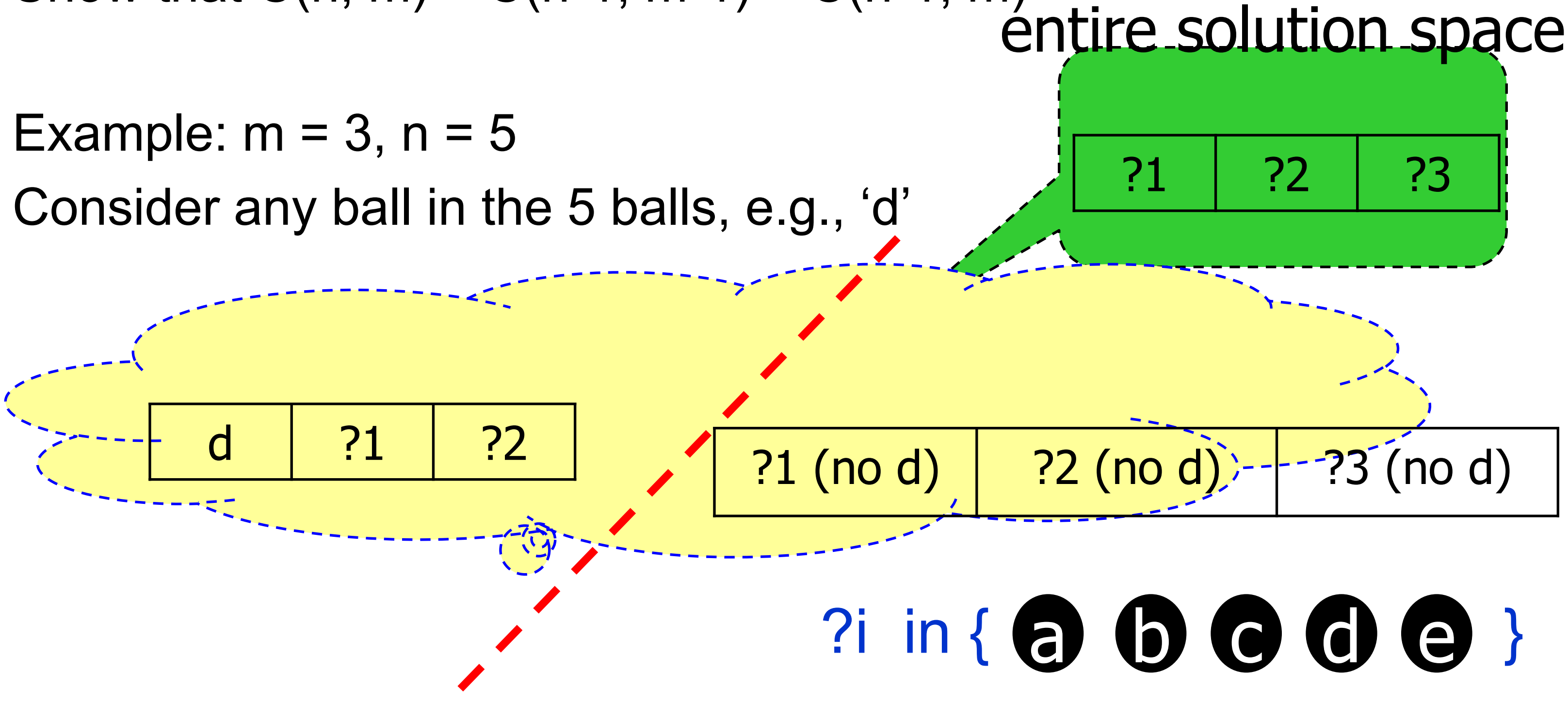
f(0) = 0! = ???

List comprehension in Haskell or python

# Understanding Recursion /2

- Let C(n, m) be the number of ways to select m balls from n numbered balls
- Show that C(n, m) = C(n-1, m-1) + C(n-1, m)

- Example: m = 3, n = 5
- Consider any ball in the 5 balls, e.g., ‘d’



# Key Points

- Sub-problems need to be "**smaller**", so that a simple/trivial boundary case can be reached
- Divide-and-conquer
  - There may be multiple ways the entire solution space can be divided into **disjoint** sub-spaces, each of which can be conquered **recursively**.

