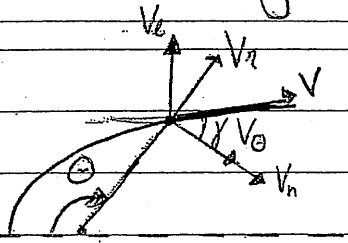
Question 2: Velocity components august 29 '02



a) $$r = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{H^2/\mu}{1+e\cos\theta}$$

$$\Rightarrow \dot{r} = \frac{H^2/\mu}{(1+e\cos\theta)^2}\, e\sin\theta\,\dot{\theta}\cdot\frac{r^2}{r^2}$$

$$= r^2\frac{H^2/\mu}{(1+e\cos\theta)^2}\, e\sin\theta\,\dot{\theta}\,\frac{(1+e\cos\theta)^2}{H^4/\mu^2}$$

$$= r^2\frac{\mu}{H^2}\, e\sin\theta\,\dot{\theta}$$

$$= \frac{\mu}{H}\, e\sin\theta$$

$$\dot{r} = V\sin\gamma$$
$$r\dot{\theta} = V\cos\gamma$$
$$H = rV\cos\gamma$$

$$r = \frac{H^2/\mu}{1+e\cos\theta} = \frac{H/\mu}{1+e\cos\theta}\left[r^2\dot{\theta}\right]$$

$$\Rightarrow r\dot{\theta} = \frac{1+e\cos\theta}{H/\mu} = \frac{\mu}{H}(1+e\cos\theta)$$

$$\Rightarrow V_\ell = \frac{\dot{r}}{\sin(180-\theta)} = \frac{\dot{r}}{\sin\theta} = \frac{\dot{r}}{\dot{r}H/\mu e} = \frac{\mu e}{H}$$

$$V_n = r\dot{\theta} + \frac{\dot{r}}{\tan(180-\theta)} = r\dot{\theta} + \frac{\dot{r}}{\tan\theta}$$

$$\cancel{= r\dot{\theta} + \frac{\dot{r}}{\sin\theta}\cos\theta = r\dot{\theta} + \frac{\mu}{H}e\cdot\frac{1}{e}\left[\frac{r^2\dot{\theta}}{H} - 1\right]}$$

$$\cancel{= r\dot{\theta} + \dot{r}}$$

$$= \frac{\mu}{H}(1+e\cos\theta) - \frac{\dot{r}}{\sin\theta}\cos\theta = \frac{\mu}{H}(1+e\cos\theta) - \frac{\mu}{H}e\cos\theta$$

$$= \frac{\mu}{H}$$

c) $E - e\sin E = M$

$dE - e\cos E \, dE = dM$

$(1 - e\cos E) = \dfrac{dM}{dE} = \left(\cancel{dM}\,\dfrac{dE}{dM}\right)^{-1}$

Substitution in

$r = a(1 - e\cos E) = a\dfrac{dM}{dE}$

So

$\left(\dfrac{r}{a}\right) = \left(\dfrac{dE}{dM}\right)^{-1}$

d) $E = M + e\sin M + e^2 \frac{1}{2}\sin(2M)$

$dE = dM + e\cos M \, dM \cancel{+} - \cancel{\tfrac{4}{4}}\, e^2 \cos(2M)\, dM$

$\dfrac{dE}{dM} = 1 + e\cos M - e^2\cos(2M) + \ldots$

$(1+y)^{-1} = 1 - y + y^2 -$ $\quad y = e\cos M \cancel{+} - e^2\cos(2M)$

$(1 + e\cos M - e^2\cos(2M)) = 1 - e\cos M + e^2\cos(2M) + e^2\cos^2 M$

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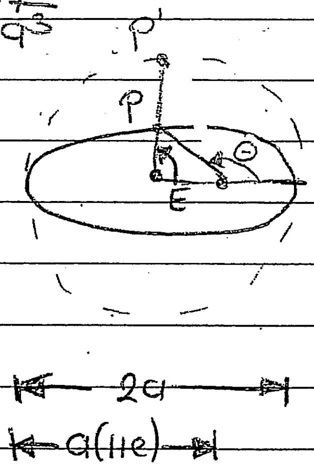
Question ~~3~~ 4 Keplers equation

aug 29 '02
aug 24 '01

a) $E - e\sin E = n(t-T) = M$

$r = a(1 - e\cos E)$

E: eccentrin anomaly
e: anomaly
n: mean angular velocity $n = \sqrt{\frac{\mu}{a^3}}$
t: time
T: time of last perigee
r: distance
a: semi-major axis
M: mean anomaly



b) $x = E - M = e\sin E = e\sin(x+M)$
$= e\sin x\cos M + e\cos x\sin M$
$= e\cos M\left(x - \frac{1}{6}x^3 + \dots\right) + e\sin M\left(1 - \frac{1}{2}x^2 + \dots\right)$

$x_1 e + x_2 e^2 + x_3 e^3 + \dots = \sin M e + \cos M x_1 e^2 + O(e^3)$

$x_1 = \sin M$
$x_2 = x_1 \cos M = \sin M \cos M = \frac{1}{2}\sin(2M)$

$E = x + M = M + e\sin M + e^2 \frac{1}{2}\sin(2M) + O(e^3)$

relateff

Question 3 Relativistic effects 29 august 02

a) $$\frac{d^2u}{d\varphi^2} + u = \frac{\mu}{H^2}$$

u: distance^{-1} sattelite to gravitational body
$u = 1/r$
φ: angular ~~velocity~~
μ: gravitational constant of body
H: angular velocity

$$\frac{d^2u}{d\varphi^2} + u = \frac{\mu}{H^2} + 3\frac{\mu}{c^2}u^2$$

c = speed of light

b) $$= \frac{\mu}{H^2}\left[1 + 3\frac{H^2u^2}{c^2}\right] = \frac{\mu}{H^2}\left[1+3\frac{V_\varphi^2}{c^2}\right]$$

$$c^2 \gg V_\varphi^2$$

c) $$u = u_0 + \alpha\frac{\mu^2}{H^4}\left[1+\frac{1}{2}e^2 + \underbrace{e\varphi\sin(\varphi-\omega)}_{\text{always increasing}} - \frac{1}{6}e^2\cos 2(\varphi-\omega)\right]$$

- increase u with constant term
- fluctuating term growing constantly with φ
- pure oscilation with constant amplitude

d) only taking long term in account

$$u = \frac{\mu}{H^2}\left[1 + e\cos(\varphi-\omega) + \alpha\frac{\mu}{H^2}e\varphi\sin(\varphi-\omega)\right]$$

$$u = \frac{\mu}{H^2}\left[1 + e\cos(\varphi-\omega) + \beta e\varphi\sin(\varphi-\omega)\right]$$