ESA Sysnova System Studies - Lunar Caves

Verification Voxeliser scripts IGMAS modelling

Voxeling_Main script shape verification

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July 28, 2020

Verification

Sphere

Shown in figure 1 is the verification procedure for a spherical underground cavity. This gives the same signal as a lone sphere with the density equal to the density difference between the rock and the cavity, so this configuration was used. Following Standard integrals, we get the following formula for the theoretical gravity anomaly signal of a sphere:

$$g_z = \frac{4\pi \cdot G \cdot \Delta\rho R^3}{3} \cdot \frac{z}{(r^2 + z^2)^(3/2)} \quad (1)$$

Here, g_z is the vertical component of the gravitational acceleration due to the anomaly in ms^{-2}, G is the gravitational constant, equal to $6.67430 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, $\Delta\rho$ is the density difference between rock and cavity in kgm^{-3}, R is the radius of the spherical cavity in m, z is the vertical distance between the station and the center of the cavity in m and r is the horizontal distance between the station and the center of the cavity in m, equal to $\sqrt{x^2 + y^2}$. x and y are station coordinates, where the elevation of the stations is assumed to be 0.
Next, this formula is applied to a situation with an underground cavity were the center is buried 20 meters beneath the surface with a radius of 10 meters. The density difference is equal to 2500 kgm^{-3}. The grid size in IGMAS is set to 1 meter, and the spherical mesh is set to 400 faces, the default value. The station resolution is 2500 stations in total. The results of this procedure are shown in Figure 1. Note that the maximum signal strength is 0.175 mGal; the maximum difference is then only 3% of the signal value. This difference is dependent on some variables, discussed later. However, the general conclusion is that the differences between these experimental and theoretical results are small enough to use IGMAS reliably in this case.

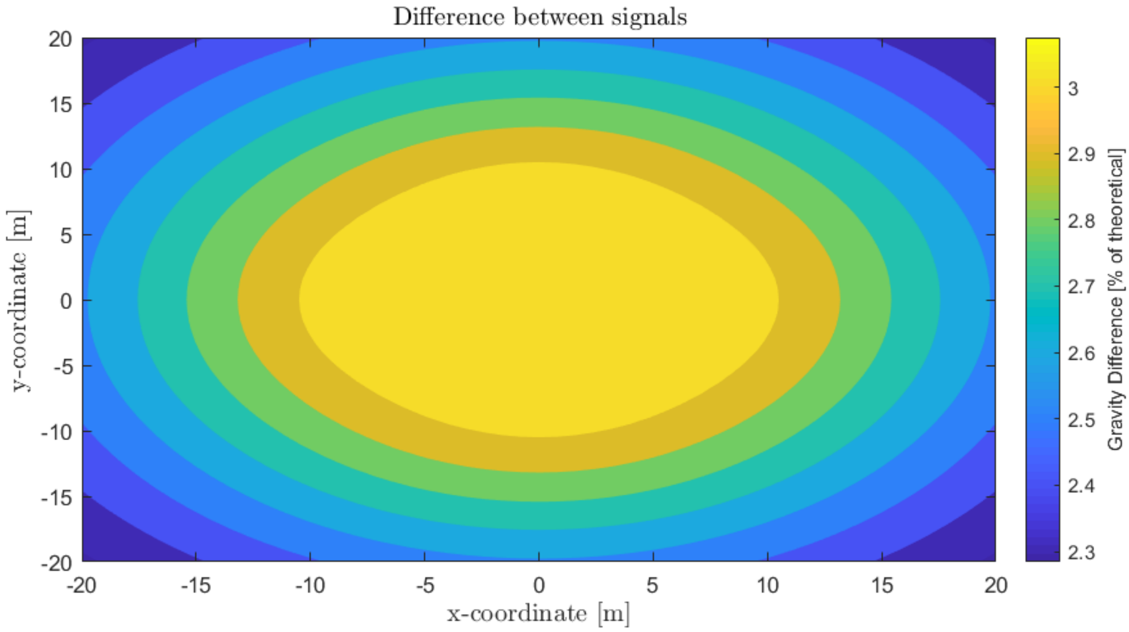

Figure 1: Difference theoretical signal and IGMAS signal, percentages.