# Geometric Intuition /1

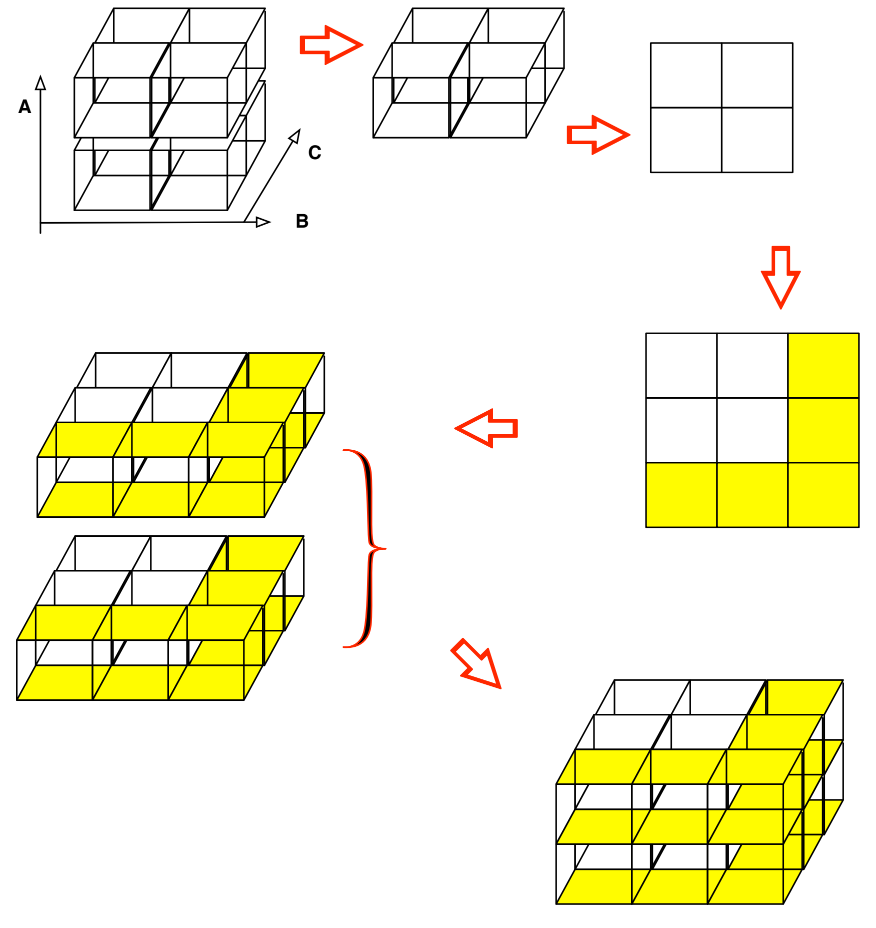
- Reduce Cube(in 2D) to Cube(in 1D)

| | b1 | b2 | b3 | |
|---|---|---|---|---|
| a1 | M11 | M12 | M13 | [Step 1] |
| a2 | M21 | M22 | M23 | [Step 1] |
| | [Step 2] | [Step 2] | [Step 2] | [Step 3] |

| | b1 | b2 | b3 | |
|---|---|---|---|---|
| [a1] × | M11 | M12 | M13 | [Step 1] |
| [a2] × | M21 | M22 | M23 | [Step 1] |
| [*] × | [Step 2] | [Step 2] | [Step 2] | [Step 3] |

