Exam Solution

Course: AE4870B Re-Entry Systems
Exam Source:Brightspace
Exam Date: 2013-04-18

Author:
collaborative effort

06-09-2019

0 Introduction

This contains an exam solution. If you wish to contribute to this exam solution:

1. Create a github account, (you can create an "anonymous" one).
2. git clone ...
3. edit your changes in the document.
4. open cmd, and browse to inside the folder you downloaded and edited
5. git pull (updates your local repository=copy of folder, to the latest version in github cloud)
6. git status shows which files you changed.
7. git add "/some folder with a space/someFileYouChanged.tex"
8. git commit -m "Included solution to question 1c."
9. git push

It can be a bit initimidating at first, so feel free to click on "issue" in the github browser of this repository and ask :) (You can also use that to say "Hi, I'm having a bit of help with this particular equation, can someone help me out?")

If you don't know how to edit a latex file on your own pc iso on overleaf, look at the "How to use" section of `https://github.com/a-t-0/AE4872-Satellite-Orbit-Determination`.

0.1 Consistency

To make everything nice and structured, please use very clear citations:

1. If you copy/use an equation of some slide or document, please add the following data:
   (a) Url (e.g. if simple wiki or some site)
   (b) Name of document
   (c) (Author)
   (d) PAGE/SLIDE number so people can easily find it again
   (e) equation number (so people can easily find it again)
2. If you use an equation from the slides/a book that already has an equation number, then hardcode that equation number in this solution manual so people directly see which equation in the lecture material it is, this facilitates remembering the equations.
3. Here is an example is given in eq. (10.32[1]) (See file references.bib [1]).

$$R_n^E = \sum_{t=1}^{n} l_m(p_t, z_t) - \underset{i}{min} \sum_{t=1}^{n} {z_t}^i \tag{10.32[1]}$$

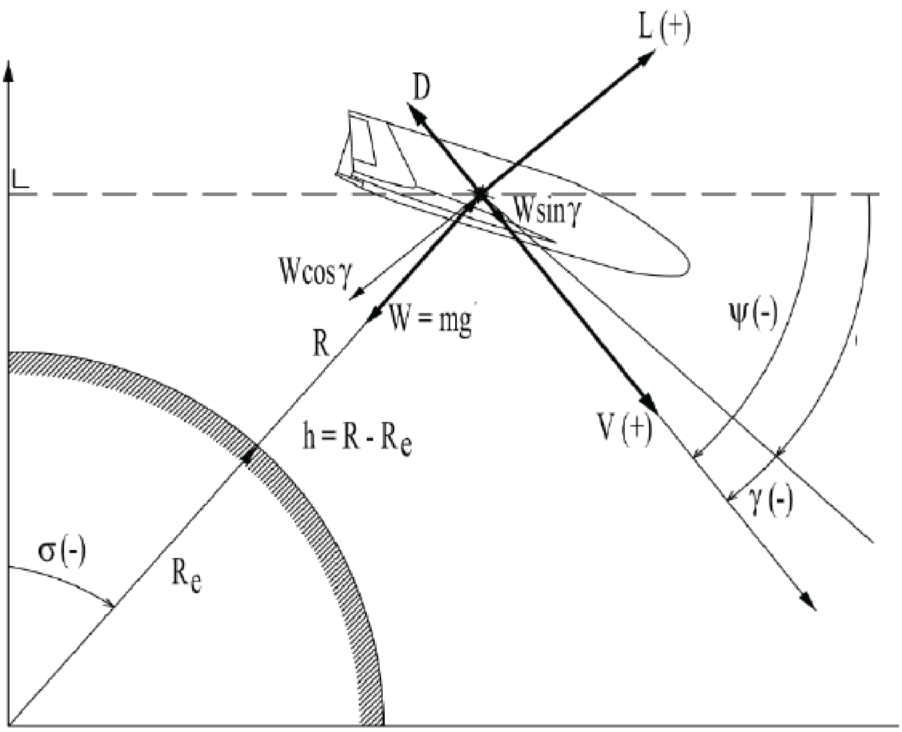
1 Introduction: True/False

- a) False ($\frac{d\gamma}{dt} = 0$ for ballistic flight)
- b) False (Should be for one of the first skips)
- c) False (this can be used with pilot chutes as well)
- d) True
- e) False (Sometimes the reference frames are nicely oriented)
- f) False (it only assumes constant temperature)
- g) True
- h) False (can also be done for smaller payload masses, for cost or redundancy reasons)
- i) False (everything can have a bias and has a bias)
- j) False (maximum deceleration independent of drag coefficient)

2 Ballistic Entry: Gliding Re-entry

a.