Horizontal Cylinder

Shown in figure 2 is the verification procedure for a horizontal cylindrical underground cavity. This gives the same signal as a lone horizontal cylinder with the density equal to the density difference between the rock and the cavity, so this configuration was used. Following Standard integrals, we get the following formula for the theoretical gravity anomaly signal of a sphere:

$$g_z = \pi \cdot G \cdot R^2 \cdot \Delta\rho \cdot z \cdot \left(\frac{L-y}{(z^2+x^2)\cdot\sqrt{(L-y)^2+x^2+z^2}} + \frac{y}{\sqrt{x^2+y^2+z^2}} \right) \tag{2}$$

Here, g_z is the vertical component of the gravitational acceleration due to the anomaly in ms^{-2}, G is the gravitational constant, equal to $6.67430 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, $\Delta\rho$ is the density difference between rock and cavity in kgm^{-3}, R is the radius of the cylindrical cavity in m, L is the length of the cavity in m, z is the vertical distance between the station and the center of the cavity in m and r is the horizontal distance between the station and the center of the cavity in m, equal to $\sqrt{x^2+y^2}$. x and y are station coordinates, where the elevation of the stations is assumed to be 0.
Next, this formula is applied to a situation with an underground cavity were the center is buried 25 meters beneath the surface with a radius of 15 meters. The density difference is equal to 2500 kgm^{-3}. The grid size in IGMAS is set to 1 meter, and the cylinder mesh is set to 22 faces, the default value. The station resolution is 2500 stations in total. The results of this procedure are shown in Figure 2. Note that the maximum signal strength is 0.844 mGal; the maximum difference is then 11% of the signal value. While this difference is quite significant, it is also dependent on some variables, discussed later. Changing these variables can result in much lower differences. The general conclusion is then that the differences between these experimental and theoretical results are small enough to use IGMAS reliably in this case.

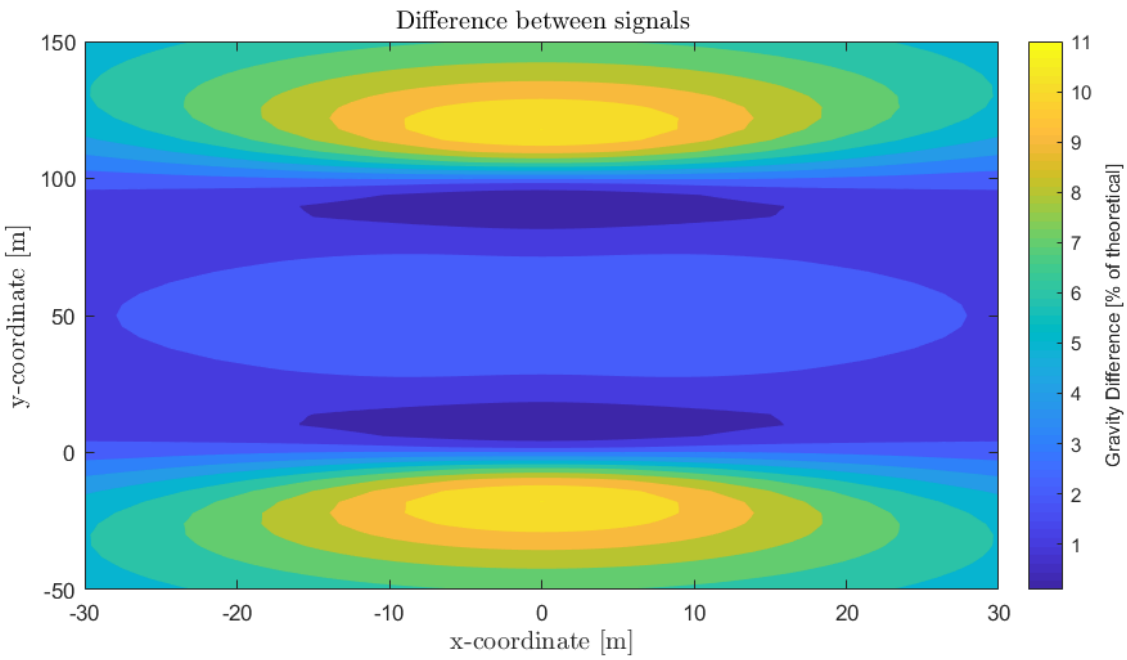

Figure 2: Difference theoretical signal and IGMAS signal, percentages.

