In this novel basal inversion method, the bed roughness is adapted over time during a forward simulation, with the rate of change of the bed roughness determined according to the difference between the modelled and the target ice geometry and velocity. This approach is based on the implementation in CISM (ref?), with one important addition being that the difference between the modelled and the target state is no longer determined locally, but is instead found by integrating both upstream and downstream along the flowline. The rationale behind this approach is that changing the bed roughness at any location will affect the ice geometry and velocity not just at that location, but also upstream and downstream. Reducing the basal roughness will increase the ice velocity along the entire flowline, causing the ice both locally and upstream to become thinner. By including these effects in the inversion procedure, numerical stability is improved, and artefacts arising from differences in the flotation mask between the modelled and the target state are reduced.

Let be a point on the ice sheet. We divide the flowline passing through into an upstream part and a downstream part , which can be found by integrating the ice surface velocity field :

|  |  |  |
| --- | --- | --- |
|  |  | (1a) |
|  |  | (1b) |
|  |  | (1c) |

In the upstream (downstream) direction, the integral is terminated at the ice divide (ice margin), i.e. when (), so that:

|  |  |  |
| --- | --- | --- |
|  |  | (2a) |
|  |  | (2b) |

In order to calculate the rate of change of the bed roughness , the following expressions are evaluated:

|  |  |  |
| --- | --- | --- |
|  |  | (3a) |
|  |  | (3b) |
|  |  | (3c) |

The linear scaling function serves to assign more weight to anomalies close to , decreasing to zero at the end of the flowline, as well as to normalise the integral:

|  |  |  |
| --- | --- | --- |
|  |  | (4a) |
|  |  | (4b) |

The three line integrals from Eqs. 3a-c are then added together, and scaled with the local ice thickness and velocity. This reflects the fact that bed roughness underneath slow-moving, thin ice has less effect on the large-scale ice-sheet geometry than does the roughness underneath fast-flowing, thick ice:

|  |  |  |
| --- | --- | --- |
|  |  | (5a) |
|  |  | (5b) |

Finally, the rate of change of the bed roughness can be calculated:

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

The various constants used in the above expressions are listed below.

|  |  |
| --- | --- |
| **Constant** | **Value** |
|  | 100 m |
|  | 250 m/yr |
|  | 3,000 m/yr |
|  | 300 m |