General question, from slides lecture 3



b.

General equations of motion:

$$
\begin{aligned}
m\frac{dV}{dt} &= -D - mg\sin(\gamma) \\
mV\frac{d\gamma}{dt} &= L - mg\cos(\gamma)(1 - \frac{V^2}{V_c^2}) \\
\frac{dr}{dt} = \frac{dh}{dt} &= V\sin\gamma
\end{aligned} \tag{1}
$$

c.

For gliding entry, we have that $\gamma \approx 0$ and $\frac{d\gamma}{dt} \approx 0$. If we use this approximations, we get the following form the above equations:

$$\begin{aligned} m\frac{dV}{dt} &= -D \\ 0 = L - mg(1 - &\frac{V^2}{V_e^2}) \\ \frac{dr}{dt} = \frac{dh}{dt} &= 0 \end{aligned} \tag{2}$$

Now also:

$$\frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt} = V\frac{dV}{ds} = \frac{-D}{m} = \frac{-D}{L}g\frac{L}{mg} \tag{3}$$

This then yields:

$$\frac{dV}{ds} = \frac{-\frac{D}{L}g(1 - \frac{V^2}{V_c^2})}{V} \tag{4}$$

This gives then:

$$VdV = -\frac{D}{L}g(1 - \frac{V^2}{V_c^2})ds \tag{5}$$

For integration, the variable $x = \frac{V^2}{V_c^2}$ can be used, with $dx = \frac{2V}{V_c^2}dV$ giving $dV = \frac{V_c^2}{2V}dx$ and then

$$\frac{V_c^2}{2}dx = -\frac{D}{L}g(1 - x)ds \tag{6}$$

Which can then be written as:

$$-\frac{L}{D}\frac{1}{g}\frac{V_c^2}{2}\frac{1}{1 - x}dx = ds \tag{7}$$

Integration gives:

$$\begin{aligned} \int_0^{R_f} ds = \int_{x_1}^{x_2} (-&\frac{L}{D}\frac{1}{g}\frac{V_c^2}{2}\frac{1}{1 - x}dx \\ R_f = -\frac{L}{D}\frac{1}{g}\frac{V_c^2}{2}&[-\ln(1 - x)]_{x_1}^{x_2} \\ R_f = \frac{L}{D}\frac{1}{g}&\frac{V_c^2}{2}\ln\frac{(1 - x_2)}{1 - x_1} \end{aligned} \tag{8}$$

Now $x_1 = \frac{V_E^2}{V_c^2}$ and $x_2 = \frac{V_F^2}{V_c^2}$, so:

$$R_f = \frac{L}{D}\frac{1}{g}\frac{V_c^2}{2}\ln\frac{(1 - \frac{V_F^2}{V_c^2})}{1 - \frac{V_E^2}{V_c^2}} \tag{9}$$

Now $V_F \approx 0$ and $V_c^2 \approx g \cdot R_e$, so ultimately:

$$\frac{R}{R_e} = -\frac{1}{2}\frac{L}{D}\ln 1 - \frac{V_E^2}{V_c^2}) \tag{10}$$

d.

Minimum range to cover: 6000 km. This means that

$$\frac{L}{D} = -2 \cdot \frac{R}{R_e}\frac{1}{\ln 1 - \frac{V_E^2}{V_c^2})} = -2 \cdot 0.9408\frac{1}{\ln 1 - \frac{V_E^2}{V_c^2})} \tag{11}$$

Now since $V_E = 0.8V_c$, we have:

$$\frac{L}{D} = -1.881\frac{1}{\ln 1 - 0.64} = 1.922 \tag{12}$$

A minimum lift-to-drag ratio of 1.922 is required for the entirety of this flight.

e.

When the entry velocity V_E approaches the circular velocity V_c, the flight range approaches infinity, which is the actual flight range if $V_E = v_c$. This is of little surprise, since this means that the vehicle is actually in orbit and will never land. This limiting case with a non-physical result is thus actually based on physical results.

3 Aerodynamics:Parachutes

a.

