[illegible]

Question 2 Newton's law of motion & gravitation

januari 08 '01

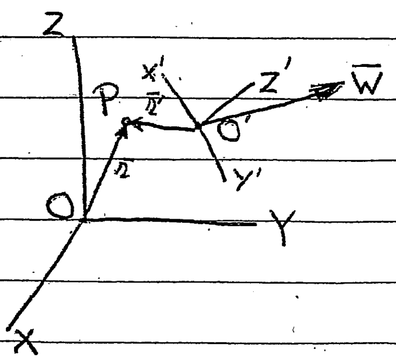
a) Three laws of motion

1) Every particle continue in its state of uniform motion in a straight line (or rest) relative to an inertial reference frame, unless compelled to change that state by forces acting upon it.
2) The time rate of change of linear momentum of a particle relative to an inertial reference frame is proportional to the resultant of force acting upon that particle and is collinear with and in the direction of that resultant force
3) If two particles exert forces on each other, these forces are equal in magnitude and opposite in direction

b) 1^{st} law could be used to define "inertial reference frame" but than there is a circular reasoning.

- In real inertial reference frame, no apperent forces are needed/present

c) $\vec{r}' = \vec{r} - \vec{W}(t - t_0)$; $t' = t + T$



$$\vec{V} = \frac{d\vec{r}}{dt} \qquad \vec{V}' = \frac{d\vec{r}'}{dt'}$$

$$\vec{V}' = \frac{d\vec{r}'}{dt}\frac{dt}{dt'} = \left(\frac{d\vec{r}}{dt} - W\right) \cdot 1 = V - W$$

V and W constant $\Rightarrow \vec{V}'$ constant. So x'y'z' also inertial reference frame

d) Forces in both reference frames

$$\vec{F} = \frac{d}{dt}(m\vec{V})$$

$$\vec{F}' = \frac{d}{dt'}(m\vec{V}') = \frac{d}{dt'}(m(\vec{V}-\vec{W})) = \frac{d}{dt}(m\vec{V}) - \vec{W}\frac{dm}{dt}$$

Only the same if $\frac{dm}{dt} = 0$. For rocket motion apparent forces are needed according to Solidification principle.

e) $\vec{F} = -G\frac{m_1 m_2}{r_{12}^3}\vec{r}_{12}$

f) field strenght $\vec{g}_2 = \frac{\vec{F}}{m_2} = -G\frac{m_1}{r_{12}^2}\left(\frac{\vec{r}_{12}}{r_{12}}\right)$

potential $\vec{g}_2 = -\nabla U_2 \Rightarrow U_2 = -G\frac{m_1}{r_{12}} + \overset{=0}{U_{2_0}}$

g) See previous Question 1

rest 3 bo 2

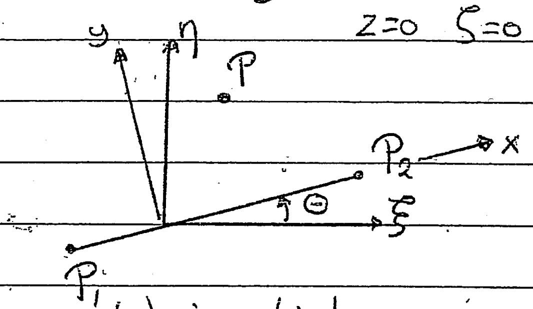
Question 3: Circular Restricted three-body problem

januari 08 '01

a) Assumptions

- $m_1, m_2 \gg m_3$ (neglect gravitational attraction of m_3 on m_1 & m_2)
- two heavy bodies move in circular orbits around the center of mass of the system

$P_1: (\xi_1, \eta_1, 0)$
$P_2: (\xi_2, \eta_2, 0)$
$P: (\xi, \eta, \zeta)$



$\xi\eta\zeta$ reference frame is (pseudo) inertial
xyz reference frame rotates

in inertial frame:

$$\frac{d^2\vec{r}}{dt^2} = -G\frac{m_1}{r_1^3}\bar{r}_1 - G\frac{m_2}{r_2^3}\bar{r}_2$$

- in xyz we should include coriolis and centrifugal force

b)
$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}$$
$$\ddot{y} + 2\dot{x} = \partial U/\partial y \quad ; U = \frac{1}{2}(x^2+y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$
$$\ddot{z} = \partial U/\partial z$$