Vertical Cylinder

Shown in figure 3 is the verification procedure for a vertical cylindrical underground cavity. This gives the same signal as a lone vertical cylinder with the density equal to the density difference between the rock and the cavity, so this configuration was used. Following Standard integrals, we get the following formula for the theoretical gravity anomaly signal of a sphere:

$$g_z = G \cdot \pi R^2 \cdot \Delta\rho \cdot \left[\frac{1}{\sqrt{x^2+y^2+z^2} - \frac{1}{\sqrt{x^2+y^2+(z+L)^2}}} \right] \quad (3)$$

Here, g_z is the vertical component of the gravitational acceleration due to the anomaly in ms^{-2}, G is the gravitational constant, equal to $6.67430 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$, $\Delta\rho$ is the density difference between rock and cavity in kgm^{-3}, R is the radius of the cylindrical cavity in m, z is the vertical distance between the station and the center of the cavity in m and x and y are the station coordinates in m, where the elevation of the stations is assumed to be 0.
Next, this formula is applied to a situation with an underground cavity were the center is buried 60 meters beneath the surface with a radius of 10 meters. The density difference is equal to 2500 kgm^{-3}. The grid size in IGMAS is set to 1 meter, and the cylinder mesh is set to 22 faces, the default value. The station resolution is 2500 stations in total. The results of this procedure are shown in Figure 3. Note that the maximum signal strength is 1.072 mGal; the maximum difference is then 30% of the signal value. While this difference is significant, it is also dependent on some variables, discussed later. Changing these variables can result in much lower differences. The general conclusion is then that the differences between these experimental and theoretical results are small enough to use IGMAS reliably in this case.

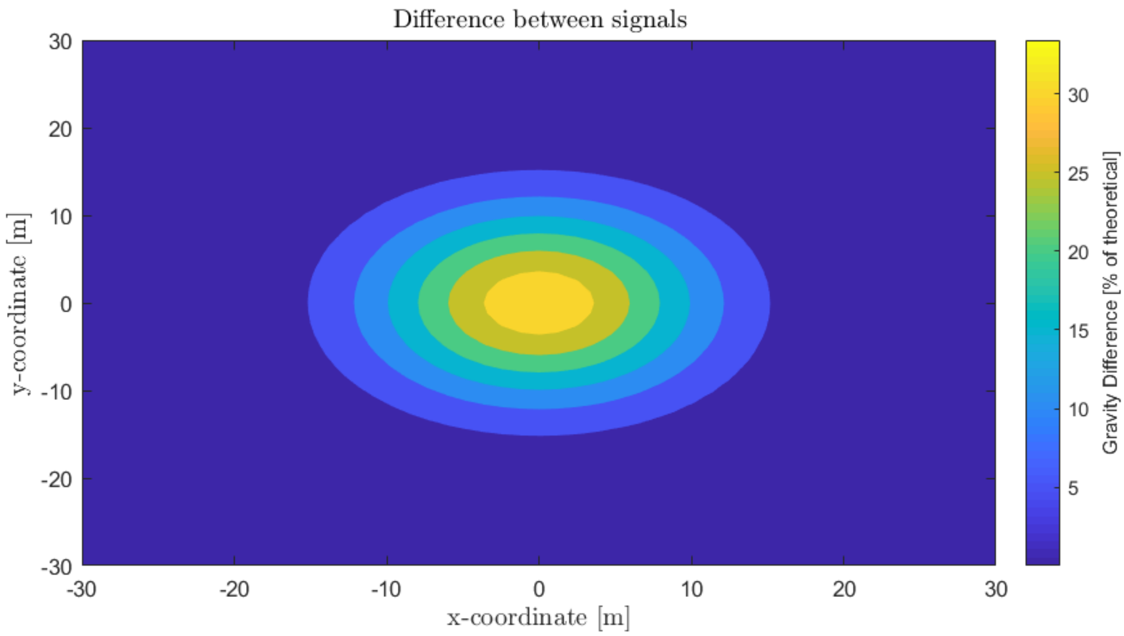

Figure 3: Difference theoretical signal and IGMAS signal, percentages.

0.1 Parameter: mesh size alphaShape

First, we look into the parameter determining the mesh size of the alphashape created. For the cylinder, this number of points is the number of points which the cylinder is defined by, which is by default 20+2. Varying this parameters makes the cylinder representation less or more realistic. In this test, the value is changed to 5+2 and to 100+2 to see the influence. The default situation is shown in Figure 10, the one for a small face value is shown in Figure 5 and the one for a large face value is shown in Figure 6. Clearly, having too few faces has a detrimental results on the realism, and increasing the number of faces compared to the default value shows some improvement, but not much.