Two main requirements which may require parachute system:

- Maximum mechanical load / limit impact speed
- Observation time of the atmosphere / control the descent duration

b.

The pilot parachute is to give initial deceleration or force for the deployment of the main parachute. The drogue parachute is attached to the payload and is used to provide stabilisation and initial deceleration. The main parachute lastly gives the largest contribution to deceleration of the payload.

c.

Parachute reefing is the controlled restriction of the canopy at the skirt. It is used to control the total drag area and in this way the deceleration, flight time and mechanical loads.

d.

Parachute clusters are used to increase drag area, to make fabrication easier, for redundancy reasons, for adjustment of drag area during flight and for shorter inflation time.

e.

Vertical equilibrium means:

$$D_p + D_{EV} = mg \tag{13}$$

Here, P indicates the parachute and EV the entry vehicle.

The general formula for drag is given by $D = \frac{1}{2}SC_d\rho V^2$, so this gives:

$$\frac{1}{2}\rho V^2(S_pC_{D_p} + S_{EV}C_{D_{EV}}) = mg \tag{14}$$

We want to solve for the impact velocity, thus:

$$V^2 = \frac{2mg}{\rho}\frac{1}{S_pC_{D_p} + S_{EV}C_{D_{EV}}} \tag{15}$$

f.

Now we want to solve for S_p. First assume that the drag due to the entry vehicle is negligible compared to the drag of the parachute:

$$\frac{1}{2}\rho V^2 S_p C_{D_p} = mg \tag{16}$$

Next, we solve for S_p:

$$S_p = \frac{2mg}{\rho V^2 C_{D_p}} \tag{17}$$

Lastly, assume that the parachute has a circular shape, such that:

$$R_p = \sqrt{\frac{2mg}{\rho V^2 C_{D_p}\pi}} = \frac{45.16m^2/s}{V} \tag{18}$$

Now fill in the values. Obtained is:

- $V_f = 8$ m/s gives $R_p = 5.64$ m

- $V_f = 10$ m/s gives $R_p = 4.52$ m
- $V_f = 12$ m/s gives $R_p = 3.76$ m

4 Fundamentals of Motion: Skipping Re-Entry

a.

$$\begin{aligned} m\frac{dV}{dt} &= -D - mg\sin(\gamma) \\ mV\frac{d\gamma}{dt} &= L - mg\cos(\gamma)(1 - \frac{V^2}{V_c^2}) \\ \frac{dr}{dt} = \frac{dh}{dt} &= V\sin(\gamma) \end{aligned} \tag{19}$$

For skipping flight, we have that the weight is much smaller than the lift and the drag. Thus, we can approximate the above equations with:

$$\begin{aligned} m\frac{dV}{dt} &= -D \\ mV\frac{d\gamma}{dt} &= L \\ \frac{dr}{dt} = \frac{dh}{dt} &= V\sin\gamma \end{aligned} \tag{20}$$

Now divide the first equation by the second to obtain:

$$\frac{\frac{dV}{dt}}{V\frac{d\gamma}{dt}} = -\frac{D}{L} \tag{21}$$

This gives then:

$$\frac{dV}{d\gamma} = -V\frac{D}{L} \tag{22}$$

Or, alternatively:

$$\frac{1}{V}dV = -\frac{D}{L}d\gamma \tag{23}$$

Integration gives then:

$$\begin{aligned} \int_{V_E}^{V}\frac{1}{V}dV &= \int_{\gamma_E}^{\gamma} -\frac{D}{L}d\gamma \\ \ln(\frac{V}{V_E}) &= -\frac{D}{L}(\gamma - \gamma_E) \\ \frac{V}{V_E} &= \exp(-\frac{(\gamma - \gamma_E)}{\frac{L}{D}}) \end{aligned} \tag{24}$$

For the latter, we have:

$$V\frac{d\gamma}{dt} = V\frac{\partial\gamma}{\partial p}\frac{\partial p}{\partial h}\frac{\partial h}{\partial t} = \frac{L}{m} \tag{25}$$

This then leads to:

$$\begin{aligned} \frac{\partial p}{\partial t} &= \frac{L}{mV}\frac{\partial p}{\partial\gamma} \\ \frac{-L}{\rho gmV}\frac{\partial p}{\partial\gamma} &= V\sin(\gamma) \end{aligned} \tag{26}$$

This was done via $\frac{dh}{dt} = V\sin(\gamma)$ and $\frac{dp}{dh} = -\rho g$.