Relations fluxes and double fluxes valid w.r.t. rotating reference frame xyz. U includes gravitational and centrifugal force (not Coriolis). Potential is non-central but is conservative

c) $$\ddot{x}\dot{x} - 2\dot{y}\dot{x} = \frac{\partial U}{\partial x}\dot{x}$$

$$\ddot{y}\dot{y} + 2\dot{x}\dot{y} = \frac{\partial U}{\partial y}\dot{y}$$

$$\ddot{z}\dot{z} = \frac{\partial U}{\partial z}\dot{z} \quad +$$

$$\ddot{x}\dot{x} + \ddot{y}\dot{y} + \ddot{z}\dot{z} = \frac{\partial U}{\partial x}\dot{x} + \frac{\partial U}{\partial y}\dot{y} + \frac{\partial U}{\partial z}\dot{z}$$

$$= \frac{dU}{dt}$$

Integration gives

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2U + C_{onst} = 2U - C$$

$$V^2 = 2U + C_{onst} = 2U - C$$

Which is Jacobi's integral with C the Jacobian constant. If the velocity of the small body is around zero (V=0) we can write

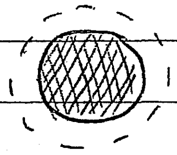
$$2U = C$$

$$x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} = C$$

pockorb

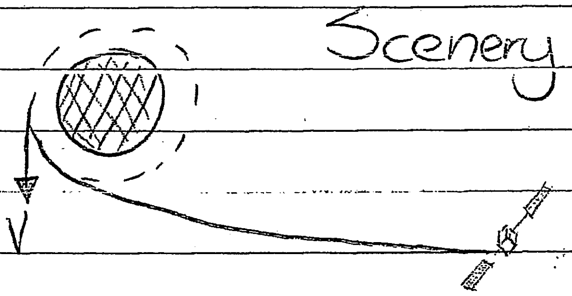
Question 1: Parking Orbit   jan 8'01

$h = 500$ [km] $e = 0$

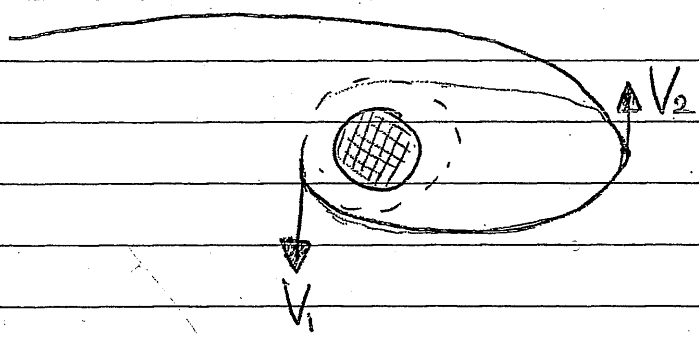


a)

Scenery 1



Scenario 2



b)

$$V_c = \sqrt{\frac{\mu}{r}} \qquad V_{esc} = \sqrt{2\frac{\mu}{r}} = \sqrt{2}\, V_c$$

$$\Delta V = V_{esc} - V_c = (\sqrt{2}-1) V_c = (\sqrt{2}-1)\sqrt{\frac{398600.4}{500+R_\oplus}}$$

$$= (\sqrt{2}-1) \cdot 7{,}6 = 3{,}15$$

c) $h_a = 5000$ [km] $r_a = 11\,378$ [km]

$h_p = 500$ [km] $r_p = 6875$ [km]

$$e = \frac{r_a - r_p}{r_a + r_p} = 0{,}247 \ [-]$$

$$V_p^2 = V_{cp}^2 (1+e)$$
$$= \left(\sqrt{\frac{\mu}{r_p}}\right)^2 (1+e)$$
$$= \left(\sqrt{\frac{398600.4}{6875}}\right)^2 (1+0,247) \cancel{= 7,37^2 \cdot 1,247}$$
$$V_p = \cancel{8,95}\ 7,61 \cdot \sqrt{1,247} = \cancel{9,50}\ 8,50 \ [km/s]$$
$$\Delta V_1 = V_p - V_{cp} = \sqrt{1+e}\, V_{cp} - V_{cp} = \left(\sqrt{1+e} - 1\right) V_{cp}$$
$$= 0,888 \ [km/s]$$
$$V_a = \frac{1-e}{1+e} V_p = 5,13 \ [km/s]$$

~~V_{esc}~~

$$\Delta V_2 = V_{esc} - V_a = \sqrt{2}\, V_{c_A} - V_A = \sqrt{2}\sqrt{\frac{\mu}{r_A}} - V_A$$
$$= 8,37 - 5,13 = 3,24 \ [km/s]$$
$$\Delta V_{tot} = 0,89 + 3,24 = 4,128 \ [km/s]$$