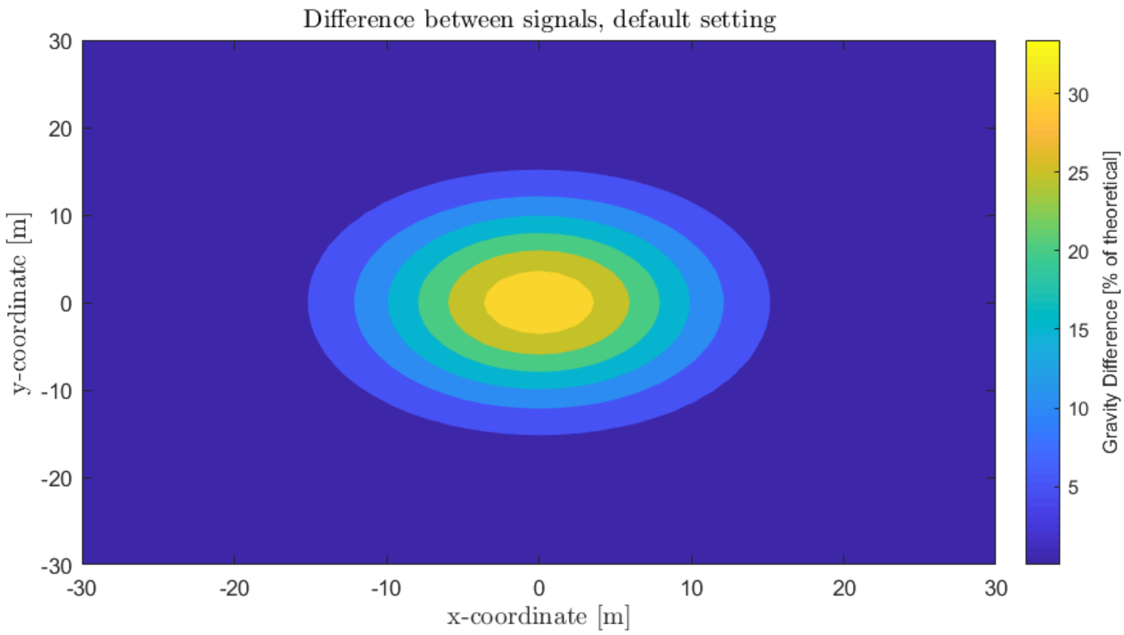

Figure 4: Difference theoretical signal and IGMAS signal, percentages. Default settings

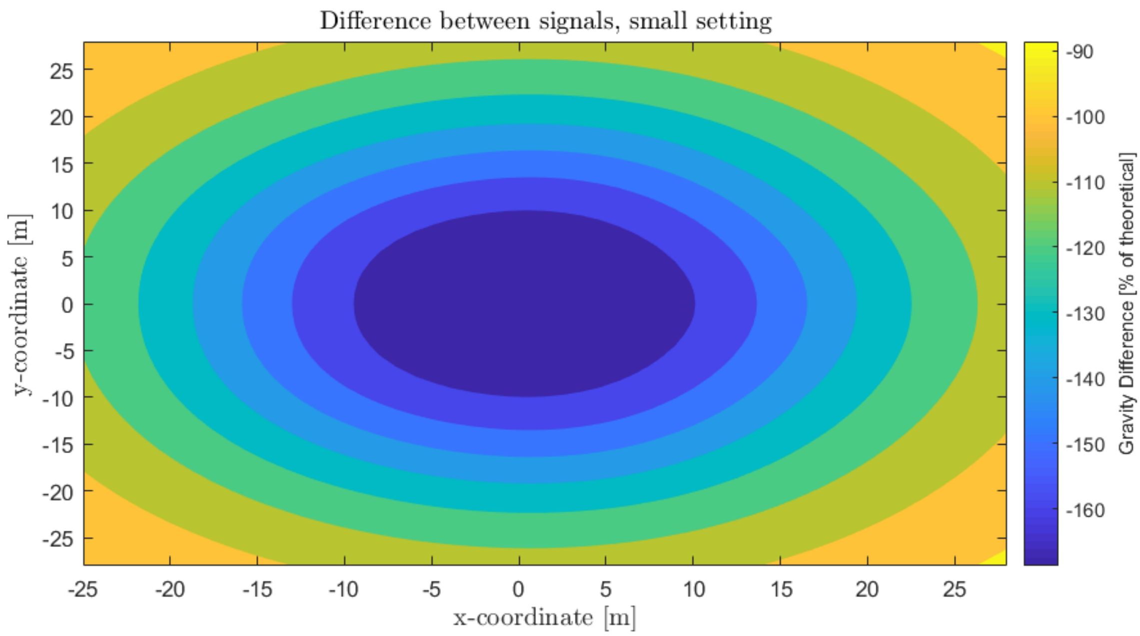

Figure 5: Difference theoretical signal and IGMAS signal, percentages. Small face value