We then get:

$$\begin{aligned}
\sin(\gamma)d\gamma &= -\frac{L}{\rho g m}\frac{1}{V^2}dp \\
\sin(\gamma)d\gamma &= -\frac{\frac{1}{2}\rho C_L V^2 S}{\rho g m}\frac{1}{V^2}dp \\
\sin(\gamma)d\gamma &= -\frac{\rho C_L S}{2W}dp
\end{aligned} \tag{27}$$

Integration then gives:

$$\begin{aligned}
\int_{\gamma_E}^{\gamma}\sin(\gamma)d\gamma &= \int_{p_E}^{p} -\frac{\rho C_L S}{2W}dp \\
\cos(\gamma-\gamma_E)d\gamma &= \frac{\rho C_L S}{2W}(p-p_E)
\end{aligned} \tag{28}$$

Note that due to the ideal gas law, $p - p_E = \frac{g}{\beta}(\rho - \rho_E)$. Also, on the top of the atmosphere $\rho_E \approx 0$. Together, this finally gives:

$$\cos(\gamma) - \cos(\gamma_E) = \frac{g}{2\beta}\frac{1}{\frac{W/S}{C_L}}\rho \tag{29}$$

b.

On the lowest point of the first skipping trajectory, we have that $\frac{dh}{dt} = 0 = V\sin(\gamma)$ and thus that $\gamma = 0$. Then we have:

$$\begin{aligned}
V &= V_E \cdot \exp\left(-\frac{(\gamma-\gamma_E)}{\frac{L}{D}}\right) \\
V = 8000 \cdot \exp -\frac{(0 - -0.262rad)}{1} &= 6157m/s
\end{aligned} \tag{30}$$

For the exit angle of the first skipping trajectory, we have $\rho \approx 0$, thus $\cos(\gamma) - \cos(\gamma_E) = 0$, leading to $\gamma = -\gamma_E$. This gives for V:

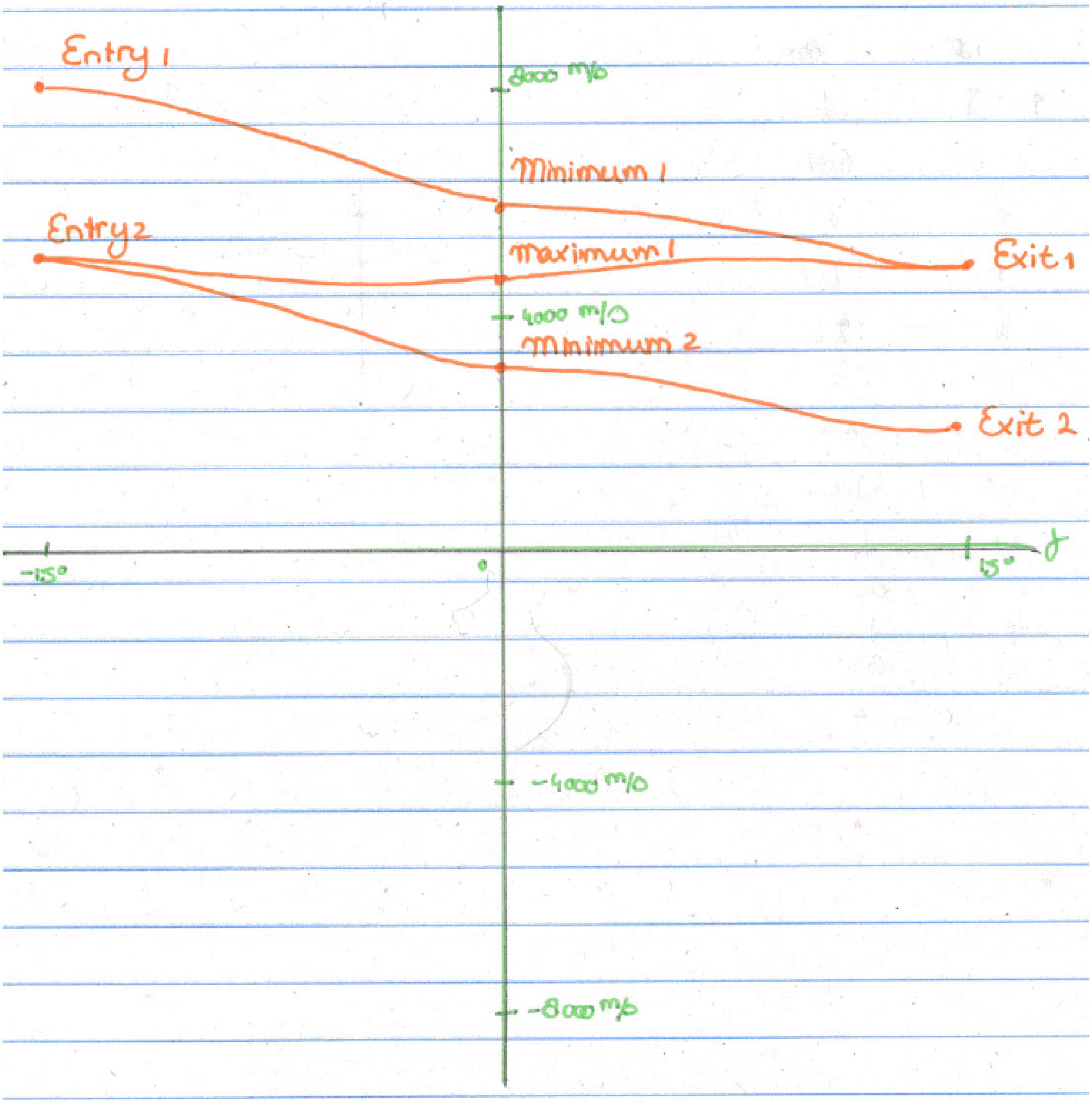
$$\begin{aligned}
V &= V_E \cdot \exp\left(-\frac{(\gamma-\gamma_E)}{\frac{L}{D}}\right) \\
V_{F,1} = 8000 \cdot \exp -\frac{(0.262 - -0.262rad)}{1} &= 4737m/s
\end{aligned} \tag{31}$$

