The University of Manchester

PHYS20762 Computational Physics   Semester 2, 2022-2023

Project 3: Monte Carlo Techniques
Neutron Transport and Scattering Through a Shielding Layer

Project Description

The task in this project is to develop a simulation of penetration of neutrons through a slab of shielding of thickness $\boldsymbol{L}$, considering only thermal neutrons, and the processes of absorption and scattering. Your notebook must be organised as follows:

- Confirm that you can generate arrays of numbers using the Python in-built random number generator **numpy.random.uniform()**, and that the numbers produced are uniform over a chosen range. (*Week 1 of project*)
- Write a script that generates an array of randomly distributed points in three dimensions $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ using **numpy.random.uniform()**.Display the generated points using an interactive scatter plot and confirm there are no spectral issues. Next, try using *randssp.ipynb* (available on the Blackboard page of Project 3) and demonstrate it causes spectral issues. (*Week 1 of project*)
- Create a random number generator that produces samples distributed according to an exponential function $\mathbf{exp}\,(-\boldsymbol{x}/\boldsymbol{\lambda})$, where $\boldsymbol{\lambda}$ is the mean free path. Use this generator to demonstrate in the absence of scattering, the characteristic attenuation length for water is about $\mathbf{45}\,\boldsymbol{cm}$, utilising the data provided in Table 1 below. (Hint: Use **numpy.histogram** to bin the generated points and perform a properly weighted linear **numpy.polyfit** on the binned data to obtain the attenuation length and its expected uncertainty). Include representative plots in your report. (*Week 1 of project*)
- Write a function that produces isotropic unit vectors $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\widehat{\boldsymbol{k}}$ such that $|\vec{\boldsymbol{r}}| = 1$ (*Hint*: use spherical polar coordinates and then do a trigonometric conversion). Visually assess the generated points using an interactive scatter plot to confirm they are uniformly distributed over a sphere. (*Week 2 of project*)
- Utilizing the function above, create a new function that generates isotropic steps with lengths distributed as $\mathbf{exp}\,(-\boldsymbol{x}/\boldsymbol{\lambda})$. Include representative plots in your report. (*Week 2 of project*)

Note: For an interactive scatter plot, you may set **import matplotlib.pyplot as plt** and **from mpl_toolkits.mplot3d import Axes3D** followed with **%matplotlib notebook**.

Table 1: Thermal neutron data for water, lead and graphite.

	Water	Lead	Graphite
Absorption, σ_a $(barn)$	0.6652	0.158	0.0045
Scattering, σ_s $(barn)$	103.0	11.221	4.74
Density, ρ (g/cm^3)	1.00	11.35	1.67
Molar Mass, M (g/mol)	18.0153	207.2	12.011

Once the above tasks are completed, create a programme that simulates the behaviour of neutrons as they travel through water, lead, and graphite under various conditions. (*Week 3-5 of project*)

Organise this programme as follows:

- Firstly, start by determining the macroscopic absorption and scattering cross-sections, as well as the total mean free path ($\boldsymbol{\lambda}$), for each material.

- Next, visualise the random walk of a few neutrons through each material. For this, consider a slab of thickness $\boldsymbol{L}$ placed between $\boldsymbol{x = 0}$ and $\boldsymbol{x = L}$. Each neutron should enter the slab perpendicularly from its left side at $\boldsymbol{x = 0}$ (i.e., it is initially along $+\boldsymbol{x}$ direction), and then perform a random walk according to the distribution $\mathbf{exp}\,(-\boldsymbol{x_i}/\boldsymbol{\lambda})$, where $\boldsymbol{x_i}$ is the $\boldsymbol{x}$-coordinate of the neutron at step $\boldsymbol{i}$. Note that for each step, you need to check whether the neutron is absorbed, scattered within the slab, or escaped from it ($\boldsymbol{x < 0}$ or $\boldsymbol{x > L}$). The random walk should only continue if the neutron is found to be scattered inside the slab.

- In a separate section, repeat this process (without random walk visualisation) for $\boldsymbol{N}$ neutrons to determine the absorption ($\boldsymbol{N_A/N}$), reflection ($\boldsymbol{N_R/N}$), and transmission ($\boldsymbol{N_T/N}$) rates by counting the number of neutrons absorbed ($\boldsymbol{N_A}$), reflected ($\boldsymbol{N_R}$), and transmitted ($\boldsymbol{N_T}$). Take $\boldsymbol{L = 10\,cm}$. Note that for each process, you must also determine the resulting uncertainty.

- Plot the variation in neutron transmission, reflection, and absorption rates with slab thickness for the three materials.

- Determine the characteristic attenuation lengths and their estimated errors for the three materials from their corresponding transmission rates. Include a summary of numerical data and representative graphs.

Note: When fitting the transmitted neutron data, you may encounter some thickness values giving zero transmitted neutrons, aborting **`np.polyfit`** with a **`NaN`** error. To avoid this, make sure to remove these data points before fitting.

Bonus: Use the Woodcock method to calculate the fraction of neutrons transmitted through two slabs of different materials in contact with each other, with each slab having a thickness of $\boldsymbol{10\,cm}$. Describe how you can validate the obtained results to ensure their accuracy.

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