Parameter: cylinder radius

The cylinder radius is not something that makes the model more or less realistic, but since the theoretical model is based on this, it is something that makes the approximation of the theoretical model more or less realistic. This is only true for the cylinders, as the sphere does not involve any approximations for the theoretical model. For the cylinders, the radius is often neglected compared to the length of the cylinder, meaning that a small radius leads to a smaller approximation and thus a smaller error than a larger radius. This is also exactly what we see in the results in Figure 8 and Figure 9

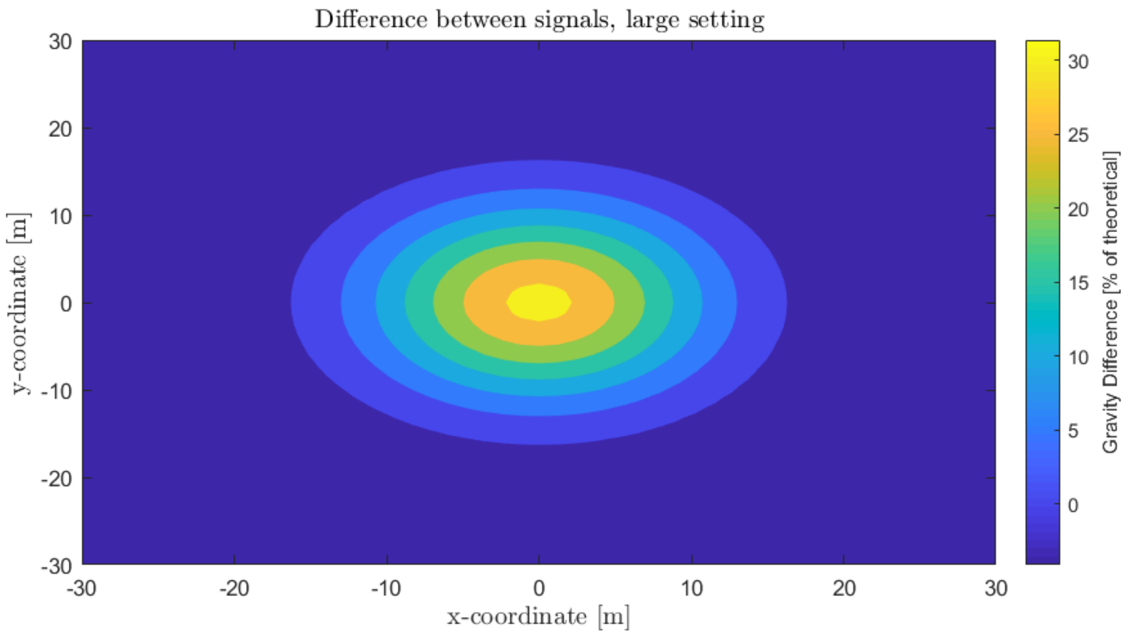

Figure 6: Difference theoretical signal and IGMAS signal, percentages. Large face value