For the top of the ballistic flight after the first skipping trajectory, we have that the vertical velocity is zero and the horizontal velocity is equal to that when the atmosphere was left, thus $V_h = V_{F,1}\cos(\gamma_{F,1}) = 4737 \cdot 0.966 = 4576m/s$. Of course, on the top also $\gamma = 0$.

For the next point, we assume that above the atmosphere no friction occurs and thus that the force field is conservative. Then $\gamma_{E,2} = -\gamma_{F,1} = -15^o$ and $V_{E,2} = V_{F,1} = 4737m/s$.

For the lowest point of the second skipping trajectory and the exit of the atmosphere of the second skipping trajectory, equal considerations hold and analogously, we obtain $\gamma = 0$ and $V = 3645m/s$ for the lowest part of the second skip and $\gamma_{F,2} = 15^o$, $V_{F,2} = 2805m/s$ for the exit of the atmosphere at the end of the second skipping trajectory.

The resulting graph can be seen underneath:



c

In the lowest point, we have that $\frac{dh}{dt} = 0 = V \sin(\gamma)$, thus that $\gamma = 0$. Then

$$1 - \cos(\gamma_E) = \frac{g}{2\beta} \frac{1}{\frac{W/S}{C_L}} \rho \tag{32}$$

Using $\sin(\gamma_E/2)^2 = 1/2 - 1/2\cos(\gamma_E)$, we get then:

$$\begin{aligned} \rho_p &= 2\sin(\gamma_E/2)^2 \frac{2\beta}{g} \frac{W/S}{C_L} \\ \rho_p &= \sin(\gamma_E/2)^2 \frac{4\beta}{g} \frac{W/S}{C_L} \end{aligned} \tag{33}$$

As can be seen, this is independent of the entrance velocity. Now since

$$\rho = \rho_0 \exp(-\beta h) = \rho_0 \exp(-\frac{h}{H}) \tag{34}$$

We get:

$$h_p = -H \ln(\frac{\rho_p}{\rho_0}) \tag{35}$$

Now $\rho_p = 0.0078 kg/m^3$ for both lowest points of the dual skipping entry. This then leads to a value of $h_p = 35392m$ for both lowest points of the dual skipping entry.

Conclusion

This seems like a representative exam for the course material. I had little problems with it, and many questions are repeated in other exams as well.

References

[1] Some author. *Advanced tree dynamics*, volume lecture 5 of *AE2344 Some course*, page 15. Accessed: 2019-04-27.