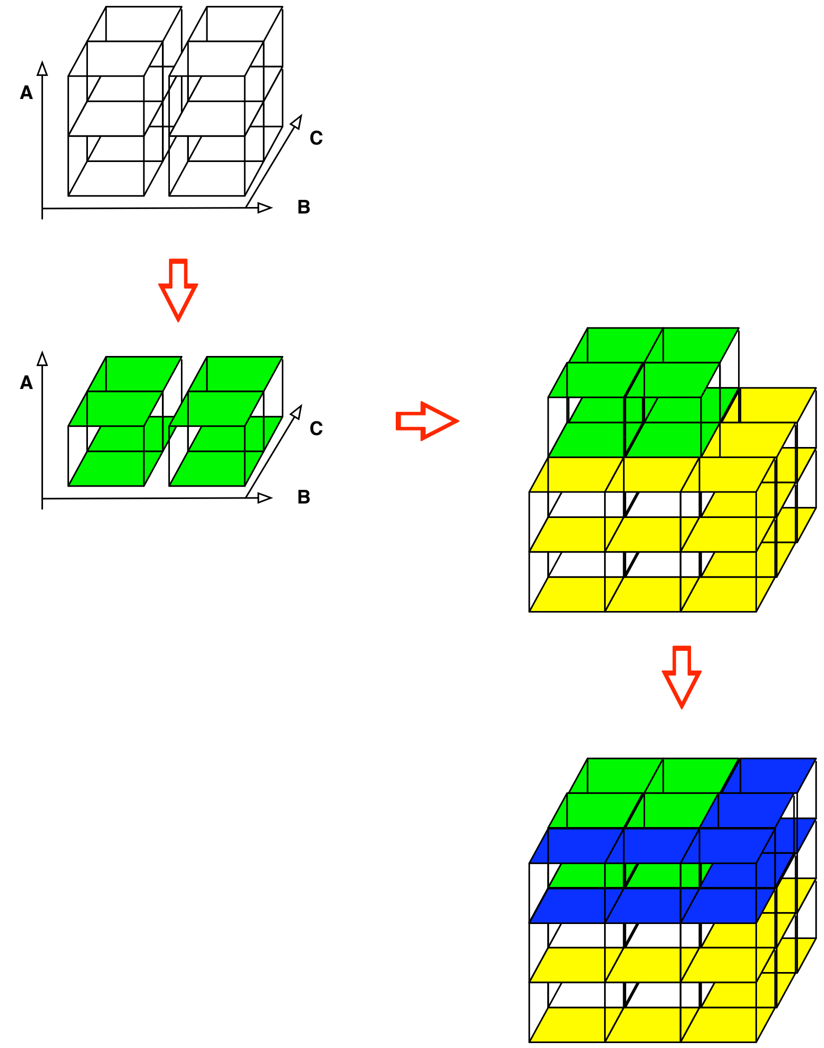
# Geometric Intuition /2

- Reduce Cube(in 3D) to Cube(in 2D)



# Geometric Intuition /3

- Reduce Cube(in 3D) to Cube(in 2D)



You need to fix the incomplete output part of the algorithm!

# BUC-SR (Simple Recursion)*

- BUC-SR(data, dims)
  - If (dims is empty)
    - Output (sum(data))
  - Else
    - Dims = [dim1, rest_of_dims]
    - For each distinct value v of dim1
      - slice_v = slice of data on "dim1 = v"
      - BUC-SR(slice_v, rest_of_dims)
    - data' = Project(data, rest_of_dims)
    - BUC-SR(data', rest_of_dims)

Boundary case:
data is essentially a list of measure values

General case:
1)Slice on dim1. Call BUC-SR recursively for each slice

2)Project out dim1, and call BUC-SR on it recursively

# Example

Input | Internal Output | Output

| | 1 | 2 | 3 | * |
|---|---|---|---|---|
| 1 | 30 | 30 | 40 | 100 |
| 2 | 50 | | | 50 |
| * | 80 | 30 | 40 | 150 |

[{r1-r5}, AB]

| | A | B | M |
|---|---|---|---|
| r1 | 1 | 1 | 10 |
| r2 | 1 | 1 | 20 |
| r3 | 1 | 2 | 30 |
| r4 | 1 | 3 | 40 |
| r5 | 2 | 1 | 50 |