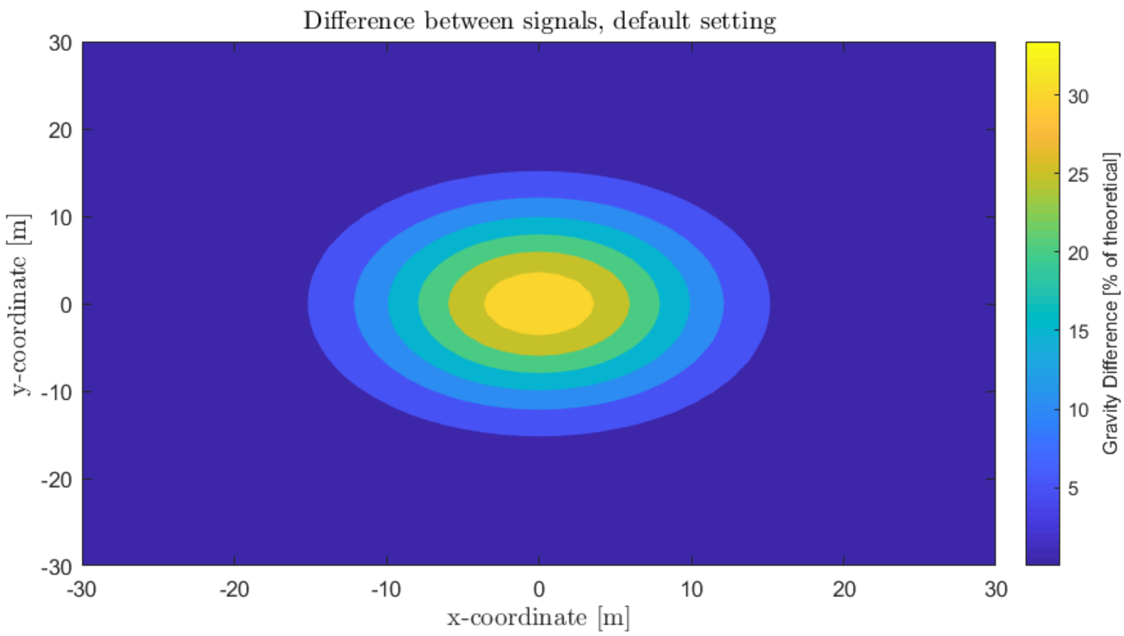

Figure 7: Difference theoretical signal and IGMAS signal, percentages. Default settings

Parameter: voxel grid spacing

Lastly, there is the grid spacing of the models. The larger the grid spacing, the larger the voxels and the more 'blocky' the shape becomes. In general, it is expected that the approximations become smaller when the shape is modelled more realistic, so with smaller grid spacings. In this case, very little difference is seen, as shown in Figure 11 and Figure 12, as apparently the straight vertical section of the cylinder is modelled accurately enough already.

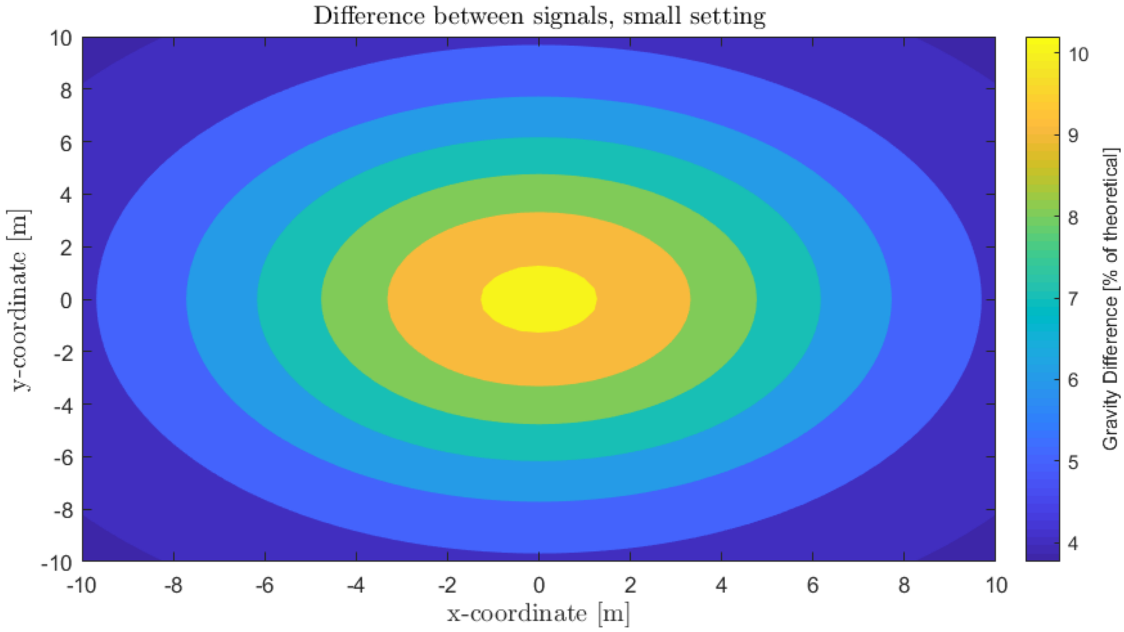

Figure 8: Difference theoretical signal and IGMAS signal, percentages. Small radius

