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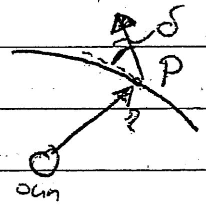
Chapter 5
Question 1: Poynting - Roberson effect

~~januari 15 '04~~
januari 14 '03

a) $\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2} + \frac{F}{m}\sin\delta$

$\frac{d}{dt}(r^2\dot{\theta}) = \frac{F}{m} r\cos\theta$

r: distance body - sun
$\dot{\varphi}$: angular velocity body around sun
μ: gravitational constant o/t sun
F: radiation force on body
δ: angle between force and normal of $\vec{r}$



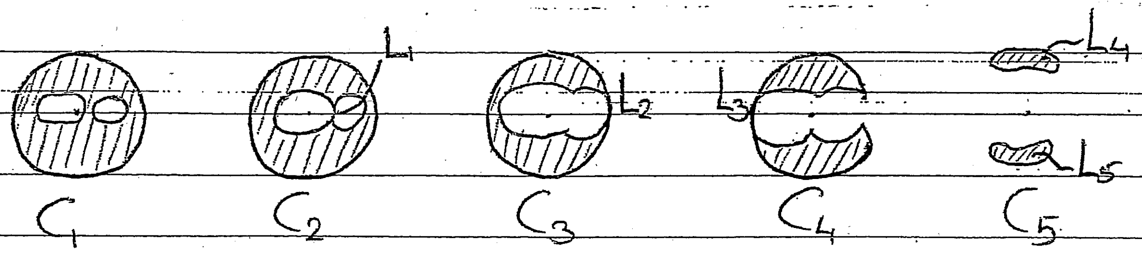
$F = C_R \frac{WS}{c}$

C_R: objects reflectivity
W: Power density
S: cross-sectional area object
c: speed of light

$\frac{F}{m} = \frac{3}{4} \frac{C_R W_S R_S^2}{c\rho R} \frac{1}{r^2}$

R_S: Radius sun
R: Radius body
ρ: Density body
m: mass body

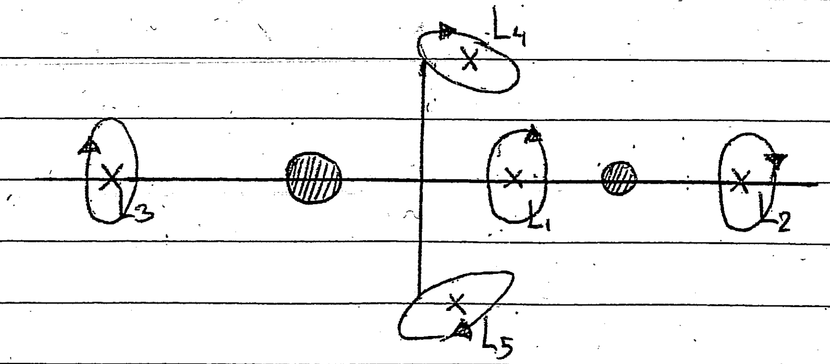
b)



$C_1 > C_2 > C_3 > C_4 > C_5$

c) In the Lagrange points $\frac{\partial U}{\partial x} = \frac{\partial U}{\partial y} = \frac{\partial U}{\partial z} = 0$ from this it follows that $\ddot{x} = \ddot{y} = \ddot{z} = 0$. So a body located in these points do not experience acceleration.
However in L_1, L_2, L_3 there is an unstable equilibrium and in L_4, L_5 an infinitesimal stability (due to linearization)

d)



Orbit always opposite to direction of rotation of x-y-z-frame. Elliptical motion (L_1, L_2: $\frac{a}{b} \approx 3$ L_3: $\frac{a}{b} \approx 2$) only small ΔV needed to remain in orbit

e) Example: ISEE-C. International Sun-Earth explorer placed in L_1 of sun earth system. meausement of high energy particles before reaching earth

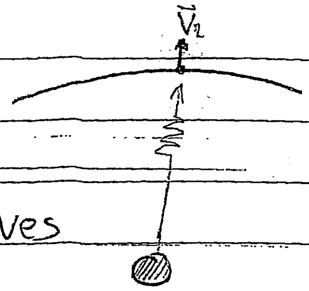
d) $\alpha = \frac{3}{4} \frac{W_s R_s^2}{c} \cdot \frac{C_R}{\rho R}$

$= \text{const} \cdot \frac{C_R}{\rho R}$

So low density and small particles are mush more affected than large high density objects like planets

e) dimension of particles blown out of the solar system are same order of magnitude as the wavelength. This gives a scattering of the light. During the history of the sun these particles are blown away and so we see the sun as a clear source

b) Due to radial velocity the power ~~density of the radiation is~~ lowered by a factor $\dot{r}/c$, this gives

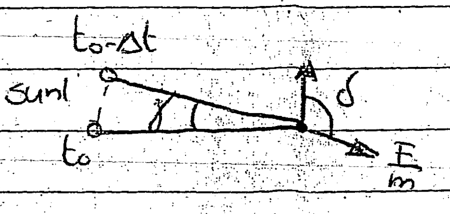


$$W' = W - W\frac{\dot{r}}{c} = W(1 - \dot{r}/c)$$

Sunlight intercepted by the body is actually emitted at time $t_0 - \Delta t$. In this time interval sunlight travelled $c\Delta t$ while the body moved $r\dot{\varphi}\Delta t$. The angle

$$\gamma \simeq \frac{r\dot{\varphi}\Delta t}{c\Delta t} = \frac{r\dot{\varphi}}{c}$$

c) $\sin\delta = \sin(90^\circ + \gamma) = \cos\gamma \simeq 1$



$$\cos\delta = \cos(90^\circ + \gamma) = -\sin\gamma$$
$$\simeq -\gamma = -\frac{r\dot{\varphi}}{c}$$

$$\frac{F}{m} = \frac{3}{4}\frac{C_R W_0 R_s^2}{c\rho R}\frac{1}{r^2} = \frac{\alpha}{r^2}$$

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2} + \frac{F}{m}\sin\delta$$
$$= -\frac{\mu}{r^2} + \frac{\alpha}{r^2}(1 - \dot{r}/c)$$

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu - \alpha}{r^2} - \frac{\alpha\dot{r}}{r^2 c}$$

$$\frac{d}{dt}(r^2\dot{\theta}) = \cancel{\alpha\dot{\theta}}\ \frac{F}{m} r\cos\delta$$
$$= -\frac{\alpha}{r^2}(1 - \dot{r}/c)\, r\frac{r\dot{\varphi}}{c} \qquad \text{neglect } 1/c^2$$

$$\frac{d}{dt}(r^2\dot{\theta}) = -\frac{\alpha\dot{\theta}}{c}$$

+ explanation

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$r = \frac{2}{V^2/\mu + 1/a} = 8359.946\ [km]$$

$$H = r V_\Theta = 59946.58\ [km^2/s]$$

~~$0.13481 = 7.254326\ e\ \sin\Theta$~~

$$\frac{\mu}{H} = 7.25433$$

e	Θ
0.25	-0.1372

$$\Theta = \mathrm{Arcsin}\left(\frac{\dot r}{e}\frac{H}{\mu}\right) =$$

$$e = \frac{V_\Theta \frac{H}{\mu} - 1}{\cos\Theta} =$$

$$-0.034200122 = e \sin\Theta$$

$$-0.093975598 = e \cos\Theta$$

$e = 0.10 \qquad \Theta = -20°$

$\tan\Theta = 20°$ or $200°$