[{r1-r4}, B]

| B | M |
|---|---|
| 1 | 10 |
| 1 | 20 |
| 2 | 30 |
| 3 | 40 |

| B | M |
|---|---|
| 1 | 30 |
| 2 | 30 |
| 3 | 40 |
| * | 100 |

| A | B | M |
|---|---|---|
| 1 | 1 | 30 |
| 1 | 2 | 30 |
| 1 | 3 | 40 |
| 1 | * | 100 |

[{r5}, B]

| B | M |
|---|---|
| 1 | 50 |

| B | M |
|---|---|
| 1 | 50 |
| * | 50 |

| A | B | M |
|---|---|---|
| 2 | 1 | 50 |
| 2 | * | 50 |

[{r1'-r5'}, B]

| B | M |
|---|---|
| **1** | **80** |
| 2 | 30 |
| 3 | 40 |

| B | M |
|---|---|
| 1 | 80 |
| 2 | 30 |
| 3 | 40 |
| * | 150 |

| A | B | M |
|---|---|---|
| * | 1 | 80 |
| * | 2 | 30 |
| * | 3 | 40 |
| * | * | 150 |

# Try a 3D-Cube by Yourself

[{r1-r5}, ABC]

| | A | B | C | M |
|---|---|---|---|---|
| r1 | 1 | 1 | 1 | 10 |
| r2 | 1 | 1 | 2 | 20 |
| r3 | 1 | 2 | 1 | 30 |
| r4 | 1 | 3 | 1 | 40 |
| r5 | 2 | 1 | 1 | 50 |

# MOLAP

- (Sparse) array-based multidimensional storage engine
- Pros:
  - small size (esp. for dense cubes)
  - fast in indexing and query processing
- Cons:
  - scalability
  - conversion from relational data

# Multidimensional Array

Step 2: An **injective** map from cell to offset

f(time, item) = 4*time + item

| time | item | dollars_sold |
|---|---|---|
| Q1 | home entertainment | 605 |
| Q2 | home entertainment | 680 |
| Q3 | home entertainment | 812 |
| Q4 | home entertainment | 927 |
| Q1 | computer | 825 |
| Q2 | computer | 952 |
| Q3 | computer | 1023 |
| Q4 | computer | 1038 |
| Q1 | phone | 14 |
| Q2 | phone | 31 |
| Q3 | phone | 30 |
| Q4 | phone | 38 |
| Q1 | security | 400 |
| Q2 | security | 512 |
| Q3 | security | 501 |
| Q4 | security | 580 |

Step 1

Mappings

| time | *value* |
|---|---|
| Q1 | 0 |
| Q2 | 1 |
| Q3 | 2 |
| Q4 | 3 |

| item | *value* |
|---|---|
| home entertainment | 0 |
| computer | 1 |
| phone | 2 |
| security | 3 |

| time | item | dollars_sold | offset |
|---|---|---|---|
| 0 | 0 | 605 | 0 |
| 1 | 0 | 680 | 4 |
| 2 | 0 | 812 | 8 |
| 3 | 0 | 927 | 12 |
| 0 | 1 | 825 | 1 |
| 1 | 1 | 952 | 5 |
| 2 | 1 | 1023 | 9 |
| 3 | 1 | 1038 | 13 |
| 0 | 2 | 14 | 2 |
| 1 | 2 | 31 | 6 |
| 2 | 2 | 30 | 10 |
| 3 | 2 | 38 | 14 |
| 0 | 3 | 400 | 3 |
| 1 | 3 | 512 | 7 |
| 2 | 3 | 501 | 11 |
| 3 | 3 | 580 | 15 |

# Multidimensional Array

Step 3': If **sparse**

| offset | dollars_sold |
|---|---|
| 0 | 605 |
| 1 | 825 |
| 2 | 14 |
| 3 | 400 |
| 4 | 680 |
| 5 | 952 |
| 6 | 31 |
| 7 | 512 |
| 8 | 812 |
| 9 | 1023 |
| 10 | 30 |
| 11 | 501 |
| 12 | 927 |
| 13 | 1038 |
| 14 | 38 |
| 15 | 580 |

Step 3: If dense, only need to store sorted slots

| Dense MD array |
|---|
| 605 |
| 825 |
| 14 |
| 400 |
| 680 |
| 952 |
| 31 |
| 512 |
| 812 |
| 1023 |
| 30 |
| 501 |
| 927 |
| 1038 |
| 38 |
| 580 |

- Think: how to decode a slot?
- Multidimensional array is typically sparse
  - Use sparse array (i.e., offset + value)
  - Could use chunk to further reduce the space
- Space usage:
  - (d+1)*n*4 vs 2*n*4
- HOLAP:
  - Store all non-base cuboid in MD array
  - Assign a value for ALL