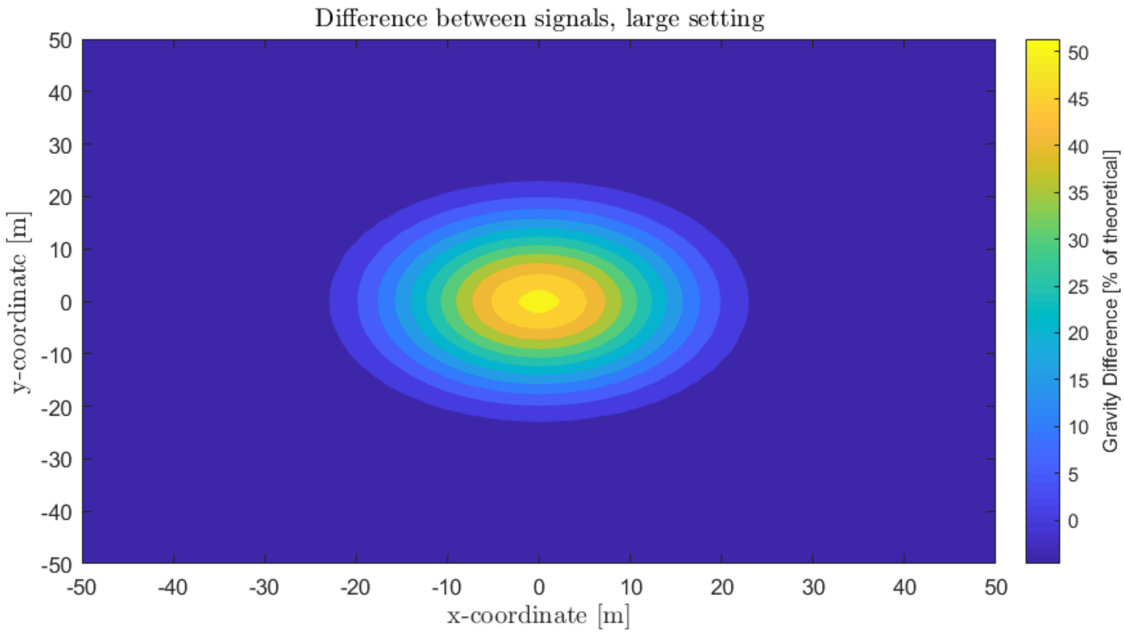

Figure 9: Difference theoretical signal and IGMAS signal, percentages. Large radius

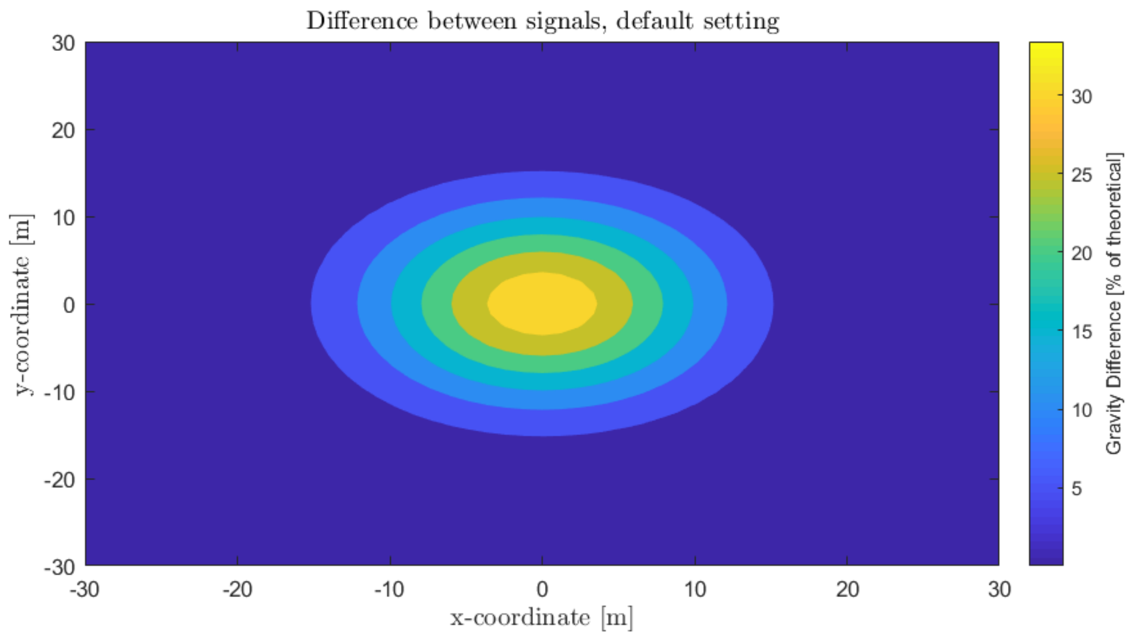

Figure 10: Difference theoretical signal and IGMAS signal, percentages. Default settings

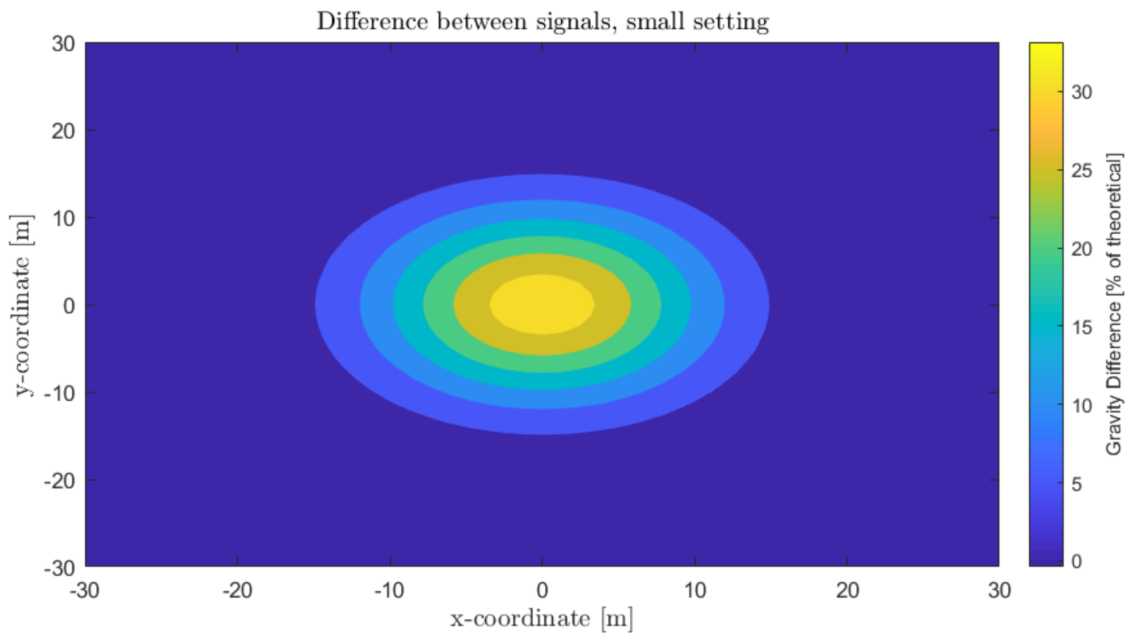

Figure 11: Difference theoretical signal and IGMAS signal, percentages. Small grid size

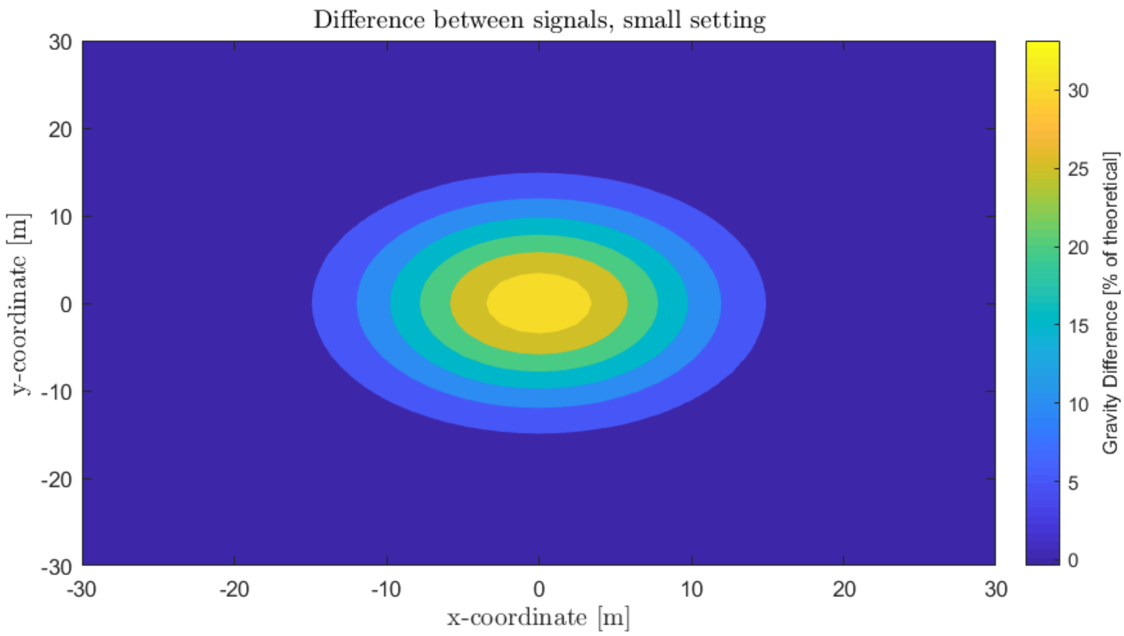

Figure 12: Difference theoretical signal and IGMAS signal, percentages. Large grid size